Dynamic models with non clearing markets

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Dynamic Models with Non Clearing Markets\textsuperscript{*}

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Abstract

This article studies a new class of models which synthesize the two traditions of general equilibrium with nonclearing markets and imperfect competition on the one hand, and dynamic stochastic general equilibrium (DSGE) models on the other hand. This line of models has become a central paradigm of modern macroeconomics for at least three reasons: (a) it displays solid microeconomic foundations, (b) it is a highly synthetic theory, which combines in a unified framework general equilibrium, nonclearing markets, imperfect competition, growth theory and rational expectations, (c) it is also an empirical success, leading to substantial progress towards matching real world statistics.


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This article studies a new class of models which synthesize the two traditions of general equilibrium with nonclearing markets and imperfect competition on the one hand, and dynamic stochastic general equilibrium (DSGE) models on the other hand. Although this line of models is still recent, it has clearly become in a short time a central paradigm of modern macroeconomics. The reasons are at least threefold.

The first is that it displays solid microeconomic foundations. This is quite natural since from the two constituent fields above this one inherited a strong general equilibrium framework where all agents (households or firms) maximize their respective objectives subject to well defined constraints.

The second is that it is a highly synthetic theory, which combines in a unified framework general equilibrium, nonclearing markets, imperfect competition, growth theory and rational expectations, so that it can appeal to macroeconomists with very different backgrounds.

The third reason is empirical. A key motivation for DSGE models is to compare the “statistics” generated by these models with the real world ones. In that respect the addition of nonclearing markets and imperfect competition has led to substantial progress in matching these statistics, and this has certainly been an important factor in the success of these models.

Now such a wide synthesis did not come all at once. So we shall begin by recalling briefly a little history and some of the antecedents of the field.

We then present a series of models with explicit solutions. These will demonstrate analytically how the introduction of non clearing markets allows to substantially improve the ability of DSGE to reproduce a number of macroeconomic facts.

1 History

1.1 Early times

At the time when many of the developments leading to these models were initiated, there was a profound split between microeconomics and macroeconomics. On the one hand microeconomics, in its general equilibrium version, was dominated by Walras’ (1874) model, as developed by Arrow-Debreu (1954), Arrow (1963), and Debreu (1959). In these models all adjustments are carried out via fully flexible prices, and agents never experience any quantity constraint. On the other hand in the standard macroeconomic model in
the Keynes (1936) and Hicks (1937) tradition, as exemplified by the IS-LM model, there are price and wage rigidities, unemployment is present and most adjustments are carried out through variations in real income, a quantity, not a price.

Confronted with this inconsistency, the strategies of macroeconomists turned out to be quite diverse and they took two different routes.

1.2 General equilibrium with nonclearing markets

On the one hand a first set of authors aimed at achieving a synthesis between the then existing microeconomics and macroeconomics. This was achieved by generalizing the traditional general equilibrium model, by introducing nonclearing markets, introducing quantity signals into demand and supply functions, and endogenizing prices in a framework of imperfect competition.

Patinkin (1956) and Clower (1965) showed that the presence of quantity constraints in nonclearing markets would drastically modify the demands for labor and goods, an insight further emphasized by Leijonhufvud (1968). Barro and Grossman (1971, 1976) combined these insights into a fixprice macromodel. Drèze (1975) and Bénassy (1975, 1982) constructed full general equilibrium concepts with price rigidities, where price movements are partially replaced by endogenous quantity constraints. Bénassy (1976) linked these concepts with general equilibrium under imperfect competition à la Negishi (1961). This link was furthered with the construction of a full general equilibrium concept of objective demand curve based on quantity constraints (Bénassy 1988. See also Gabszewicz and Vial, 1972, for a Cournotian view). All these developments are reviewed in the dictionary entry “Nonclearing markets in general equilibrium”.

1.3 Dynamic market clearing macroeconomics

A second set of authors achieved consistency between microeconomics and macroeconomics by importing into macroeconomics the basic assumption of the then dominant general equilibrium microeconomic models, market clearing. At the same time they paid strong attention to the issues of dynamics and expectations. A central part of these developments was the use of “rational expectations” in the sense of Muth (1961). This was an important addition, as in the Keynesian system it was sometimes difficult to disentangle the results due to price or wage rigidity from those due to incorrect expectations.
Rational expectations allowed the suppression of the second type of results. It appeared also that, even with rational expectations and market clearing, it was possible to build rigorous models displaying fluctuations (Lucas, 1972, Kydland and Prescott, 1982, Long and Plosser, 1983).

1.4 Non Walrasian cycles

Starting in the mid-eighties authors began combining elements of the two paradigms described above, achieving the synthesis that is the subject of this article. Svensson (1986) studies a dynamic stochastic general equilibrium monetary economy subject to supply and demand shocks. Prices are preset one period in advance by monopolistically competitive firms, so we have both imperfect competition and sticky prices. Because of price presetting the model has multiple regimes.

Various types of rigidities have been then introduced in dynamic models, leading to different patterns of cycles. Andersen (1994) reviews various causes and consequences of price and wage rigidities.


Models with nominal rigidities study situations where the nominal prices themselves (and not relative prices) are sluggish. Several devices have been used. The first, following the early works on wage and price contracts by Gray (1976), Fischer (1977), Phelps and Taylor (1977), Taylor (1979, 1980) and Calvo (1983), assumes that there is a system of contracts expiring at deterministic or stochastic dates. For that reason they are called “time dependent”. Such contracts have been integrated in DSGE models by Cho (1993), Cho and Cooley (1995), Bénassy (1995, 2002, 2003a,b), Yun (1996), Cho, Cooley and Phaneuf (1997), Andersen (1998), Jeanne (1998), Ascani (2000), Chari, Kehoe and McGrattan (2000), Collard and Ertz (2000), Ascarì and Rankin (2002), Huang and Liu (2002), Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005), to name only a few.

Another type of price rigidity, called “state dependent”, is based on costs of changing prices. Two specifications are favorite in the literature: quadratic
costs of changing prices (Rotemberg, 1982a,b), which have been implemented, for example, in Hairault and Portier (1993), and fixed costs of changing prices (Barro, 1972), often renamed “menu costs”. Clearly these costs should be interpreted as surrogates for other unspecified causes, and identifying these causes is a challenge that faces this line of research.

Now most of the contributions of this field are based on numerical evaluations of various models. So we shall present next a number of models with explicit solutions which will make clear why this line of models has been successful in solving problems that were difficult to solve in market clearing models.

2 An analytical illustration

We shall now show in this section in a series of explicitly solved models how the introduction of nominal rigidities in DSGE models allows to considerably improve the capacities of these models to reproduce the dynamic evolutions of actual economies.

We first present a basic model and compute as a reference its Walrasian equilibrium and dynamics. Then we introduce a first nominal rigidity, one-period wage contracts. This improves some correlations, but cannot create strong persistence as in reality. We next introduce multiperiodic wage contracts, and show that this allows to obtain a persistent response of output to demand shocks. Finally simultaneous rigidities of wages and prices are considered, and we show that one can obtain in this way with fairly realistic values of the parameters a persistent and humpshaped response of both output and inflation.

2.1 The basic model

We shall study a dynamic monetary economy à la Sidrauski (1967) and Brock (1975), where goods are exchanged against money at the (average) price $P_t$ and work against money at the (average) wage $W_t$. There are two types of agents: households and firms. Firms have a simple technology:

$$Y_t = Z_t N_t^\alpha$$

(1)

where $N_t$ is the quantity of labor used by firms and $Z_t$ a technological shock common to all firms. Note that we do not introduce capital in this model.
Because its rate of depreciation is low, it would not add much to our argument, and would substantially complicate the results and exposition.

The representative household works \( N_t \), consumes \( C_t \), and ends period \( t \) with a quantity of money \( M_t \). It maximizes the expectation of its discounted utility:

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t + \omega \log \frac{M_t}{P_t} - \xi N_t^{\nu} \right]
\] (2)

At the beginning of period \( t \) the household faces a monetary shock à la Lucas (1972), whereby the quantity of money \( M_{t-1} \) coming from \( t-1 \) is multiplied by \( \mu_t \), so that its budget constraint for period \( t \) is:

\[
C_t + \frac{M_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{\mu_t M_{t-1}}{P_t}
\] (3)

There are thus two shocks in this economy, the technology shock \( Z_t \) and the monetary shock \( \mu_t = M_t/M_{t-1} \). As an illustration we shall use below the following traditional processes (in all that follows lowercase letters represent the logarithm of the variable represented by the corresponding uppercase letter):

\[
m_t - m_{t-1} = \frac{\varepsilon_{mt}}{1 - \rho L} \quad z_t = \frac{\varepsilon_{zt}}{1 - \varphi L}
\] (4)

where \( \varepsilon_{zt} \) and \( \varepsilon_{mt} \), the innovations in \( z_t \) and \( m_t \), are uncorrelated white noises with:

\[
\text{var}(\varepsilon_{zt}) = \sigma_z^2 \quad \text{var}(\varepsilon_{mt}) = \sigma_m^2
\] (5)

### 2.2 Walrasian dynamics

As a benchmark we shall first study here the case where both labour and goods markets are in Walrasian equilibrium in each period, as in the first traditional RBC (real business cycles) models, and we shall see how this economy reacts to technological and monetary shocks. Solving the model we find that money holdings are a multiple of consumption:

\[
\frac{M_t}{P_t C_t} = \frac{\omega}{1 - \beta}
\] (6)

and that employment \( N_t \) is constant:
\[ N_t = N = (\alpha/\xi)^{1/\nu} \]  
\[ n_t = n \quad y_t = z_t + \alpha n \quad w_t - p_t = y_t - n \]

Using (1) and (7) we find (we eliminate some irrelevant constant terms):

Although we will not do any real calibration in this article, we can note at this stage a few issues that posed a problem to researchers in the RBC domain.

First, real wages are much too procyclical in this Walrasian model. From (8) we see that the real wage-output correlation is equal to 1. Even though this correlation is lower than 1 in calibrated models where \( N_t \) varies, it is always quite above what is observed in real economies.

A second problem concerns the inflation-output correlation, a problem related to the literature on the Phillips curve. Whereas it is generally considered that this correlation is positive, the above Walrasian model yields a negative correlation:

\[ \text{cov} (\Delta p_t, y_t) = -\frac{\sigma^2}{1 + \varphi} < 0 \]

Finally an important and recurrent critique of RBC type models has been that they do not generate any internal propagation mechanism, and that the only persistence in output movements is that already present in the exogenous process of technological shocks \( z_t \) (see, for example, Cogley and Nason, 1993, 1995). This appears here in equation (8) where the dynamics of output \( y_t \) is exactly the same as that of the technological shock \( z_t \).

We shall now introduce wage contracts, first lasting one period, and then multiperiod overlapping contracts, and we shall see that the above problems find a natural solution in this framework.

2.3 Single period wage contracts

Let us thus assume (Bénassy, 1995, and Bénassy, 2002 for microfoundations), that the wages are predetermined at the beginning of each period at the expected value of the Walrasian wage (in logarithms), and that at this contractual wage the households supply the quantity of work demanded by firms (this type of contracts was introduced by Gray, 1976).
Combining (6) and $C_t = Y_t$ we find that the Walrasian wage $w_t^*$ is, up to an unimportant constant, equal to $m_t$, so that the preset wage $w_t$ is given by:

$$w_t = E_{t-1}w_t^* = E_{t-1}m_t$$  \hspace{1cm} (10)

where $E_{t-1}m_t$ is the expectation of $m_t$ formed at the beginning of period $t$, before shocks are known.

The difference with the Walrasian case is that employment $N_t$ is now variable and demand determined. Equations (8) become:

$$y_t = z_t + \alpha n_t \quad w_t - p_t = y_t - n_t$$  \hspace{1cm} (11)

while $n_t = n$ is replaced by (10). So we first obtain the level of employment in period $t$:

$$n_t = n + m_t - E_{t-1}m_t = n + \varepsilon_{mt}$$  \hspace{1cm} (12)

since $m_t - E_{t-1}m_t = \varepsilon_{mt}$. Contrarily to what happened in the Walrasian version of the model, unanticipated monetary shocks now have an impact on the level of employment, and therefore output. We shall now use the preceding formulas to show that the hypothesis of preset wages allows to substantially improve some correlations relative to the Walrasian model.

Let us start with the real wage which, in the Walrasian model, has a much too high positive correlation with output. Let us combine (11) and (12), to obtain the values of output and real wage:

$$y_t = z_t + \alpha \varepsilon_{mt} \quad w_t - p_t = z_t - (1 - \alpha) \varepsilon_{mt}$$  \hspace{1cm} (13)

We see that supply shocks create a positive correlation between the real wage and output. However monetary shocks create a negative correlation. Our model thus allows us to combine this last characteristic, typical of traditional Keynesians models, with the usual results of RBC models. If one considers the technological and monetary shocks (4), one obtains the following correlation:

$$\text{corr}(w_t - p_t, y_t) = \frac{\sigma_y^2 - (1 - \varphi^2) \alpha (1 - \alpha) \sigma_m^2}{[(\sigma_y^2 + (1 - \varphi^2) \alpha^2 \sigma_m^2)]^{1/2} [\sigma_y^2 + (1 - \varphi^2) (1 - \alpha)^2 \sigma_m^2]^{1/2}}$$  \hspace{1cm} (14)
We see that the real-wage-output correlation is equal to 1 if there are only technological shocks. But this correlation diminishes as soon as there are monetary shocks, and it can even become negative. One can thus reproduce the correlations observed in reality by adequate combinations of technological and monetary shocks.

Let us now consider the relation between inflation and output, which are generally considered to be positively correlated, at least in Keynesian tradition. If we assume again the monetary and technological shocks (4), we find:

\[
\text{Covariance}(\Delta p_t, y_t) = \alpha (1 - \alpha) \sigma_m^2 - \frac{\sigma_z^2}{1 + \varphi}
\]  

(15)

Formula (15) shows us that the positive covariance (and thus correlation) between inflation and output is linked to the presence of demand shocks, and that the sign of this correlation may change if there are sufficiently strong technological shocks.

So we just saw that one-period contracts allow us to improve some important correlations. We shall now naturally ask a question already posed for the standard RBC model: Is the response to shocks, and in particular to demand shocks, sufficiently persistent? Let us recall equation (13):

\[
y_t = z_t + \alpha \varepsilon_{mt}
\]  

(16)

We see that monetary shocks now have an immediate effect on output (and employment), but that, starting with the second period, the effect of these shocks is completely dampened. One period contracts allow us to solve the puzzle raised by some correlations, but certainly not the persistence problem. We shall see in the next two sections that multiperiodic contracts allow us to solve that problem.

2.4 Multiperiodic wage contracts

The models that we have examined so far share with traditional RBC models the defect of having an extremely weak internal propagation mechanism. In particular the response of output to monetary demand shocks is almost entirely transitory. But several empirical studies (see, for example, Christiano, Eichenbaum and Evans, 1999, 2005) have pointed out that in reality the response to monetary shocks not only was persistent, but also had a
hump-shaped response function. We shall now introduce multiperiodic wage contracts in rigorous stochastic dynamic models, and show that they allow to reproduce these features. Models with such multiperiodic wage or price contracts have been studied notably by Yun (1996), Andersen (1998), Jeanne (1998), Ascari (2000), Chari, Kehoe and McGrattan (2000), Collard and Ertz (2000) and . Bénassy (2002, 2003a,b).

In order to make our demonstration analytically, we shall use a contract, inspired by Calvo (1983) and developed in Bénassy (2002, 2003a), which has three advantages: (a) the average duration of contracts can take any value from zero to infinity, (b) an analytical solution can be found with both wage and price contracts, (c) it has explicit microfoundations.

In this framework in each period $s$ a contract is made for wages at period $t \geq s$. As in the Gray contract the contract wage is the expectation of the market clearing wage in period $t$. So if we denote as $x_{st}$ the contract wage made in $s$ for period $t$:

$$x_{st} = E_s (w_t^*)$$  \hspace{1cm} (17)

Now, as in Calvo (1983), each wage contract has a probability $\gamma$ to stay unchanged, and a probability $1 - \gamma$ to be broken. If the contract is broken, a new contract is immediately renegotiated on the basis of current period information. So for $\gamma = 0$, wages are totally flexible, for $\gamma = 1$ they are totally rigid.

It is easy to compute the average duration of these contracts. The probability for a contract to be still valid $j$ periods after the date it was concluded is equal to $(1 - \gamma) \gamma^j$. The expected duration of the contract is thus:

$$\sum_{j=0}^{\infty} (1 - \gamma) j \gamma^j = \frac{\gamma}{1 - \gamma}$$  \hspace{1cm} (18)

We thus see that varying $\gamma$ from 0 to 1 the average duration of the contract varies from zero to infinity.

The average wage $w_t$ is the mean of past $x_{st}$’s weighted by the probability for the corresponding contract to be still in effect. Because of the law of large numbers, and since the probability of survival of wage contracts is $\gamma$, the proportion of contracts coming from period $s \leq t$ is $(1 - \gamma) \gamma^{t-s}$. Therefore the average wage in the economy is given by:
\[ w_t = (1 - \gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} x_{st} \]  

(19)

If we now solve the model with the shocks (4) we find that the dynamics of employment is characterized by (Bénassy, 2002, 2003a):

\[ n_t = n + \frac{\gamma \varepsilon_{mt}}{(1 - \gamma L)(1 - \gamma \rho L)} \]  

(20)

where \( L \) is the lag operator: \( L^j X_t = X_{t-j} \). The response of output is deduced from that of employment through:

\[ y_t = \alpha n_t + z_t \]  

(21)

Formula (20) shows clearly that, contrarily to the case of one period contracts, the response to a monetary shock can be quite persistent. We can have an idea of the temporal profile of this response by computing the response function of output and employment to a monetary shock. The value of \( \rho \) most often found in the literature is \( \rho = 0.5 \). As for \( \gamma \), we saw above (formula 18) that the average duration of wage contracts is equal to \( \gamma/(1 - \gamma) \). One considers generally that the average duration of wage contracts is about one year (see for example Taylor, 1999), which corresponds to \( \gamma = 4/5 \). Figure 1 shows the response of employment (output is derived via 21) to a monetary shock for \( \gamma = 4/5 \).

**Figure 1**

We see that the response function displays persistence in the effects of monetary shocks, and has even a hump shaped response. If we plot, however, the response function of inflation, we find that it is steadily decreasing after the initial jump, whereas it seems to have a delayed hump shaped response in reality.

### 2.5 Wage and price multiperiodic contracts

We shall now enlarge our model by considering simultaneously wage and price multiperiodic contracts (see Bénassy, 2003b, for such a model with explicit microfoundations). Numerically solved models with both wage and
Figure 1
price multiperiodic contracts are found in Christiano, Eichenbaum and Evans

Wage contracts are exactly the same as in the preceding section: each
contract is maintained with probability $\gamma$, or renegotiated with probability
$1 - \gamma$. Symmetrically price contracts are maintained with probability $\phi$, or
break down and are renegotiated with probability $1 - \phi$. The average price
$p_t$ is given by:

$$p_t = (1 - \phi) \sum_{s=-\infty}^{t} \phi^{t-s} q_{st}$$

(22)

where $q_{st}$ is the price contract negotiated in period $s$ for period $t$. Using again
the shock processes (4), and taking $\nu = 1$, we find the following dynamics
for output and inflation:

$$y_t = z_t - \frac{\phi \epsilon_{zt}}{1 - \phi \varphi L} + \frac{\alpha \gamma \epsilon_{mt}}{(1 - \gamma L)(1 - \gamma \rho L)}$$

$$+ \frac{\phi \epsilon_{mt}}{(1 - \phi L)(1 - \phi \rho L)} - \frac{\alpha \gamma \phi \epsilon_{mt}}{(1 - \gamma \phi L)(1 - \gamma \phi \rho L)}$$

(23)

$$\pi_t = (1 - L) p_t = (1 - L) (m_t - y_t)$$

(24)

As in the preceding section we shall take as an illustration $\alpha = 2/3$, $\rho = 1/2$
and $\gamma = 4/5$ (one year wage contracts). As for prices we want
to take a rather low duration of contracts, so we shall take $\phi = 1/2$ (one
quarter). Simulations show that in that case we obtain a persistent and
humpshaped response for both output and inflation.

So we see that with only reasonable nominal rigidities we obtain some
realistic response functions. Clearly the adjunction of “real” rigidities would
allow to reproduce even better the actual dynamic macroeconomic patterns.
References


