Individual Behaviors and Collective Welfare: Ramsey's "microfoundations" of "macro-equilibrium"

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I. Introduction

Since the end of 1970's -and the success of the New Classical School –, the face of macroeconomics has changed profoundly. On the one hand, the research has turned gradually away from the analysis of the short-term fluctuations to favor a study of the long term determining factors of macroeconomic variables. One has studied the determinants of growth and has interpreted the cycles as fluctuations around these long-term values. On the other hand and in the same movement, one saw setting itself a general equilibrium macroeconomics. One of the weapons of these alterations is what is called in a generic way the "à la Ramsey models". In its simplest shape, this type of model considers an economic agent with infinite life expectancy, who decides on the intertemporal allocation of its income and work. Under certain very restrictive conditions, this economic agent can be interpreted as a "representative agent", whose choices represent those of an economy of general equilibrium.[1] The "à la Ramsey" models are thus used to describe macroeconomic phenomena, such as growth or fluctuations, as the result of individual procedures of intertemporal optimization. Although this use rests on controversial hypotheses - rational expectations, market clearing, and the figure of the representative agent itself - it is often considered as a possible way to provide macroeconomics with consistent microeconomic foundations.

The current success of the "à la Ramsey" models confers in return on Frank Ramsey (1903-1930) a role in the history of economic thought: that of an illustrious precursor, who would have, as early as 1928, established the tools privileged by the contemporary macroeconomists. Such is for example Robert Barro and Xavier Sala-I-Martin's vision (1995).

But the tributes often do not stop there, and temptation is strong to make Ramsey the precursor of the current theoretical representations, by qualifying him as precursor of "the modern theory of the growth" (Barro and Sala-I Martin 1995, 10): one in this case describes Ramsey's economic agent with infinite life expectancy as the first representative agent in macroeconomics, and one describes Ramsey's contribution as "the prototype for studying the intertemporal allocation of resources" (Blanchard and Fisher 1989, 21) or "the battery of small [contemporary] models" (Blanchard 2000, 1381) as many applications of Ramsey's work.

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[1] Two theoretical justifications support this interpretation. The first one resorts to the welfare theorems, the intertemporal Pareto-optimal choice of the representative agent being associated with a decentralized allocation of resources. See for instance Blanchard and Fisher (1990, 21 and 50-51). The second justification is bound to the results of the literature on the aggregation conditions. The representative agent represents fictitiously a general equilibrium economy, in which individuals are identical, or, in a less caricature way, in which individuals are endowed with identical and homothetic preferences, but with different initial endowments. In this "*hicksian* economy" (Arrow and Hahn 1971, 220), the choices of the representative agent are the same as the ones of a general equilibrium economy. See for instance Jacques and Rebeyrol (2001).

Such paternity recognitions arouse naturally curiosity. Ramsey indeed wrote in Cambridge in the 1920's, where Marshallian thought was still dominating and impervious to the theory of general equilibrium. Especially, macroeconomics was not yet an autonomous research field, and it was not naturally even time to wonder about its microeconomic foundations. Therefore, the young Cambridge mathematician could not have envisaged the present stakes of the "à la Ramsey" models and the current interpretations of Ramsey's work should probably suffer from a retrospective slant.

Incursion into the original text allows testing these two kinds of assertion: that of the macroeconomist, looking for pre-Keynesian years heroes, and that of the historian, looking for the initial theoretical framework, in order to rediscover the issue at stake in 1928. The experience is then relatively surprising, and leads to two kinds of results.

1. The first one concerns the stakes in Ramsey's initial contribution. More than the single determination of a mathematical optimal saving's rule, the matter was to understand the consequences of individual savings behaviors on collective welfare. The challenge was therefore, and more profoundly, to state a theoretical representation allowing connecting a micro- and a macro level. 2. The second result concerns Ramsey's theoretical solution. In contrast with some retrospective interpretations, he exactly avoided any representative agent logic, in the double sense we give today to this concept in macroeconomics: a way to pass over the individual idiosyncrasy in constructing a fictional economic agent whose choices represent those of an underlying decentralized economy or a way to use an agent that reflects the aggregation of individual behaviors. Ramsey's challenge was rather to state results concerning macroeconomic equilibrium by bypassing the partial equilibrium framework he inherited from Cambridge.

Such statements rest logically on two kinds of inquiry I successively lead in the following pages.

The next section first concentrates on the origin of Ramsey's inaugural question: "how much of its income should a nation save" (Ramsey 1928, 543). It shows that Ramsey's theory has to be read in light of a striking question in the 1920's: is a *laissez-faire* system able to warrant a maximum of collective economic welfare? As regards the question of saving, the problem arises from a noticeable fact: individuals tend naturally to prefer present satisfactions. The resulting global saving appears insufficient if we consider the prosperity of the nation, conceived as a transgenerational entity. Therefore, the 1928 essay deals first of all with a problem of divergence between the pursuit of the individual interest and the realization of the collective interest.

Taking such a redefinition of the « Ramsey Problem » into account imposes then that our glance settles on the whole 1928 contribution, as I propose in sections III and IV. One usually retains in Ramsey's paper only the first part, dedicated to the mathematical demonstration of the famous rule of optimal saving. However, this demonstration takes place into a very specific framework - that of a cooperative-nation refusing to privilege the present moments - and we guess that Ramsey's attempt to enlarge his approach in the remainder of the 1928 article was at least of equal importance in Ramsey's eyes. In a least known part of his work, he indeed handles the individual behavior of saving, introduces a discount rate in the problem of optimization, and discusses some consequences of his analysis on the determinants of the interest rate. A thorough perusal of Ramsey's global argumentative logic reveals that he is in fact building, step by step, a careful study of the consequences of the discount rate on the national welfare and that progressing in the same movement towards a theoretical representation of a decentralized economy. His solution rests then on an original concept, which allows him to bind the individual and the collective levels, the concept of dynasty.

II. Ramsey's Problem: origin and stakes

Two hypotheses could explain the origin of the 1928 essay. The first assumption relies on the intellectual path of Ramsey himself: the difficulties he met in the writing of his article on optimal

taxation, "A Contribution to the Theory of Taxation" (1927) could be the roots of the 1928 reflections. In that paper, published in the March issue of the *Economic Journal*, Ramsey tried to define the best way to distribute a global amount of tax, so that "the decrement of [global] utility could be a minimum" (Ramsey 1927, 47). We shall not remind here of his demonstrations but simply note that in the developments of the fourth part of the 1927 article, dedicated to some applications of his mathematical result, Ramsey mentions the possibility of a dynamic analysis, by wondering notably about the opportunity of a differentiated taxation on savings. He does not succeed in handling this problem, and indicates that it would require "a mathematical theory considerably more difficult than anything in [the present article]" (Ramsey 1927, 59). He may have pursued this objective by approaching the drafting of his "Mathematical Theory of Saving". The editorial correspondence between Maynard Keynes and Ramsey (Keynes 1983, 784) informs us indeed that Ramsey had initially planned to deal with taxation on saving in the 1928 paper. He again gives up[2], but this project seems to have partially motivated his research on optimal saving.

The second assumption is bound to the theoretical background, which ruled over the publication of the 1928 article. Ramsey's will "to apply the utilitarianism to saving" (Ramsey, June 1928, in Keynes 1983, 784) would have been suggested by Pigou's reflections on economic welfare (Chakravarty 1987). More widely, it is advisable to understand Ramsey's theoretical ambition through a general reflection on the determinants of welfare and on the ability of the regime of *laissez-faire* to guarantee a maximum of collective welfare.

We elsewhere have noticed (Gaspard 2003) that Ramsey began his economist's short career in the *Apostles Society*. In a contribution dated 1923 "Socialism and Equality of Income" (Ramsey 1923) he notably proposes an analysis of the comparative merits of a free market economy and a socialist economy. On the occasion of what appears as a plea for economic planning, Ramsey draws up a picture of the crisis of the 1920's and a list of the advantages a bigger control from the State in the economic sphere would bring.[3] Such reflections join more globally in the anxieties of the Cambridge economists, as for the duration of the 1920's retraining crisis naturally, but also as for the suitability of the Marshallian theory to grasp correctly the English economy after the First World War. More exactly, the question discussed at that time is that of the failures of liberalism, a principle underlying the philosophy of Marshall's *Principles*.

In this background, two emblematic reactions are emerging. That of Keynes, who begins to express the necessity to change the theory, by indicating that it does not correspond any more to the actual economic organization.[4] That of Pigou, who tries to amend Marshallian theory by estimating the places where State intervention turns out to be necessary for the improvement of national welfare. Pigou's research program, started with *Wealth and Welfare* (1912) and pursued in multiple editions of *The Economics of Welfare* (1920 and following ones), consists explicitly in studying:

> "[The] obstacles that prevent a community's resources from being distributed among different uses or occupations in the more effective way. […] [The] purpose of this study is essentially practical. It seeks to bring into clearer light some of the ways in which it now is, or eventually

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[2] In his submission letter to the *Economic Journal*, Ramsey comments on his own work:

> "The mathematics is all very elementary, and the beginning part is fully explained for the sake of those who will read a little way. Although the matter is terribly oversimplified, the equations must arise in any attempt to apply utilitarianism to saving and so far as I know they've never been treated before. The difficulty is to find simple results of sufficient generality to be interesting and yet not obvious. In this I think I've succeeded surprisingly well. To prove them is never hard, except when it comes to taxation which is very complicated and in which I've wasted a lot of time with only one achievement of any sort" (Ramsey, June 1928, in Keynes 1983, 784).

The section devoted to taxation disappeared in the published draft.

[3] He in particular underlines the defects of coordination of the individual decisions, just as much as the intrinsically connected tares of a system based on the search for immediate profit (or for secure income).

[4] Such an analysis is led notably in his political essays (Keynes 1925, 1926).

> may become, feasible for the governments to control the play of economic forces in such wise as to promote the economic welfare, and through that, the total welfare, of their citizens as a whole" (Pigou 1932, 129).

Ramsey learns economic theory with these two personalities. He is personally very close to Keynes, with whom he shares certain convictions about the roots of the economic crisis. However, he follows, on the strict theoretical plan, in Pigou's steps: the 1927 article appears as an attempt at economic and fiscal policy, in which are assumed the concepts of Pigou. Hence, the 1928 essay could be included as an element of the general program mentioned above.

As far as saving is concerned, the problem arises from a general observation: individuals tend to overvalue present satisfactions. Pigou underlines this fact in chapter II of *The Economics of Welfare*. Pigou's foremost concern in this chapter does not directly deal with saving, since he is looking for a way to measure subjective economic welfare[5]. It is notwithstanding the occasion to deal with the question of the intertemporal choice.

Pigou's analysis is well-known: he suggests using the quantity of money one is prepared to supply for a given quantity of commodities as a measure of utility, that is, as a cardinal index of the individual "satisfaction" or "desire". Proposing here to confuse desire and satisfaction, Pigou admits an exception: when individuals estimate future utility, they consider their *desire*, and underestimate the future satisfaction brought by consumption or by the ownership of a certain amount of income. This distinction will not intervene any more in Pigou's later work, which never really returns to the question of the intertemporal allocation of resources. It is however the occasion for a discussion about the economic implications of this individual propensity to discount future utilities.

Unlike Fisher (1907), Pigou defines - as Ramsey also will do – the discount rate as the result of a pure time preference, that is independent from the expected stream of income[6]. Pigou brings out two explanations for such a pure time preference. The first explanation is that of a "defective telescopic faculty" (Pigou 1932, 25). Individuals are *shortsighted*, what leads them to behaviors that Pigou considers as "irrational" (Pigou 1932, 25). Under the hypothesis according to which the individual temperament - or the utility function - is invariant over the time, the argument is that the satisfaction connected to the consumption of a given amount of commodities in the next period is exactly the same as would be felt today. The pure preference for present consumption appears so as a distortion with regard to a standard, frequently mentioned - by Rae as well as by Böhm-Bawerk or by Fisher - : that of an equal treatment of present and future satisfactions[7]. The second explanation lies in the fact that individuals have a finite life. Two factors intervene here: *uncertainty*, which carries the risk that the individuals will never take advantage of the fruits of their saving; and the *egoism* of the individuals, who express a preference for themselves over their descendants.

Pigou underlines afterwards two important economic consequences of the tendency to discount the future. On one hand, the individual efforts (here, the labor supply) intended for future pleasures are insufficient (Pigou 1932, 25-26). On the other hand, uncertainty and egoism limit the savings, intended to finance long-term projects of investment. Moreover, said Pigou, intergenerational altruism and the progress of the stock exchange do not suffice to compensate for the egoism of the individuals (Pigou 1932, 27). Many investment projects, which could be essential for the community, risk never to be undertaken if one abandons them to the private initiative, because of a too long wait for dividends. The overvaluation of the present satisfactions will therefore have a fatal consequence on the creation of fresh

[5] Pigou has already defined the objective measure of welfare by the size of the national income.
[6] It then corresponds to Böhm-Bawerk's (1889) psychological motive for the existence of a positive rate of interest.
[7] Such a tradition is particularly well studied by Elster and Loewenstein (1992).

capital:

> "The practical way in which these discrepancies between desire and satisfaction work themselves out to the injury of economic welfare is by checking the creation of new capital an encouraging people to use up existing capital to such a degree that larger future advantages are sacrificed to smaller present ones" (Pigou 1932, 27).

Many analytical shifts punctuate here Pigou's argument. He for instance deviates from the idea of a simple report of consumption to the possibility to enhance the value of such a report thanks to investment. He also slides from the description of the individual behavior to an evaluation of the collective welfare, without any explanation of the underlying economic mechanisms. However, the diagnostic is clear: private initiative, because of the discount rate of utility, is not sufficient to maximize the welfare of humanity, present and future. Logically, and as for the rest of Pigouvian work, there would be here a place for the State (or the government) "which is the trustee for unborn generations as well as for its present citizens" (Pigou 1932, 29). In particular, there would be a place for a fiscal legislation encouraging saving.

However, as Collard (1996) notes, a fiscal policy in favor of saving contradicts the criteria of welfare increase established elsewhere by Pigou[8]. An economic policy aiming at favoring saving (for instance a lowering in the income tax or in the tax on successions) would have contradictory effects. It could admittedly favor a better temporal distribution among generations and so increase intertemporal welfare. But it also will increase inequalities among people of the same generation and according to Pigou, will decrease economic welfare, by virtue of the law of the decreasing marginal utility, tend to increase the welfare of the community at a given moment. Pigou therefore hesitates to settle the question.

Notwithstanding this analysis allows to identify clearly the stakes of Ramsey's question in the 1928 article: it is because individual behaviors do not lead to a maximum of collective welfare – that is because individual interests and national interests diverge - that the theoretical question of knowing "how much of its income should a nation save?" (Ramsey 1928, 543) makes sense. Beyond the dilemma of the arbitrage between present and future consumption, defined usually as the "Ramsey Problem", hides thus the problem of the divergence between the actual behavior of individuals and the desirable behavior for collective welfare.

III. The rule and its variations: some unpleasant things to know about the 1928 essay

To systematize Pigou's analysis thanks to a mathematical theory of saving, supposed so three stages. The first one consists in defining what should ideally save the national community. The second stage consists in formalizing what implies, in term of cost for the collective welfare, the existence of a discount rate in the individual preferences. The third one consists then in defining the foundations of a fiscal policy.

As we have noticed, Ramsey finally abandoned this third stage when publishing the article. Both first stages are on the contrary successively handled in the 1928 essay, but without any reference to something like an aggregation of the individual decisions of saving, and without allusion to any fictive agent whose choices could represent the underlying individual decisions. Before considering the nature of his solution, it is necessary to remind here the contents of Ramsey's paper. This exercise, although mainly descriptive, shows that the most neglected part of Ramsey's work is exactly the one that brings light on Ramsey's attempt. The synopsis proposed in the appendix to this paper can help the reader find a way in the maze of Ramsey's arguments**.**

[8] According to Pigou's analysis there are two ways to increase the national economic welfare: an increase in the national dividend itself; a better distribution of this national dividend; Collard shows that Pigou had first mentioned a third criterion: the egalitarian distribution in time.

The 1928 essay consists, according to the author's himself, in three parts. Each seems to answer different theoretical and technical objectives, and each rests on a set of different mathematical and institutional assumptions.

The first part: Ramsey's normative solution

The first part of Ramsey's theory is well known: he approaches the question of optimal saving through an economic planning point of view. Formally, the nation is represented by:

- A collective net utility function: $U(x(t)-V(a(t))$, where $U(x)$ stands for the utility derived from the global consumption (x) at the moment t ($t\in\Re$), and $V(a)$ stands for the disutility of labor (a). Such a function is assumed invariant in time.

- An aggregate production function $f(a(t),c(t))$, where $c(t)$ represents the capital available in the economy at the moment t. The marginal productivities of capital and labor are assumed positive and decreasing.

The population is constant, and there is no technical progress (Ramsey 1928, 543). The community is endowed with a substantive rationality. Its aim of the community is to achieve what Ramsey calls a state of bliss: a long-term stationary state, where the community reaches (or tends close to) a maximum level of net satisfaction, "the maximum obtainable rate of enjoyment"(Ramsey 1928, 549)**.** According to Ramsey, such a bliss path is due either to the saturation of the net satisfaction itself (case (**i**))[9], or to the saturation of the capital accumulation (the marginal productivity of capital decreases and finally nullifies (case **ii**).

Let us recall that in the first part of his essay, and according to a now famous formula, Ramsey refuses to consider any discount rate of utility, a practice that he qualifies as "ethically indefensible" and arising from the "poverty of our imagination" (Ramsey 1928, 543).

Under such restrictive assumptions, Ramsey's rule rests on the transposition, at the collective or global level, of the marginalists' equilibrium conditions inherited from Marshall. Since the model is unisectorial, the equilibrium of the nation is characterized by three equations.

The first equation is an accounting identity. Labor (a) and capital (c) produce a flow of output $f(a,c)$, which is used either for consumption (x) or for saving **(1)**. Saving is identified with capital accumulation ($\dot{c}$). The second one equates the marginal satisfaction drawn from consumption and the marginal effort due to work **(2)**. The third equation expresses the intertemporal balance of the "rational community:" With $u(x)$ standing for the marginal utility of consumption,[10] we have:

$$\frac{d}{dt}u(x) = -f_c(a,c)\cdot u(x)\ ^{11} \qquad \textbf{(3)}$$

By a game of derivations, integral calculus and variables substitutions, and thanks to a mathematical definition of bliss (B), Ramsey states the following income allocation rule:

$$\dot{c} = \frac{B-\left(U(x)-V(a)\right)}{u(x)} \qquad \textbf{(5)}$$

The rule indicates the level of saving, or investment, the community should adopt at every date:

> "The rate of saving multiplied by marginal utility of consumption should always be equal bliss minus actual rate of utility enjoyed" (Ramsey, 1928, 547)

It is the Keynes-Ramsey's rule, such as it was named in the 1960's[12]. In absence of a discount

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[9] The nation tends in this case to the greatest level of satisfaction "conceivable" (Ramsey 1928, 544).

[10] We have: $u(x) = U'(x)$

[11] Equations (2) and (3) are in fact Euler's equations.

[12] Koopmans (1965) re-states this rule when demonstrating the existence of an optimal path of capital accumulation aiming towards Phelp's golden rule. He summaries Ramsey's argument and observes that in the absence of discount

rate of future satisfactions, the determination of the saving level is purely teleological: the community takes her decision by considering the distance that still separates her from the ideal state of the end of times (the bliss path). This rule defines a unique optimal path of capital accumulation, as a suite of optimal decisions at every date. Through the definition of B indeed, Ramsey characterizes a final state of the trajectory of capital accumulation, which exempts from any transversality condition in infinite horizon[13].

The remainder of the 1928 essay consists in a set of theoretical tracks, which amend gradually the hypotheses of the first part.

The second part: Community vs. individual

The second part of Ramsey's article splits into three paragraphs. After having proposed a graphic illustration of his savings rule (*§ IIa*)[14], Ramsey proposes a transposition of his rule in the case of an individual who lives a finite life (*§ IIb*). He then introduces a positive discount rate and deals successively with the community behavior, then with the individual behavior (*§ IIc)*. The difficulty is that each paragraph introduces modifications in the initial set of assumptions.

The first modification deals with the shape of the production function. On the occasion of the graphic representation, Ramsey in fact assumes that:

> "returns to capital and labor are constant and independent, so that: $f(a,c) = pa + rc$, where *p*, the rate of wages, and *r* the rate of interest are constants" (Ramsey 1928, 549).

The assumption consists *a priori* simply in the definition of a particular shape for the production function. This peculiar shape should allow a simple graphical representation, which rests on a change of variable (Ramsey 1928, 550). To assume that the productivity of capital is constant makes also the equation (2) a differential equation simple to resolve. Such a specification is however curious, because it means that the production factors are perfectly substitutable[15]. Especially, capital accumulation does not imply the decline of the marginal productivity any more and there is therefore no more technical limit to the to the increase of the global product (or income). From then on, if the marginal productivity of capital is constant, alone the definition (i) of the bliss path has a sense: the long-term steady state corresponds to a state of utility saturation. Such consequences are not discussed by Ramsey. Let us moreover note that it appears**,** with this specification, an identification between the marginal productivity of factors and their prices (interest and wage), which suggests that the production factors receive a constant payment, according to their (constant) contribution to production. In that case, Ramsey seems to use an equilibrium condition (factors are paid their marginal productivity) to express an equation of product exhaustion.

The study of the individual behavior (*§ IIb*) rests exclusively on a graphical demonstration, and preserves the new assumptions on the production function. Ramsey must however modify his analysis to integrate the following fact: an individual lives only a finite life, and cannot hope to

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rate of utility, the only optimal path among all possible paths beginning with a given amount of capital verifies Ramsey's final rule. Koopmans restates the equation (5) (Koopmans, 1965, 243 and 272-275) and names it the « Keynes-Ramsey's condition » (1965, 243). Cass (1965) uses optimal control to define existence conditions of an optimal path of capital accumulation. He introduces a discount rate of utility but analyzes the case without discount rate and finds the same rule as Ramsey, which he calls "Keynes-Ramsey's rule" (Cass, 1965, 239). We do not know when the expression "Keynes-Ramsey condition" was used to refer to the Euler's equation of the problem, that is the equation (3) of Ramsey's paper. Even today, this expression refers alternately to equation (3) or to Ramsey's final rule.

[13] See Chiang (1992, 115).

[14] These notations are Ramsey's ones.

[15] Ramsey's formulation means indeed a production function like $f(a,c) = \alpha a + \beta c$, where α and β are positive real numbers.

achieve one day the state of bliss. The rule of optimal saving should be redefined, because *B* has now no meaning. In order to define the maximum level of satisfaction that an individual could achieve, Ramsey indicates "we must know how much capital our man feels it necessary to leave his heirs" (Ramsey 1928, 552). He argues after that in a purely graphical way, by distinguishing two cases, as the bequeathed capital is superior or inferior to the level of initial capital. We shall not clarify here his reasoning. Let us simply retain that with the equations (1), (2) and (3), and if we know the borders of the capital accumulation trajectory ($c(0)$ and $c(T)$), we can define a unique path of consumption or saving, which is the solution of a maximization program of utility under an intertemporal constraint.

The following paragraph introduces a third modification. Ramsey tries to know "How our results must be modified when we no longer reckon future utilities and disutilities as equal to present ones, but discount them at a constant rate ρ" (Ramsey 1928, 553). He is here anxious to precise first, that it is necessary to distinguish the objective discount rate (the interest rate) from the subjective discount rate and second, that the discount rate must be assumed constant:

> "This is the only assumption we can make, without contradicting our fundamental hypothesis that successive generations are actuated by the same system of preferences. " (Ramsey 1928, 553).

Ramsey specifies in fact that his theoretical framework (in particular the hypothesis of unchanged preferences over time) implies the assumption of constant discount rate. The constancy of the subjective discount rate does not appear as a simplifying or realistic hypothesis, but as a way to maintain the consistency of the rational agent's behavior. In fact, the definition of intertemporal preferences should integrate a property of dynamic consistency: the community cannot be incited to modify the initially chosen trajectory of consumption and saving during the capital accumulation process. Ramsey chooses so an exponential discount function, which is characterized by a constant discount rate[16]. Strotz (1956) will show some decades later that it is there the only discount function compatible with temporal consistency, at least in certain environment.

Ramsey first envisages the behavior of the community. Two scenarios are possible.

The first case consists in supposing a strictly positive discount rate lower than the interest rate *r*, which is still assumed constant. This situation leads to save, because the rate of payment for saving is superior to the subjective rate of valuation of future utilities. The rhythm of saving will be defined by use of the equations (**1**) and (**2**), and by a modified intertemporal balance equation:

$$\dot{u}(x) = -u(x) \cdot \left\{ f_c(a,c) - \rho \right\} = -u(x)(r - \rho) \qquad \textbf{(3')}$$

The marginal utility of consumption should decrease at a rate given by the difference between the interest rate (a constant rate, explicitly identified here with the marginal productivity of capital) and the discount rate. If the interest rate is superior to the discount rate, the marginal utility of the consumption decreases at a constant rate and the consumption describes a continuously increasing path, as long as the marginal utility of consumption does not nullify. Ramsey uses then the same method of replacements as in the first part of the article, by using the equations (**1**), (**2**) and (**3'**) to determine a new rule of saving, which makes this time intervene a level of "modified bliss" (Ramsey 1928, 554).

The second case corresponds to a situation where $r < \rho$. The agent is in that case little

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[16] The discount function is: $F(\tau) = e^{-\rho\tau}$, where τ stands for the distance between the current and the future periods of consumption. The discount rate is the opposite of the decreasing rate of the discount function on the horizon τ : $-F'(\tau)/F(\tau) = \rho$, which is constant. These expressions are not given in the 1928 essay, but they appear in a letter from Ramsey to Keynes dated July 1928 (Keynes 1983, 788).

inclined to saving. The equation (**3'**) indicates that marginal utility of consumption rises at the constant rate $-(r-\rho)$. The consumption path follows a continuously decreasing profile and the net satisfaction, instead of tending to bliss falls gradually towards "the barest subsistence level at which its marginal utility may be taken as infinite, if we disregard the possibility of suicide" (Ramsey 1928, 555).

Next comes briefly the study of the individual case, with discount rate of utility. It does not raise any peculiar problem for Ramsey who only notices that " as in [*II.*]*b*, we can adapt our solution to the case of an individual with only a finite time to live"(Ramsey 1928, 555). Ramsey alludes here to a graphic resolution.

The third part: the family and the new long-term equilibrium

The third part of the essay pursues the investigation by studying theoretical implications of the previous paragraphs: Ramsey will this time evoke classes of individuals with finite life expectancy and indifferent to their heirs (*§ III*χ), these classes being differentiated by their discount rate of future utilities. He finally will scrutinize the implications of a differentiation of "families" with infinite life expectancy on the long-term equilibrium (*§ III*β).

Previously, he deals for a while with the macroeconomic determination of the real interest rate (*§ III*α). He comes back to the initial theoretical framework of the community, in which the marginal productivity of the production factors is positive and continuously decreasing, but he now assumes that "everyone discounts future utility for himself or his heirs at the same rate ρ" (Ramsey 1928, 556). Ramsey affixes this time a magnifying glass on what happens on the capital market, the only market envisaged in the article. With a Marshallian method, he treats this market in partial equilibrium, confronting a virtual demand function for capital stemming from the productive side of the community, with the supply of saving stemming from this same community.

As in Marshallian analysis, the interest rate is in the very short period determined by the demand price for capital. If the stock of capital is given c_0 the demand price for capital is the maximal price accepted by the community as a producer to get this stock of capital. It is equal to the marginal productivity of capital. If the marginal productivity of capital for c_0 is strictly superior to the discount rate of future utilities, $f_c(a,c_0) > \rho$, the community would save. The saving level (or the capital accumulation $\dot{c}$) is determined by the Keynes-Ramsey's rule as defined in paragraph *IIc*. For a given level of the marginal productivity of capital, the economic agent defines a certain rate of saving, which is going to increase the stock of current assets. In the next period, the interest rate remains determined by the marginal productivity of this new stock of global capital. The level of this capital has however been made endogenous by the saving's rule. The accumulation process stops when the marginal productivity of capital has enough decreased as to reach a value equal to ρ. The community has reached a stationary state, in which it enjoys the *modified bliss*. In this stationary state, there is neither saving, nor accumulation, and the whole income is devoted to consumption. As in Marshallian theory, the ultimate supply of capital ($\hat{c}$) (which corresponds to a null supply of accumulation $\dot{c} = 0$) determines the long-term price of capital (the long-term interest rate). The demand determines this time the quantity of capital finally accumulated $\hat{c}$, which should be such as $f_c(a,\hat{c}) = \rho$.

Therefore, during the process of capital accumulation, the interest rate is determined by the marginal productivity of capital. Once the long-term steady state achieves, the equilibrium interest rate is fixed by the subjective discount rate. The economy stays in a stationary state, called modified bliss (in fact what is nowadays called the path of "modified golden rule"). Let us note that if the discount rate is null, the community accumulates capital until the marginal productivity nullifies: the community achieves in this case the state of bliss (or the golden rule path). The distinction between the short- and the long period governs the last two paragraphs of

the 1928 essay.

Ramsey sets first the action on short term, considering the individual level. He stages this time classes of individuals who are differentiated by their discount rate. He then approaches a long-term analysis with the introduction of new economic agents: the families.

In paragraph *IIIβ,* he considers individuals who live a finite life, are perfectly indifferent to the lot of their heirs and who "start [their] working life without any capital and end it without any, having spent [their] savings on an annuity" (Ramsey 1928, 557). The individuals react to an interest rate, which is "supposed constant" (Ramsey 1928, 557). If, for a given individual $\rho > r$, this individual is going to save according to the rule revealed in both first and second parts of the article: "If we neglect variations in his earning power, his action can be calculated by modifying the equations of *II.c* to apply to a finite life as in *IIb*" (Ramsey 1928, 558). Such an individual will accumulate capital during a time and will then spend everything. Now

> "Besides this man, we must suppose there to be in our community other men, exactly like him except for being born at different times. The total capital possessed by *n* men of this sort whose birthdays are spread evenly through the period of a lifetime will be *n* times the *average* capital possessed by each in the course of his life. The class of men of this sort will, therefore, possess a constant capital depending on the rate of interest, and it will be the amount of capital supplied by them at this price. (If $\rho > r$, it may be negative, as they may borrow when young and pay back when old). We can then obtain the total supply curve of capital by adding together the supplies provided at a given price by each class of individual"(Ramsey 1928, 558).

On one hand, we are so in presence with classes of identical (except their age) individuals who accumulated and supply on average a given amount of capital. For a given value of the real interest rate, the behavior of saving gives birth to an average stock of supplied capital. We have on the other hand classes characterized by different discount rates, which offer each a stock of capital for a given interest rate. The sum of these stocks fixes the global supply of capital. The meeting of this supply function with the demand function determines an equilibrium interest rate, which is given by the marginal productivity of capital. This interest rate will be equal to the discount rate of a "marginal saver" (1928, 558), in fact a marginal class of savers.

The classes of individuals that have a discount rate superior to this equilibrium interest rate are going to save. The others are on the contrary looking for wasting their capital: Ramsey can finally return to an analysis in infinite horizon by considering either individuals but families with infinite life expectancy, which discount their utility at a constant rate but each at her own rate.

He once again simplifies his production function by neglecting the labor contribution. The total income is then a function of c, $f(c)$. The rate of interest is still determined at the moment *t* by the marginal productivity of capital $f'(c)$, which is positive but decreasing. He adds then that one can suppose that

> "Every individual could attain the maximum conceivable utility with a finite income x_1, and that no one could support life on less than x_2" (Ramsey 1928, 558).

Ramsey sets then the action at once in a long-term equilibrium state, keeping silent about the way of reaching it. We can guess that one leaves an initial situation, where the short-term interest rate of equilibrium is equal to the discount rate of the marginal class of savers, and that a differentiated accumulation process takes place. Ramsey implies that, as in the part dedicated to the determination of the interest rate, the accumulation process of a community, so formed by families, is going to stop for a given amount of accumulated capital $\hat{c}$, for which the marginal productivity of capital is equal to a certain value $\hat{r}$. In the final state of equilibrium, he says,

> "Families, say $m(r)$ in number, whose rate of discount is less than r must have attained bliss or they would still be increasing their expenditure […]. Consequently, they have between them an

> income $m(r) \cdot x_1$. The other families, $n - m(r)$ in number (where m is the total number of families), must be down to the subsistence level, or they would be still decreasing their expenditure. […]In such a case, therefore, equilibrium would attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level" (Ramsey 1928, 559) [17]

In the stationary state, the total income of the community will be from then on shared between two classes of savers according to the equation:

$$f(\hat{c}) = m(\hat{r})x_1 + (n - m(\hat{r}))x_2$$

The global equilibrium values would be so defined with this equation of final distribution of income and an equation indicating that the real interest rate of long-term equilibrium is equal to the marginal productivity of the accumulated capital: $\hat{r} = f(\hat{c})$.

Such a conclusion seems to close curtly, from a normative point of view, Pigou's interrogation on the judiciousness to consider or not any discount rate in the analysis of the intertemporal behavior. By comparing the mathematical results of each of these developments, the message seems clear: when the nation with perfect forecast – and refusing to discount future utilities- follows the initial rule, it is promised to bliss. The introduction of the discount rate, associated with the hypothesis of temporal consistency eases this happy perspective and can only lead to a level of "modified bliss". Finally, if the individuals are differentiated by their discount rates, the same rule indicates that some take the risk of eternal poverty. Therefore, if one tries to define "how much of its income should a nation save", it seems that it is better to conform to the ideal described in the first part of the article, either by the adoption of a centralized regime, or by the intervention of economic policy. But what are finally the springs of such a demonstration?

IV. Splendors and misery of a theory of saving

Having now reached the term of a linear presentation of the theoretical developments in the remainder of the 1928 essay, I would like to complete it by three general comments.

1. First of all, Ramsey's text surprises the contemporary reader by the number of handled situations, and by the establishment, or at least the intuition, of results that are not usually associated with the 1928 contribution. That contribution is in fact often reduced to its first part, which is considered as a departure point for the reflections on optimal growth. Two points are then retained: the stating of the mathematical rule of optimal saving on one hand; the choice of a utilitarian criterion to express the intertemporal welfare, that is the refusal of time preference on the other hand. One indicates then the progress of techniques of intertemporal optimization, or one discusses the aptness of the refusal to discount utilities - both points being sometimes bound in the reflections on the existence conditions of an optimal intertemporal path[18]. Nevertheless, and in spite of doubtless outmoded mathematical tools, a closer reading reveals a less evident ethical choice, and the definition, with the introduction of the time preference, of a long term equilibrium – the modified bliss- which corresponds fundamentally to what is nowadays called the modified golden rule. We could furthermore spot in Ramsey's text the intuition of a life cycle saving's theory. Finally, it appears clearly that there is no need to wait for Barro's (1974) justification of the consideration of economic agents with infinite life expectancy, to see appearing the notion of family or dynasty as relevant theoretical concept to analyze the intertemporal choices.

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[17] We don't really know how such a stationary state is reached. Is everything decided at the initial moment, the initial value of the interest rate (the marginal productivity of the initial capital) deciding between families intended to bliss or those destined to the subsistence level? Or does Ramsey envisage a more complex process, where accumulation of the first type of families and debts of the second type would counterbalance only partially, what would make vary the stock of aggregate capital, the marginal productivity of capital, and would redefine a border between savers' families?

[18] For a survey of contributions to optimal growth theory since 1928, see for instance Chakravarty (1987).

2. The asymmetry in the posterity of Ramsey's work corresponds doubtless to the initial asymmetry that characterizes the care brought by the author himself to the first part of the essay and the vagueness that presides on the contrary over the following developments. While the first part is completely specified, Ramsey then is particularly sparing of comments. The paragraphs of parts II and III adopt changeable, sometimes implicit, often *ad hoc* hypotheses, in the sense that they are sometimes used only to help in the demonstration being then quickly given up. Ramsey takes however the previous results for granted and do not feel necessary to re-demonstrate them when the hypotheses vary. Such is for example the choice of the production function with constant marginal productivities: Useful for the graphic representation, and for the description of the behavior of saving with discount rate, it disappears then in the third part of the text. It would not present particular problems if the results established in this framework were not used in the third part.

Further, Ramsey seems often worried about technical precision but does not condescend to justify crucial assumptions: when he introduces for instance the discount rate, he dedicates a big part of his analysis to the justification of the fact that this rate should be constant. He on the contrary forgets to explain why, thoroughly having described this practice as indefensible, he finally agrees to consider it, even when nothing has changed in the analysis except the assumption of constant marginal productivity.

3. Finally, Ramsey's text is characterized by an oscillation, or a permanent tension between the treatment of the community and that of the individual, both decision-making figures involved in Ramsey's problem such as we have re-defined it in the second section of this paper. Ramsey explains nothing on the links that unite these two entities, except one thing: he does not envisage any aggregation of the behaviors of identical individuals. When dealing in the first part with the national behavior, he does not try to justify the use of global" functions, such as the global utility and disutility function and *a fortiori,* he carefully avoids referring to any kind of aggregating procedure. When introducing the case of the individual choice, he in fact precise that he applies the same method to the treatment of two different questions: the optimal allocation of the national income and that of an individual income. As he explains for instance "In assuming the rate of discount constant, I do not mean that it is the same for all individuals, since we are at present concerned with one individual or community […]" (Ramsey 1928, 553). He also reminds a few paragraphs later that "the method of resolution, both for a community and for an individual is […] the same as [in the first part]" (Ramsey 1928, 554), but he nowhere claims that the optimal saving behavior of the "nation" can arise from the sum of optimal behavior of individuals identical in their preferences**.**

He does not either suggest that the national behavior could appear as such from the underlying individual choices: while the behavior of the nation supposes exactly the absence of discount rate, the individuals them discount their future satisfactions. Finally, when he introduces classes of individuals, it is then to burst the nation by differentiating these classes by the discount rates, as intrinsic element of the preference. We therefore cannot attribute to Ramsey the idea of a possible "representative agent", in contrast with the temptation of some retrospective analyses. Community and individual conceptually stay two separate entities.

However, Ramsey's argument is not free from ambiguity. On this point too, there is a blatant asymmetry between the attention carried to the formal conditions of extension of his initial rule on one hand, and the doubt he on the contrary lets hang over the institutional framework or the status of the economic entity who decides to save on the other hand. While he defines exactly the implications of the finite life expectancy – and studies many configurations depending on whether the individual leaves inheritance or not- Ramsey clarifies nothing on the economic organization in which takes place the individual decision. We guess that we are leaving a framework of planned economy but we do not really know what could replace it. Should one consider an individual in autarky, who would be both consumer and owner of his production

structure, what suggests the simple transposition of the initial rule, established in the framework of the community? Alternatively, should one consider an *individual in society*, that is in interaction with other individuals? Ramsey does not enlighten us on this point.

Put simply, Ramsey's text distinguishes by mysteries, which surround notably the conception of the economic organization in which takes place the saving decision, and by a choice of moving *ad hoc* hypotheses. It is characterized on the other hand by the will to distinguish the micro- and the macro-order, and by preservation of an intertemporal consistency assumption as for the behavior of the decision-making entity is concerned. By exploiting what is explicit in Ramsey's paper, it is therefore possible to understand the implicit, and to exhume the global consistency of the argument. The consistency appears in fact more clearly if we scrutinize more specifically the status of the decision-making entities which inhabit the 1928 essay.

V. The faces of the economic agent

The first decision-making entity appearing in the article of 1928 is so the nation, considered as a transgenerational entity. As we have already said, the first part of the essay is enough precise so that we can clarify the corresponding economic representation. Two points should be raised, which are profoundly connected: the implicit organization of the cooperative-economy and the refusal to consider a discount rate.

In the first part of the 1928 article, Ramsey's opts at once for a macroeconomic glance, by choosing to represent the national economy as what can be described with Joan Robinson (1978) as a "cooperative". The nation, or the "community" (Ramsey 1928, 543), is embodied by an abstract economic agent – who is at the same time a producer, a consumer and a working provider – who lives forever and whose identity endures in time. This very specific representation is crucial for the demonstration of the saving rule.

It allows indeed two "tours de force". On the one hand, it allows the transposition of marginalist equilibrium equations to the collective level. In an economic tradition dominated by the theory of partial equilibrium, it was doubtless the only possibility to justify such a use of the marginalists' results, which were up to those days reserved for the study of individual behavior. On the other hand, the main advantage of this overall presentation is to allow concentrating on the decision-problem and to exempt from analyzing any question of coordination of individual decentralized decisions. It dispenses from the outset with any kind of adjustment process. These questions are solved by definition[19]. In the first part of Ramsey's article, there is thus no need to make explicit the markets' functioning (for goods, labor or loanable funds). Formally, it means that it is not necessary to formalize prices, as vectors of information and coordination. The only exogenous elements are therefore the "structural" parameters. Every equilibrium condition is expressed by Ramsey directly in terms of marginal utilities and of marginal productivity of the production factors, which could be interpreted as shadow prices.

Further, such a representation also excludes by definition the questions of wealth distribution among the members of the community, and even, with the choice of a utilitarian criterion of aggregation of the instantaneous utilities, among the different "generations". The question of the distribution (of labor or wealth) is not considered by the nation - the planner- as in the literature wondering about the possibility of comparing competitive systems or centralized systems. The community is in equilibrium all the time but this equilibrium, or this succession of equilibriums is purely postulated.

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[19] Ramsey is certainly not the first one to envisage the global behavior of an economy as a foreground question. Besides the whole classical literature, the marginalist economists begin, at the end of the nineteenth century, to reinstate problems of macroeconomic order. That is the case with J. Bates Clark (1899), who assimilates the economy to a giant firm; that is also a question for the researches dealing with the establishment of social welfare functions (Pareto 1913, Pigou 1932 for example). However these analyses preserve a study of coordination or of the aggregation conditions.

Such approach is fundamentally consistent with Ramsey's refusal to introduce a discount rate of future utilities in the first part of the article. Ramsey appeals here to two types of justification of this refusal.

The first justification –to discount future utilities is "ethically indefensible" (Ramsey 1928, 543)- rests on an interpretation of the discount rate as a social actualization rate. Ramsey's refusal joins here in a utilitarian tradition, which considers discounting as an intergenerational injustice. This point of view, which we found by Pigou, is, as Collard (1996) points out, a common consideration in the Cambridge classical or neoclassical tradition. The second justification is more directly bound to our comment: it is profoundly connected to the institutional framework retained in the first part of Ramsey's essay. The "poverty of the imagination" (Ramsey 1928, 543) echoes here Pigou's defective telescopic faculty: the impossibility to correctly represent oneself the future events[20]. Such a consideration would not have its place within the framework of perfect forecasting hold here. Besides, it is noteworthy that the representation of the nation as a unique economic agent with infinite life expectancy abolishes *formally* any idea of generation gaps: the nation as an undivided entity has no reason for favoring on principle such or such phase of its own life. Furthermore, no uncertainty presses here on the decision to consume or to save. Thus, the three motives for the discounting mentioned by Pigou in chapter II of *The Economics of Welfare* have no place in the first part of Ramsey's essay. The nation is in a way the opposite of the individual.

Why, from then on, agree finally to introduce a discount rate, even when Ramsey is still interested in the national behavior of saving, as it is the case in the paragraph *IIb* of the 1928 essay? What is today a natural or usual assumption contradicts on the contrary Ramsey's initial comments and could relegate our analysis towards the over interpretation. If we prolong our reasoning, and if we accept filiations between Pigou's analysis and Ramsey's work, such motives (uncertainty, egoism and nearsightedness) can indeed only appear in a theory of the individual behavior of saving. The contradiction can be raised if one agrees to consider this counter natural association, according to terms of Ramsey himself, as a provisory stage in a wider demonstration. It so appears as a temporary step, amending gradually the initial framework in order to progress toward the consideration of the individual behavior. Our hypothesis, admittedly quite surprising *a priori*, seems consolidated if one considers this time all the hypotheses introduced into the second part of the essay.

Besides the consideration of life expectancy (and of the discount rate), the individual can not indeed be considered as a cooperative or as an amoeba any more. Now, if Ramsey says nothing on the economic organization supposed during the essay, a footnote, concerning the shape of the function of production, delivers deciding indications.

Having introduced the hypothesis of the constant marginal productivities, Ramsey comments in a footnote:

> "[These assumptions] are less absurd if the state is one among others which are only advancing slowly, so that the rates of interest and wage are largely independent of what our particular state saves and earns" (Ramsey 1928, 549).

This quotation concerns in fact more the assumption of a constant rate of interest and rate of wage than the generic shape of the production function. It suggests a new interesting idea: the nation could be seen as a "small opened economy", whose choices, or fate, do not influence the prices, in peculiar the interest rate. Ramsey would then assimilate or confuse a hypothesis of

[20] The word "imagination", stemming from psychology, appears frequently in the literature to describe the power of abstraction from the present situation: John Rae is talking about " deficit of imagination" to appoint the cultural valuation of the present pleasures (see Rothbard 1987, 645). Böhm-Bawerk (1889) uses the term of imagination systematically and exclusively in the book V of *The positive theory of capital*, dedicated to the subjective apprehension of the future.

constant prices with a hypothesis concerning the decisional entity, that is a hypothesis of a "price-taker" agent. Ramsey leaves us however with this mysterious comment and concentrates on the explanation of the graphical analysis. But we can note first that with the assumption of constant prices appears also the first allusion to the existence of prices in the analyses. And if we pursuit the idea according to which Ramsey would associate constant price and given prices, this modification takes all its sense when dealing with the individual behavior. Therefore, Ramsey would not have in mind, when transposing his rule to the individual case, a representation of an individual in autarky, but indeed the idea of a price-taker individual. Assumptions on the shape of the production function would then be an institutional assumption as much as a formal assumption. Once again, the assumption would anticipate the study of the individual behavior. This gliding in the analysis appears then clearly when, in the third part of the essay, Ramsey considers the market of capital, the only market evoked in the 1928 text. The individuals confront their subjective discount rate with a given interest rate, and of this confrontation emerges the short-term value of the interest rate. In all these discussions, Ramsey deals with "constant" interest rate.

Therefore, whereas the shape of the production function, the introduction of a discount rate, and the ghostly apparition of a market for loanable funds invite criticism or perplexity within the framework of the cooperative economy, they seem more adequate to the treatment of the individual behavior. The finite life expectancy of the individual can explain the introduction of a discount rate as well as the consideration of a price-taker individual can justify the hypothesis of constant prices, that is, by Ramsey, given prices. We can then interpret the variations of the second part as many preparatory stages in the formalization of the individual behavior. Nevertheless, as we have noticed, these developments are punctuated with the reaffirmation that the community and the individual are two separate theoretical entities.

A similar reading of the third part of Ramsey's essay allows taking the measure of such an interpretation. In this third part, Ramsey introduces a new economic agent, which appears as a midway figure between the individual and the undivided community: the "family", corresponding exactly to what we today understand by "dynasty".

The concept of family allows indeed two ways to split the community, in order to give her a theoretical representation connecting with individual characteristics. It first of all allows to split the national entity in the time, and to think an economic agent with infinite life expectancy as a succession of agents with finished life expectancy. Ramsey puts so an agent with infinite life expectancy, for which it not inconceivable any more to associate with a discount rate of the future utilities. It opens logically the door to an analysis of the consequences of the individual inclination to discount future utilities on the long-term equilibrium of the economy.

The notion of family allows as well splitting the cooperative community at every date. The national economy is this time compound, all the time, with individuals differentiated by their discount rate. Those individuals apply the rule defined in *IIb* and their behavior of saving allows Ramsey to endogeneize the rhythm of capital accumulation. They are price-taker individual, or at least, their behavior is not enough strong to make vary the value of the interest rate. But such (short term) interest rate of equilibrium engages the movement of separation of the community into two classes that defines the long-term macro equilibrium.

The family is so an intermediate figure, mixing in a consistent way individual and national characteristics, which allows him, in an admittedly artificial way, to think the characteristics of a long-term macroeconomic equilibrium by bypassing the yoke of the partial equilibrium framework.

The general movement of the 1928 article is then that of a partial explosion of the initial cooperative community, thanks to the family agent, [21] which indicates the search for a theoretical representation that could represent an alternative to the initial cooperative representation. Such a solution to study the effects of the individual behavior of saving on the collective welfare rests notwithstanding on a kind of subterfuge. It bases indeed on a change in the status of the decision entity and on the use of the market of capital as a control lever to examine the macroeconomic consequences of the individual tendency to discount. Ramsey so avoids analyzing the mechanisms that give concrete expression to the interaction of the individual behaviors (the supply side remains for instance represented by a global production function).

VI. Conclusion

The question of the coexistence of a utility discount rate with the concern for the lot of future generations is a "classic" by Cambridge economists. The first part of Ramsey's article evoked a clear-cut position; the practice that consists in discounting is simply indefensible. This refusal of a discount rate is logically implied by the representation of the economy as a cooperative. This community could suffer neither from a split of the personality (leading it to favor such or such phase of his life) nor from failing telescopic faculties. On the contrary, the identity split of the community over the time, through the definition of families constituted by mortal individuals - certainly bound together by blood relationships but having different identities - can bring a consistent justification for the consideration of a discount rate. If the identity of the agent with infinite life expectancy is segmented, there can be no more moral or rational reason to justify the fact that present generations carry the complete weight of saving, whereas future generations are bursting with bliss. While the second part of the article showed an association community - discounting, contradictory with the assertions of part I, the family entity enlightens this temporary association.

The concept of family also introduces a split of the community at every date. The society is now composed with different individuals. We do not really know what is the modality of their social interactions – because the individuals are only perceived through their supply of saving – but their saving decisions, confronted globally, determine the real interest rate of equilibrium (and the evolution of the product *via* the capital accumulation). In retrospect, the third part therefore justifies the formalization of the saving behavior of economic agents considering the interest rate as constant, but also the fact that the distance between the discount rate and the marginal productivity of the global capital can stay constant, if the saving behavior of every individual is not powerful enough to make it vary. What was meaningless in the second part of the article – except if we assimilate the community to a small opened economy – meets ultimately a partial justification in the consideration of this dynastic behavior.

So, while the first part of the article can find an autonomous logic, which has doubtless make its success some decades later, the developments of the second and third parts by spreading out multiple ways of reflection only take sense in the last paragraph of the article. This last paragraph marks then the resolution of an underlying tension inside the whole Ramsey's argumentation: between representation of the behavior of the cooperative community with infinite life expectancy, and that of the individual behavior of the price-taker agent with finite

[21] There is thus no need to wait for Barro (1974) and his definition of dynasties, to see appearing in an implicit but completely coherent way, a logical association between family and discount rate of utility. But while in Barro's article, the problem is to pass from the individual figure to that of an agent with infinite life and to justify, by an intergenerationnal incomplete altruism, the coexistence of a discount rate (due to the nearsightedness of the initial agent) and the consideration of the future generations (which are integrated into the initial utility function), Ramsey's construction leaves the national figure without discount rate (where there is no need to resort to an altruistic hypothesis because the individual who saves and the one that profit of the capital accumulation capital is the same), then to justify the discount rate by the dis-aggregation of the community.

live slides an intermediate figure, that of the family.

In these thought processes, the study of the cooperative economy endowed with perfect forecasts fulfills a heuristic as well as a normative function. It allows the establishment of the initial mathematical rule, by authorizing the use of total production and total utility functions, and by allowing to concentrate on a simple question, that of the equilibrium economy. It then darkens the question of the coordination of the underlying individual decisions, and allows concentrating on the convergence towards the long period stationary path. The main result of the first part, the Keynes-Ramsey rule, defines so a rule of decision of saving for every moment, which allows attaining bliss as soon as possible. At the same time, the framework of the cooperative economy allows treating the community as a rational agent, that is a maximizating agent endowed with consistent intertemporal preferences, the behavior of which is by definition the best possible. It establishes a standard by virtue of which to estimate or to direct the real behavior, by virtue of which one can also compare the alternate results proposed in the following paragraphs of the article. It then offers a first answer to Pigou's dilemma about the legitimacy of a political economy in favor of saving.

Once established, the mathematical rule of saving serves as an *Ariadne's clew* in the consideration of more complex questions. During the article, Ramsey seems more careful with the formal conditions of transposition of his rule than with the successive modifications of the theoretical and institutional framework that these variations imply. Ramsey's argumentation, and the way he brings us gradually towards the conclusion of the article, takes partially the shape of a subterfuge. In substance, he indeed says simply: "I take the rule demonstrated in *I*; I argue by analogy with the *IIb* to establish the transposition of the rule of the *IIc* in the individual case and integrate it into the *IIIβ*. I prolong then this *IIIβ* by means of the reasoning defined in *IIIα*, and I arrive, with some supplementary and simplifying hypotheses, in the conclusion of the *IIIγ*". Now, the underlying economic representation in each of the paragraphs of the article, is never either completely the same, or completely an other one, and the theoretical links between the elements finally assembled are not inevitably as evident as Ramsey claims.

These variations take nevertheless sense in the movement that one operates, from the figure of the nation towards that of the family with infinite life expectancy. This movement is in our sense a conscious objective in the second and third parts of the 1928 essay. They reveal theoretical results which are not always associated with the 1928 essay, such as the widening of the analysis to the consideration of a subjective discount rate and the definition of the modified bliss, or the distinction between the determinants of a positive real rate of interest in the accumulation period (marginal productivity of capital) and in the stationary state (discount rate of utility). Furthermore, the whole article tends clearly to show the superiority, in terms of collective welfare, of the economic representation chosen in the first part of the article. If we follow Ramsey's argument step by step, it appears that he introduces after all an intermediate figure between the individual and the nation, which allows to bound the individual behavior with the collective welfare: the figure of the family or the dynasty. We find so in germ at Ramsey the justification of consideration of individuals with infinite life expectancy through the dynasty, the justification New Classical macroeconomics has adopted since Robert Barro's (1974) article.

Ramsey's 1928 paper so appears as a quite consistent construction, in spite of the numerous hesitations that sometimes limit the meaning of its statements.

Some of these hesitations can be imputed to Ramsey's argument itself: the introduction of the article and the implementation of the corpus of hypotheses do not warn us at all for the forthcoming variations. The other waverings hold in the fact that Ramsey strays widely from the beaten track. Numerous theoretical problems, more or less identified in 1928, not always resolved today, appear constantly on the fringe of his theory. Besides the problem of aggregation of goods or behaviors, that of the distribution of income at the aggregate level, that of the definition of the

capital[22] appears the question of the formalization of the discount rates of utility, or moreover that of the representation of an underlying decentralized economy. Ramsey seems to have the intuition of these problems. He tries to protect himself from it by simplifying assumptions, darkening widely the underlying economical mechanism of his representation. He notwithstanding suggests possible tracks for solution[23].

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[22] Absent in the analysis.

[23] In doing this, Ramsey bequeaths to the economists numerous intuitions: intuitions of the problems just as much as of the possible theoretical solutions. The definition of an optimal path of capital accumulation and the use of the intertemporal optimization; the theoretical necessity of thinking a global economy in equilibrium all the time; and the substitutability of the production factors which is associated to it; the allocation, justified by the economic planning, of a substantial rationality to this community, which allows to make saving endogenous; the attempt finally, to decentralize partially his community, and the met difficulties (places where Ramsey's eclipses are the most blatant) make from his article a strangely modern article.

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Synopsis

	Objective	Assumptions on the production function	Discount rate	Long-term equilibrium
Part I. Community	Determination of Ramsey's rule with infinite life expectancy	$f_a(a,c) \geq 0$ and $f_{aa}(a,c) \leq 0$ $f_c(a,c) \geq 0$ and $f_{cc}(a,c) \leq 0$	$\rho = 0$	Bliss
Part II. §a. Community	Graphical illustration	$f_a(a,c) = \text{cst}$ $f_c(a,c) = \text{cst}$ $f_{ac}(a,c) = f_{ca}(a,c) = 0$	$\rho = 0$	Bliss
Part II. §b Individual	Graphical representation. Individual rule with finite live expectancy.	$f_a(a,c) = \text{cst}$ $f_c(a,c) = cs$	$\rho = 0$	
Part II. §c Community, then individual	Discounting, with infinite live, then finite live expectancy	$f_a(a,c) = \text{cst}$ $f_c(a,c) = \text{cst}$	$\rho > 0$	Modified bliss
Part III. §α Community	Determination of the interest rate	$f_a(a,c) \geq 0$et $f_{aa}(a,c) \leq 0$ or $a = \text{cst}$ $f_c(a,c) \geq 0$et $f_{cc}(a,c) \leq 0$	$\rho > 0$	
Part III. §β Classes of individuals	Very short-term equilibrium	$a = \text{cst}$ $f_c(a,c) = \text{cste}$	$\rho_i > 0$ $i \in \{1,2,....,n\}$	
Part III. §γ Dynasties	Long-term equilibrium	$a = \text{cst}$ $f_c(a,c) \geq 0$et $f_{cc}(a,c) \leq 0$	$\rho_i > 0$ $i \in \{1,2,....,n\}$	Division in two classes: bliss or subsistence level