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# Income Inequality, Mobility, and the Accumulation of Capital

The Role of Heterogeneous Labor Productivity

Cecilia García-Peñalosa  
Stephen J. Turnovsky

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**Income Inequality, Mobility, and the Accumulation of Capital:  
The Role of Heterogeneous Labor Productivity\***

Cecilia García-Peñalosa  
Aix-Marseille University, Marseille

Stephen J. Turnovsky  
University of Washington, Seattle, WA 98195

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**Abstract:** We examine the determinants of income inequality and mobility in a Ramsey model with elastic labor supply. Individuals differ both in their initial capital endowment and productive ability (labor endowment). With two sources of heterogeneity, initially poorer agents may catch up with the income and wealth of initial richer ones, implying that the Ramsey model is compatible with rich distributional dynamics. We show that the elasticity of the labor supply plays a key role in the extent of mobility in the economy. Capital-rich individuals supply less labor while ability-rich agents tend to work more. The more elastic the labor supply is, the stronger these effects tend to be and hence the greater the degree of income mobility is.

**JEL Classification Numbers:** D31, O41

**Key words:** inequality; income mobility; endogenous labor supply; transitional dynamics.

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Contacts: [cecilia.garcia-penalosa@univmed.fr](mailto:cecilia.garcia-penalosa@univmed.fr) and [sturn@u.washington.edu](mailto:sturn@u.washington.edu)

## 1. Introduction

The Ramsey (1928) model is one of the cornerstones of modern macrodynamics, and has served as an important tool for evaluating the consequences of structural changes and various aspects of economic policy. While the basic model treats all agents as identical – the representative agent paradigm – macroeconomists have increasingly introduced alternative sources of heterogeneity, such as different time discount rates and heterogeneous initial endowments of capital.<sup>1</sup> Using a simple model with inelastic labor supply, Caselli and Ventura (2000) show that if preferences are homothetic the model can also accommodate heterogeneity in ‘labor endowments’ or ability. They demonstrate that the simultaneous introduction of two sources of heterogeneity raises the possibility of wealth and income mobility, meaning that over time less wealthy, but more skilled, agents may overtake wealthier, but less skilled, agents in the distributions of wealth and income.

In this paper we examine the determinants of income mobility in a Ramsey model without any external shocks. We consider an economy with endogenous labor supply, where agents differ in both their endowments of ability and of physical capital, and examine how changes in factor prices affect the rewards to each of the two sources of inequality and hence induce changes in the ranking of individuals in terms of income. Allowing for the endogeneity of labor supply is critical for two reasons. First, as we have shown in previous work, the adjustment of labor (or leisure) to wealth is a key determinant of the distribution of wealth and income.<sup>2</sup> This becomes even more crucial in an economy with skill heterogeneity, where agents of varying skill levels receiving differential wages will have different incentives to adjust their respective labor supplies in response to the evolving returns on capital and labor. Second, the endogeneity of the labor supply implies heterogeneity in work time and hence the distribution of earnings can change with the evolution of macroeconomic aggregates even if the underlying dispersion of abilities is assumed to be constant.

The analytical framework we employ is the one-sector model developed in Turnovsky and García-Peñalosa (2008), to which we add a (time-invariant) initial distribution of labor endowments

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<sup>1</sup> For example Becker (1980) and Becker and Foias (1987) allow for heterogeneity in the time discount rate, while Chatterjee (1994), Sorger (2000, 2002), Mailar and Mailar (2001), Turnovsky and García-Peñalosa (2008), and García-Peñalosa and Turnovsky (2011) allow for heterogeneity of capital endowments. See Bertola, Foellmi, and Zweimüller (2006) for a general discussion of income distribution in macroeconomic models.

<sup>2</sup> See e.g. Turnovsky and García-Peñalosa (2008).

that may, or may not, be correlated with the agents' initial endowments of capital (wealth). Representing preferences by a utility function that is homogeneous in consumption and leisure facilitates aggregation as in Gorman (1953) or Eisenberg (1961), and generates a representative-consumer characterization of the macroeconomic equilibrium. This homogeneity assumption allows the analysis to proceed sequentially.<sup>3</sup> First, we determine the dynamics of aggregate magnitudes, which are independent of distribution. We then obtain the dynamics of the wealth and income distributions as functions of the aggregate capital stock and labor supply.

In this context there are three mechanisms that we must consider to understand the distributional changes. First, an agent's relative income at any point in time depends on his relative endowments of effective labor and capital, with the latter evolving endogenously in response to changes in aggregate magnitudes. Second, the relative importance of ability and wealth depends on the endogenous evolution of factor prices. Third, because we assume an endogenous supply of hours of work (labor time), labor income will depend not only on the agent's ability but also on his decision of how much to work. As in our previous work, we derive a negative relationship between agents' relative wealth (capital) and their relative allocation of time between work and leisure.<sup>4</sup> Wealthier agents have a lower marginal utility of wealth, and hence choose to consume more of all goods, including leisure, thus reducing their labor supply.<sup>5</sup> In contrast, more able workers have a higher opportunity cost of leisure, and this creates a positive correlation between individual labor supply and their skill endowment. These two opposing responses create a complex relationship between the agent's relative income and his supply of labor that will depend on factor prices.

The introduction of two sources of heterogeneity radically alters the implications of existing analyses of heterogeneity in the Ramsey model. In the earlier models with heterogeneous discount

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<sup>3</sup> Without the homogeneity assumption, aggregate behavior and distribution become simultaneously determined and analysis of the transitional dynamics becomes analytically intractable. One example of a departure from this assumption is Sorger (2002). In most cases, the literature has resorted to numerical analysis in order to keep track of the effect of distribution on aggregate magnitudes when preferences are non-homothetic; see, for example, Krusell and Smith (1998).

<sup>4</sup> See García-Peñalosa and Turnovsky (2006, 2007) and Turnovsky and García-Peñalosa (2008).

<sup>5</sup> There is substantial empirical evidence in support of this negative relationship between wealth and labor supply. Holtz-Eakin, Joulfaian, and Rosen (1993) provided evidence to suggest that large inheritances decrease labor participation. Cheng and French (2000) and Coronado and Perozek (2003) use data from the stock market boom of the 1990s to study the effects of wealth on labor supply and retirement, finding a substantial negative effect on labor participation. Algan, Chéron, Hairault, and Langot (2003) employ French data to analyze the effect of wealth on labor market transitions, and find a significant wealth effect on the extensive margin of labor supply.

rates, the most patient agent ends up holding all the capital, irrespective of policy changes or technology shocks; the distribution of wealth therefore degenerates. When agents differ only in their initial capital endowments, the growth process may expand or contract the distributions of wealth and income, but the rankings of agents according to either wealth or income remain unchanged over time. In contrast, with two sources of heterogeneity, both wealth and income mobility become possible during the transition, and given the prevalence of such mobility in practice, being able to incorporate it in our analysis, in our view, represents a significant advance.

The joint analysis of inequality and mobility is also important because not all forms of inequality are perceived in the same way. In particular, rewarding ability is often seen as a ‘fairer’ source of inequality than are differences in income due to initial wealth endowments; see Roemer (1998). As a result, one’s perception of the fairness of an economy with a certain level of inequality will also depend on the degree of income mobility that is associated with that level of inequality.

Our analysis takes the form of a combination of theoretical propositions setting out conditions under which wealth and income catch-up may, or may not occur, and the corresponding implications for overall wealth and income inequality. Our results contrast the implications of the two sources of heterogeneity on inequality and mobility. In general, in an expanding economy, heterogeneity arising from initial capital endowments will reduce wealth inequality, while heterogeneity due to ability will exacerbate it. In contrast, income inequality tends to respond non-monotonically, with an initial jump in one direction, followed by a subsequent offsetting gradual transition. As is found for wealth inequality, the two sources of heterogeneity influence these adjustments in opposite ways.

The extent of mobility in an economy depends both on the type of shock and on the fundamentals. First, in an expanding economy it is the ability-rich that catch-up with those with larger initial capital, implying that in the new steady state distribution those at the top are more likely to be ability-rich than in the original steady state. In contrast, during a contraction it is those with large initial wealth endowments that catch-up and end up having higher incomes than those with greater ability but smaller initial wealth endowments. Second, the elasticity of labor plays a key role in determining the degree of mobility. To understand this think of an expanding economy in which

high ability individuals are upwardly mobile. Their capacity to catch up with wealthier agents depends both on their labor endowment but also on their labor supply, since both determine earnings. A larger elasticity implies a stronger response to own ability, thus increasing their (relative) labor supply and thus their earnings, thereby facilitating income catch-up.

Our analysis also shows that income inequality and mobility need not move together. The reason for this is the behavior of the aggregate labor supply. As we have discussed, the evolution of income inequality is driven by an initial jump in labor and a subsequent gradual adjustment towards its steady state. It is then possible that these two effects are largely offsetting and lead to small changes in steady state inequality. In contrast, the degree of income mobility depends only on the transition, as it is during that phase that agents may change their relative positions. Consequently, it is possible for a shock or policy change to induce a small change in steady-state income inequality coupled with a high degree of income mobility, so that those at the top of the distribution are different individuals in the initial and in the new steady state.

Our paper contributes to the recent literature characterizing distributional dynamics in growth models, an issue first examined by Stiglitz (1969) using a form of the Solow model. One approach to this question has examined economies with ex-ante identical agents and uninsurable, idiosyncratic shocks, where inequality emerges as a result of these shocks and can persist over time, as in Castañeda, Díaz-Giménez, and Rios-Rull (1998), Díaz, Pijoan-Mas, and Rios-Rull (2003), Maliar and Maliar (2003), and Wang (2007). This class of models has the advantage that it generates the possibility of income mobility, as individual shocks may reverse the relative positions of agents over time. However, their complexity implies that analytical solutions are not possible and hence the analysis is entirely based on numerical computations. An alternative approach has been to consider economies without shocks in which inequality results from the assumption that agents are initially heterogeneous along a single dimension. Several sources of heterogeneity have been considered, such as different discount rates as in Becker (1980) and Becker and Foias (1987), although recently this literature has focused mainly on differences in initial capital endowments, an aspect we consider

in our model.<sup>6</sup>

Our analysis makes two main contributions to this latter literature. The first is that allowing for two sources of heterogeneity generates the possibility of income mobility even in a model with no idiosyncratic shocks. This contrasts both with our previous work and the papers just cited, where the evolution of macroeconomic aggregates would lead to an expansion or contraction of the distribution of income, but would produce no change in the ranking of individuals. The second novelty of the paper is to combine the introduction of heterogeneous ability with an endogenous supply of labor. As argued above, this is a central element determining the degree of mobility that takes place after a shock. Closest to our work is Maliar, Maliar, and Mora (2005), who also consider a setup with dispersion in initial capital and ability. Nevertheless, the focus of their analysis is different, since they are interested in the effect of business cycle fluctuations on the distribution of income. As result, although their model can potentially generate income mobility they do not study this aspect, but focus instead only on changes in an inequality index. Moreover, they assume a fixed labor supply and hence do not incorporate the labor supply responses that, as we will see, are a crucial element in determining the extent of income catch-up. In contrast our analysis provides a benchmark setup with which to assess the implications of shocks and policy changes, both in terms of the steady-state distribution, but also in terms of the degree to which, these changes allow individuals to alter their relative position within this distribution.

Following this introduction, Section 2 describes the economy and section 3 derives the macroeconomic equilibrium. Section 4 characterizes the dynamics of relative wealth and relative income, while Section 5 derives the consequences for wealth and income inequality. These two sections derive the main analytical results. The effects of changes in fundamentals and tax rates on the long-run distributions of wealth and income are then illustrated in Section 6 with a number of numerical examples. Section 7 concludes, while insofar as possible technical details are relegated to an Appendix.

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<sup>6</sup> See Chatterjee (1994), Chatterjee and Ravikumar (1999), Sorger (2000), Maliar and Maliar (2001), Alvarez-Peláez and Díaz (2005), Obiols-Homs and Urrutia (2005), Borissov and Lambrecht (2009), and Bosi, Boucekkine and Seegmuller, (2010), as well as our previous work.

## 2. The analytical framework

We begin by setting out the components of the model.

### 2.1. Consumers

The economy is populated by  $N$  individuals, each indexed by  $i$ . There are two sources of heterogeneity: agents' relative skill levels, denoted by  $a_i$ , and their initial endowments of capital,  $K_{i,0}$ . By defining  $a_i$  in terms of relative skills, the average economy-wide skill level is simply  $\sum_i a_i / N = 1$ . The heterogeneity of relative skill across agents is described by its (constant) standard deviation,  $\sigma_a$ . Relative capital (wealth) is defined by  $k_i(t) \equiv K_i(t) / K(t)$ , where  $K(t)$  is the average economy-wide capital stock at time  $t$ . At any point of time the relative capital stock has mean 1, while its dispersion across agents is given by the standard deviation,  $\sigma_k(t)$ , with the initial (given) dispersion being  $\sigma_{k,0}$ . The correlation between initial capital endowments and skills is denoted by  $\chi$ , and may be  $\geq 0$ . The initial distribution may be of any arbitrary form, the only restriction being that the largest initial wealth endowment is less than the level,  $\bar{k}$ , that would induce that individual to withdraw entirely from the labor market (i.e. supply zero labor).<sup>7</sup>

Two remarks are in order at this point. First, there are several measures of inequality, each having its advantages and drawbacks. Measuring the underlying sources of inequality in terms of *relative* deviations, which are effectively measures of coefficients of variation (CV), is one very natural inequality measure. At the same time, having more than one source of inequality, an interesting issue concerns its decomposition into its underlying components, in which case alternative measures are more convenient; see Bourguignon (1979). Like the widely used Gini coefficient, to which it is dimensionally equivalent, the CV is not decomposable in this way. In contrast, the squared coefficient of variation (SCV), defined as the variance over the square of the mean, is a convenient member of the class of decomposable inequality measures identified by Bourguignon. In what follows, we will focus on this measure of inequality.

Each individual is endowed with a unit of time that can be allocated either to leisure,  $l_i$ , or to

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<sup>7</sup> The value of this upper bound can be obtained from the expressions from labor supply and steady state capital that we derive below; see footnote 14.

supplying labor,  $1 - l_i \equiv L_i$ . The agent maximizes lifetime utility, assumed to be an isoelastic function of consumption and leisure plus an additively separable function of government expenditure

$$\max \int_0^{\infty} \left[ \frac{1}{\gamma} (C_i(t) l_i(t)^\eta)^\gamma + v(G(t)) \right] e^{-\beta t} dt, \quad \text{with} \quad -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1, \gamma(1+\eta) < 1 \quad (1)$$

where  $G(t)$  is per capita government expenditure and  $v' > 0$ .<sup>8</sup> This maximization is subject to the agent's capital accumulation constraint

$$\dot{K}_i(t) = [(1 - \tau_k)r(t) - \delta]K_i(t) + (1 - \tau_w)w_i(t)(1 - l_i(t)) - C_i(t) + T_i \quad (2)$$

where  $r(t)$  is the return to capital,  $w_i(t)$  the wage received by the individual,  $\delta$  the capital depreciation rate,  $\tau_k$  and  $\tau_w$  are the tax rates on capital income and labor income, respectively, and  $T_i$  are the transfers received by agent  $i$ .

## 2.2. Technology and factor payments

Aggregate output is produced by a single representative firm, using a standard neoclassical production function

$$Y = F(K, L) \quad F_L > 0, F_K > 0, F_{LL} < 0, F_{KK} < 0, F_{LK} > 0 \quad (3)$$

where  $K$ ,  $L$  and  $Y$  denote respectively the per capita stock of capital, effective labor supply, and per capita output. Since labor productivity is heterogeneous, the effective labor employed by the firm is

$$L = \frac{1}{N} \sum_i a_i L_i$$

Firms pay capital and labor according to their marginal physical products,

$$r(t) \equiv r(K, L) = F_K(K, L) \quad (4a)$$

$$w_i(t) = w_i(K, L) = F_{L_i}(K, L) = a_i F_L(K, L) \quad (4b)$$

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<sup>8</sup> The assumption of additive separability is made simply for convenience, allowing government spending to have a positive role, without introducing the complications arising from its interaction with private decisions. These have been considered elsewhere, in the case where the only source of heterogeneity arises from initial endowments of capital; see García-Peñalosa and Turnovsky (2011)

where the wage received by agent  $i$ ,  $w_i$ , reflects his skill level. Letting the average wage rate be

$$w(t) \equiv w(K, L) = F_L(K, L) \quad (4b')$$

the wage paid to individual  $i$  is  $w_i(K, L) = a_i w(K, L)$ . Thus, we immediately see that the distribution of relative wage rates,  $w_i(t)/w(t)$ , is given and unchanging, and simply reflects the given distribution of skill levels across agents.

### 2.3. Government

We assume that the government sets its expenditure and transfers as fractions of per capita output, in accordance with  $G = gY(t)$ ,  $T = \tau Y(t)$ , so that  $g$  and  $\tau$  become the policy variables together with the tax rates. We also assume that it maintains a balanced budget expressed as

$$\tau_k rK + \tau_w wL = G + T = (g + \tau)F(K, L) \quad (5)$$

This means that, if  $\tau_w$ ,  $\tau_k$ , and  $g$  are fixed, as we shall assume, then along the transitional path, as economic activity and the tax/expenditure base is changing, the rate of lump-sum transfers must be continuously adjusted to maintain budget balance. To abstract from any direct distribution effects arising from lump-sum transfers (which are arbitrary), we shall set  $T=0$  in steady state, and assume that during the transition  $T_i(t)/T(t) = K_i(t)/K(t)$  which ensures that  $\int_0^N T_i di = (T/K) \int_0^N K_i di = T$ , consistent with the government budget constraint. The role of transfers is then only to ensure a balanced budget during the transition.<sup>9</sup>

## 3. Derivation of the macroeconomic equilibrium

### 3.1. The individual problem

We begin by considering the individual's maximization problem, which is to choose consumption, leisure, and the rate of capital accumulation to maximize the utility function, (1), subject to the budget constraint, (2). The following standard first-order optimality conditions obtain

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<sup>9</sup>An alternative (and virtually identical) approach would be to introduce debt financing during the transition.

$$C_i^{\gamma-1} l_i^{\eta\gamma} = \lambda_i \quad (6a)$$

$$\eta C_i^{\gamma} l_i^{\eta\gamma-1} = (1 - \tau_w) a_i w \lambda_i \quad (6b)$$

$$(1 - \tau_k) r - \delta = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \quad (6c)$$

where  $\lambda_i$  is agent  $i$ 's shadow value of capital, together with the transversality condition

$$\lim_{t \rightarrow \infty} \lambda_i K_i e^{-\beta t} = 0 \quad (6d)$$

From these optimality conditions it is possible to show that (see appendix A.1)

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C}; \quad \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \quad \text{for all } i \quad (7)$$

That is, all agents will choose the same growth rate for consumption and leisure, implying further that average consumption,  $C$ , and average leisure,  $l$ , will also grow at the same common growth rates, as in Turnovsky and García-Peñalosa (2008). In particular,

$$l_i = \rho_i l \quad (8)$$

where  $\rho_i$  is the relative leisure of agent  $i$ , which by (7) is constant over time, and,  $\sum_i \rho_i / N = 1$ .

### 3.2 Macroeconomic equilibrium

In general, we shall define economy-wide averages as  $X(t) = \sum_i X_i(t) / N$ . Thus, summing  $K_i, l_i$  over all agents, equilibrium in the capital and labor markets is described by

$$K(t) = \frac{1}{N} \sum_i K_i(t) \quad (9a)$$

$$L(t) = (1 - \Omega) l(t) \quad \text{where } \Omega \equiv \frac{1}{N} \sum_i a_i \rho_i \quad (9b)$$

Likewise, aggregating (A.1) in the Appendix over all agents, the economy-wide consumption is

$$C(t) = (1 - \tau_w) \frac{w l(t)}{\eta} \frac{1}{N} \sum_i a_i \rho_i \equiv (1 - \tau_w) \frac{w}{\eta} l(t) \Omega = (1 - \tau_w) \frac{w}{\eta} (1 - L(t))$$

Before proceeding further, we need to interpret  $\Omega$ . First, note that since  $L(t) = (1 - \Omega l(t))$ , and assuming that effective labor supply is positive,  $L(t) > 0$ , we have that

$$1 > \Omega l(t) \quad (10)$$

where  $\Omega l(t)$  is effective leisure. Thus  $\Omega$  measures the labor lost through leisure, with the losses incurred by each individual being weighted by their level of ability. We may note  $\Omega - 1 = \sum_i (a_i - 1)(\rho_i - 1)/N$ , so that  $\Omega - 1 = \text{cov}(a_i, \rho_i)$ . Hence, if more talented people choose more leisure than those with less ability, the covariance between ability and leisure will be positive, implying  $\Omega > 1$ . Alternatively, when more talented people work more we will have  $\Omega < 1$ . With homogeneous labor productivity,  $\Omega = 1$  which yields the case examined in Turnovsky and García-Peñalosa (2008). Moreover, note that because ability is given and  $\rho_i$  is constant during the transition to a steady state,  $\Omega$  does not change over time, implying that the dynamics of effective leisure,  $\Omega l(t)$ , will reflect the dynamics of  $l(t)$ .

The procedure we follow to solve the model is analogous to that employed by Turnovsky and García-Peñalosa (2008), the only difference being that, due to the presence of the term  $\Omega$  which involves the aggregation of the differential labor productivities, it is more transparent to express the dynamics in terms of labor rather than leisure, as was done previously. In Appendix A.1 we show that the dynamic equations governing aggregate behavior are just those of the standard (aggregate) Ramsey model with endogenous labor supply, implying that the evolution of the aggregate capital stock and labor supply are *independent* of any distributional characteristics.

Assuming that the economy is stable, aggregate quantities converge to a steady state characterized by a constant average per capita capital stock, labor supply, and effective leisure time, denoted by  $\tilde{K}$ ,  $\tilde{L}$  and  $\tilde{\Omega}l$ , respectively. Setting  $\dot{K} = \dot{L} = 0$ , the steady state is summarized by

$$(1 - \tau_k)F_K(\tilde{K}, \tilde{L}) = \beta + \delta \quad (11a)$$

$$(1 - g)F(\tilde{K}, \tilde{L}) - \delta\tilde{K} = (1 - \tau_w)F_L(\tilde{K}, \tilde{L})\frac{(1 - \tilde{L})}{\eta} \quad (11b)$$

$$\tilde{L} + \tilde{\Omega}l = 1 \quad (11c)$$

The first two equations jointly determine per capita steady-state values of capital and labor, with  $\Omega\tilde{l}$  being determined by (11c). In fact, (11c) implies only effective, but not average, leisure, which requires knowledge of  $\Omega$  and hence of the distribution of ability and capital.

Rewriting equation (11b) in the form

$$(1-g)F(\tilde{K}, \tilde{L}) - \delta\tilde{K} - (1-\tau_w)F_L(\tilde{K}, \tilde{L})\tilde{L} + (1-\tau_w)F_L(\tilde{K}, \tilde{L})\frac{1+\eta}{\eta}\left[\tilde{L} - \frac{1}{1+\eta}\right] = 0, \quad (12)$$

we see that if the share of private consumption expenditure,  $[(1-g) - \delta(\tilde{K}/\tilde{F})]$ , exceeds the after-tax share of labor income,  $(1-\tau_w)(\tilde{F}_L\tilde{L}/\tilde{F})$ , then (12) imposes the restriction<sup>10</sup>

$$\frac{1}{1+\eta} > \tilde{L} > 0 \quad (13)$$

As we will see below, this condition plays a critical role in characterizing the dynamics of the wealth distribution. It can be expressed equivalently as

$$1 > \Omega\tilde{l} > \frac{\eta}{1+\eta} \quad (13')$$

These inequalities yield an upper (lower) bound on the steady-state time allocation to labor supply (leisure) that is consistent with a sustainable equilibrium.

In Appendix A.1 we show that the (locally) stable path for  $K(t)$  and  $L(t)$  in the neighborhood of steady state can be expressed as

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t} \quad (14a)$$

$$L(t) = \tilde{L} + \frac{b_{21}}{\mu - b_{22}}(K(t) - \tilde{K}) = \tilde{L} + \frac{\mu - b_{11}}{b_{12}}(K(t) - \tilde{K}) \quad (14b)$$

where  $\mu < 0$  is the stable eigenvalue and  $b_{11}, b_{12}, b_{21}, b_{22}$  are the coefficients of the linearized system. As we will see below, the evolution of average labor supply over time is an essential determinant of the time path of wealth and income inequality. To determine the slope of the stable saddle path we need to consider the likely signs of the coefficients in (14b). Since  $b_{12} > 0$ , the locus is negatively

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<sup>10</sup> This restriction, which we impose, is in fact relatively weak and is satisfied for plausible choices of parameters.

sloped if and only if  $\mu < b_{11}$ . This expression reflects two offsetting influences of capital on the dynamics of labor supply. On the one hand, a greater capital stock reduces the return to capital and hence to future consumption, thus decreasing desired labor. On the other, greater  $K$  increases wages and thus increases the growth rate of labor. Which effect dominates depends crucially upon the elasticity of substitution in production,  $\varepsilon$ . In Appendix A.1 we show that a necessary and sufficient condition for  $\mu < b_{11}$  is that this elasticity exceeds a certain lower bound, which is easily satisfied for reasonable parameter values. Henceforth, we shall restrict ourselves to what we view as the more plausible case of a negatively sloped stable locus, (14b).

In addition, for expositional convenience we shall focus on situations in which the economy is subject to an expansionary structural shock that results in an increase in the steady-state average per capita capital stock relative to its initial level ( $K_0 < \tilde{K}$ ). From (14b) this will lead to an initial positive jump in labor supply, such that  $L(0) > \tilde{L}$ , so that thereafter, labor supply will decrease monotonically during the transition; an analogous relationship applies if  $K_0 > \tilde{K}$ .

## 4. The dynamics of relative wealth and income

### 4.1. Relative capital stock (wealth)

To derive the dynamics of individual  $i$ 's relative capital stock,  $k_i(t) \equiv K_i(t)/K(t)$ , we use the individual's budget constraint (2) together with the aggregate one. With transfers set such that  $T_i / K_i = T / K$  this leads to<sup>11</sup>

$$\dot{k}_i(t) = \frac{w(K, L)(1 - \tau_w)}{K} \left[ \left( a_i - a_i \rho_i l \frac{1 + \eta}{\eta} \right) - \left( 1 - \Omega l \frac{1 + \eta}{\eta} \right) k_i(t) \right] \quad (15)$$

where initial relative capital  $k_{i,0}$  is given from the initial endowment and the aggregate magnitudes  $K$  and  $\Omega l = 1 - L$  change over time.

To solve for the time path of the relative capital stock, we first note that (15) implies the following relationship between agent  $i$ 's allocation of time to labor, his steady-state relative

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<sup>11</sup> For more of the details see Turnovsky and García-Peñalosa (2008) and Appendix A.2. We have also considered an alternative lump-sum transfer rule  $T_i = T$ , with very small differences in results from those we are reporting here.

holdings of capital, and his (given) ability:

$$\tilde{L}_i - \tilde{L} = \left( \tilde{L} - \frac{1}{1+\eta} \right) \left( \frac{\tilde{k}_i}{a_i} - 1 \right) \quad \text{for each } i \quad (16)$$

where  $\tilde{k}_i$  is the steady state relative capital of agent  $i$ . Equation (16) asserts that individual  $i$ 's long-run allocation of time to labor, relative to the economy-wide average, decreases with his relative wealth and increases with his relative ability. Using the definition and constancy of  $\rho_i$  we can show that an analogous equation to (16) holds at all points of time

$$L_i(t) - L(t) = \frac{l(t)}{\tilde{l}} \left( \tilde{L} - \frac{1}{1+\eta} \right) \left( \frac{\tilde{k}_i}{a_i} - 1 \right) \quad (16')$$

This equation captures one of the critical elements determining the evolution of the distributions of wealth and income and explains why the dynamics of the aggregate quantities are unaffected by distributional aspects. The reason is simply that each agent's labor supply is a linear function of the ratio of his relative capital to ability, with this sensitivity being common to all agents and depending upon the aggregate economy-wide labor/leisure allocation. Moreover, recalling (13), equation (16') implies that the greater this ratio, the more leisure the agent consumes and the less labor he supplies. This has two effects, an equalizing effect that partly offsets the impact of wealth inequality on the distribution of income and an unequalizing effect that magnifies the effect of differences in ability, since the more able supply more labor.

Clearly the elasticity of labor plays a key role in determining the relative labor supply responses of agents, as can be seen by (16) and (16'). The direct effect of a higher value of  $\eta$  is to make the agent's labor supply more responsive to endowments. A higher elasticity of labor will also have an indirect impact through its effect on the aggregate labor supply,  $\tilde{L}$ , and which, if  $d\tilde{L}/d\eta < 0$  (as in the case for a Cobb-Douglas), will partially offset the direct impact of  $\eta$ .

To analyze the evolution of the relative capital stock, we linearize equation (15) about the steady-states,  $\tilde{K}, \tilde{L}, \tilde{l}$ . From (13'), the coefficient on  $k_i$  in (15) is positive. In Appendix A.2 we show that for agent  $i$ 's relative stock of capital to remain bounded, and therefore to yield a non-degenerate steady-state wealth distribution,  $k_i(t)$  must follow the stable path:

$$k_i(t) = \frac{1+\theta(t)}{1+\theta(0)} k_{i,0} + \frac{\theta(0)-\theta(t)}{1+\theta(0)} a_i \quad (17)$$

where

$$\theta(0) \equiv \frac{F_L(\tilde{K}, \tilde{L})(1-\tau_w)/\tilde{K}}{\frac{F_L(\tilde{K}, \tilde{L})(1-\tau_w)}{\tilde{K}} \left( \frac{1+\eta}{\eta} \right) \left( \frac{1}{1+\eta} - \tilde{L} \right) - \mu} \left( \frac{L(0) - \tilde{L}}{1 - \tilde{L}} \right) \quad (18)$$

$$\theta(t) = \theta(0)e^{\mu t} \quad (18')$$

At any point of time, an agent's relative capital is a weighted average of his initial capital stock and his relative skills. Equations (17) and (18') imply that the weights of the two endowments change over time. This is because as the economy converges to a new steady state, factor prices change, altering the relative contributions of wealth and skill endowments to the individual's income, and hence to his savings. In an expanding economy,  $L(0) > \tilde{L}$ , and from (18) and (18') we see that  $\theta(0) > \theta(t) > 0$ ,  $\dot{\theta}(t) = \mu\theta(t) < 0$ , so that over time the relative weight shifts from the endowment of capital toward skills.

Using (17) we can write the difference between an agent's relative capital and the mean as<sup>12</sup>

$$k_i(t) - 1 = \frac{1+\theta(t)}{1+\theta(0)} (k_{i,0} - 1) + \frac{\theta(0)-\theta(t)}{1+\theta(0)} (a_i - 1) \quad (19)$$

$$\tilde{k}_i - 1 = \frac{1}{1+\theta(0)} (k_{i,0} - 1) + \left( \frac{\theta(0)}{1+\theta(0)} \right) (a_i - 1) = \frac{1}{1+\theta(t)} [(k_i(t) - 1) + \theta(t)(a_i - 1)] \quad (20)$$

From these expressions we see the potential for agents to change their relative wealth positions. That is, if an agent begins with above-average capital (i.e.  $k_{i,0} > 1$ ), but is endowed with below-average skills (i.e.  $a_i < 1$ ), he may end up with below-average capital. This is because there are two offsetting forces driving the accumulation of capital.<sup>13</sup> On the one hand, those with large initial wealth accumulate capital more slowly (during an expansion), which tends to deteriorate their

<sup>12</sup> Having determined  $\tilde{k}_i$  from (20), agent  $i$ 's (constant) relative leisure,  $\rho_i$ , can be derived from (16).

<sup>13</sup> This expression can be used, together with the individual's budget constraint, to show that for all agents to supply a strictly positive amount of labor in the steady state the initial distribution of capital must be such that an agent with ability  $a_i$  has an endowment below  $\bar{k} \equiv a_i [(1+\theta_0)s_L / (\eta L s_K) - \theta_0]$ .

relative position. On the other, those with more ability have higher incomes, *ceteris paribus*, and hence accumulate more capital, which tends to improve their relative position. As a result, the potential for wealth mobility exists.

This contrasts with the case considered in our previous work where, with homogeneous ability, if agent  $i$  begins with above-average capital, he ends up with above average-capital, although his relative wealth may change over time. This can be seen by setting  $a_i = 1$  in (20) to obtain  $\tilde{k}_i - 1 = (k_{i,0} - 1)/(1 + \theta(0))$ . Note also that in this case during the transition to a new steady state with higher capital (i.e. for  $\theta(0) > 0$ ), those with initial capital above average will experience a reduction in their relative capital, while those with initial capital below average will improve their relative position. The intuition lies on the fact that accumulating capital implies a falling interest rate and a rising wage rate, hence the income of the capital rich grows more slowly than that of those for whom labor income is more important and hence their rate of accumulation of wealth is slower.

Clearly  $\theta(0)$  is the key element driving long-run relative wealth as given by (20). The larger  $\theta(0)$  is, the smaller is the weight of initial capital in steady state capital and the larger is that of ability. The result that the relative capital stock of an agent is a weighted sum of his initial wealth and his ability is also obtained in Maliar, Maliar and Mora (2005). The advantage of our approach is that we can immediately see which are the determinants of the relative weight of these two elements. The intuition for the effect of  $\theta(0)$  can be obtained from equation (18). There are two key aspects determining the size of  $\theta(0)$ . The first is the net wage, as given by  $F_L(\tilde{K}, \tilde{L})(1 - \tau_w)$ . A higher steady-state wage implies higher income for the ability-rich and thus allows them to accumulate wealth faster, leading to a lower weight on initial capital (from (18) and all other things constant, a higher value of  $F_L(\tilde{K}, \tilde{L})(1 - \tau_w)$  results in a larger  $\theta(0)$ ). The second are the transitional dynamics. Both a lower absolute value of the eigenvalue  $\mu$  and a greater distance between  $L(0)$  and the steady state value  $\tilde{L}$  imply a longer transition period. Since during the transition the wealth distribution - conditional on ability- becomes less dispersed, this reduces the weight of initial wealth.

## 4.2 The dynamics of relative income

With distortionary taxes, before-tax and after-tax relative incomes will generally not

coincide. We consider the evolution of before-tax income, and relegate the discussion of the latter to Appendix A.3. Agent  $i$ 's before-tax relative income is given by

$$y_i(t) = s_K(t)k_i(t) + s_L(t)a_i \frac{L_i(t)}{L(t)} \quad (21)$$

where  $s_K \equiv F_K K/F$ ,  $s_L = 1 - s_K$  denote the shares of capital and labor income. Thus, the first term reflects the individual's (relative) income derived from wealth, and the second is his relative income derived from labor –more simply “earnings”– defined by  $y_i^e(t) \equiv (a_i w L_i(t)) / (w L(t))$ , which substituting for (16'), can be written as

$$y_i^e(t) = a_i + \frac{l(t)}{\tilde{l}} \left( \frac{1}{1+\eta} - \tilde{L} \right) \frac{1}{1+\theta(t)} \frac{a_i - k_i(t)}{L(t)} \quad (22)$$

This expression highlights how whether an agent's relative earnings exceed or are less than his relative ability depends on his comparative position in the wealth and ability distributions. If he is more endowed (relatively) in ability, his relative earnings exceed his relative skill level, his labor supply will be above average, and this will tend to raise his relative income. The opposite applies if he is more endowed with capital. Note that if the labor supply were inelastic, relative earnings would be unchanged over time and equal to relative ability.

Combining (22) and (21), pre-tax income can be expressed as

$$y_i(t) = \varphi(t)k_i(t) + (1 - \varphi(t))a_i \quad (23)$$

where

$$\varphi(t) \equiv \left[ s_K(t) + s_L(t) \frac{l(t)}{\tilde{l}L(t)} \left( \tilde{L} - \frac{1}{1+\eta} \right) \frac{1}{1+\theta(t)} \right] \quad (24)$$

represents the weight in current relative income due to the agent's *current* relative capital (wealth). From (23) and (24) we see that the dynamics of  $y_i(t)$  are driven by those of the aggregate variables,  $K(t), L(t)$ , both directly and through their effect on factor shares, as well as by the agent's relative rate of capital accumulation,  $k_i(t)$ . Note also that the elasticity of labor supply plays a key role, and with  $\tilde{L} < 1/(1+\eta)$  the weight on capital will be smaller than the share of capital ( $\varphi(t) < s_K(t)$ ), while that on ability will be greater than the labor share. The reason for this is the opposite signs of

laborsupply responses to increases in wealth and ability seen in equation (16').<sup>14</sup>

In steady state, the share of income due to capital is  $\tilde{\varphi} \equiv 1 - \tilde{s}_L / ((1 + \eta)\tilde{L})$ . For the Cobb-Douglas production function employed in our numerical examples  $s_L = 0.67, \eta = 1.75, \tilde{L} = 0.28$ , implying that in the long run 87% of current income is due to skills and 13% to relative capital, roughly consistent with existing evidence on factor decompositions of household income.<sup>15</sup>

Using (18) we can express current relative income as a weighted average of *initial* relative capital and skills

$$y_i(t) = \varphi(t) \frac{1 + \theta(t)}{1 + \theta(0)} k_{i,0} + \left( 1 - \varphi(t) \frac{1 + \theta(t)}{1 + \theta(0)} \right) a_i \quad (25)$$

Since  $\varphi(t) < s_K$  and  $\theta(0) > \theta(t) > 0$  in a growing economy, at any point of time relatively more of current income is attributable to endowed skill rather than to initial capital, as compared to the determinant of current wealth. Over time, the change in the relative weights in (25) will reflect the decline in  $\theta(t)$ , together with the change in the relative importance of capital due to  $\varphi(t)$ . The latter reflects the change in factor shares, and for the Cobb-Douglas function is negative, reinforcing the increasing relative importance of skills.

In Appendix A.3 we summarize the dynamics of relative income, showing how they depend upon two factors. The first is the gap between the agent's initial endowments of skills and physical capital, the second is the change in aggregate labor (leisure). The effect of endowments, in turn, is determined by the evolution of factor prices and hence depends crucially on the elasticity of substitution in production, while the labor supply response comprises both the initial response and the dynamics along the transitional path. As a result, relative income dynamics depend on endowments, parameters and the nature of the shock.

To give an example, consider an economy that is accumulating capital as a result of some

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<sup>14</sup>Although we cannot rule out  $\varphi(t) < 0$  at some point along the transitional path, in steady state  $0 < \tilde{\varphi} < \tilde{s}_K$  if and only if,  $1/(1 + \eta) > \tilde{L} > \tilde{s}_L / (1 + \eta)$  a condition that is met for the benchmark calibrations summarized in Table 1. Note also that we can write  $y_i^e(t) = (1 - s_K)^{-1} \{ (\varphi - s_K) k_i(t) + (1 - \varphi) a_i \}$ . With  $\varphi < s_K$  the agent's relative wealth has a negative effect on relative earnings, in contrast to the positive effect it has on total relative income. As a result, earnings inequality evolves very differently from income inequality, as our numerical simulations illustrate.

<sup>15</sup>See, for example, García-Peñalosa and Orgiazzi (2010) who find that the share of earnings in household income in industrial economies ranges between 70 and 85 percent.

external shock. If the production function is Cobb-Douglas, the relative income of agent  $i$  will increase along the transitional path if and only if his relative skill exceeds his relative initial wealth. However, because of an initial jump in relative income, this need not be associated with a long-run increase in relative income. A sufficient condition for this to be so is that the accumulation of capital be associated with a long-run reduction of labor supply.<sup>16</sup>

### 4.3 Wealth and income mobility

We can now compare two individuals  $i, j$ , and express their wealth gap at time  $t$  as

$$k_i(t) - k_j(t) = \frac{1 + \theta(t)}{1 + \theta(0)} \Delta k + \frac{\theta(0) - \theta(t)}{1 + \theta(0)} \Delta a \quad (26)$$

where  $\Delta a \equiv a_i - a_j$  and  $\Delta k \equiv k_{i,0} - k_{j,0}$ . This expression indicates that there are two offsetting forces influencing this gap, the differences in initial capital and the differences in ability. In a growing economy,  $\theta(0) > \theta(t)$ ,  $\dot{\theta}(t) < 0$ , implying that the term multiplying the capital gap is less than one and declining over time. As in Turnovsky and García-Peñalosa (2008), when the economy is accumulating capital, savings behavior and the dynamics of factor returns reduce capital inequality. At the same time, the coefficient of the skill gap is positive and growing over time, and this tends to increase wealth differentials. This is because the more able agents have higher labor incomes and will accumulate capital faster than those having lesser ability.

Consider now wealth mobility, which we define to be the possibility that an individual having an initial small wealth endowment overtakes some other initially richer agent. From (26) and (18') the initially less wealthy individual, agent  $j$  say, will catch up to the richer one, agent  $i$ , at time  $\hat{t}$ , determined by

$$\hat{t} = \frac{1}{\mu} \ln \left( \frac{(a_i - a_j) + (k_{i,0} - k_{j,0}) / \theta(0)}{(a_i - a_j) - (k_{i,0} - k_{j,0})} \right) \quad (27)$$

Clearly, catch-up will occur if and only if  $\hat{t} > 0$ .

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<sup>16</sup> More details on the dynamics of the response of relative income can be found in our previous work, see Turnovsky and García-Peñalosa (2008) and García-Peñalosa and Turnovsky (2011). Although there we did not consider heterogeneity in ability, the short term responses are qualitatively the same as in the present set up.

**Proposition 1:** In an economy that is accumulating capital [  $\theta(0) > 0$  ],

- (i) if individual  $j$  is initially endowed with less wealth than is individual  $i$ , the poorer agent will catch up in wealth if and only if  $-\Delta a \cdot \theta(0) > \Delta k$  ;
- (ii) if individual  $j$  is initially endowed with both less wealth and less ability than individual  $i$ , the poorer agent will never catch up.

**Proof:** Catch-up will occur if and only if  $\hat{t} > 0$ , which since  $\mu < 0$ , will be so if and only if

$$0 < \frac{(a_i - a_j) + (k_{i,0} - k_{j,0}) / \theta(0)}{(a_i - a_j) - (k_{i,0} - k_{j,0})} < 1$$

In the case of a growing economy, i.e. when  $\theta(0) > 0$ , these inequalities imply that for  $k_{i,0} > k_{j,0}$  there will be catch up if and only if  $-\Delta a \cdot \theta(0) > \Delta k$ . ■

The proposition indicates that the poorer agent will catch up in wealth if and only if he has sufficiently superior ability. It captures the conflict between the two forces discussed above: both more wealth and greater ability imply, other things equal, higher income and more savings. An initially less wealthy individual can catch up only if he is sufficiently able, so that he accumulates faster than does the wealthier, but less able, individual. If in addition to having less capital he also has less ability, he will never catch up.

With income subject to an initial jump, the potential for income mobility is more complex in that if it occurs, it may do so on impact, or along the subsequent transitional path. To examine this further, we compare two individuals  $i, j$ , in an initial steady state, where  $i$  has greater initial income, i.e.  $y_{i,0} > y_{j,0}$ . Thus, in the initial equilibrium

$$y_{i,0} - y_{j,0} = \tilde{\varphi}_0(k_{i,0} - k_{j,0}) + (1 - \tilde{\varphi}_0)(a_i - a_j) > 0 \quad (28)$$

where  $\tilde{\varphi}_0 \equiv 1 - (\tilde{s}_{L,0} / \tilde{L}_0)(1/(1+\eta))$ . Clearly,  $i$  may have higher initial income either because he has more ability than  $j$ , because he is initially wealthier, or both, but having more initial wealth, alone, does not suffice to ensure higher income.

There are two ways in which a shock can result in income catch-up. If  $y_i(0) < y_j(0)$ , then

following a shock agent  $j$  immediately overtakes agent  $i$  in income. Alternatively, if  $y_i(0) > y_j(0)$  and  $\tilde{y}_i < \tilde{y}_j$ , agent  $j$  overtakes agent  $i$  along the transition. It is even possible for agent  $j$  to overtake agent  $i$  on impact, but then for their relative incomes to revert to their original positions during the subsequent transition. Since instantaneous catch-up is unlikely, we shall focus attention on the more plausible case where it occurs along the transitional path. The following proposition specifies the circumstance under which such mobility is possible:

**Proposition 2:** Individual  $i$  may initially be richer than individual  $j$  because of higher initial wealth, higher ability, or both. If that is the case, then

- (i) if  $i$  has a larger endowment both of ability and wealth,  $j$  cannot catch up to  $i$ 's income level;
- (ii) if individual  $j$  is initially endowed with less wealth than is individual  $i$ , the poorer agent will catch up in income along the transitional path if and only if

$$\frac{\tilde{\varphi}}{1 + \theta(0) - \tilde{\varphi}} < -\frac{\Delta a}{\Delta k} < \frac{\varphi(0)}{1 - \varphi(0)} \quad (29a)$$

and the economy satisfies  $\varphi(0) > \tilde{\varphi} / (1 + \theta(0))$ ;

- (iii) if individual  $j$  is initially endowed with less skill than is individual  $i$ , the poorer agent will catch up in income if and only

$$\frac{\tilde{\varphi}}{1 + \theta(0) - \tilde{\varphi}} > -\frac{\Delta a}{\Delta k} > \frac{\varphi(0)}{1 - \varphi(0)} \quad (29b)$$

and the economy satisfies  $\varphi(0) < \tilde{\varphi} / (1 + \theta(0))$ .

**Proof:** At any point of time following a shock

$$y_i(t) - y_j(t) = \varphi(t) \frac{1 + \theta(t)}{1 + \theta(0)} (k_{i,0} - k_{j,0}) + \left( 1 - \varphi(t) \frac{1 + \theta(t)}{1 + \theta(0)} \right) (a_i - a_j) \quad (30a)$$

implying that

$$y_i(0) - y_j(0) = \varphi(0)(k_{i,0} - k_{j,0}) + (1 - \varphi(0))(a_i - a_j) \quad (30b)$$

$$\tilde{y}_i - \tilde{y}_j = \frac{\tilde{\varphi}}{1 + \theta(0)}(k_{i,0} - k_{j,0}) + \left(1 - \frac{\tilde{\varphi}}{1 + \theta(0)}\right)(a_i - a_j) \quad (30c)$$

We consider in turn the two cases in which income mobility is possible. Suppose first that  $i$  has a greater capital endowment than  $j$  but a lesser skill endowment, so that  $\Delta k > 0$  and  $\Delta a < 0$ . From (30b,c),  $y_i(0) - y_j(0) > 0$  and  $\tilde{y}_i < \tilde{y}_j$  hold if and only if (29a) holds. A necessary condition for (29a) to hold is  $\varphi(0) > \tilde{\varphi} / (1 + \theta(0))$ . Consider now the case where agent  $i$  is initially richer due to a greater skill endowment but  $j$  has greater initial wealth:  $\Delta a > 0$  and  $\Delta k < 0$ . From (30), income mobility is possible if and only if (29b) holds. Moreover, satisfying (29b) requires  $\varphi(0) < \tilde{\varphi} / (1 + \theta(0))$ . ■

Proposition 2 indicates that, as the economy converges to a new steady state, income mobility is possible only for one type of agent, either the skill-rich or the capital-rich, but not both. The reason for this is that income mobility depends on the behavior of factor prices. If wages are growing fast, then skill-rich agents will be able to catch-up but capital-rich individuals will not, and vice versa. The behavior of factor prices will, in turn, depend on both the structure of the aggregate economy and the nature of the shock, which is captured by the sign of  $[\varphi(0) - \tilde{\varphi} / (1 + \theta(0))]$ . For the Cobb-Douglas production function, in a growing economy  $\varphi(0) > \tilde{\varphi} / (1 + \theta(0))$  always holds.<sup>17</sup> It is then the skill-rich that may catch up in income; in contrast to a contracting economy it is the capital-rich for whom this is possible.

Note that equation (29a) has a simple interpretation. The right-hand side inequality is the condition for the wealthy individual to have a higher initial income, and simply requires that agent  $j$  does not have a sufficiently high skill endowment, relative to the initial wealth gap. The left-hand side inequality says that, given that  $i$  has initially higher income,  $j$  can catch-up only if his ability gap is sufficiently high. Similarly, from equation (29b) we can see that the right-hand inequality is the condition for  $i$  to be initially richer, and the left-hand inequality asserts that mobility can occur only

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<sup>17</sup> Substituting for  $\varphi(0)$  and  $\tilde{\varphi}$  from (24) we can show for the Cobb-Douglas production function

$$\varphi(0) - \tilde{\varphi} / (1 + \theta(0)) = (1 + \theta(0))^{-1} \left[ s_k \theta(0) + s_L \left( (1 + \eta)^{-1} - \tilde{L} \right) (L(0) - \tilde{L}) (L(0) \tilde{L} (1 - \tilde{L}))^{-1} \right]$$

Examining this expression, we immediately see that  $\varphi(0) - \tilde{\varphi} / (1 + \theta(0))$  is positive in response to an expansionary shock and negative following a contractionary shock.

if, given the initial wealth gap, the ability gap is not too large.

We can now construct measures of wealth and income mobility in the economy.

**Definition 1.** Let  $\Delta\hat{a}$  be the minimum ability gap required for  $j$  to catch up to  $i$ 's wealth, given their initial wealth gap,  $\Delta k$ . Let also  $\Delta\bar{a}$  (alternatively  $\Delta\bar{k}$ ) be the minimum ability (wealth) gap required for  $j$  to catch-up in income when it is the ability-rich (capital-rich) that may experience income mobility. We then define the extent of wealth mobility, denoted  $\omega_k$ , as  $\omega_k \equiv -(\Delta\hat{a} / \Delta k)^{-1}$ .

Our measure of wealth mobility is the inverse of the minimum ability gap required for catch-up. That is, the larger the ability gap required in order to catch up to a given wealth gap, the lower is mobility.<sup>18</sup> Using (27) we obtain

$$\omega_k = \theta(0) = \frac{F_L(\tilde{K})(1-\tau_w) / \tilde{K}}{\frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}} \left( \frac{1+\eta}{\eta} \right) \left( \frac{1}{1+\eta} - \tilde{L} \right) - \mu} \left( \frac{L(0) - \tilde{L}}{1 - \tilde{L}} \right) \quad (31)$$

The degree of wealth mobility depends on both the structural characteristics of the aggregate economy and the specific change generating the initial jump in aggregate labor supply. From (20), we see that a larger weight of ability in an agent's steady-state relative wealth will be associated with greater wealth mobility.

**Definition 2.** Whenever the skill-rich can catch-up with the capital-rich, we define the measure of income mobility to be  $\omega_y^a \equiv -(\Delta\bar{a} / \Delta k)^{-1}$ ; whenever it is the capital-rich that are catching up, we define it to be  $\omega_y^k \equiv -(\Delta\bar{k} / \Delta a)^{-1}$ .

Our definition implies that we measure the degree of income mobility by the endowment gap required for the poorer agent to be able to catch up to the richer one during the transition, where income mobility depends on which agent is doing the catching up. From the definitions of  $\omega_y^a$  and

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<sup>18</sup> While this measure is very natural in our context, it is not the measure of mobility commonly found in the literature. Both sociologists and economists usually examine mobility across successive generations, and define it as the probability that an individual is in an income/wealth class above that of his parents; see Piketty (2000) for a survey. In our model, agents are infinitely-lived which does not allow us to use such a measure.

$\omega_y^k$  we can write

$$\omega_y^a = \frac{1 + \theta(0) - \tilde{\varphi}}{\tilde{\varphi}} \quad (32a)$$

$$\omega_y^k = \frac{\tilde{\varphi}}{1 + \theta(0) - \tilde{\varphi}} \quad (32b)$$

A higher value of  $\omega_y^a$  or  $\omega_y^k$  implies that, for given distributions of initial wealth and skills, a greater fraction of the population will change their relative position along the distribution of income.

**Proposition 3:** In a growing economy if agent  $i$  catches up to agent  $j$ 's level of wealth he will do so only after he has caught up to agent  $j$ 's level of income. It is also possible that he will catch up to his level of income, but not to his level of wealth.

**Proof:** The time at which the income of two agents is the same, denoted  $\bar{t}$ , is defined by

$$\varphi(\bar{t})(1 + \theta(\bar{t})) = \frac{[1 + \theta(0)](a_i - a_j)}{(a_i - a_j) - (k_{i,0} - k_{j,0})} \quad (33)$$

implying that there is catch-up if and only if  $\bar{t} > 0$ . Combining (33) and (27) we see that  $\varphi(\bar{t})(1 + \theta(\bar{t})) = 1 + \theta(\hat{t})$ . Since  $\varphi(\bar{t}) < 1$  in a growing economy, this equality implies that  $\theta(\bar{t}) > \theta(\hat{t})$  which, given the definition of  $\theta(t)$  in (18), in turn implies  $\hat{t} > \bar{t}$ . ■

The intuition of Proposition 3 is straightforward. Since agents save a fraction of their income strictly less than one and given that  $i$  had a higher initial stock of capital,  $j$  will manage to accumulate as much wealth as  $i$  only if he has a higher level of income. Hence, he must catch up  $i$ 's income level before he can catch-up to his wealth.

## 5. Wealth and income inequality

Because of the linearity of the expression for relative wealth, (19), we can immediately transform these expressions into corresponding measures of aggregate wealth inequality, expressed either as the CV,  $[\sigma_k(t)]$ , or the SCV,  $[\sigma_k^2(t)]$ . While both have qualitatively similar implications, they have different advantages;  $\sigma_k(t)$  is dimensionally equivalent to the Gini coefficient, while

$\sigma_k^2(t)$  is decomposable.<sup>19</sup> Thus, recalling (19) and the definitions of  $\sigma_{k,0}$  (initial distribution of capital) and  $\sigma_a$  (distribution of skills), we can write  $\sigma_k^2(t)$  as

$$\sigma_k^2(t) = \frac{1}{[1+\theta(0)]^2} \left( [1+\theta(t)]^2 \sigma_{k,0}^2 + [\theta(0) - \theta(t)]^2 \sigma_a^2 + 2(1+\theta(t))[\theta(0) - \theta(t)] \sigma_{k,0} \sigma_a \chi \right) \quad (34)$$

where  $\chi$  is the correlation coefficient between initial capital endowments and skills. Letting  $t \rightarrow \infty$  in (34) yields

$$\tilde{\sigma}_k^2 = \frac{1}{[1+\theta(0)]^2} \left( \sigma_{k,0}^2 + \theta^2(0) \sigma_a^2 + 2\theta(0) \sigma_{k,0} \sigma_a \chi \right) \quad (35)$$

Using (34) and (35) we can (i) decompose the asymptotic dispersion of capital,  $\tilde{\sigma}_k^2$ , into its components, (ii) compare  $\tilde{\sigma}_k^2$  to its initial distribution,  $\sigma_{k,0}^2$ , and (iii) determine its initial response.

We summarize these in

**Proposition 4:** Consider an economy that is accumulating capital as a result of an expansionary external shock. In general, this can be associated with an increase or decrease in wealth inequality, depending upon the relative dispersions of the initial endowments of capital and skills and their correlation. More specifically, we find:

- (i) If  $\sigma_a = 0$ , long-run wealth inequality will decline
- (ii) If  $\sigma_{k,0} = 0$ , long-run wealth inequality will increase.
- (iii) Positive (negative) correlation  $\chi$  between endowments of capital and skills will increase (decrease) long-run wealth inequality.
- (iv) If the two endowments are independently distributed,  $\theta^2(0)$  measures the relative contribution of skills and capital endowments to long-run wealth inequality, as measured by its SCV. This will increase across steady states in response to an expansionary shock if and only if  $\sigma_a^2 / \sigma_{k,0}^2 > 1 + 2/\theta(0)$ .

Three interesting implications of Proposition 4 merit highlighting. First, if skill endowments

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<sup>19</sup> While  $\sigma_k^2, \sigma_k$  are qualitatively similar, they yield very different quantitative measures. By putting greater weight on extreme observations, changes in  $\sigma_k^2$  imply larger percentage changes in equality than do changes in  $\sigma_k$ .

dominate sufficiently to generate a long-run increase in wealth inequality, this increase will occur non-monotonically unless wealth and skill endowments are strongly positively correlated. Second, the effect of relative skill endowments depends crucially upon  $\theta(0)$ , which in turn depends upon how close labor supply jumps to its steady-state. The effects of skills on wealth inequality manifest themselves along the transitional path. Finally, (35) implies how wealth inequality can emerge from differences in skill endowments alone. In that case any structural shock induces transitional dynamics during which agents accumulate capital at different rates. Those with higher ability will accumulate capital faster and hence the new steady state will be one of wealth inequality.

Analogously, we can express income inequality in terms of its SCV. Using equation (25) and defining  $\phi(t) \equiv \varphi(t)(1 + \theta(t))/(1 + \theta(0))$ , the SCV of (pre-tax) income can be written as

$$\sigma_y^2(t) = \phi(t)^2 \sigma_{k,0}^2 + (1 - \phi(t))^2 \sigma_a^2 + 2\phi(t)(1 - \phi(t))\sigma_{k,0}\sigma_a\chi \quad (36)$$

Consider now an economy that is initially in steady state and is subject to a structural change. The changes in income inequality between the two steady states is given by

$$\begin{aligned} \tilde{\sigma}_y^2 - \tilde{\sigma}_{y,0}^2 = & \left( \frac{\tilde{\varphi}}{1 + \theta(0)} - \tilde{\varphi}_0 \right) \\ & \left\{ \left( \frac{\tilde{\varphi}}{1 + \theta(0)} + \tilde{\varphi}_0 \right) [\sigma_{k,0}^2 + \sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi] - 2[\sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi] \right\} \quad (37) \end{aligned}$$

where  $\tilde{\varphi}_0$  and  $\tilde{\varphi}$  are, respectively, the values of  $\varphi(t)$  in the initial and in the new steady states. The overall change in income inequality is the result of the change immediately following the shock and caused by the reaction of factor prices and the labor supply, and the change along the subsequent transitional path to the new steady state are. Although it is not possible to sign these changes in general, results can be obtained in the case of Cobb-Douglas production.

**Proposition 5:** Consider an economy with a Cobb-Douglas production technology. If the economy experiences an expansionary external shock that leads to an accumulation of capital and does not cause a long-run decline in employment, we obtain the following:

- (i) If  $\sigma_a = 0$ , income inequality, as measured by its SCV, initially increases and then declines unambiguously during the transitional phase.
- (ii) For  $\sigma_{k,0} = 0$ , income inequality, as measured by its SCV, initially declines and then increases unambiguously during the transitional phase.

**Proof:** See Appendix A.3. ■

The first part of Proposition 5 captures the equalizing effect of the transition: because the capital-rich accumulate more slowly than the capital poor, then inequality in wealth is reduced. If this is the only source of heterogeneity, then income inequality falls too. The second result captures the unequalizing effect of the transition. This is in turn the result of two complementary forces. On the one hand, accumulating capital implies that the wage increases, magnifying the effect of unequal abilities. On the other, because those with high ability have higher incomes they will also save more, adding to the inequality in ability an inequality in capital. As a result inequality increases during the transition.

The last aspect we consider is the relationship between income inequality and mobility. Using (32) and (36) we can express the change in income inequality following a shock as

$$\tilde{\sigma}_y^2 - \tilde{\sigma}_{y,0}^2 = \left( \frac{1}{1 + \omega_y^a} - \tilde{\varphi}_0 \right) \left\{ \left( \frac{1}{1 + \omega_y^a} + \tilde{\varphi}_0 \right) \left[ \sigma_{k,0}^2 + \sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi \right] - 2 \left[ \sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi \right] \right\} \quad (38)$$

The interesting implication of this equation is that inequality and mobility need not move together. It is possible for a shock to generate substantial income mobility (i.e. result in a large value of  $\omega_y^a$ ) and yet engender small changes in inequality, which would occur if  $\left[ 1/(1 + \omega_y^a) - \tilde{\varphi}_0 \right]$  is close to zero. The intuition for this result is that shocks, by affecting factor prices, change who is at the top of the income distribution. A shock that results in a large increase in wages and a large reduction in the interest rate would give rise to substantial mobility. At the same time, because ability is unequally distributed, the increase in the wage would imply an increase in earnings inequality thus offsetting the equalizing effect that a reduction in the interest rate has. If the increase in earnings dispersion is sufficiently large, high mobility could even be associated with greater income inequality, as captured

by the non-monotonicity of the expression in (38) with respect to  $\omega_y^a$ . In other words, equation (38) implies that high mobility can be associated with increases or decreases in inequality, and that a given change in steady-state inequality may be accompanied by different degrees of mobility.

## 6. Numerical Simulations

To obtain further insights into the dynamics of wealth and income distribution we employ numerical simulations. These are based on the following functional form and parameter values, characterizing the benchmark economy:

Production function:	$Y = A(\alpha K^{-\rho} + (1-\alpha)L^{-\rho})^{-1/\rho}$
Basic parameters:	$A = 1.5, \alpha = 0.33$
	$\rho = 0$ (elast of sub $\varepsilon = 1$ )
	$\beta = 0.04, \gamma = -1.5, \eta = 1.75, \delta = 0.07$
Fiscal parameters:	$\tau_k = \tau_w = g = 0.22$
Distributions:	$\sigma_{k0}^2 = 14, \sigma_a^2 = 0.4, \chi = 0.33$

Preferences are summarized by an intertemporal elasticity of substitution  $1/(1-\gamma) = 0.4$ , rate of time preference of 4%, while the benchmark elasticity of leisure in utility is 1.75. The production function is CES with distributional parameter  $\alpha = 0.33$  and with an elasticity of substitution,  $\varepsilon = 1/(1+\rho)$ , of 1, while  $A = 1.5$  scales the level of productivity.<sup>20</sup> The depreciation rate is 7% per annum.<sup>21</sup> These parameters are all standard and typical of those found in the literature.<sup>22</sup>

The choice of tax rates is less straightforward and has generated debate, due to the difficulty of mapping the complexities of the real world tax structure into a simple one-sector growth model.

<sup>20</sup>Berndt's (1976) early comprehensive study has long been used to justify the Cobb-Douglas function as a plausible benchmark. For the preferred methods of estimation, using superior data, he finds estimates of the elasticity of substitution to range from around 0.8 to 1.2. However, more recent authors have argued that the treatment of technological change has biased the estimates toward unity, and that modifying the econometric specification leads to significantly lower estimates of the elasticity, in the range 0.5-0.7, thus rejecting the Cobb-Douglas specification; see e.g. Antràs (2004), Klump, McAdam, and Willman (2004). Duffy and Papageorgiou (2000) estimate the elasticity of substitution using cross-sectional data and find that the Cobb-Douglas production function is an inadequate representation of technology across countries. Their evidence suggests that the elasticity of substitution exceeds unity for rich countries, but is less than unity for developing countries. By letting  $\varepsilon$  range between 0.75-1.15 we are covering most of the plausible estimates.

<sup>21</sup>For simplicity we assume that depreciation costs are not tax deductible.

<sup>22</sup>For example, the intertemporalelasticity of substitution of 0.4 is well within the range summarized by Guvenen (2006), while the relative weight on leisure in utility is close to the conventional value of the real business cycle literature; see Cooley (1995). The production elasticity  $\alpha = 0.33$  is also well within the conventional range.

Recently, McDaniel (2007) has computed effective tax rates that can be readily used in macroeconomic models. Her tax rates indicate substantial fluctuations of tax rates in the US, with the tax rate on capital and labor income varying within the rather wide range of 15% to 30%. In our benchmark numerical examples we set a uniform tax on the two types of income of 22%, even though the two tax rates have tended to differ. This has the advantage that the tax system has no direct distributive effects (i.e. pre- and post-tax inequality are the same) and hence we can focus on the indirect distributive effects caused by changes in factor rewards. Later we consider how differences between tax rates affect distribution. Finally, we set the government consumption expenditure rate at  $g = 0.22$ , implying that it is entirely financed by the income tax.

We also require estimates of the distributions of ability and initial wealth, together with their correlation. To choose these we use the figures reported in García-Peñalosa and Orgiazzi (2010), who decompose overall income inequality into its factor components. For the US, their figures give dispersions (as measured by the SCV) for capital of 13.17, for earnings of 0.93, and for gross income of 0.58, for the year 1979. In 2004 these three inequality measures were, respectively, 16.10, 1.34, and 0.82, capturing the well-known increase in both income and earnings inequality.<sup>23</sup> We set  $\sigma_{k_0}^2 = 14$  which approximates the large dispersion of capital income observed in the data. The dispersion of ability is assumed to be  $\sigma_a^2 = 0.4$  and the initial correlation of the two endowments is set at  $\chi = 0.33$ . As can be seen in Table 1, for both the benchmark case and that of a “low” elasticity of leisure,  $\eta = 1$ , these parameters will generate dispersions of wealth, earnings and income of the same magnitudes as those observed in the data.

Table 1 reports the benchmark steady-state equilibrium (shown in bold) for the chosen parameters, as well as the long-run responses to changes in technology and preferences. The benchmark case is reported in the first panel. There we see that the baseline setup, reported on the first line, yields an equilibrium allocation of labor of 27.7%. The dispersion of earnings is 1.422 and that of income 0.676. The second panel reports the case of a low elasticity of leisure ( $\eta = 1$ ). The dispersion of earnings is now much smaller, 0.990, as a result of weaker labor supply responses.

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<sup>23</sup>See also Sierminska, Brandolini and Smeeding (2006) for recent estimates of the distribution of wealth. They obtain Gini coefficients, a different inequality index that tends to give less weight to extreme observations.

Recall that those endowed with higher ability tend to work more, thus making earnings more dispersed than ability. Since the impact of ability on working time is now smaller, so is earnings inequality. In contrast, income inequality is higher than in the benchmark case, taking a value of 0.875. The reason for this is the equalizing effect of labor supply reactions to wealth differences. As we have seen, richer agents tend to work less, which reduces, other things constant, their earnings and hence their income. With a lower elasticity of labor, this response is milder and thus ‘less equalizing’.

The first line of the third panel indicates that with a higher elasticity of substitution in production ( $\varepsilon = 1.15$ ) the labor supply is lower than in the benchmark case, 0.256, which results in a greater degree of income inequality since, as we have seen, a lower labor supply results in greater dispersion of working hours and hence of earnings; see equation (20). For the same distributions of ability and initial wealth, we find a dispersion of earnings of 2.534 and of income of 0.779. The former is 65 percent higher than in the benchmark, but the increase in income inequality is much smaller due to the positive correlation between leisure and wealth. The level of earnings inequality is implausibly large, the reason for this is that earnings inequality is very sensitive to  $\varepsilon$  and that we have chosen the distribution of ability to match the data for low values of  $\varepsilon$ .

We now consider some examples of shocks and how they affect distribution and mobility. We begin by examining the impact of a productivity shock and then consider changes in the fiscal structure.<sup>24</sup>

### 6.1. Increase in the level of technology

Consider first the effect of a technological shock, parameterized by an increase in productivity  $A$  from 1.5 to 2. As noted previously, the transitional adjustment of  $\tilde{L} - L(0)$  is a critical determinant of the response of wealth inequality, hence both the initial response and the steady state value of labor are reported. The last two columns of all three panels report our measures of mobility following a shock.

In all cases, steady state capital and output increase, while the labor supply is lower than or

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<sup>24</sup> The formal expressions describing the responses of the aggregates are provided in Appendix A.4.

(in the case of a Cobb-Douglas) the same as in the initial equilibrium. In all cases there is an increase of labor on impact, with the labor supply then falling until it reaches its new steady state. With the dispersion of wealth endowment dominating that of ability, the transitional adjustment of labor supply leads to a long-run, gradual and monotonic reduction in wealth inequality, consistent with Proposition 4 (iv). Moreover, with most of the adjustment in employment taking place on impact, we see from the three panels that in all cases the changes in the distribution of wealth that occur during the transition are also moderate, with the eventual reduction of wealth inequality ranging from 1.28% to 3.03%.<sup>25</sup> Because of the moderate changes in wealth accumulation that occur during the transition, wealth mobility is extremely low. Recalling our definition of wealth mobility,  $\omega_k \equiv -\Delta k / \Delta \hat{a}$ , the figure of 0.016 for the Cobb-Douglas case implies that for agent  $j$  (the more able individual) to catch up with the wealth of agent  $i$  (the initially wealthier agent) their gap in wealth must be less than 0.016 of their ability gap. In other words, the ability gap of  $j$  with respect to  $i$  has to be at least 62 times as large as their initial wealth gap!

In contrast to wealth inequality, earnings and income inequality and the degree of income mobility are highly sensitive to the preference and production parameters. Consider first the case of a Cobb-Douglas production function reported in the top two panels. In this case neither the labor supply nor factor shares change, and hence all distributional changes are due to the evolution of the distribution of wealth and to differential labor supply response across agents. In both cases, the productivity shock results in a reduction in both earnings and income inequality of rather similar magnitudes (earnings fall by between 2 and 2.5 percent, while income inequality falls by between 1.1 and 1.4 percent).

There are nevertheless substantial differences in terms of income mobility with a high elasticity of labor resulting in a much higher degree of mobility than in the case of a low elasticity (7.471 and 5.157, respectively). Our results for  $\omega_y^a$  indicate that, for our benchmark case, agent  $j$  will catch up the income of all those agents who were initially richer and for whom the wealth gap between the two is less than 7.471 times their ability gap. In the case of a low labor elasticity the

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<sup>25</sup> The fact that most of the adjustment in labor supply occurs on impact is characteristic of this class of model; see Turnovsky (2004). It reflects the fact that there is no cost to adjusting labor supply.

corresponding figure for mobility is 5.157. Although this number still implies high mobility, it is substantially lower than in the case of  $\eta = 1.75$ . The intuition can be obtained from equation (20'). Mobility is possible because of heterogeneity in ability, and the direct effect of ability is reinforced by the fact that more able agents also supply more labor. With a low elasticity, there is a weak response of individuals' labor supplies to ability. As a result it is harder for the ability-rich to catch up and mobility is low; a higher elasticity implies a stronger labor supply reaction thus reinforcing the direct effect of ability and making it easier to catch up in income.

The bottom panel considers the case of a high elasticity of substitution in production,  $\varepsilon = 1.15$ . In this case the long-run effect of the shock on income inequality is reversed, with inequality increasing by 3.1%. The reason for this is the labor supply: the reduction in the labor supply results in a sharp increase in wages and thus in earnings inequality which more than offsets the fall in wealth inequality, as a result income inequality increases. The last two columns report the mobility measures. Faster convergence implies lower wealth and income mobility than in the benchmark case. Note, however, that income mobility is 5.662, i.e. roughly of the same magnitude as in the case of Cobb-Douglas production and low elasticity of labor. Comparing the two bottom panels we can derive two conclusions. First, a similar degree of mobility can be compatible with a reduction in income inequality (case of  $\varepsilon = 1$  and  $\eta = 1$ ) as well as with an increase in inequality (case of  $\varepsilon = 1.15$  and  $\eta = 1.75$ ). Second, low mobility (relative to the benchmark) can be the result of either of two effects. A low elasticity of labor tends to reduce mobility because it implies a small labor supply response to ability, and hence lower differences in earnings between those with different degrees of ability. Alternatively, it can be the result of a high elasticity of substitution in production, since with a higher elasticity, factor prices are less responsive to changes in the capital-labor ratio. As a result, the wage (interest rate) declines (increases) more in response to the shock, making income more sensitive to ability and less so to wealth endowments.

## **6.2 Tax changes and mobility**

The effects of fiscal changes are reported in the three panels of Table 2, corresponding to the benchmark case as well as those with low elasticity of the labor supply and high elasticity of

substitution in production. In all cases the first line in each panel reports the magnitudes for the initial steady state.

As a first example of the distributional dynamics arising from a change in the fiscal structure, we consider the effect of a balanced reduction in the (common) tax and expenditure rates from 22% to 17%. The aggregate responses are qualitatively identical to those resulting from an increase in the level of technology. In all three cases the changes in wealth inequality are extremely small, in line with our previous work where we found that the transitional dynamics following a tax change are much milder than those after a productivity change; see García-Peñalosa and Turnovsky (2011). In contrast, both earnings and income inequality may exhibit substantial changes once we move away from the Cobb-Douglas case. Despite much smaller changes in inequality, the degree of income mobility is about the same as that generated by a productivity change. This is the result of the direct impact of tax changes on income and labor supply, which is absent in the case of a productivity change. The consequence of this is that although the reduction in taxes has a small impact on income inequality, those at the top of the income distribution are more likely to be ability-rich than they were before the tax reduction.

Our second exercise is to consider the effects of changing the tax structure to finance a given rate of expenditure,  $g$ .<sup>26</sup> These effects are summarized in the third and fourth lines of the three panels in Table 2. We consider two initially identical economies with uniform tax rates,  $\tau_w = \tau_k = g = 0,22$ , and suppose that they shift the respective tax burdens in opposite directions. One reduces the tax on capital income by 5 percentage points, from 22% to 17%, offsetting this with an appropriate increase in the tax on labor income. The other reduces the labor income tax by the same magnitude, from 22% to 17%, and compensates this by a higher capital income tax. Since the share of labor is much higher than that of capital, in the first case the required increase in the labor tax is mild (between 2 and 4 percentage points), while in the second case capital income taxes increase sharply (up to 32% in the case of a low elasticity of labor).

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<sup>26</sup>Tax structures, and not just tax rates, differ substantially across countries, as documented by McDaniel (2007). Her results indicate that a key feature of the US economy is  $\tau_k > \tau_w$ , a characteristic that holds uniformly since 1953. For example, average values of these tax rates for the decade 1991-2000 were  $\tau_k = 0.276$  and  $\tau_w = 0.224$ . In contrast, European economies have tended to have a higher effective tax rate of labor than on capital.

The aggregate effects of such compensated tax changes have been extensively studied, and are summarized in Appendix A.4.<sup>27</sup> There we see that substituting a tax on labor income for a tax on capital income will reduce long-run employment, while increasing the long-run capital stock, and output (and consumption). Since  $\tilde{K} > K_0$ , then  $L(0) > \tilde{L}$  (slightly) and hence we have  $\theta(0) > 0$  and  $\tilde{\varphi} < \varphi(0)$ . In addition, the fact that labor declines in the long run implies that  $\tilde{L} < \tilde{L}_0$ . The opposite occurs when the capital tax substitutes for a labor tax.

The distributional responses are substantial, certainly much larger than the responses to an increase in the common income tax rate, the reason being that they elicit sharp labor supply responses.<sup>28</sup> Several general results emerge. First, wealth responses are mild, and wealth mobility requires phenomenally large ability gaps. Second, wealth and earnings inequality move in opposite directions. This is the result of the opposite effects of tax changes on capital and labor. For example, the reduction in the tax on capital income increase the steady-state capital stock and during the transition wealth inequality becomes less dispersed. At the same time, the tax change reduces the labor supply, increasing the dispersion of earnings. These two forces have opposite effects on the distribution of income. Third, the economy with the low capital income tax exhibits much lower income inequality than that with the high capital income tax. Although it seems puzzling that lower earnings inequality is associated with higher income inequality, the force driving this result is the negative correlation between wealth dispersion and labor supply dispersion for a given level of ability. As a result, inequality in earnings partly offsets the inequality in capital incomes, and the greater earnings dispersion is, the lower income inequality becomes. In our tax exercise, a reduction in the capital income tax results in both lower capital income inequality and a more dispersed distribution of earnings (which has an equalizing effect), thus leading to lower income inequality. The opposite happens in the case of an increase in the capital income tax.

Lastly, inequality and mobility move together. In order to see this, consider the Cobb-

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<sup>27</sup>See Chamley (1986) and Judd (1985) for early analyses that concluded that capital income should not be taxed and Prescott (2004) and Turnovsky (2004), among others, for the effects of taxation on labor supply responses.

<sup>28</sup>Using the “idiosyncratic shock model” to generate inequality, Domeij and Heathcote (2004) reach a similar qualitative conclusion regarding the effect of reducing capital income taxes, suggesting that changing the balance between capital and income taxes is likely to have very significant distributional consequences insofar as welfare inequality is concerned. Although space limitations preclude us from investigating welfare issues, it is a direction in which the present analysis could easily be extended, using the approach of García-Peñalosa and Turnovsky (2011)

Douglas case with  $\eta = 1.75$ . The economy with the low tax on capital exhibits a level of income inequality of 0.601 and income mobility of 9.106. This last figure implies that agent  $i$  will catch up with agent  $j$  if their ability gap is 11% or more of their wealth gap. For the economy with a high capital tax, inequality is greater (0.847) and poor agents need more ability in order to catch up in income – at least 19% of the wealth gap, corresponding to mobility of 5.281.

Similar relationships appear in the case of our other parameters. Note, however, that there are sharp differences in income mobility across the three panels. With a high elasticity of labor, the degree of mobility responds sharply to the type of fiscal change (both for the Cobb-Douglas case and the high value of  $\varepsilon$ ). In contrast, for a low elasticity of labor (i.e.  $\eta = 1$ ) the values of  $\omega_y^a$  are much more similar across the three tax experiments. The reason is simply that in this case mobility is mainly driven by factor price changes as the low value of  $\eta$  implies that labor supply responses are small.

## 7. Conclusions

In this paper we have studied the dynamics of wealth and income distributions in a Ramsey model with heterogeneous endowments of wealth and ability. As has been shown previously, the homogeneity of the utility function facilitates aggregation and leads to a macroeconomic equilibrium having a simple recursive structure. First, the aggregate dynamics are determined, independently of distribution. Then, the evolution of aggregate capital and labor drive the distributional dynamics. Because the aggregate behavior collapses to that of a representative-consumer setup, existing results of conventional representative-agent growth models with homogeneous preferences are robust with respect to the introduction of these two sources of heterogeneity.

In contrast to existing work with only one source of heterogeneity, our setup generates rich distributional dynamics that are highly responsive to structural and policy changes. For example, in our previous work with differences in only initial wealth, while such changes could expand or contract the distribution of income, agents' relative positions remained unchanged over time. In contrast, for heterogeneity in discount rates, an agent's relative position can change, yet the extent of mobility and the degree of wealth inequality are unaffected by changes in technology or taxes since

the distribution of wealth ultimately degenerates to one in which the most patient individual holds all the capital. In our current framework, both inequality and mobility respond sharply to the macroeconomic environment, implying that changes in fundamentals or policy affect aggregate magnitudes, distribution, and the extent to which agents' position in the income distribution depend on their ability endowment.

The dynamics of income inequality are driven by three factors: the dynamics of wealth inequality, of factor shares, and of labor supply/leisure. To appreciate the underlying driving forces consider a growing economy. First, as the economy accumulates capital, wealthier agents enjoy more leisure and accumulate more slowly than the average, while those who have more ability earn a higher income, save more, and accumulate assets faster. This implies that the distribution of wealth changes over time and that its correlation with ability increases. Second, earnings inequality changes as the economy converges towards a steady state. This is due to the change in the wage rate (per efficiency unit of labor), as well as to the labor supply responses of individuals, which induce those with greater ability to work longer hours. These mechanisms create a complex relationship between aggregate magnitudes and distributional variables, which imply that different patterns of distributional dynamics may obtain during the transition to the steady state.

Our numerical examples highlight the key role played by the elasticity of the labor supply. We found that although the percentual changes in inequality were roughly the same in the case of a high and a low elasticity, mobility differed substantially in the two cases, with a greater elasticity of labor being associated with a higher value of our mobility index. The reason for this difference lies in the role played by the endogenous labor supply. Wealthier people supply less labor while more able people supply more, and these two effects drive, together with changes in factor prices, the possibility of income mobility. With a high elasticity of the labor supply, these responses are large, allowing the ability-rich to catch up more easily with the capital-rich and this results in greater mobility than for low values of this elasticity.

When we consider the effect of a reduction in government expenditure (and the required income tax rate), our analysis highlights the different behavior of inequality and mobility. The policy change results in much smaller changes in inequality than in the case of a productivity change, yet

the degree of income mobility is about the same. The reason for these responses is that there is now a direct impact of tax changes on income and labor supply, which is absent in the case of a productivity change. The policy thus barely affects the overall degree of inequality yet there is substantial movement of individuals along the income distribution, so that, in the long-run, there is a stronger correlation between ability and income than before the policy change. Moreover, when we compare various tax changes we find that although mobility is highly sensitive to the particular tax change in the case of highly elastic labor supply, with a low elasticity our index of mobility varies much less across the various policy changes examined.

Our results have two important general implications. On the one hand, they indicate that – under certain assumptions – analyzing the distributive implications of macroeconomic policies and shocks does not require an entirely new framework of analysis. Rather, distributional responses can be examined with existing aggregate models. On the other, they emphasize that although substantial progress has been made in understanding the behavior of income distribution in macroeconomic models, focusing only on changes in an inequality index is insufficient to understand the implications of policy changes, since behind a given degree of inequality there may lay very different patterns of individual income mobility.

**Table 1: Increase in productivity**Baseline: Cobb-Douglas ( $\rho = 0, \varepsilon = 1$ ) and labor supply elasticity of  $\eta = 1.75$ 

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $A = 1.5$	<b>0.277</b>	<b>1.804</b>	<b>0.771</b>	<b>14</b>	<b>1.422</b>	<b>0.676</b>	-	-
$A = 2$	$L(0)$ 0.280 $\tilde{L}$ 0.277 (0%)	2.771 (+53.6%)	1.184 (+53.6%)	13.575 (-3.03%)	1.386 (-2.57%)	0.669 (-1.10%)	0.016	7.471

Low elasticity of the labor supply:  $\eta = 1.0$  (and Cobb-Douglas production)

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $A = 1.5$	<b>0.401</b>	<b>2.614</b>	<b>1.117</b>	<b>14</b>	<b>0.990</b>	<b>0.875</b>	-	-
$A = 2$	$L(0)$ 0.405 $\tilde{L}$ 0.401 (0%)	4.015 (+53.6%)	1.716 (+53.6%)	13.590 (-2.93%)	0.970 (-2.05%)	0.862 (-1.44%)	0.016	5.157

High elasticity of substitution in production:  $\rho = -0.13, \varepsilon = 1.15$  (and labor supply elasticity of  $\eta = 1.75$ )

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $A = 1.5$	<b>0.256</b>	<b>2.472</b>	<b>0.876</b>	<b>14</b>	<b>2.354</b>	<b>0.779</b>	-	-
$A = 2$	$L(0)$ 0.252 $\tilde{L}$ 0.250 (-2.18%)	4.222 (+70.8%)	1.433 (+63.6%)	13.820 (-1.28%)	2.651 (+12.6%)	0.803 (+3.11%)	0.007	5.662

**Table 2: Fiscal changes**

Baseline: Cobb-Douglas ( $\rho = 0, \varepsilon = 1$ ) and labor supply elasticity of  $\eta = 1.75$

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $\tau_w = \tau_k = g = 0.22$	<b>0.277</b>	<b>1.804</b>	<b>0.771</b>	<b>14</b>	<b>1.422</b>	<b>0.676</b>	-	-
Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$	$L(0)$ 0.278 $\tilde{L}$ 0.277	1.979	0.795	13.890	1.413	0.674	0.004	7.368
Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.245, g = 0.22$	$L(0)$ 0.271 $\tilde{L}$ 0.270	1.933	0.776	13.920	1.650	0.601	0.003	9.106
Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.322, \tau_w = 0.17, g = 0.22$	$L(0)$ 0.288 $\tilde{L}$ 0.289	1.536	0.753	14.240	1.066	0.847	-0.009	5.281

Low elasticity of the labor supply:  $\eta = 1.0$  (and Cobb-Douglas production)

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $\tau_w = \tau_k = g = 0.22$	<b>0.401</b>	<b>2.614</b>	<b>1.117</b>	<b>14</b>	<b>0.991</b>	<b>0.875</b>	-	-
Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$	$L(0)$ 0.402 $\tilde{L}$ 0.401	2.868	1.152	13.894	0.985	0.872	0.004	5.085
Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.245, g = 0.22$	$L(0)$ 0.394 $\tilde{L}$ 0.394	2.813	1.130	13.919	1.129	0.795	0.003	5.746
Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.321, \tau_w = 0.17, g = 0.22$	$L(0)$ 0.414 $\tilde{L}$ 0.416	2.202	1.082	14.250	0.775	1.048	-0.009	4.077

**Table 2 (continued): Fiscal changes**

High elasticity of substitution in production:  $\rho = -0.13, \varepsilon = 1.15$  (and labor supply elasticity of  $\eta = 1.75$ )

	Labor	$\tilde{K}$	$\tilde{Y}$	$\tilde{\sigma}_k^2$	$\tilde{\sigma}_e^2$	$\tilde{\sigma}_y^2$	$\omega_k$	$\omega_y^a$
<b>Base:</b> $\tau_w = \tau_k = g = 0.22$	<b>0.256</b>	<b>2.472</b>	<b>0.876</b>	<b>14</b>	<b>2.354</b>	<b>0.779</b>	-	-
Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$	$L(0)$ 0.255 $\tilde{L}$ 0.255	2.771	0.914	13.926	2.408	0.783	0.003	5.863
Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.254, g = 0.22$	$L(0)$ 0.247 $\tilde{L}$ 0.246	2.681	0.884	13.953	2.929	0.666	0.002	7.527
Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.298, \tau_w = 0.17, g = 0.22$	$L(0)$ 0.269 $\tilde{L}$ 0.270	2.136	0.854	14.170	1.701	0.964	-0.006	4.494

## Appendix

### A.1 Derivation of the macroeconomic equilibrium and linearization of the aggregate system

From the optimality conditions (6) we obtain

$$\eta \frac{C_i}{l_i} = a_i w (1 - \tau_w) \quad (\text{A.1})$$

Taking the time derivatives of (6a) and (A.1) (with  $a_i$  constant over time), and combining the former with (6c), yields

$$(\gamma - 1) \frac{\dot{C}_i}{C_i} + \eta \gamma \frac{\dot{l}_i}{l_i} = \frac{\dot{\lambda}_i}{\lambda_i} = \beta + \delta - r(K, l)(1 - \tau_k) \quad (\text{A.2a})$$

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{\dot{w}}{w} \quad (\text{A.2b})$$

With all agents facing the same tax rates and factor returns, (A.2a) and (A.2b) imply  $\dot{C}_i / C_i = \dot{C} / C$  and  $\dot{l}_i / l_i = \dot{l} / l$  for all  $i$ . Substituting (A.1) into (2), we may write the individual's accumulation equation in the form

$$\frac{\dot{K}_i}{K_i} = [(1 - \tau_k)r - \delta] + (1 - \tau_w) \frac{w}{K_i} a_i \left( 1 - l_i \frac{1 + \eta}{\eta} \right) + \frac{T_i}{K_i} \quad (\text{A.3})$$

To derive the macroeconomic equilibrium and its dynamics, we sum this equation over  $i$ , together with other components of the individual agent's optimality conditions.

Aggregating (A.1) over all agents, the aggregate economy-wide consumption is

$$C(t) = (1 - \tau_w) \frac{w l(t)}{\eta} \frac{1}{N} \sum_i a_i \rho_i \equiv (1 - \tau_w) \frac{w}{\eta} l(t) \Omega = (1 - \tau_w) \frac{w}{\eta} (1 - L(t)) \quad (\text{A.1}')$$

Substituting (7) into (A.2a) and aggregating (A.3) over all agents, we obtain the aggregate (average) Euler and capital accumulation equations, respectively

$$(\gamma - 1) \frac{\dot{C}}{C} + \eta \gamma \frac{\dot{l}}{l} = \beta + \delta - (1 - \tau_k)r(K, l) \quad (\text{A.2a}')$$

$$\frac{\dot{K}}{K} = (1-\tau_k)r - \delta + (1-\tau_w)\frac{w}{K}\left(1-\Omega l\frac{1+\eta}{\eta}\right) + \frac{T}{K} \quad (\text{A.3}')$$

These can be reduced to a pair of dynamic equations in  $K$  and  $L$  that are independent of the distributional aspects. The procedure we follow is analogous to that employed by Turnovsky and García-Peñalosa (2008), the only difference being that due to the presence of the term  $\Omega$ .

Using (A.2a'), (A.3a'), the government budget constraint, (5), the equilibrium factor returns, (4a) and (4b'), and the labor market clearance condition, (9b), the aggregate dynamic system can be summarized by

$$\dot{K} = (1-g)F(K, L) - (1-\tau_w)F_L(K, L)\frac{(1-L)}{\eta} - \delta K \quad (\text{A.4a})$$

$$\dot{L} = \frac{1}{Z(L)} \left\{ (1-\gamma)\frac{F_{KL}}{F_L} \left( (1-g)F - (1-\tau_w)F_L\frac{(1-L)}{\eta} - \delta K \right) + [(\beta+\delta) - F_K(1-\tau_k)] \right\} \quad (\text{A.4b})$$

where

$$Z(L) \equiv \frac{1-\gamma(1+\eta)}{1-\tilde{L}} - \frac{(1-\gamma)F_{LL}}{F_L} > 0$$

These equations are just a representation of the standard (aggregate) Ramsey model with endogenous labor supply. The dynamic equations (A.4a), (A.4b) highlight how the evolution of the aggregate capital stock and labor supply are *independent* of any distributional characteristics. Moreover, knowing  $L(t)$ , (A.1') and (9b) imply that the same applies to aggregate consumption,  $C$ , and effective leisure  $\Omega l$ , respectively.

We can now examine the dynamics of the aggregate system. Linearizing eqs. (A.4a) and (A.4b) around the steady state (11a) and (11b) yields the local dynamics for  $K(t)$  and  $L(t)$ :

$$\begin{pmatrix} \dot{K}(t) \\ \dot{L}(t) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} K(t) - \tilde{K} \\ L(t) - \tilde{L} \end{pmatrix} \quad (\text{A.5})$$

where

$$b_{11} \equiv (1-g)F_K - (1-\tau_w)F_{KL}\frac{(1-\tilde{L})}{\eta} - \delta; \quad b_{12} \equiv F_L \left( (1-g) + \frac{1}{\eta}(1-\tau_w) \right) - (1-\tau_w)F_{LL}\frac{(1-\tilde{L})}{\eta} > 0;$$

$$b_{21} \equiv \frac{1}{Z(L)} \left[ \frac{(1-\gamma)F_{KL}}{F_L} b_{11} - F_{KK}(1-\tau_k) \right]; \quad b_{22} \equiv \frac{F_{KL}}{Z(L)} \left[ \frac{(1-\gamma)}{F_L} b_{12} - (1-\tau_k) \right]$$

By direct calculation and using (11b) we can show that

$$\Gamma \equiv b_{11}b_{22} - b_{12}b_{21} = \frac{(1-\tau_k)(1-\tau_w)F_L F_{KK}}{Z(L)\tilde{L}\eta} < 0 \quad (\text{A.6})$$

which implies that the equilibrium is a saddle point.

To determine the slope of the stable saddle path given by (14b) note, first, that  $b_{12} > 0$ . Thus the slope will depend upon  $\text{sgn}(\mu - b_{11})$ . Solving for  $\mu$  yields

$$\mu - b_{11} = \frac{1}{2} \left[ (b_{22} - b_{11}) - \sqrt{(b_{22} - b_{11})^2 + 4b_{12}b_{21}} \right]$$

Thus, knowing  $b_{12} > 0$ , a necessary and sufficient condition for  $\mu < b_{11}$  is that  $b_{21} > 0$ . To consider this condition further it is useful to express the elements in terms of dimensionless quantities such as the elasticity of substitution in production,  $\varepsilon \equiv F_K F_L / F F_{KL}$  and  $s_K \equiv F_K K / F$ , capital's share of output. Thus, using the steady-state equilibrium conditions, we may write

$$b_{11} = \frac{(1-g)(\varepsilon-1)(\beta+\delta) - \delta(\varepsilon - \tilde{s}_K)(1-\tau_k)}{\varepsilon(1-\tau_k)};$$

$$b_{21} = -\frac{F_{KK}}{ZF_K} \left[ \beta + \delta + b_{11} \frac{\tilde{s}_K(1-\gamma)}{1-\tilde{s}_K} \right]; \quad b_{22} = -\frac{1}{Z} \left[ F_{KL}(1-\tau_k) + \frac{(1-\gamma)F_{LK}}{F_L} b_{12} \right]$$

The condition  $b_{21} > 0$  involves tradeoffs between  $\varepsilon$  and other parameters. For example, if  $\delta = 0$ , it is equivalent to

$$\varepsilon > \frac{\tilde{s}_K(1-g)(1-\gamma)}{\tilde{s}_K(1-g)(1-\gamma) + (1-\tilde{s}_K)(1-\tau_k)} \quad (\text{A.7})$$

This imposes a lower bound on the elasticity of substitution and is certainly met by the Cobb-Douglas. Taking  $\gamma = 0$ ,  $\tau_k = g$ , this reduces to  $\varepsilon > \tilde{s}_K$ , which holds in all plausible circumstances.

## A.2 Derivation of the dynamics of individual relative capital

The derivation of the dynamics of the relative individual capital stock follows Turnovsky and

García-Peñalosa (2008). Linearizing (15) around the steady state yields

$$\dot{k}_i(t) = \frac{w(\tilde{K}, \tilde{L})(1-\tau_w)}{\tilde{K}} \left[ \left( \Omega \tilde{l} \frac{1+\eta}{\eta} - 1 \right) (k_i(t) - \tilde{k}_i) + \frac{1+\eta}{\eta} [\Omega \tilde{k}_i - a_i \rho_i] (l(t) - \tilde{l}) \right] \quad (\text{A.8})$$

The stable (bounded) solution to this equation is

$$k_i(t) = \tilde{k}_i + \frac{1}{\mu - \frac{F_L(\tilde{K}, \tilde{L})(1-\tau_w)}{\tilde{K}} \left( \Omega \tilde{l} \frac{1+\eta}{\eta} - 1 \right)} \frac{F_L(\tilde{K}, \tilde{L})(1-\tau_w)}{\tilde{K}} \frac{1+\eta}{\eta} [\Omega \tilde{k}_i - a_i \rho_i] (l(t) - \tilde{l}) \quad (\text{A.9})$$

Rewriting (16) in the form

$$\Omega \tilde{k}_i - a_i \rho_i = \left( \frac{\eta}{1+\eta} \right) \frac{(\tilde{k}_i - a_i)}{\tilde{l}}$$

and recalling (9b), we may express (A.9) in the form

$$k_i(t) = \tilde{k}_i + \frac{1}{\frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}} \left( \Omega \tilde{l} \frac{1+\eta}{\eta} - 1 \right) - \mu} \frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}} [\tilde{k}_i - a_i] \left( \frac{L(t) - \tilde{L}}{1 - \tilde{L}} \right)$$

With  $k_i(0) = k_{i,0}$  given, we have

$$k_{i,0} = \tilde{k}_i + \frac{1}{\frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}} \left( \Omega \tilde{l} \frac{1+\eta}{\eta} - 1 \right) - \mu} \frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}} [\tilde{k}_i - a_i] \left( \frac{L(0) - \tilde{L}}{1 - \tilde{L}} \right)$$

which can be expressed in the form of equations (17) and (18) in the text.

### A.3 Before-tax and after-tax relative income

We characterize the dynamic adjustments of before-tax relative income and then obtain an expression for after-tax relative income. In contrast to wealth, which always evolves gradually, relative income undergoes a discrete change whenever a structural change occurs. To consider this, and the subsequent change in agent  $i$ 's relative position during the transition, we recall the following

- (i) Initial pre-shock steady state:  $\tilde{y}_{i,0}(0) = \tilde{\varphi}_0 k_{i,0} + (1 - \tilde{\varphi}_0) a_i$  where

$$\tilde{\varphi}_0 \equiv \tilde{s}_{K,0} + \frac{\tilde{s}_{L,0}}{\tilde{L}_0} \left( \tilde{L}_0 - \frac{1}{1+\eta} \right) = 1 - \frac{\tilde{s}_{L,0}}{\tilde{L}_0} \frac{1}{1+\eta}$$

(ii) Initial post-shock relative income:  $y_i(0) = \varphi(0)k_{i,0} + (1 - \varphi(0))a_i$  where

$$\varphi(0) \equiv s_K(0) + s_L(0) \frac{l(0)}{\tilde{L}(0)} \left( \tilde{L} - \frac{1}{1+\eta} \right) \frac{1}{1+\theta(0)}$$

(iii) Post shock steady state: which is given by

$$\tilde{y}_i = \frac{\tilde{\varphi}}{1+\theta(0)} k_{i,0} + \left( 1 - \frac{\tilde{\varphi}}{1+\theta(0)} \right) a_i$$

$$\tilde{\varphi} \equiv 1 - \frac{\tilde{s}_L}{\tilde{L}} \frac{1}{1+\eta}$$

Agent  $i$ 's relative income undergoes the following changes in response to a structural change:

(i) **Impact effect**

$$y_i(0) - \tilde{y}_{i,0} = (\varphi(0) - \tilde{\varphi}_0) (k_{i,0} - a_i) \quad (\text{A.10a})$$

(ii) **Transitional effects**

$$\tilde{y}_i - y_i(0) = \left( \frac{\tilde{\varphi}}{1+\theta(0)} - \varphi(0) \right) (k_{i,0} - a_i) \quad (\text{A.10b})$$

(iii) **Overall effects**

$$\tilde{y}_i - \tilde{y}_{i,0} = \left( \frac{\tilde{\varphi}}{1+\theta(0)} - \tilde{\varphi}_0 \right) (k_{i,0} - a_i) \quad (\text{A.10c})$$

The signs of these expressions depend upon both relative endowments of skills to initial capital and changes in factor shares and leisure, and no general patterns can be established.

To examine the dynamics of income inequality recall (36). Consider now an economy that is initially in steady state and is subject to a structural change. The changes in income inequality immediately following the shock, and along the subsequent transitional path to the new steady state are:

$$\sigma_y^2(0) - \tilde{\sigma}_{y,0}^2 = (\varphi(0) - \tilde{\varphi}_0) \left\{ (\varphi(0) + \tilde{\varphi}_0) [\sigma_{k,0}^2 + \sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi] - 2[\sigma_a^2 - \sigma_{k,0}\sigma_a\chi] \right\} \quad (\text{A.11a})$$

$$\tilde{\sigma}_y^2 - \sigma_y^2(0) = \left( \frac{\tilde{\varphi}}{[1 + \theta(0)]} - \varphi(0) \right) \cdot \left\{ \left( \frac{\tilde{\varphi}}{[1 + \theta(0)]} + \varphi(0) \right) [\sigma_{k,0}^2 + \sigma_a^2 - 2\sigma_{k,0}\sigma_a\chi] - 2[\sigma_a^2 - \sigma_{k,0}\sigma_a\chi] \right\} \quad (\text{A.11b})$$

where  $\tilde{\varphi}_0$  and  $\tilde{\varphi}$  are, respectively, the values of  $\varphi(t)$  in the initial and in the new steady states. After the impact response, inequality will move towards its new steady state, with the difference between the two steady states being given by (37).

Finally, we consider relative after tax income, which is given by

$$y_i^a = \frac{(1 - \tau_k)s_K k_i + (1 - \tau_w)s_L a_i (L_i/L)}{(1 - \tau_k)s_K + (1 - \tau_w)s_L} \quad (\text{A.12})$$

Note that this after-tax income measure ignores the direct distributional impacts of lump-sum transfers, which are arbitrary. Using (16') and defining

$$\psi(t) = \frac{(\tau_w - \tau_k)s_K(t) + (1 - \tau_w)\varphi(t)}{(1 - \tau_k)s_K(t) + (1 - \tau_w)s_L(t)} = \varphi(t) + \frac{(\tau_w - \tau_k)s_K(t)(1 - \varphi(t))}{(1 - \tau_k)s_K(t) + (1 - \tau_w)s_L(t)}$$

We can write after-tax relative income as

$$y_i^a(t) = \psi(t)k_i(t) + (1 - \psi(t))a_i \quad (\text{A.13})$$

which again is a weighted average of current capital and ability. Note that if the two tax rates are the same, then pre- and after-tax relative incomes coincide.

The SCV of after-tax income is given by

$$\sigma_y^2(t) = \psi'(t)^2 \sigma_{k,0}^2 + (1 - \psi'(t))^2 \sigma_a^2 + 2\psi'(t)(1 - \psi'(t))\sigma_{k,0}\sigma_a\chi \quad (\text{A.14})$$

where  $\psi'(t) \equiv \psi(t)(1 + \theta(t))/(1 + \theta(0))$ .

#### A.4 Comparative statics for aggregate magnitudes

Assuming a common tax rate for all income,  $\tau_y$ , steady-state  $\tilde{K}, \tilde{L}, \tilde{Y}$  are determined by

$$(1 - \tau_y)AF_K(\tilde{K}, \tilde{L}) = \beta \quad (\text{A.15a})$$

$$F(\tilde{K}, \tilde{L}) = \frac{F_L(\tilde{K}, \tilde{L})(1 - \tilde{L})}{\eta} \quad (\text{A.15b})$$

$$\tilde{Y} = F(\tilde{K}, \tilde{L}) \quad (\text{A.15c})$$

(i) **Productivity shock:** Effect of increase in  $A$  on these aggregate magnitudes is:

$$\frac{d\tilde{K}}{dA} = \frac{\tilde{K}}{A} \frac{1}{s_L} \left[ s_K(1 - \tilde{L}) + \varepsilon(1 + \eta)\tilde{L} \right] > 0 \quad (\text{A.16a})$$

$$\frac{d\tilde{L}}{dA} = \frac{\tilde{L}}{A} \frac{s_K}{s_L} (1 - \tilde{L})(1 - \varepsilon) \quad (\text{A.16b})$$

$$\frac{d\tilde{Y}}{dA} = \frac{F(\tilde{K}, \tilde{L})}{A} \frac{s_K}{s_L} \left[ \varepsilon\tilde{L} + (1 - \tilde{L}) \right] > 0 \quad (\text{A.16c})$$

(ii) **Tax financed change in government expenditure:** Writing the government budget constraint, (5), in the form  $\tau_k s_K + \tau_w s_L = g + \tau$ , implies  $s_K d\tau_k + (1 - s_K) d\tau_w + (\tau_k - \tau_w) ds_K = dg + d\tau$ . In the case that the initial tax rates and the tax changes are uniform, ( $\tau_w = \tau_k \equiv \tau_y$ ;  $d\tau_w = d\tau_k \equiv d\tau_y$ ) we obtain  $dg / d\tau_y = 1$ , which allows us to write the aggregate effects of the change in taxes as

$$\left. \frac{d\tilde{K}}{d\tau_y} \right|_{dg=d\tau_y} = -\frac{\tilde{K}}{1 - \tau_y} \frac{1}{s_L} \left[ s_K(1 - \tilde{L}) + \varepsilon(1 + \eta)\tilde{L} \right] < 0 \quad (\text{A.17a})$$

$$\left. \frac{d\tilde{L}}{d\tau_y} \right|_{dg=d\tau_y} = -\frac{\tilde{L}}{1 - \tau_y} \frac{s_K}{s_L} (1 - \tilde{L})(1 - \varepsilon) \quad (\text{A.17b})$$

$$\left. \frac{d\tilde{Y}}{d\tau_y} \right|_{dg=d\tau_y} = -\frac{F(\tilde{K}, \tilde{L})}{1 - \tau_y} \frac{s_K}{s_L} \left[ \varepsilon\tilde{L} + (1 - \tilde{L}) \right] < 0 \quad (\text{A.17c})$$

(iii) **Shift in tax burden:** Suppose that capital and labor income are initially taxed at the uniform rate, and consider the effect of shifting the tax burden, while maintaining  $g$  constant. In this case the required change in the tax rates is

$$\frac{d\tau_k}{d\tau_w} = -\frac{1-s_K}{s_K} \quad (\text{A.18})$$

We can then show that the aggregate effects are given by

$$\frac{d\tilde{K}}{d\tau_w} = \frac{\varepsilon\tilde{K}(1-\tilde{L})}{1-\tau_w} \frac{s_L}{s_k} \frac{1+\eta}{\eta} > 0 \quad (\text{A.19a})$$

$$\frac{d\tilde{L}}{d\tau_w} = -\frac{\varepsilon\tilde{L}(1-\tilde{L})}{1-\tau_w} < 0 \quad (\text{A.19b})$$

$$\frac{d\tilde{Y}}{d\tau_w} = \frac{\varepsilon(1-\tilde{L})}{1-\tau_w} \frac{F_L\tilde{L}}{\eta} > 0 \quad (\text{A.19c})$$

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