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Reading Between the Number Lines

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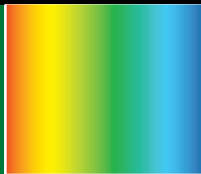
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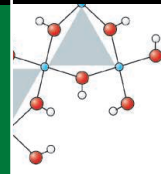
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LETTERS

edited by Jennifer Sills

Reading Between the Number Lines

IN THEIR REPORT “LOG OR LINEAR? DISTINCT INTUITIONS OF THE number scale in Western and Amazonian indigene cultures” (30 May, p. 1217), S. Dehaene *et al.* investigate how Mundurucu Indians of the Amazon map numerosity judgments on a line segment. They conclude that the concept of a linear number line is a product of culture and formal education and that “the mapping of numbers onto space is a universal intuition and that this initial intuition of number is logarithmic.” While I fully agree with the former statement—which I have defended elsewhere (1)—I disagree with the latter.

First, if the intuition of mapping numbers onto space is as fundamental as the authors claim, we should expect ubiquitous

Ancient math. By providing insight into the math used in ancient cultures, artifacts such as this 4000-year-old clay tablet can help to distinguish learned mathematical concepts from those that are intuitive.



manifestations of number lines—linear or logarithmic—in early arithmetic in Mesopotamia, Egypt, China, and Mesoamerica. But no such evidence exists. Indeed, 4000-year-old clay tablets show that Babylonians developed a sophisticated knowledge of arithmetical bases, fractions, and operations apparently without the slightest reference to number lines (see photograph) (2). The number line is a complex idea that appears to have been introduced as late as 1685 by John Wallis (3). Second, if humans’ initial intuition of “number” is logarithmic, we wouldn’t have had to wait until the 17th century to see the invention of logarithms through Napier’s painstaking work. These late inventions are inconsistent with the authors’ claim that “mathematical” objects may find their ultimate origin in basic intuitions internalized through millions of years of evolution. The Report seems to be less about mathematical concepts and more about the role a line segment can play in reporting a person’s impressions of numerosity.

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Response

WE AGREE WITH NÚÑEZ THAT THE MUNDURUCU do not master the formal properties of number lines and logarithms, but as the term “intuition” implies, they spontaneously experience a logarithmic mapping of number to space as natural and “feeling right.”

Contrary to Núñez’s claims, mappings of numbers onto space are omnipresent in ancient mathematics. Systems of measurement in which numbers are applied in a systematic linear manner to measure lengths (as well as a variety of physical continua) date back at least to the third or fourth millennia BCE in Egypt, Mesopotamia, and the Indus Valley (1). Babylonians were engaged in the measurement of lengths, as illustrated, for example, by a clay tablet giving the length of the diagonal of the square up to the sixth deci-

mal (see photograph) (2); the realization that the diagonal and the side of the square were incommensurate led to a major and fruitful crisis in Pythagorean mathematics. We did not find a precise date for the introduction of formalized number lines, but writings from the 17th century indicate that mathematicians had conceptualized number lines by that time (3). Wallis’s *Treatise on Algebra*, cited by Núñez, was written years after the introduction of coordinate systems in 1637 by Descartes and Fermat. Wallis only uses a number line metaphor to set the ground for the notion of negative numbers, en route to introducing complex numbers.

The logarithmic scale for mapping numbers onto space is highly resistant to change, as we observed the production of logarithmic scales even among educated Mundurucus,

who were proficient in Portuguese. U.S. children of European ancestry show a similar behavior, both with the present task (4) and in a slightly different version in which they were instructed to divide a rectangle in a given number of parts, which should have favored a linear scale (5). This resistance could explain the late introduction of formal linear number lines in mathematics, despite a solid intuition for number-space mapping. In the absence of a formalized concept of logarithms, a logarithmically compressed scale presents limited computational utility, in contrast to a linear scale, which embodies properties of addition and subtraction and can be used as a ruler. Initially, only linearly scaled number lines were pursued by mathematicians, yet introducing these linear scales required overcoming the robust logarithmically shaped intuition

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that numbers should be mapped onto space based on their ratio similarity. Today, the universal use of logarithmic axes, even in newspaper graphs, and the extraordinary spread of slide rules before the advent of calculators testify to the status of the logarithm as both an abstract mathematical concept and an intuitive mental tool.

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The Risks of Pigging Out on Antibiotics

THE NEWS STORY "THE BACTERIA FIGHT BACK" by G. Taubes (Special Section on Drug Resistance, 18 July, p. 356) highlights the growing health threat from antibiotic-resistant bacteria, especially methicillin-resistant *Staphylococcus aureus* (MRSA), and the need to rein in medical uses of antibiotics to curb resistance.

But reining in health care uses alone is insufficient to address the resistance epidemic. As recommended by the Institute of Medicine (1), World Health Organization (2), American Academy of Pediatrics (3), and other health organizations, routine and widespread use of medically important antibiotics in animal agriculture also must be ended to effectively address resistance. Recent evidence showing that some human MRSA infections are associated with animal agriculture underscores this point.

In Europe, MRSA has been shown to be transmitted from pigs to farmers and their families, veterinarians, and hospital staff (4, 5). One MRSA strain, once found only in pigs, is associated with serious human illness, including skin, wound, lung, and heart infections (6, 7). This new pig strain is linked to more than 20% of human MRSA infections in the Netherlands (8).

Researchers have only begun to examine

MRSA from North American livestock. Both Canadian pig farmers and swine are commonly colonized by MRSA (9). A recent study found that 70% of the tested pigs in Iowa and Illinois carried MRSA (10).

Extensive use of antibiotics in livestock operations can select for resistant bacteria such as MRSA, just as in health care settings. By one estimate, more than 70% of all antibiotics and related drugs used in the United States are used as feed additives for livestock (11). Dutch pig farms that routinely use antibiotics are more likely to have MRSA than farms with limited antibiotic use (12).

According to the World Health Organization, "Our grandparents lived during an age without antibiotics. So could many of our grandchildren" (13). Overuse of antibiotics in agriculture as well as in human medicine could result in this frightening outcome.

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Letters to the Editor

Letters (~300 words) discuss material published in *Science* in the previous 3 months or issues of general interest. They can be submitted through the Web (www.submit2science.org) or by regular mail (1200 New York Ave., NW, Washington, DC 20005, USA). Letters are not acknowledged upon receipt, nor are authors generally consulted before publication. Whether published in full or in part, letters are subject to editing for clarity and space.

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Battle of the Bugs

IN THE NEWS STORY "THE BACTERIA FIGHT BACK" (Special Section on Drug Resistance, 18 July, p. 356), G. Taubes describes the ongoing war between bacteria and antibiotics, which the bacteria appear to be winning. Against this backdrop, scientists are struggling to uncover viable therapeutic alternatives to these erstwhile wonder drugs.

One such alternative, probiotic therapy, has become the focus of considerable research effort in recent times (1). Indeed, several clinical trials have attributed impressive health-promoting effects to probiotics—so-called "good bugs"—including effective antagonistic activities against a variety of microbial pathogens by competitive exclusion and bacteriocin production (2).

Furthermore, a new generation of probiotics termed "designer probiotics" has been engineered to express proteins that mimic cell surface receptors, which adsorb toxins and specifically target enteric infections by blocking ligand-receptor interactions between pathogen and host cells (3). Blocking bacterial adherence reduces infection, while toxin neutralization ameliorates symptoms until the pathogen is eventually overcome by the immune system.

Indeed, McFarland (4), in her seminal review on the control of antibiotic-resistant *Clostridium difficile*, proposed that effective treatment must "reduce the burden of *C. difficile* and its toxins in the intestine, restore the normal colonic microflora and assist the host's immune system." Designer probiotics, satisfying all of these criteria, provide an ideal alternative for the treatment of not only *C. difficile* but also other multidrug-resistant pathogens.

Perhaps the only hope of winning the war against "bad bugs" will be achieved by recruiting "good bugs" as our allies.

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DOE Should Keep Education in Mind

I BELIEVE A. CHO'S NEWS OF THE WEEK STORY "Two U.S. labs vie for long-delayed exotic nuclei source" (18 July, p. 328) misses a few important points about the competition between Michigan State University (MSU) and Argonne National Laboratory (ANL) to build the Department of Energy-funded Facility for Rare Isotope Beams (FRIB). I know both institutions, having received a Ph.D. in nuclear physics from MSU in 1967 and having worked as a contractor during the

1970s at ANL, where I taught courses on the assay of nuclear materials to members of the International Atomic Energy Commission.

Cho frames this issue as a David-versus-Goliath contest, comparing the size of MSU's existing nuclear science facility, the National Superconducting Cyclotron Laboratory (NSCL), with ANL in its entirety. A more apt, apples-to-apples comparison would have considered the relative scale of MSU (\$1.7 billion annual budget, 11,700 employees) and ANL (\$530 million annual budget, 2,800 employees) (1). Clearly, both institutions are capable of managing large, complex operations. And of the two, only MSU, the nation's eighth largest university, is host to a lab designated as one of the nation's flagship nuclear

physics facilities by the Nuclear Science Advisory Committee (2).

However, size is not the critical issue in this competition. Although the national labs provide many training opportunities, education is of primary importance only in a university setting. MSU has the second-best U.S. nuclear physics graduate program (behind MIT) and trains 10% of the nation's nuclear physics Ph.D.'s (3). The MSU lab is currently training about 100 graduate and undergraduate students.

Cho points out that the MSU lab might close if FRIB lands elsewhere. So, as the competition proceeds, decision-makers must consider whether expanding our national lab complex at the expense of jeopardizing a successful university-based educational and scientific center is in the national interest.

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CORRECTIONS AND CLARIFICATIONS

News of the Week: "Full-genome sequencing paved the way from spores to a suspect" by M. Enserink (15 August, p. 898). The affiliation of microbial genomicist Claire Fraser-Liggett should have been the University of Maryland School of Medicine in Baltimore.

Table of Contents: (25 July, p. 457). In the description of the Report "Did cooling oceans trigger Ordovician biodiversification? Evidence from conodont thermometry" by J. A. Trotter *et al.*, the time of the cooling trend through the Early Ordovician was incorrect. The sentence should read: "About 470 million years ago, ocean temperatures dropped to values near those of today after being much higher for many millions of years, coeval with a sharp jump in biodiversity."

Reports: "Did cooling oceans trigger Ordovician biodiversification? Evidence from conodont thermometry" by J. A. Trotter *et al.* (25 July, p. 550). In the References and Notes, reference 27, which is a duplicate of reference 4, should have been deleted. References and notes 28 to 39 should have been renumbered 27 to 38. The citations to these references are correct. In the Fig. 3 legend, the citation to Chen *et al.* should be 29 (not 31).

Perspectives: "Tracking corrosion cracking" by A. Stierle (18 July, p. 349). The last sentence on p. 349 incorrectly stated that "King *et al.* have found that the grain boundaries in stainless steel ... are more sensitive to carbon segregation and the formation of chromium carbides, which makes them more sensitive to corrosion." Instead, the sentence should state that "King *et al.* have found that the grain boundaries in stainless steel ... are more sensitive to corrosion, which might be related to enhanced carbon segregation at these grain boundaries and the formation of chromium carbides."

Reviews: "Rise of the Andes" by C. N. Garzzone *et al.* (6 June, p. 1304). A minus sign was missing from an equation in the second paragraph of the third column on p. 1305. The correct equation should read " $h = -472.5\delta^{18}\text{O}_{\text{rainfall}} - 2645$."

Reports: "Hidden neotropical diversity: Greater than the sum of its parts" by M. A. Condon *et al.* (16 May, p. 928). The first sentence of the main text included a misplaced reference citation. The sentence should read: "The diversity of neotropical herbivorous insects, ranging in number from 3 million to 30 million species (1), has been hypothesized to be a function of plant diversity (2), but the degree to which specialization shapes that function is contentious (3)."

TECHNICAL COMMENT ABSTRACTS

COMMENT ON "Fire-Derived Charcoal Causes Loss of Forest Humus"

Johannes Lehmann and Saran Sohi

Wardle *et al.* (Brevia, 2 May 2008, p. 629) reported that fire-derived charcoal can promote loss of forest humus and belowground carbon (C). However, C loss from charcoal-humus mixtures can be explained not only by accelerated loss of humus but also by loss of charcoal. It is also unclear whether such loss is related to mineralization to carbon dioxide or to physical export.

Full text at www.sciencemag.org/cgi/content/full/321/5894/1295c

RESPONSE TO COMMENT ON "Fire-Derived Charcoal Causes Loss of Forest Humus"

David A. Wardle, Marie-Charlotte Nilsson, Olle Zackrisson

We find the suggestion that substantial charcoal loss occurred in the humus-charcoal mixtures implausible and discuss why complexing of soluble carbon released from the mixtures by underlying mineral soil should be minor. This exchange highlights our limited knowledge about charcoal effects on native soil carbon, indicating that strong advocacy for charcoal addition to offset CO₂ emissions remains premature.

Full text at www.sciencemag.org/cgi/content/full/321/5894/1295d

Call for an Objective DOE Decision

AFTER READING A. CHO'S NEWS OF THE WEEK story "Two U.S. labs vie for long-delayed exotic nuclei source" (18 July, p. 328), I can't help but wonder: How can anyone be sure that the U.S. Department of Energy (DOE) will make this decision objectively? Our National Laboratories have outlived the reasons for which they were established, and, as would any large and aging organization, they are vying to find a new *raison d'être*.

Our universities are hurting for American students, particularly in technical fields. Universities, not national labs, have been a fountain of fresh, competent, and cost-effective personnel that will provide leadership and allow us to regain our technological edge in the coming decades. Support for universities is sorely needed at this time, and DOE must bear this in mind when they make their decision, particularly when the facts point them in a direction that is against their own interests.

CONSTANTINE CASSAPAKIS

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