Is employee ownership so senseless?*

Nicolas AUBERT†, Bernard GRAND‡, André LAPIED§, Patrick ROUSSEAU‡

Abstract

Since Enron and the ruin of thousands of its employees, employee ownership is harshly criticized. Investing savings in employer’s stock would be equivalent to bet on only one asset. Moreover, employee ownership’s debated efficiency would not justify employers to grant company stock to their employees. Still, employee ownership is put in place by thousands of companies and withheld by millions of employees throughout the world. This paper considers a moral hazard setting where a risk neutral entrepreneur grants company stock to its risk averse employee as an incentive. We show that there is an optimal transfer of employee ownership that satisfies employee’s risk preference and has an incentive effect. We thus bring about rational argument in favor of employee ownership.

Key words: Employee ownership, moral hazard, company stock, perfect Nash equilibrium in sub-game.

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† Corresponding author. nicolas.aubert@univmed.fr, Université de la Méditerranée, DEFI, Château La Farge - Route des Milles 13290 Les Milles (France).
‡ Université Paul Cézanne, CERGAM, Clos Guiot Purylicard - CS 30063, 13089 Aix en Provence Cedex 2 (France).
§ Université Paul Cézanne, GREQAM, 15-19 allée C. Forbin, 13627 Aix-en-Provence Cedex 1 (France).
1. **Introduction**

When employees invest both their human capital and a substantial proportion of their wealth in their company, they expose themselves to ill-considered risk. Indeed, such a strategy contradicts the basic advises of portfolio theory regarding diversification’s benefits. The risk increases if employee ownership serves as retirement assets. Several sadly famous American firms such as Enron and WorldCom went bankrupt in the early 2000s blowing away the savings of thousands of workers who invested massively in their company stock. These events put employee ownership into light and made policy makers aware of the risks associated with undiversified 401(k) plans\(^1\). The most striking was that contributions to the 401(k) plans were discretionary. Employees decided themselves to put a substantial proportion of their retirement savings in their company stock with no regard to the basic diversification rules. Studies focusing on employee ownership and diversification tried to understand or to justify employees’ behavior regarding investment in company stock. These contributions rely on the same assertion, that is: employee owners’ behavior is senseless because it contradicts the basic advises of modern portfolio theory. Consequently, employee ownership should be very limited. However, employee ownership is put in place by companies worldwide and millions of workers hold their company stock.

The literature on employee ownership and diversification has two shortcomings. The first is that it is mainly focused on a very specific form of employee ownership, namely employee ownership in American 401(k). But, although employee ownership within American 401(k) has been the target of the critics these latter years, it is not the only way of holding employee ownership in the United States. In fact, the diversification problem is more likely to affect American workers in large firms whose 401(k) is massively invested in company stock. Other Employee Stock Ownership Plans (ESOPs) are more widespread in smaller companies where employees often have a diversified retirement plan. In most western countries, retirement system differs significantly from the American one. The United States rely less on their Social Security system to provide for retirement income than European countries. Moreover, some countries restrict or prohibit employee ownership within retirement plans. In both cases, employee ownership has little to do with retirement and the diversification problem concerns a shorter period of time. A second drawback of the literature on employee ownership and

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\(^1\) 401(k) is the most popular US retirement savings scheme. The term 401(k) refers to the US Internal Revenue Code. A 401(k) plan is a defined contribution plan that permits employees to have a portion of their salary deducted from their paycheck and contributed to an account. Federal (and sometimes State) taxes on the employee contributions and investment earnings are deferred until the participant receives a distribution from the plan (typically at retirement). Employers may also make contributions to a participant’s account.
The major contribution of the paper is based on the introduction of the argument of diversification in the usual principal-agent model. Our model investigates both employee ownership incentive effects and diversification problem. We thus put the emphasis on the company’s contribution policy. In order to study the interaction of incentive mechanisms with the diversification problem, we consider employee ownership as a reward system within a moral hazard setting. This interaction has never been implemented so far. The main result of our analysis is to show that there is an optimal transfer of company stock that satisfies employee’s risk preference and has an incentive effect. This result brings about rational argument in favor of employee ownership.

The paper is organized as follows. After a discussion of our contribution relative to the existing literature on employee ownership and diversification in section 2, we present the model set-up in section 3. Section 4 proves the existence of a general solution which gives the optimal transfer of employee ownership as a perfect Nash equilibrium in sub-game. Under standard hypotheses on probability distribution and utility, we develop some simulations which illustrate the properties of the solution in section 5. Section 6 offers concluding remarks. All proofs are presented in the appendix.

2. Literature
Investing in only one security implies throwing away the benefit of diversification. From a rational perspective, this lack of diversification should be compensated. A first rational motivation of this apparently irrational behavior might be that employees receive compensation of this risk from their employer. Such compensation can take the form of a discount on stock price or a matching contribution in company stock conditional on employees’ investment decision. The idea of compensating the risk of holding company stock with employers’ contribution was first evoked by one of the founders of the American Economic Association in 1886. John Bates Clark considered company’s contribution as a way
to stimulate employee ownership without putting personal worker saving at risk. As a matter of fact, matching contribution in company stock is a strong motivation for employees. Benartzi (2001), Holden and VanDerhei (2001) and Brown et al. (2006) find that employees’ investment in company stock is higher in firms where the employer directs matching contribution into company stock. According to these authors’ computations, the percent of employees’ own contribution allocated to company stock is nearly ten percent points higher on average. Interestingly, Brown et al. (2006) find a relationship between company’s matching policy and their stock risk.

A second rational explanation is the advantageous tax treatment associated to employee stock ownership. Numerous western countries put in place tax benefit in order to stimulate employee ownership. In the United States, most of rules about these tax benefits are found in the Employee Retirement Income Security Act (ERISA) and the Internal Revenue Code (IRC). But, surprisingly, Benartzi et al. (2007) show that employees are not fully aware of these tax advantages and conclude they cannot be the main motivation to invest in their company stock.

A third rationale for employee owners’ holdings is the private information hypothesis. As insiders, employees would have better knowledge of their firm and would be able to earn abnormal returns. Yet, Benartzi (2001) and Huberman and Sengmuller (2004) report no significant correlation between investment choices and subsequent returns and rejected the information based explanation. Cohen (2008) does not find evidence that employees have superior information about future company stock returns either.

According to Harbaugh (2005), employee stock ownership increases the efficiency of wage bargaining. Without employee ownership plans, bargaining can lead to over or under employment depending on the business cycle. So employee ownership would be a mean to reduce the drawback of fixed wages.

If the above mentioned explanations apply, the diversification cost may be compensated. But how much it will take to compensate this cost? Several contributions have estimated it, on a risk-adjusted basis, holding a single stock is worth less than holding a market indexed portfolio. According to Poterba (2003), Meulbroek (2005) and Brennan and Torous (1999), investors holding a single-stock portfolio sacrifice half the equivalent amount invested in a diversified portfolio. Option pricing techniques allow Ramaswamy (2003) to find that the cost of insuring the extra risk of company stock is prohibitively expensive. In spite of the high cost

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2 11 percent for Benartzi (2001; p. 1753) and 8 percent for Brown et al. (2006; p. 1324).

3 This reference was suggested by an anonymous referee.
of this strategy, employees hold their company stock and seem to miss the true risk of lack of diversification. This apparently irrational behavior has motivated scholars to investigate psychological factors.

Huberman (2001) argues that employees’ financial well-being is closely tied to their company stock because this investment is very familiar to them. Because of “familiarity’s bias”, employees consider their company stock as a safe investment. Survey data from Benartzi et al. (2007) shows that 39 percent of employees think their company stock is as risky as a diversified stock fund, whereas 25 percent believe it is safer. Cohen (2008) argues that loyalty to one’s employer is another psychological factor leading employees to hold their company stock. Using divisional employee status as a loyalty proxy, Cohen finds support of the “loyalty hypothesis”.

Benartzi and Thaler (2001) find that employees’ saving behavior is altered if employee ownership is offered as a choice within their retirement plan. They conclude: “It appears that the mental accounting of these investments involves putting the company stock into its own category separate from other equities” (Benartzi and Thaler, 2001; p. 595).

Benartzi (2001) extends the investigation further by observing workers’ behavior in large corporate defined contribution retirement plans. He found that employees interpret abnormally high past performance as representative of future performance, even if stock returns are largely unpredictable. Benartzi (2001) shows that employees whose employer stock experienced the best returns during the last ten years invested nearly 40% of their 401(k) plans in company stock. Conversely, for the same period of time, employees whose firms experienced the worst results invested only 10% of their plan in company stock. This investment behavior is called “excessive extrapolation”. He concluded that employee ownership is a “dubious strategy” and that “individuals do not fully understand the risk of company stock” (Benartzi, 2001; p. 1760).

While insightful, these latter studies disregarded the potential incentive power of employee ownership. They did not investigate the trade-off between risk and incentive that employee ownership incurs. In order to explain employees’ behavior, these recent studies either adopted a rational viewpoint or they focused on psychological factors without considering employee ownership as a reward system. However, several studies emphasized that employee ownership affects performance at the individual and corporate level. Kruse (2002) reviewed most of them and concluded that “studies are split between favorable and neutral findings on the relationship between employee ownership and firm performance”.
Klein (1987) shows that employee ownership is especially effective if it is financially rewarding. According to her, this finding “is consistent with the economic literature on principal and agent relations which suggests that financial incentives such as merit pay, gainsharing, and — by extension — employee ownership, may make agents utility interests compatible with those of the principal” (Klein, 1987; p. 321). At the corporate level, Blasi et al. (1996) show that companies with Employee Stock Ownership Plans report 6 percent higher productivity holding other factors constant.

More recently, Blasi et al. (2008) investigated if employee ownership is consistent with proper diversification. They used traditional portfolio analysis techniques to conclude that holding company stock does not significantly annihilate diversification benefits. Our model reinforces this result bridging portfolio analysis technique with employee ownership’s incentive effect. The analysis is based on a sequential game in which the manager motivates the employee to expand effort by granting company stock. So far, the literature has focused either on the portfolio diversification problem or on the link between employee ownership and performance separating the two questions. On one hand, diversification cost of investing in company stock was estimated assuming perfect information whereas empirical studies about employees’ decision relied on behavioral approach. To the best of our knowledge, this first body of literature never regarded imperfect information problems as a major determinant of employee ownership. On another hand, empirical studies investigating the incentive effects of employee ownership disregarded the risk on employees’ wealth that comes with it. Our paper is an attempt to fill in this theoretical gap. We have found an analytical solution that shows that, under some assumptions, it is possible to find an optimal level of company stock distribution that ensures an optimal level of profit for the company.

3. Model description

The one-period model with two steps consists of a risk neutral entrepreneur and a risk averse employee who is representative of all company employees. We thus consider all employees to have homogeneous preferences.

At the first stage, the entrepreneur designs the compensation system aimed at motivating the employee to expand his desired level of effort. The compensation system we investigate only consists in employee ownership. We thus consider a setup where the entrepreneur has to decide whether or not to offer company stock to his employee and the amount of it. The entrepreneur’s wealth $W_d$ is totally invested in company stock. When employee ownership is
offered to employee, it takes the form of a contribution in company stock that diminishes entrepreneur’s wealth. To simplify the exposition, suppose $c$ to be a proportion of $W_s$, the initial employee’s wealth. $cW_i$ is the money value of company stock granted to employee. Indeed, the money value of the contribution granted to the employee is very often commensurate with the amount invested by the employee. For instance, it takes the form of a discount or a matching contribution. Degeorge et al. (2004) confirmed empirically that wealthier employees are more willing to take a firm exposure.

At the second stage, the employee observes the compensation system and chooses $e$, the level of effort that affects return on company stock $r_e$. Let $F(r_e | e^H)$ and $F(r_e | e^L)$ be the cumulative distribution functions of company stock return conditional on high level of effort $e^H$ and low level of effort $e^L$, we assume $F(r_e | e^H) < F(r_e | e^L)$. Therefore, the agent effort is productive in the sense of first-order stochastic dominance. The effort is personally costly to the employee that endures $\psi(e)$ the disutility that is increasing with effort. Employee’s initial wealth $W_s$ is totally invested in the market portfolio. If employee ownership is granted, employee’s wealth increases by an amount $cW_i$ of company stock. We explicit the means of the conditional distributions of $r_e$ with respect to $e^H$ and $e^L$ respectively as $\mu^H_e$ and $\mu^L_e$. The return of company stock is then $(1 + r_e + \mu^j_e)$ for a level of effort $j=\{H,L\}$ with $f(r_e)$ the centered density function. Similarly, $\mu_m$ is the mean of the distribution of $r_m$, the return of the market portfolio is then $(1 + r_m + \mu^j_m)$, with a centered density function $f(r_m)$.

The entrepreneur’s and employee’s utilities at the end of the game depend on the strategies they adopt. If, at some period of the game, one of the players decides to not participate, their expected utility at the end of the game is null. The agents’ expected utilities at the end of the game depend on whether $c$ is granted or not and on the agent’s level of effort $e^H$ or $e^L$. Let’s denote $V^{i,j}$ the entrepreneur expected utility if a contribution $i=\{0,c\}$ is granted and a level of effort $j=\{H,L\}$ is implemented. We suppose the entrepreneur is risk neutral. Therefore, he maximizes the expected value of his wealth $V^{0,j}$ or $V^{c,j}$ depending on whether $c$ is granted or not:

$$V^{0,j} = \int_{-\infty}^{+\infty} W_d(1 + r_e + \mu_e^j) f(r_e) dr_e = W_d[1 + \mu_e^j]$$  \hspace{1cm} (1)

$$V^{c,j} = \int_{-\infty}^{+\infty} (W_d - cW_s)(1 + r_e + \mu_e^j) f(r_e) dr_e = (W_d - cW_s)[1 + \mu_e^j]$$  \hspace{1cm} (2)
The employee’s expected utility is $U^{0,j}$ or $U^{c,j}$ and also depends on $c$, with $u(.)$ the risk averse utility function:

$$ U^{0,j} = \int_{-\infty}^{\infty} u[W_t (1 + r_m + \mu_m)] f(r_m) dr_m - \psi(e^j) $$

$$ U^{c,j} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u[W_t (1 + r_m + \mu_m) + cW_e (1 + r_e + \mu_e)] f(r_e, r_m) dr_e dr_m - \psi(e^j) $$

Figure 1 presents the structure of the game and the gain matrix.

**Assumption A1:** The employee’s expected utility is required to be positive even when company stock is not granted.

$$ U^{0,L} = \int_{-\infty}^{\infty} u[W_t (1 + r_m + \mu_m)] f(r_m) dr_m - \psi(e^L) > 0 $$ (A1)

The first assumption ensures the existence of the firm.

**Assumption A2:** The employee’s expected utility is higher with employee ownership for a given level of effort.

$$ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u[W_t (1 + r_m + \mu_m) + cW_e (1 + r_e + \mu_e)] f(r_e, r_m) dr_e dr_m - \psi(e^j) \geq \int_{-\infty}^{\infty} u[W_t (1 + r_m + \mu_m)] f(r_m) dr_m - \psi(e^j) $$ (A2)

Without any counterpart, for a given level of effort, granting company stock to the employee increases his expected utility.

**Assumption A3:** The employee is cautious. His relative risk aversion is smaller than one.

$$ \forall x : \alpha = -x \frac{u''(x)}{u'(x)} \leq 1 $$ (A3)

The usual justification of this assumption is that, in this case, the increase of the risk free rate fosters savings (Gollier, 2008).

**Assumption 4:** The market portfolio return’s distributions are symmetric.
This assumption avoids hedging effects between the market portfolio and company stock due to left and right asymmetry (antimonotonic assets) (Chateauneuf et al., 2001).

4. Derivation of an optimal relation contract

Before presenting the main result, we need to develop some machinery and intuition. Lemmas 1, 2, 3 and 4 and 5 describe respectively the entrepreneur’s and employee’s optimal strategies.

**Lemma 1**: When the entrepreneur does not grant company stock \((c=0)\), the employee exerts low level of effort.

**Proof**: See the appendix.

**Lemma 2**: Entrepreneur’s exit does not lead to a perfect Nash equilibrium in sub-game.

**Proof**: See the appendix.

Granting a positive amount of company stock \((c > 0)\) and exerting a high level of effort \((V^{c.H}, U^{c.H})\) is a perfect Nash equilibrium in sub-game when several conditions are satisfied.

The manager plays \(c > 0\) if \(V^{c.H} \geq V^{0.L}\) and \(V^{c.H} \geq V^0\).

The employee chooses to work harder if \(U^{c.H} \geq U^{c.L}\) and \(U^{c.H} \geq U^0\).

**Lemma 3**: The condition \(V^{c.H} \geq V^{0.L}\) implies that the condition \(V^{c.H} \geq V^0\) is satisfied.

**Proof**: See the appendix.

**Lemma 4**: The condition \(U^{c.H} \geq U^{c.L}\) implies that the condition \(U^{c.H} \geq U^0\) is satisfied.

**Proof**: See the appendix.
According to lemmas 3 and 4, a perfect Nash equilibrium in sub-game is fulfilled if $V^{c,H} \geq V^{0,L}$ and $U^{c,H} \geq U^{c,L}$. The first condition ($V^{c,H} \geq V^{0,L}$) that leads the manager to grant employee ownership can be written as follows.

$$c \leq \bar{c} = \frac{W_s \mu_e^H - \mu_e^L}{W_s + \mu_e^H}$$ (5)

Where $\bar{c}$ is the threshold above which granting company is too costly to the entrepreneur.

The second condition ($U^{c,H} \geq U^{c,L}$) that ensures a high level of effort exerted by the employee can be expressed as $\omega(c) \geq \psi(e^H) - \psi(e^L)$ where:

$$\omega(c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u[W_s(1 + r_m + \mu_m) + cW_s(1 + r_e + \mu_e^H)]f(r_e, r_m)dr_e dr_m - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u[W_s(1 + r_m + \mu_m) + cW_s(1 + r_e + \mu_e^L)]f(r_e, r_m)dr_e dr_m \geq \psi(e^H) - \psi(e^L)$$ (6)

$\omega(0)=0$, if the difference $\psi(e^H) - \psi(e^L)$ is sufficiently small and $\omega(c)$ is an increasing function, an optimal level of company stock $c^*$ exists, $c^* \in ]0, \bar{c}].$

**Lemma 5:** $\omega(c)$ is an increasing function from 0.

**Proof.** See the appendix.

Given lemmas 1 to 5, the following proposition establishes the existence of an optimal transfer of company stock that satisfies employee’s risk preference and has an incentive effect.

**Proposition:** Under assumptions A1 to A4, given a small disutility of effort difference $\psi(e^H) - \psi(e^L)$, an optimal transfer of company stock $c^* \in ]0, \bar{c}]$ exists. It satisfies the conditions of a perfect Nash equilibrium in sub-game (Selten, 1965) given:

$$\omega(c^*) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u[W_s(1 + r_m + \mu_m) + c^*W_s(1 + r_e + \mu_e^H)]f(r_e, r_m)dr_e dr_m - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u[W_s(1 + r_m + \mu_m) + c^*W_s(1 + r_e + \mu_e^L)]f(r_e, r_m)dr_e dr_m = \psi(e^H) - \psi(e^L)$$

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$V^{c,H} \geq V^{0,L}$ can be written as $c \leq \bar{c}$ (see equation 5). $\omega(c^*) = \psi(e^H) - \psi(e^L)$ and $c^* = \omega^{-1}[\psi(e^H) - \psi(e^L)] \leq \bar{c}$, with $\omega$ monotinous and increasing. Consequently, $\psi(e^H) - \psi(e^L) \leq \omega(\bar{c})$ that explains what is called a sufficiently small difference.
with \( \bar{c} = \frac{W_d \mu_e^H - \mu_e^L}{W_s \left( 1 + \mu_e^H \right)} \)

Figure 2 illustrates the proposition. The entrepreneur selects the minimum level of \( c \) that insures a high level of effort. Indeed, the higher the amount of company stock, the lower the entrepreneur’s wealth. The additional disutility bore by the employee due to a higher effort is exactly compensated by a supplementary utility of wealth. Figure 2 shows that the difference between the two levels of disutility of effort cannot be compensated above \( \bar{c} \). Beyond \( \bar{c} \), employee ownership becomes too costly to the entrepreneur.

5. Application

To illustrate how relational contract can be used, this section provides numerical results of the model. The calibration needs the specification of the distributions of company stock and market portfolio returns and the utility function of the employee. We take the usual assumption of normality for the distribution and negative exponential for the utility function. Given the conditions of the perfect Nash equilibrium in sub-game expounded in section 4 and the form of the negative exponential utility function, the analytical solution of \( c^* \) is given by\(^5\):

\[
\omega(c^*) = \psi(e^H) - \psi(e^L) \Leftrightarrow \exp \delta^L - \exp \delta^H = \frac{\psi(e^H) - \psi(e^L)}{a}, a > 0
\]

with:

\[
\delta = \frac{1}{2 |\Delta|} \left[ \left( \sigma_e^2 - \sigma_m^2 \right) m_y^2 + \left( \sigma_m^2 - \sigma_e^2 \right) m_x^2 + \left( \sigma_e^2 + \sigma_m^2 - 2 \sigma_{em} \right) m_x m_y \right] - \alpha W_s \left[ 1 + \mu_m + c^* (1 + \mu_e) \right]
\]

The joint distribution of \( r_e \) and \( r_m \) is normal with a covariance matrix \( \Delta = \begin{pmatrix} \sigma_e^2 & \sigma_{em} \\ \sigma_{em} & \sigma_m^2 \end{pmatrix} \). \( \delta \) takes the values of \( \delta^H \) and \( \delta^L \) when \( \mu_e = \mu_e^H \) and \( \mu_e = \mu_e^L \) respectively and where \( m_x \) are \( m_y \) given by expressions (21) and (22) computed in the appendix.

**PROOF.** See the appendix.

Traditional comparative statics analysis makes it possible to emphasize several properties of the solution. Some results are intuitive and do not raise a debate. Conversely, other major results clarify the specificities of employee ownership contracts.

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\(^5\) A graphical illustration of the solution is given by figure 2.
Major results:

1) Figure 3(a) displays how the correlation coefficient between the company stock and the market returns interacts with $c^*$. This first result is the most interesting. $c^*$ decreases as the correlation of company stock with the market increases. It is in line with Blasi et al. (2008) argument according to which “a small but meaningful employee stock ownership level will not significantly deteriorate the diversification of employee portfolios” (p. 17). Here, as far as the return on company stock is highly correlated with the market, incentive effect associated with employee ownership is obtained for a lower level of $c^*$.

2) Figure 3(b) shows the positive relationship between the company stock mean return obtained with a low level of effort and $c^*$. In figure 2, increasing $\mu_c^L$ moves $\omega(c)$ to the bottom and increases $c^*$. This second result is appealing because it highlights the relative situation of firms according to their level of productivity. Ceteris paribus, the marginal efficiency of one dollar distributed to the employee is more important for a low level of productivity ($\mu_c^L$) than for a higher level. The compensation plan will be more costly for a highly productive firm than for a lower one.

Minor results

3) The remaining figures show evidence of more intuitive results. Figure 3(c) exhibits a positive link between the market portfolio mean return and $c^*$. As the market return is higher, the opportunity cost of investing in company stock whose return remains constant is higher. It becomes more costly to the employee to hold a smaller part of his wealth in the market portfolio. To induce a high level of effort, the manager must compensate this cost by increasing $c^*$. Figure 3(c) shows to what extent this cost must be compensated by a higher $c^*$.

4) Figure 3(d) shows a positive relationship between the market portfolio volatility and $c^*$. This result is due to the construction of the correlation coefficient. For a given level of this correlation coefficient, increasing the market portfolio volatility always results in increasing the company stock standard deviation. The additional risk of holding company stock has to be compensated by a higher $c^*$ up to the threshold $\overline{c}$.

5) Figure 3(e) investigates the link between entrepreneur’s wealth and $c^*$. As entrepreneur’s wealth increases, $\overline{c}$ expands to a certain level where it becomes high
enough to compensate employee’s effort. Behind this level, the conditions of the perfect Nash equilibrium in sub-game are not satisfied. The dashed increasing part of the figure 3(e) illustrates this situation which corresponds to values of $c$ between 0 and $c^*$ in figure 2. Once $c^*$ is reached, increasing the amount of company stock becomes useless since the employee always selects a high level of effort.

6) Figure 3(f) presents the negative relationship between employee’s wealth and $c^*$. The employee’s cost of exerting a high level of effort remains constant, so is the money value of company stock he is granted $c^*W_s$. $c^*$ must decrease to keep this amount constant when employee’s wealth increases. Then the relation between $W_s$ and $c^*$ is hyperbolic.

6. Concluding remarks
An important idea in the literature is that employee ownership is too costly to firms and employees to be implemented. From the management’s point of view, employee ownership would be ineffective. Regarding employees’ point, holding company stock would not be an optimal investment strategy. Our analysis identifies circumstances under which employee ownership is an optimal strategy, even though it is costly to entrepreneur and employee. The main result is that there is an optimal transfer of employee ownership that satisfies employee’s risk preference and has an incentive effect. So far, only behavioral arguments have explained the existence of employee ownership as an investment strategy. We thus bring about rational arguments in favor of employee ownership and show how several variables included in the model interact with the optimal transfer of company stock. In this paper, we are able to give an analytical expression of the optimal level of company stock as a function of several variables. It appears that if the correlation between the market return and the company return is high, then the employee’ risk connected to under diversification is reduced and so, the optimal level of share to be distributed is not as high as when there is no correlation between market return and firm return. In addition, it is also possible to conclude on a decreasing return of the incentive effect.

Future work could examine the impact of different aspects of the employee ownership contract. For instance, firms often offer employee ownership within retirement plan. Conditions under which company stock can be included in a long run investment strategy could be emphasized.
Appendix

Figure 1: Extensive form of the game and the payoff matrix

Figure 2: Determination of the $c^*$

Notes. Thick lines represent (a) entrepreneur’s wealth $V^{c,H}$ with the optimal contract; (b) the maximum threshold of company stock granted $\bar{c}$; (c) the difference of employee’s utility wealth due to a high level of effort $\omega(c)$. Dash line represents the disutility of effort bore by the employee.
Figure 3: Relationships between $c^*$ and the variables of the model

(a) $\rho_{em}$

(c) $\mu_m$

(e) $\sigma_m$

(b) $\mu_L^c$

(d) $\sigma_w$

Notes. Values of the parameters are: $\mu_L^c = 0.05$; $\mu_L^{\mu} = 0.3$ (panel B=1); $\mu_m = 0.5$; $\sigma_m = 1$; $\sigma = 7.5$; $\rho_{em} = 0.2$; $W_s = 0.01$; $W_d = 1$; $\alpha = 0.5$. 
**Proof of Lemma 1.**
According to A1, the employee has always an incentive to participate to the contract. Indeed, his utility is always positive \((U_{0.1} > 0)\) if he participates whereas it equals zero \((U^0 = 0)\) when he does not participate. A1 insures employee’s participation even without employee ownership. Furthermore, exerting a high level of effort is more costly to the employee \((\psi(e_H) > \psi(e_L))\). Consequently, if \(c=0\), employee’s optimal action is to exert low level of effort \((U_{0.1} < U_{0.1})\).

**Proof of Lemma 2.**
Manager’s participation gives him a higher wealth than exit \((V_{0.1} \geq V^0)\) even if the employee selects low level of effort. More over, lemma 1 insures that \(V_{0.1}\) is always reachable.

**Proof of Lemma 3.**
Manager’s wealth is smaller if he does not participate whatever the level of effort selected by the employee.

**Proof of Lemma 4.**
According to A2 and A1, the employee’s utility is (i) higher if he is granted company stock than if he is not \((U_{0.1} \geq U_{0.1})\) and (ii) always positive in case of participation to the contract \((U_{0.1} > U^0)\). Consequently, the employee’s utility is higher when he chooses the low level of effort and he is granted company stock than when he does not participate to the contract \((U_{0.1} > U^0)\).

**Proof of Lemma 5.**
For \(c=0\), \(\varphi(0)=0\). Furthermore, as shown below, \(\frac{\partial \varphi(c)}{\partial c} > 0\).

\[
\frac{\partial \varphi(c)}{\partial c} = \int_{-\infty}^{\infty} \left[ W_s(1 + r_x + \mu_x^u)u'[W_s(1 + r_m + \mu_m) + cW_s(1 + r_x + \mu_x^u)]f(r_x, r_m)dr_x dr_m - \int_{-\infty}^{\infty} W_s(1 + r_x + \mu_x^l)u'[W_s(1 + r_m + \mu_m) + cW_s(1 + r_x + \mu_x^l)]f(r_x, r_m)dr_x dr_m \right]
\]

(8)

According to A3, \(\alpha \leq 1\).

\[
\alpha = -x \frac{u''(x)}{u'(x)} \leq 1 \iff u'(x) + xu''(x) \geq 0, \forall x \iff (xu'(x))' \geq 0
\]

(9)

Let

\[
X = W_s(1 + r_m + \mu_m) + cW_s(1 + r_x + \mu_x^u)
\]

\[
Y = W_s(1 + r_m + \mu_m) + cW_s(1 + r_x + \mu_x^l)
\]

Replacing \(X\) and \(Y\) in equation (7) gives:

\[
\frac{\partial \varphi(c)}{\partial c} = \int_{-\infty}^{\infty} \frac{X - W_s(1 + r_m + \mu_m)}{c}u'(X)f(r_x, r_m)dr_x dr_m - \int_{-\infty}^{\infty} \frac{Y - W_s(1 + r_m + \mu_m)}{c}u'(Y)f(r_x, r_m)dr_x dr_m
\]

(10)

We name \(I\) and \(J\) the two parts of the following expression:
We use a change of variable so we can write the

\[
\frac{\partial \omega(c)}{\partial c} = \frac{1}{c} \left[ \int \int X'(X) f(r_e, r_m) dr_e dr_m - \int \int Y'(Y) f(r_e, r_m) dr_e dr_m \right] + W_s \left[ \int \int u'(Y) (1 + r_m + \mu_m) f(r_e, r_m) dr_e dr_m - \int \int u'(X) (1 + r_m + \mu_m) f(r_e, r_m) dr_e dr_m \right] \\
+ W_s \left[ \int \int r_m u'(Y) f(r_e, r_m) dr_e dr_m - \int \int r_m u'(X) f(r_e, r_m) dr_e dr_m \right] \\
\right] 
\]

(11)

With \( c \geq 0 \), if \( I \) and \( J \) are positive, then \( \frac{\partial \omega(c)}{\partial c} \) is also positive.

Since \( xu'(x) \) is monotonously increasing and \( X > Y, I \geq 0 \).

We name \( K \) and \( L \) the two parts of \( J \):

\[
J = W_s \left[ (1 + \mu_m) \left[ \int \int u'(Y) f(r_e, r_m) dr_e dr_m - \int \int u'(X) f(r_e, r_m) dr_e dr_m \right] + \int \int r_m u'(Y) f(r_e, r_m) dr_e dr_m - \int \int r_m u'(X) f(r_e, r_m) dr_e dr_m \right] \\
\right] 
\]

(12)

With \( W_s \geq 0 \), \( (1 + \mu_m) \geq 0 \), if \( K \) and \( L \) are positive then \( J \) is also positive.

With \( X > Y \) and \( u'' < 0 \), we have \( u'(Y) > u'(X) \). Consequently, \( K > 0 \).

We write \( L \):

\[
L = \int \int r_m u'(W_s(1 + r_m + \mu_m) + cW_s(1 + r_e + \mu^I_e)) f(r_e, r_m) dr_e dr_m \\
\]

(13)

\[
\frac{\partial G(r_m, r_e)}{\partial r_m} = W_s \left[ u'(W_s(1 + r_m + \mu_m) + cW_s(1 + r_e + \mu^I_e)) - u''(W_s(1 + r_m + \mu_m) + cW_s(1 + r_e + \mu^I_e)) \right] \\
\]

With \( u''' \geq 0 \) and \( \mu^I_e > \mu^L_e \), \( \frac{\partial G(r_m, r_e)}{\partial r_m} \geq 0 \). Since \( G(r_m, r_e) \) is increasing in \( r_m \) and the distributions of \( r_m \) are symmetric (A4), we have: \( L = \int \int r_m G(r_m, r_e) f(r_m, r_e) dr_m dr_e \geq 0 \)

Since \( K > 0 \) and \( L \geq 0 \), we have \( J > 0 \).

\( I \geq 0 \) and \( J > 0 \) imply \( \omega'(c) > 0 \).

**APPLICATION WITH A NEGATIVE EXPONENTIAL UTILITY FUNCTION**

To give an analytical value of \( c^* \): (i) We use a change of variable so we can write the employee utility function multiplied by the normal distribution density function like another density function with different means multiplied by a constant term \( D \). Since the normal density integrals equal one, the constant term \( D \) appears after integration. (ii) We then normalize the utility function and (iii) compute \( c^* \).
(i) Normal distribution density function

For a given normal distribution \( N(\mu, \Sigma) \) defined in \((\mathbb{R}^n, \mathbb{R}^n)\) with \( X \) the random vector, \( \mu \) is the means vector and \( \Sigma \) the covariance matrix. If the covariance matrix is regular (\( \Sigma \neq 0 \)), the normal distribution density function is:

\[
\phi(X) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(X-\mu)'\Sigma^{-1}(X-\mu)}
\]  

(14)

Our model includes two random variables, the market portfolio and the company stock returns. Consequently, we write (14) with \( n=2 \). We write:

\[
X = \begin{pmatrix} x \\ y \end{pmatrix}, m = \begin{pmatrix} m_x \\ m_y \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}
\]

with \( |\Sigma| = \sigma_x^2 \sigma_y^2 - \sigma_{xy}^2 \neq 0 \) the covariance matrix’ determinant and \( \Sigma^{-1} = \frac{1}{|\Sigma|} \begin{pmatrix} \sigma_y^2 - \sigma_{xy} & \sigma_y^2 - \sigma_{xy} \\ \sigma_x^2 - \sigma_{xy} & \sigma_x^2 - \sigma_{xy} \end{pmatrix} \) the inverted covariance matrix. Replacing in equation (14) gives:

\[
\phi(x, y) = \frac{1}{2\pi \sqrt{|\Sigma|}} e^{-\frac{1}{2}(x-m_x \quad y-m_y) \begin{pmatrix} \sigma_y^2 - \sigma_{xy} & \sigma_y^2 - \sigma_{xy} \\ \sigma_x^2 - \sigma_{xy} & \sigma_x^2 - \sigma_{xy} \end{pmatrix} \begin{pmatrix} x-m_x \\ y-m_y \end{pmatrix}}
\]

\[
= \frac{1}{2\pi \sqrt{|\Sigma|}} e^{-\frac{1}{2}|\Sigma| \left[ (\sigma_y^2 - \sigma_{xy})x^2 + (\sigma_x^2 - \sigma_{xy})y^2 + (\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy})xy \\
- [2(\sigma_y^2 - \sigma_{xy})m_x + (\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy})m_y]x \\
- [2(\sigma_x^2 - \sigma_{xy})m_y + (\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy})m_x]y \\
+ (\sigma_y^2 - \sigma_{xy})m_x^2 + (\sigma_x^2 - \sigma_{xy})m_y^2 + (\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy})m_x m_y \right]}
\]  

(15)

We therefore assume that the market portfolio and the company stock returns are normally distributed and centered in \((\mathbb{R}^n, \mathbb{R}^n)\), \( m_e = m_m = 0 \) and that the regular covariance matrix is \( \Delta = \begin{pmatrix} \sigma_e^2 & \sigma_{em} \\ \sigma_{em} & \sigma_m^2 \end{pmatrix} \) with \( |\Delta| = \sigma_e^2 \sigma_m^2 - \sigma_{em}^2 \). After replacing in (15), the joint utility function density \( f(r_e, r_m) \) becomes:

\[
f(r_e, r_m) = \frac{1}{2\pi \sqrt{|\Delta|}} e^{-\frac{1}{2|\Delta|} \left[ (\sigma_e^2 - \sigma_{em})r_e^2 + (\sigma_e^2 - \sigma_{em})r_m^2 + (\sigma_e^2 + \sigma_m^2 - 2\sigma_{em})r_e r_m \right]}
\]  

(16)

When employee ownership is offered, the employee’s utility of wealth is given by (4) conditional on the level of effort exerted. With an exponential utility function, the employee’s utility becomes:

\[
g(r_e, r_m) = \exp(-\alpha[W_i(1+r_m + \mu_m) + e^*W_i(1+r_e + \mu_e)])f(r_e, r_m)
\]  

(17)

where \( \mu_e \) takes the values \( \mu_e^H \) and \( \mu_e^L \).

Combining (16) with (17) gives:
\[
g(r_x, r_m) = \frac{1}{2\pi \sqrt{|\Delta|}} \exp\left( -\frac{1}{2|\Delta|} \left[ (\sigma^2 - \sigma_{em}) r_x^2 + (\sigma^2 - \sigma_{em}) r_m^2 + (\sigma^2 + \sigma^2 - 2\sigma_{em}) r_x r_m \right. \right.
\]
\[
\left. + (2|\Delta| \alpha W_x) r_x + (2|\Delta| \alpha * W_x) r_m + 2|\Delta| \alpha W_x [(1 + \mu) + c^*(1 + \mu_\epsilon)] \right) \right) \right)
\]

where \( \mu_\epsilon \) takes the values \( \mu_\epsilon^H \) and \( \mu_\epsilon^L \).

We must now compute the means \( m_x \) and \( m_y \) such as \( \sigma_x = \sigma_x, \sigma_y = \sigma_m, \sigma_{xy} = \sigma_{em} \) (that is \( \Sigma = \Delta \)) to identify \( \varphi(x, y) \) and \( g(r_x, r_m) \) multiplied by a constant term \( D \) so that:

\[
D \varphi(x, y) = g(r_x, r_m)
\]

The identification of (15) multiplied by \( D \) and (18) gives the following expressions:

\[
-2(\sigma^2 - \sigma_{em}) m_x + (\sigma^2 + \sigma^2 - 2\sigma_{em}) m_y = 2|\Delta| \alpha W_x
\]

\[
-2(\sigma^2 - \sigma_{em}) m_x + (\sigma^2 + \sigma^2 - 2\sigma_{em}) m_y = 2|\Delta| \alpha W_x
\]

This system has a unique solution if:

\[
\det \left( \begin{array}{cc}
2(\sigma^2 - \sigma_{em}) & 2(\sigma^2 + \sigma^2 - 2\sigma_{em}) \\
(\sigma^2 + \sigma^2 - 2\sigma_{em}) & 2(\sigma^2 - \sigma_{em})
\end{array} \right) = - (\sigma^2 - \sigma_{em})^2 \neq 0
\]

The determinant is not null if \( \sigma_e > \sigma_m \).

The solutions of the system (20) are:

\[
m_x = \frac{2|\Delta| \alpha W_x [2c^*(\sigma^2 - \sigma_{em}) - (\sigma^2 + \sigma^2 - 2\sigma_{em})]}{(\sigma^2 - \sigma_{em})^2}
\]

\[
m_y = \frac{2|\Delta| \alpha W_x [2(\sigma^2 - \sigma_{em}) - c^*(\sigma^2 + \sigma^2 - 2\sigma_{em})]}{(\sigma^2 - \sigma_{em})^2}
\]

Identifying (15) multiplied by \( D \) to (18) finally gives:

\[
D = \exp(\delta)
\]

with:

\[
\delta = \frac{1}{2|\Delta|} \left[ (\sigma^2 - \sigma_{em}) m_x^2 + (\sigma^2 - \sigma_{em}) m_y^2 + (\sigma^2 + \sigma^2 - 2\sigma_{em}) m_x m_y \right] - \alpha W_x [(1 + \mu + c^*(1 + \mu_\epsilon)]
\]

where \( \delta \) takes the values of \( \delta^H \) and \( \delta^L \) when \( \mu_\epsilon = \mu_\epsilon^H \) and \( \mu_\epsilon = \mu_\epsilon^L \) respectively and where \( m_x \) are \( m_y \) given by expressions (21) and (22).

(ii) Normalization of the utility function

The Von Neumann Morgenstern utility function was normalized by \( u(0) = 0 \) for a given level of wealth \( W_x \). We use an affine transformation.

\[
v(W) = -e^{-\alpha W}, \alpha > 0
\]

\[
u(W) = \alpha v(W) + b, a > 0
\]

\[
\Leftrightarrow u(W) = -ae^{-\alpha W} + b, u(0) = 0 \Rightarrow b = a
\]

Consequently, the utility function becomes:

\[
u(W) = a[1 - e^{-\alpha W}], \alpha > 0, a > 0
\]

Considering equations (19) and (24), we can rewrite the condition \( \omega(c^*) \):
\[ \omega(c^*) = a \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(r_e, r_m) dr_e dr_m - D^L \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varphi(x, y) dx dy \right] \\
- \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(r_e, r_m) dr_e dr_m - D^H \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varphi(x, y) dx dy \right] \]

where \( a \) is a positive constant term.

Since the normal density integrals equal to one, \( \omega(c^*) \) becomes:

\[ \omega(c^*) = a(D^L - D^H) \]  

(25)

where \( D^H \) and \( D^L \) are the values of \( D \) given by (22) with \( \mu_e = \mu_e^H \) and \( \mu_e = \mu_e^L \) respectively.

(iii) Computation of \( c^* \)

\( c^* \) is given by:

\[ \omega(c^*) = \psi(e^H) - \psi(e^L) \iff \exp \delta^L - \exp \delta^H = \frac{\psi(e^H) - \psi(e^L)}{a} > 0 \]  

(7)

where \( \delta^H \) and \( \delta^L \) are given by (23) with \( \mu_e = \mu_e^H \) and \( \mu_e = \mu_e^L \) respectively and \( m_x \) et \( m_y \) are given respectively by (21) et (22).

References


