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THE UNITY OF SCIENCE IN THE ARABIC TRADITION: SCIENCE LOGIC EPISTEMOLOGY AND THEIR INTERACTIONS

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LOGIC EPISTEMOLOGY AND THE UNITY OF SCIENCE

**THE UNITY OF SCIENCE IN THE ARABIC TRADITION:
SCIENCE LOGIC EPISTEMOLOGY
AND THEIR INTERACTIONS**

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إذا مات ابن آدم انقطع عمله إلا من ثلاث: صدقة جارية، أو علم ينتفع به أو ولد صالح يدعو له.
حديث شريف

When a man is dead, his actions are brought to an end except in three cases:
a permanent charity, a useful knowledge or a good son that prays for him.
The Prophet

To the memory of my late father Prof. Dr Aziz ur-Rahman who kindled in
me a passion for the adventure of science and to my mother, Hilde Rahman,
for her brave vision of a world without frontiers.

Shahid Rahman

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Τρανσκριπτιον Συστημ οφ τηε Αραβιχ Αλπηαβετ

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INTRODUCTION

I THE MAJOR BREAKTHROUGH IN SCIENTIFIC PRACTICE

فإذا جُمعَ يسيرُ ما نال كل واحدٍ من النَّاتِلِينَ الحقَّ منهم، اجتمعَ من ذلك قدرٌ جليلٌ.

When, though, the little which each one of them who has acquired the truth is collected, something of great worth is assembled from this.

و ينبغي لنا ألا نستحيي من استحسان الحقِّ، واقتناء الحقِّ، من أين أتى. وإن أتى من الأجناس القاصية عنّا، والأمم المبابنة.

We ought not to be ashamed of appreciating the truth and of acquiring it wherever it comes from, even if it comes from races distant and nations different from us.

(Al-Kindī *في الفلسفة الأولى* (*On the First Philosophy*, 1974, pp. 57-58)).

الرَّجَالُ أَرْبَعَةٌ: رَجُلٌ يَدْرِي وَ يَدْرِي أَنَّهُ يَدْرِي فَسَلُوهُ،
و رَجُلٌ يَدْرِي وَ لَا يَدْرِي أَنَّهُ يَدْرِي فَذَاكَ نَاسٌ فَذَكِّرُوهُ،
و رَجُلٌ لَا يَدْرِي وَ يَدْرِي أَنَّهُ لَا يَدْرِي فَذَلِكَ مُسْتَرْشِدٌ فَعَلِّمُوهُ،
و رَجُلٌ لَا يَدْرِي وَ لَا يَدْرِي أَنَّهُ لَا يَدْرِي فَذَلِكَ جَاهِلٌ فَارْفُضُوهُ.

There are four kinds of men: men who know and know that they know; ask them.
Men who know and do not know that they know, they are forgetful; remind them.
Men who do not know and know that they do not know, they search for guidance; teach them.
And men who do not know and do not know that they do not know, they are ignorant; shun them.

(Al-Khalīl ibn A1mad al-Farāhīdī, in Ibn Qutayba *'Uyūn al-akhbār*, 1986, II, p. 142)

Knowledge was a major issue in science and philosophy in the twentieth century. Its first irruption was in the heated controversy concerning the foundations of mathematics. To justify his rejection of the use of the actual infinite in mathematical reasoning, Brouwer has made the construction of mathematical objects dependent on the knowing subject. This approach was rejected by the mainstream of analytical philosophers who feared a fall into psychologism. Several years later, the question of the progress of scientific knowledge was put forward in the thirties by the post-positivist philosophers to fill the vacuum in the philosophy of science following the demise of the logical positivism programme. The answers given to these questions have deepened the already existing gap between philosophy and the history and practice of science. While the positivists argued for a spontaneous, steady and continuous growth of scientific knowledge the post-positivists make a strong case for a fundamental discontinuity in the development of science which can only be explained by extrascientific factors. The political, social and cultural environment, the argument goes on, determine both the questions and the terms in which they should be answered. Accordingly, the sociological and historical interpretation involves in fact two kinds of discontinuity which are closely related: the discontinuity of science as such and the discontinuity of the more inclusive political and social context of its development. More precisely it explains the discontinuity of the former by the discontinuity of the latter subordinating in effect the history of science to the wider political and social history. The underlying idea is that each historical and social

context generates scientific and philosophical questions of its own. From this point of view the question surrounding the nature of knowledge and its development are entirely new topics typical of the twentieth century social context reflecting both the level and the scale of the development of science. To the surprise of modern historians of science and philosophy, the same kind of questions, which would allegedly be new topics specific to the twentieth century concerning the nature of knowledge and its progress, were already raised more than eleven centuries earlier in the context of the Arabic tradition which, as we discuss further on, developed a trans-cultural and trans-national concept of the unity of science (see the contributions of Deborah Black and Jon McGinnis which tackle the issue of the nature of knowledge). The neglect of the Arabic tradition in philosophy of science is a major gap not only in the development of science but a fundamental flaw in the writing of its history and philosophy caused by the total reduction of epistemology to political and social history of science. How has this period of the history of science and philosophy come to be ignored? In what circumstances were the questions akin to the nature of knowledge raised in the first place? What is the relation between on the one hand the questions of knowledge and its growth and on the other hand the unity of science in the Arabic tradition? The answers to some of these questions are the aim of the present volume, the first of the series *Logic, Epistemology and the Unity of Science* to be devoted to a so-called non-western tradition. Let us first highlight in a kind of overview some landmarks concerning the *timing* of the emergence of the Arabic tradition and its significance for the history of science.

1. What happened in the ninth century?

Since the beginning of the history of science in the mid-eighteenth century and its firm establishment as an independent discipline in the nineteenth century, the history of science has been largely written by western historians. The views of most historians of the nineteenth century have succeeded in shaping the standard view, still prevailing today, concerning the Arabic tradition. In this respect, the received view's approach was motivated by two main concerns: (i) to recover the lost Greek heritage extant only in the Arabic version, and in the meantime to find out to what extent Arab scientists and philosophers are proved to be capable of correctly understanding sophisticated Greek thought; (ii) to assess the contribution of the Arabic tradition to the development of the so-called western science. The focus on the relation between the Greek and the Arabic traditions reflects the major concern of this approach which consists in examining what has commonly been called the reception of the Greek scientific and philosophical works in the Arab world. While it is true that the Arabic tradition was developed against the background of Greek scientific and philosophical writings — a phenomenon which is similar in this regard to the fact that Greek philosophy had emerged against the background of the achievements of the Babylonian and Egyptian civilisations, the standard approach seems to have gone too far in its assessment of the so-called reception-role of the Arabic tradition. Indeed, according to the received view the Arabic tradition seems to be deprived of any interest of its own. Indeed the impression given is that Greek philosophical doctrines have succeeded not only in overthrowing the Babylonian and Egyptian beliefs, but they continued to dominate throughout the classical Islamic era. It is thus not surprising that the received view came to the conclusion that the importance and the relevance of the Arabic tradition to the history of science lies only in its intermediary role consisting in handing almost intact the Greek works over to the medieval Europeans. It looks as if Greek scientific and philosophical books were brought to the Arabic libraries to save them from an imminent major disaster that could strike the Greek heritage. We have here some kind of paradox: many historians make such kind of definitive judgments by considering only few materials of a tradition which reigned uniquely over the scientific and philosophical scene for up to seven

centuries. This paradox is symptomatic of the underlying epistemological approach to the approach to the history of science which is by its very nature an open system. The assumption is that the study of the Arabic tradition was sufficiently exhausted to the extent that no new findings could have any significant impact on our present state of knowledge concerning the development of knowledge. This long-lasting prevailing view has recently been challenged by a careful study of some important Arabic scientific works. From the mid-twentieth century onwards some historians have set themselves the task of translating important Arabic writings aimed at filling the gap in our understanding of the development of the Arabic tradition. It is in this context that Sabra has challenged the use of what seems to be a neutral term to describe the transmission of Greek scientific and philosophical works. He argues that “Reception” might “connote a passive receiving something being pressed upon the receiver, and this might reinforce the image of Islamic civilisation as a receptacle or repository of Greek learning” (Sabra 1987, p. 225). He stresses that Greek science and philosophy was not thrust upon but rather “invited [as a] guest” by the Arabic Islamic society (ibid., p. 236). Sabra proposes instead “appropriation” to describe the “enormously creative act ... the cultural explosion of which the translation of ancient science and philosophy was a major feature” (ibid., pp. 226-228). His argument seems to have little effect on the received view concerning the Greco-Arabic transmission. But some historians such as Willy Hartner and Gotthard Strohmaier have tried to refine their analysis of the periodisation of the development of Arabic science by admitting the existence of a second period during which the Islamic society was more productive and creative than receptive and imitative. The restriction of the application of the Reception concept to the early period of the translation movement can be seen as an important concession to the opponents of the Reception doctrine. But Dimitri Gutas, who precisely devotes a whole book to this question, rejects out of hand this compromise which consists in applying the Reception interpretation to the early period

One such prevalent misconception [about the development of Arabic science] is that the translation movement went through two major stages, a ‘receptive’ one, roughly through the time of al Ma’mūn, and a creative one subsequently. Study of the translation complexes, as the example of the Kindī circle complex of the translations shows, invalidates by itself even the very posing of the question in such a way” (Gutas 1998, pp. 149-150).

Besides its passive connotation underlined by Sabra, the misconception induced by “Reception” is that the transmission can be understood as the result of direct cultural exchanges between on the one hand the Greeks, as producers and exporters (Strohmaier actually speaks of providers) of scientific and philosophical theories, and on the other hand the Arabs as users and consumers. Unlike the transmission of science and philosophy to medieval Europe, the Greco-Arabic transmission has taken place in an entirely different climate as Gutas rightly points out (ibid., p. 4). In other words the massive translations from Greek and Arabic into Latin starting from the twelfth century reflect the powerful and profound impact that the flourishing and advanced Arabic-Islamic civilisation has had on the medieval European psyche, where there is no equivalent driving force in the case of the Greco-Arabic transmission since the social and cultural environment in which Greek science and philosophy were developed was extinguished for so many centuries. Does it mean that no driving force can be found behind the translation movement? Is there only one or more than one driving force? And in the latter case, do they have equal influence in the development of Arabic science or do some of them play a much more prominent role than others? We shall see in a moment how Gutas deals with these various questions.

While agreeing wholly with Sabra on the creative nature of the translation movement, he expresses his reservation to the use of “appropriation” to describe the process of the transmission since he finds it a “surreptitiously servile term” (ibid., p. 187). No specific term has been proposed by Gutas since he prefers simply to call it a “creation of early ‘Abbasid

society and its incipient Arabic scientific and philosophical tradition” (ibid.). It looks as if the language has run short of words since, among the many memorable moments of the history of science, this is the only particular historical moment for which no specific word could be found to mark the unprecedented large-scale scientific activity triggered by what some historians, including Gutas, call the political revolution. It seems thus that the description of the Arabic translation movement is no less problematic than the question of the assessment of the Arabic tradition itself (see Tahiri’s introduction to his paper). What did happen in the ninth century is not the recovery of Greek science but the implementation of a new idea of science, where science and the scientist are conceived as institutions and instruments of research and development.¹ Moreover, as we shall see in paragraph two and three of our introduction this new concept of science was first carried out by means of the creation in Bagdad of an institution, namely *the house of wisdom (Bayt al-ikma)* and the production of an Arabic scientific literature with a technical vocabulary in a kind of what Gutas calls *a high koiné language* fit for inter and trans-disciplinary work in a way which might be considered to be an analogue to what has been described as the role of *lingua franca* given to formal language by the French Encyclopedists (see Rahman/Symons 2004, pp. 3-16). Both projects, the house of wisdom and the production of an Arabic *koiné* language, provided the instruments with the help of which the notion of the unity of science was implemented within the Arabic tradition.

2. Science awakening and *Bayt al-ikma* (the house of wisdom)

There have been many conquests in history but few had such a direct and decisive impact on the history of science and philosophy as the Arabic conquest. One of its main features is that the expansion of the Arabic-Islamic civilisation and the development of science go hand in hand. The Arabs have not waited for science and philosophy to come to them, we have to bear in mind that the Arabic peninsula did not come under the rule of Alexander the great. They have had instead to go after knowledge. The task was very challenging since they have had to start from scratch. Gutas describes in the following passage how the scale of this ambitious intellectual project has required the unprecedented mobilisation of a huge amount of resources and energy of an entire nation for more than two centuries.

The Greco-Arabic translation movement lasted, first of all, well over two centuries; it was no ephemeral phenomenon. Second, it was supported by the entire elite of ‘Abbasid society: caliphs and princes, civil servants and military leaders, merchants and bankers, and scholars and scientists; it was not the pet project of any particular group in the furtherance of their restricted agenda. Third, it was subsidized by an enormous outlay of funds, both public and private; it was no eccentric whim of a Maecenas or the fashionable affectation of a few wealthy patrons seeking to invest in a philanthropic or self-aggrandizing cause. Finally, it was eventually conducted with rigorous scholarly methodology and strict philological exactitude — by the famous ʿUnayn ibn Isḥāq and his associates — on the basis of a sustained program that spanned generations and which reflects, in the final analysis, a social attitude and the public culture of early ‘Abbasid society; it was not the result of the haphazard and random research interests of a few eccentric individuals who, in any age or time, might indulge in arcane philological and textual pursuits that in historical terms are proven irrelevant. (ibid., p. 2)

This is modern science in the making. Modernity should be understood here not in the narrow sense which is traditionally associated with the advent of the new physics conceived as a finished product, but in the act of creating, through the close co-operation of the political power and the Arabic-Islamic society, a new and long-lasting dynamic structure. It turns out that the unstoppable growth of the new entity, which has proved to be far more outliving both the political entity which gave it birth in the first place and the social context of its formation, is designed to transform the life of the Arabic-Islamic society and with it the societies of the

rest of the world. For the first time in history science becomes a profession. Unlike in the Greek tradition where it is practised by some happy few who have the luxury thanks to their wealth to enjoy what they regarded as the supreme life by just contemplating nature. Science becomes in the Arabic-Islamic tradition a third institution with growing influence along side with the two extant most powerful institutions: the legal and the political powers. The result of this unprecedented collective hard and enduring work: by the end of the tenth century almost all non-literally and non-historical Greek books that were available were translated in Arabic. Greek science and philosophy has been transformed once and for all by “the magic translator’s pen” as is nicely expressed by Gutas.

It should be noted however that the translation movement is not confined to the Greek writings though the latter form the bulk of the works translated, it is a more global and international phenomenon since it concerns all the books that are fit to be translated. There are Arabic versions of books written in other languages such as the Persian, the Sanskrit and possibly the Chinese language.² The successful achievement of this monumental enterprise, which can at any moment be interrupted or abort altogether for a variety of reasons, falls nothing short of a great miracle, the assessment of which has not yet begun, since it opens a new era in the history of human thought. The idea of knowledge has been completely reinvented through the systematic survey of all existing scientific writings. By the turn of the eleventh century, the translation of Greek works has significantly died down reflecting the advanced level reached by Arabic science. As Gutas put it bluntly “the waning of the Greco-Arabic translation movement can only be seen due to the fact that it had nothing to offer... not in the sense that there were no more secular Greek books to be translated, but in the sense that it had no more books to offer that were relevant to the concerns and demands of the sponsors, scholars and scientists alike” (ibid., p. 152), in other words “the translated works lost their relevance and became part of the history of science” (ibid., p. 153). Consequently there was a shift in demand for more up-to-date research. Gutas further explains the major impact of the rapid spread of the Arabic scientific institution model far beyond the spatiotemporal context that gave it rise in the first place

Once the Arabic culture forged by early ‘Abbasid society historically established the universality of Greek scientific and philosophical thought, it provided the model for and facilitated the later application of this concept in Greek Byzantium and the Latin West: in Byzantium, both in Lemerle’s ‘first Byzantine humanism’ of the ninth century and in the later renaissance of the Palaeologoi; and in the west, both in what Haskins has called the renaissance of the twelfth century and in the Renaissance proper”(ibid., p. 192).

Contrary to the prevailing view according to which there is only one renaissance in history, Gutas seems to be saying that the Arabic tradition gives rise to a series of renaissances which reaches its climax in the advent of the famous south-western European Renaissance. The Renaissance proper as Gutas would like to call it now, which is recognised by the sociological doctrine as the starting point of the scientific revolution, appears to be then not the first of its kind as it is generally believed but the outcome of previous renaissances which originate in the foundation in Bagdad of the *Bayt al-Ikma* or the house of wisdom, the famous scientific institution that gives rise to the development of Arabic science by hosting the first movement of what can be called the translation project and (see below). But what about this crucial period during which the Greco-Arabic transmission took place? Can the ninth century be called a renaissance? Gutas appears to be somewhat hesitant. On the one hand he is inclined to describe it as the “real renaissance in the original sense of the revival of Greek learning” (ibid., p. 154). But on the other hand this “real renaissance” seems to be quite different from the traditional European Renaissance. He rightly points out that the “philological aspect of classical studies, which also has its modern origin in the European Renaissance, was wholly absent in the Arabic counterpart” (ibid., p. 155), for the obvious reason that the translation

activity is very selective since it is restricted only to scientific and philosophical writings excluding thus the humanities (such as literary and historical works). As a result of this methodologically worked out plan, the translation activity virtually ceased, as already mentioned, once its goal was achieved. Because of the advanced level reached by Arabic science in the eleventh century and reflected in the comprehensive philosophical and scientific work of Ibn Sīnā, there was no need to pursue Greek studies, for the “hurricane of Avicenna’s philosophy quickly swept such tendencies” (ibid., p 155, see Ardeshir, Bäck and Thom’s papers devoted to his encyclopedic thought). The second major difference is that the translation movement, as Gutas’s fascinating account demonstrates, is much more than the mere revival of Greek learning. First of all if by revival Gutas means translation then it should be reminded once more that it is not only Greek learning which was revived through the translator’s creative imagination but also the learning of other civilisations such as the Persian, Indian and even the Chinese. Second, the real intention of the translation project is not to revive the culture of previous civilisations, a task best left to the indigenous people, but the construction of knowledge according to a long term research programme.

Gutas describes the historical background of the foundation of the *Bayt al-ikma* and its later development as follows

It was a library, most likely established as a “bureau” under al-Man{ūr, part of the ‘Abbāsīd administration modelled on that of the Sasanians. Its primary function was to house both the activity and the results of translations from Persian to Arabic of Sasanian history and culture. As such there were hired translators capable to perform this function as well as book binders for the preservation of books. This was its function in Sasanian times, and it retained it throughout the time of Hārūn ar-Rashīd, i.e. the time of the Barmakids [the secretaries of the early caliphs]. Under al-Ma’mūn it appears to have gained an additional function related to astronomical and mathematical activities; at least this is what the names³ associated with the name *Bayt al-ikma* during that period would imply. We have, however, no specific information about what those activities actually were; one would guess research and study only, since none of the people mentioned was himself actually a translator (ibid., p. 58).

In this passage, Gutas wants to make the point, strongly emphasised afterwards, that the Greco-Arabic translation, the subject of his book, is not conducted in the *Bayt al-ikma*.⁴ As a result, the whole translation movement during the early ‘Abbāsīd era was conducted in two stages. (1) The first wave of translations of Persian heritage undertaken in the *Bayt al-ikma* (conducted under the ruling of al-Man{ūr (754-775)); (2) the Greco-Arabic translation represents the second wave of translations (from the time of al-Mahdī (775-785) onwards). One of the main reasons given by Gutas for denying any role of the *Bayt al-ikma* in the Greco-Arabic translation is that there is no mention of Greek works being stored on its shelves. To back his argument, he quotes 2unayn ibn Is1āq (d. ca. 873) who seems to have been complaining about the “efforts he expended in search of Greek manuscripts and again he never mentions that he looked for them right under his nose in the *Bayt al-ikma* in Bagdad” (p. 59). This might be the case. But 2unayn’s complaint might also indicate that Greek works were circulating in the society. Important manuscripts, which existed in a very limited number of copies, are not designed or expected to be stored in a public library. The absence of books from the shelves reflects their relevance to the concerns of society. This may explain why texts of humanities such as Persian, Ethiopian or Zimyarite manuscripts could be found in the *Bayt al-ikma* but not Greek ones due to their scientific nature. By excluding the *Bayt al-ikma* from playing any role in the Greco-Arabic translation, Gutas seems to create a gap between the two translation movements. A gap that he seems to narrow by appealing to the translation culture: “What the *Bayt al-ikma* did do for the Greco-Arabic translation

movement, however, is to foster a climate in which it could be both demanded and then conducted successfully” (p. 59). According to Gutas, two common points can be found between the two translation movements: (1) the obvious point is that they are both part of the translation culture widely prevailing in the region. Gutas reminds us indeed of the existence of “pre-Islamic translations into Pahlavi [the Persian language] of Greek scientific and possibly philosophical works” (p. 25). This explains the fact that the earliest translation of Greek works into Arabic are made not directly from the Greek but through Pahlavi. (2) The heavily involvement of the state apparatus though for entirely different political motivations. Actually, the contrast that Gutas is struggling to make is that the Persian-Arabic translations were temporary and narrower in scope than the Greco-Arabic translations. The first was confined to the political sphere while the second was a social phenomenon. Neither the structure of the *Bayt al-Ikma*, as was inherited from the Sasanians, nor state resources could cope with the scale of the second wave of translations. This explains the role of the private sector which seems to be absent or at least very limited in the first wave of the translations. The private sector stepped in to satisfy the growing demand for knowledge expressed by the society.

There is in fact a third point, not political but scientific one, which can indeed intimately link the Greco-Arabic translations to the Persian-Arabic translations and ultimately to the activities of the *Bayt al-Ikma*. Despite the little historical information available about the *Bayt al-Ikma*, it is known for sure that a number of astronomers and algebraists, such as al-Khwārizmī (d. 850), were employed full time in the *Bayt al-Ikma*, in the service of the caliph al-Ma'mūn (813-833). This evidence indicates that the activities undertaken in the *Bayt al-Ikma* were not confined to its original task, which consists in translating the Persian heritage, throughout its existence. The nature of such activities seems to be broadened to include research and study which prompt Gutas' suggestion made in the aforementioned passage: “Under al-Ma'mūn it [*Bayt al-Ikma*] appears to have gained an additional function related to astronomical and mathematical activities.” Research and study guess gains much assurances when we know that *Algebra* was not a Persian translated work but the result of al-Khwārizmī's studies and reflections on the Babylonian and Indian scientific practices (see Heffer's paper). In chapter V (i.e. two chapters later) devoted to Applied and theoretical Knowledge of his book, Gutas describes the circumstances (and the motivation) of the composition of *Algebra* which gives us a more specific idea on the nature of research practised by scientists of the *Bayt al-Ikma*

During early 'Abbāsīd times, however, Islamic law was also developing rapidly and algebra became an essential tool for working out all the intricate details of inheritance laws. Both of these applications are mentioned by Muḥammad ibn Mūsā al-khwārizmī himself in the introduction to his *Algebra*. Al-Ma'mūn, he says: ‘encouraged me to compose a compendious work on algebra, confining it to the fine and important parts of its calculations, such as people constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another where surveying, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned’ (ibid., p. 113).

The significance of the *Bayt al-Ikma* lies not only in the continuity of scientific research since it paves the way for more translations from both the farther eastern tradition (mainly Indian sources) and the western tradition (Greek sources), but also in setting the pattern of how future scientific activities should be conducted. By contributing to the emergence of a new scientific tradition, the translations and the scientific activities taken place in the *Bayt al-Ikma* explain Gutas' insight according to which “the translations movement should be seen [right] from the very beginning as a part of a research processes”⁵ whose aim is the construction of knowledge based on the constant interaction between theory and practice as was implemented by the early scientists working in the *Bayt al-Ikma*.

The details of such programme were clearly spelled out by the first Arabic philosopher al-Kindī (ca. d. 870)⁶, called so because his name was traditionally linked to the introduction of philosophy to the Islamic world. Its first step should be seeking to acquire it as he insists in his introduction of *On the First Philosophy*.

The knowledge of the true nature of things includes knowledge of Divinity, knowledge of Unity and knowledge of virtue and a complete knowledge of everything useful, and the way to it; and the distance from anything harmful, with precautions against it. [...] Devotion to this precious possession is, therefore, required for possessors of the truth, and we must exert ourselves to the utmost in its pursuit (al-Kindī 1974, p. 59).

The process of translations is the means to get rid of those linguistic elements that might jeopardize the universality of scientific writings, it tends to act as some sort of a filter through which only scientific thoughts are allowed to pass. The result of this process of acquisition is that knowledge becomes accessible to everybody. Because Arabic was the only global language in all walks of life and mainly in science and philosophy, knowledge is promoted to the international level. As a result, it is no longer linked to a specific culture but it becomes the property of all humanity.

The second step of the construction of knowledge is to work on its unification in the sense of putting together its various pieces which were collected from the previous civilisations.

It has been clear to us and to the distinguished philosophers before us who are not our co-linguists, that no man by diligence of his quest has attained the truth, i.e., that which the truth deserves, nor have the philosophers as a whole comprehended it. Rather, each of them has not attained any truth or has attained something small in relation to what the truth deserves. When, though, the little which each one of them who has acquired the truth is *collected* جُمِعَ, *something of great worth is assembled from this* اجْتَمَعَ مِنْ ذَلِكَ شَيْءٌ لَهُ قَدْرٌ جَلِيلٌ [...] Indeed this has been assembled only in preceding past ages, age after age, until this our time, accompanied by intensive researches, necessary perseverance and love of toil in that (our emphasis, al-Kindī 1974, p. 57).

The second step announces the following one which consists in building upon the achievements of previous civilisations. Al-Kindī tells us more precisely afterwards how the body of knowledge can be increased.

In the time of one man — even if his life span is extended, his research intensive, his speculation subtle and he is fond of perseverance — it is not possible to assemble as much as has been assembled, by similar efforts, — of intense research, subtle speculation and fondness of perseverance — over a period of time many times as long. [...] It is well for us — being zealous for the perfection of our species, since the truth is to be found in this — to adhere in this book of ours to our practice in all composition of presenting the ancients' complete statement on this subject according to *the more direct way and facile manner* على أقصد سبيله و أسهلها سلوكاً to be followed for those who take it; and completing that which they did not say completely, فيه قولاً تاماً, by following the custom of the language and contemporary usage, and insofar as is possible for us. (This) in spite of the disadvantage affecting us in this of being restrained from going into an extended discussion necessary to solve difficult, ambiguous problems (our emphasis, *ibid.*, pp. 57-58).

The third step amounts then to seeking the progress of knowledge and to facilitating its learning for younger generations and its transmission to future civilisations since it is conceived not as a finished product but as an ongoing process. As a result knowledge needs to be continually and constantly worked out and perfected by correcting and improving the inevitable shortcomings inherent to the achievements of previous civilisations for which they should not of course be blamed.

Our most necessary duty is not to blame ومن أوجب الحقَّ ألا نذمَّ anyone who is even one of the causes of even small and meagre benefits to us; how then shall we treat those who are responsible for many causes, of large, real and serious benefits to us? Tough *deficient in some of the truth* الحقَّ القاصرُ، و إن قَصُرُوا عَنْ بَعْضِ their thoughts which have become our *ways and instruments* سُبُلًا و الآتِ leading us to much

knowledge of that the real nature of which they fell short of obtaining (our emphasis, *ibid.*, p. 57, Ivry's translation is slightly modified).

According to the Arabic conception of knowledge, there is no such thing as perfect knowledge. This idea is so deeply entrenched in the Arabic-Islamic culture that it is expressed in a variety of ways by many proverbs, one of them is the following: "a man remains knowing as long as he searches for knowledge and continues to study. When he thinks he knows, he has become ignorant لا يزال المرء عالماً ما طلب العلم فإذا ظن أن قد علم فقد جهل."

Gutas is well aware of the fact that "renaissance" is not the appropriate word to describe the translation movement since the passage mentioned above is the only place where he brings it in the context of responding to other scholars. Throughout his whole book, he prefers rather to focus on the man whose vision and sagacity led to the foundation of the first scientific institution in history.

The crux of the matters seems to lie in al-Manṣūr's creation, after the 'Abbasid revolution, of a new social configuration in Bagdad through the genial idea of creating a new city. This meant, in essence, granting himself the licence to start everything anew by freeing him from constraints carried over from the previous *status quo* (Gutas 1998, p. 189).

The series of renaissances including the Renaissance proper appears to be then the result of the original creation of the famous house of wisdom from which all sprung.

In this context, al Manṣūr's adoption of a Sasanian imperial ideology becomes possible and meaningful, as does the establishment of the attendant translation movement. The process once set in motion, proceeded for over two centuries on its own (*ibid.*, p. 191).

These two crucial passages have far reaching implications on the periodisation of science. According to Gutas' analysis, it is the ninth century and not the Renaissance which should be the starting point not only of a series of renaissances but also of the scientific revolution. But he stops short from drawing such a conclusion for obvious epistemological reasons since he warns that his "book is not about Arabic science and philosophy" (*ibid.*, p. 192). The gap left by Gutas' approach between political and social history and the history of science has been precisely bridged by Tahiri's paper which provides the very badly needed epistemological backing for Gutas' underlying thesis since it reaches basically the same conclusion by analysing the history of astronomy. Further analysis of Arabic scientific and philosophical writings will provide further evidence for making the ninth century a landmark in the history of science and philosophy and will indicate how it should be viewed and remembered in the history of science.

3. The Arabic language and the unity of science

Historians of science and philosophy are usually selective in their choice of the kind of questions they seek to answer. One of the remarkable historical facts seldom noticed is that science and philosophy have been developing without interruption since the ninth century as the great French historian Pierre Duhem shows in his monumental *Le Système du Monde*. How can we explain, in the case of astronomy for example, the fact that this scientific discipline has made no progress whatsoever since the second century (and a fortiori for much older scientific disciplines like mathematics)? A particularly tempting answer is given by the recent rising tendency in the history of science: the lack of progress is due to extrascientific factors. According to the sociological interpretation of the history of science which is in fashion nowadays in the humanities, major gaps in the development of science cannot be explained intrinsically but only by appealing to the political, social and cultural context in which science and philosophy are developed. After all, according to this view, science is a social and cultural phenomenon since it is the making of human beings and its development is determined by the social environment in which scientists live and work. That is why the Dark

ages, the period during which science made no progress in Europe, has been entirely blamed on the Roman-Christian societies for its failure to generate the kind of change badly needed for the development of science. It seems thus that medieval Europe had to wait for the emergence of the Arabic Islamic culture to see the light at the end of the long tunnel. This is at least the conclusion drawn by Gutas' analysis.

Byzantine society, although Greek-speaking and the direct inheritor of Greek culture, never reached the level of scientific advancement of the early 'Abbasids and had itself later to translate *from Arabic* ideas that ultimately go back to classical Greece. In such an analysis, the contribution of individuals is also to be put in perspective. Sergius of Resh'aynā and Boethius, at the two antipodes of Greek cultural spread in the early sixth century, conceived of projects to translate and comment upon philosophy and the sciences as presented in the philosophy of Aristotle – and hence all knowledge, as understood in the Alexandrian scholarship of their age. The conception is to their credit as individuals; that they failed indicated the adverse circumstances of their environment (ibid., pp. 188-189).

Our analysis will show, however, that Gutas' conclusion is only half of the story. The other half is yet to be told. By focusing only on the extrascientific factors, there is the risk to neglect those epistemological and methodological considerations which might have influenced the lack of progress of science. Indeed, Gutas' work *Greek Thought Arabic Culture*, where he describes the political and the social factors that occasioned the translation movement, can be seen as a further support for the sociological interpretation of the history of science. Gutas justifies his approach by the fact that the translation movement as a social phenomenon has been very little investigated while “its significance for Greek and Arabic philology and the history of philosophy and science have been overwhelmingly studied to this day” (ibid., p. 2). He may have some point there but this might lead to overlook the fact that some crucial epistemological points with regard to the significance of the Arabic tradition has been missed out by most historians. Actually, while describing the political and the social context of what he calls the 'Abbasīd revolution, Gutas' work draws the attention to one of the these important central epistemological points in the development of Arabic science: namely the fundamental role played by the Arabic language in the development of science and philosophy.

The particular linguistic achievement of the Greco-Arabic translation movement was that it produced an Arabic scientific literature with a technical vocabulary for its concepts, as well as high *koiné* language that was fit to vehicle the intellectual achievements of scholarship in Islamic societies in the past and the common heritage of the Arab world today. [...] its significance lies in that it demonstrated for the first time in history that scientific and philosophical thought are international, not bound to a specific language or culture (ibid., p. 192).

This aspect of the contribution of the Arabic tradition to the history of science and philosophy has been ignored or widely underestimated. How could the progress of a major scientific discipline, like mathematics for example, be achieved had not its various parts, scattered for so many centuries from the East to the West, brought together by a unifying language? How could the awakening of science even be imagined if it was still encoded in a language no longer in use? For science to be developed the way it did, it needs the emergence of a nation that should have such an admiration for its language⁷ and a passion for knowledge⁸ that sets itself the historical mission of collecting, processing and translating all scientific data produced by previous civilisations and making the resulting systematic work worldwide available and easily accessible through the unprecedented spreading and circulation of books. Historically the Arabic language shows indeed for the first time the possibility of the construction of a unified corpus of knowledge able to work as a worldwide vehicle for transmission of scientific and philosophical thoughts from one language and science to another. As mentioned above, the production of an Arabic *koiné* language provided one of the bases of the notion of the unity of science within the Arabic tradition. This might also help to

understand why in the Arabic tradition the study of grammar and logic (see the chapter of Cornelia Schöck) — including poetics and rhetoric, was conceived as a kind of integration factor for all other fields on knowledge and science. Moreover, in the Arabic tradition grammar, poetics and rhetoric were seen as closely linked with what we would call nowadays a normative epistemic logic conceived as an extended organon for the search and transmission of knowledge. Logic and grammar were at the center of the creation of a scientific Arabic *koiné* language with precise epistemic and epistemological aims.

Rashed, one of the first distinguished historians who questions the current periodisation of science in his investigation into the development of mathematics between the ninth and the seventeenth centuries, suggests that what he calls the notion of *differential* is much more adequate in historical scientific studies than the dominant continuity/discontinuity approach, currently widely used in the history of science. Rashed argues that the notion of differential, when applied to the history of mathematics can be used as an instrument in assessing effectively the actual increase of mathematical truths by comparing the state of each mathematical branch (its results, methods and ways of reasoning) at two important times of its evolution (Rashed 1987, p. 360). Indeed this approach cannot only help us to adequately determine the *timing* of the emergence of a new scientific discipline but also to illuminate *how* science is viewed and understood by indicating the underlying motivation of the context of its development. This is the method that underlies the analysis of our introduction. More precisely, we think that Rashed's notion of differential can be fruitfully applied to study the uninterrupted development of science and philosophy since the ninth century in the Arabic tradition by comparing it with the approach of the ancient Greeks. Now, certainly this would involve us in the development of a long and difficult thesis but let us simply highlight very briefly some relevant remarks which we think will be sufficient to suggest the main lines of the analysis which follows of such a comparison.

4. Some remarks in relation to the heritage of the Greek approach to scientific inquiry

In his *Posterior Analytics* Aristotle imposes strict conditions on the definition of *episteme*. Knowledge is produced by a demonstration which, he asserts, “must proceed from premises which are true, primary, immediate, better known than, prior to, and causative of the conclusion” (71b20). It is clear for the Stagirite that the mere use of syllogism cannot produce knowledge since he insists on the fact that “syllogism will be possible without these conditions, but not demonstration; for the result *will not be knowledge*” (our emphasis). This makes it harder for disciplines other than mathematics to reach one day the episteme status since they cannot fulfill the very tough Aristotelian criteria — by the way, the axiomatics of Euclid could not be captured by syllogism. It seems thus that Aristotle actually calls knowledge, is that knowledge displayed in what we call nowadays formal sciences – some interpreters would include here metaphysics. Since by definition this kind of knowledge is of things that cannot be otherwise than they are i.e. a necessary knowledge, Aristotle introduces a sharp distinction between mathematics and empirical sciences. But when it comes to physics for example, Aristotle's task is to give a discursive and systematic explanation of all kinds of change. The problem of physics is according to him to find the “principles of perceptible bodies” (*On Coming-to-be and Passing-away*, 327b7). The main conceptual apparatus that he invents for this purpose is the famous four causes doctrine.

Now, the causes being four, it is the business of the physicist to know about them all, and if he refers his problems back to all of them, he will assign the ‘why’ in the way proper to his science (Physics II 7198a).

According to this view, knowledge in physics seems to be quite different from mathematics finding out all the four causes of any natural phenomenon. In his physical theory, he endorses Empedocles' fundamental idea that all substances are made of the four simple elements: earth, water, air and fire. Earth has some privilege in his explanation of motion. Though being made of the four elements, it is also the natural place of terrestrial objects. As for the supralunar world, the matter from which it is made, that he calls *aither*, is of a completely different order because of the eternal, circular and regular motion of the heavenly bodies.

Aristotle is indisputably the philosopher of antiquity. His conceptual apparatus lays down both what type of questions should be asked and the terms in which they should be answered. This explains why philosophers who followed closely Aristotle's framework contributed little to the development of science. Indeed the great advances in such subjects as mathematics or astronomy are the work of men who were primarily scientists and not philosophers and manage to escape his influence. However, despite important scientific achievements, Aristotle's physical doctrine remains unshaken and the domination of his philosophical system seems to be the last word of the Greek tradition. The Greek heritage is now in the hands of their successors though it seems that the Greeks did not care so much about their legacy as is suggested by the eminent classical scholar G. E. R. Lloyd's perspicuous remark: "there were many [of the ancients] who recognised that civilisation had developed in the past, there were few who imagined that it would or could progress much further in the future" (Lloyd 1972, p. 394). The lack of the idea of scientific progress in Greek culture, which has an impact on their philosophical and scientific approach, explains at least in part why we have to wait until the ninth century for the emergence of their immediate successors. In his comprehensive study, Lloyd sums up the whole ancient Greek approach to scientific inquiry as follows

Experimental method was only of very limited usefulness on the fundamental problem of physics, the question of the ultimate constituents of matter. Although quite simple experiments would have yielded useful information about the nature of certain compounds, the principal controversy between atomism and the qualitative theory of Aristotle, for example, was not one that could be settled by an appeal to either observations or experiments, since the controversy turned on the question of the type of account that was attempted. [...] A more important point is that such experiments as were performed by the Greeks were usually set out with the set purpose of supporting the writer's own theory. The appeal to experiment was an extension of the more usual notion of appealing to evidence: experimentation was a corroborative, far more than heuristic, technique. Tests were conducted to confirm the desired result, and it is only in late antiquity that we find examples where attempts were made to vary the conditions of experiments systematically in order to isolate causal relations. [...] Nevertheless the impression that much of the history of early Greek science leaves is one of the dominant roles of abstract argument (Lloyd 1970, pp. 140-141).

A second limitation is the little place given to practice in relation to theory which led to most of the philosophers after Aristotle to dramatically oppose the two activities. Theoretical studies which should be pursued for their own sake are extremely valued at the expense of practical arts which are viewed with disdain. This is true, as Lloyd explains, even for some scientific disciplines like medicine which is expected to be highly regarded for its noble cause.

Many of the most famous biologists were doctors, who were motivated in their research partly by the desire to improve the treatment of the sick, and sought to apply their knowledge to this end. Yet not even the most famous and successful doctors in antiquity entirely escaped the disdain usually felt for the craftsman. In the Greek scale of values the theorist was always superior to the technologist (Lloyd 1972, p. 395).

It is clear that empirical sciences, and with them theoretical studies, cannot flourish in a cultural context where the role of practical arts in the prosperity and the well-being of the society is heavily undermined by its top elite. Lloyd has rightly identified the huge gap

created by the Greek society between theory and practice as one of the main reasons preventing the development of scientific research.

The institutions where extensive investigations were carried out were rare throughout antiquity. The ancients lack the idea that dominates our own society, that scientific research holds the key to material progress. [...] The *raison d'être* of the Lyceum and Museum and of the many minor schools modelled on them was not any idea of *usefulness of scientific research*, but the idea of a 'liberal' higher education (our emphasis, *ibid.*).

The second main reason is the lack of co-operation and of scientific and philosophical exchanges because of the extrascientific motivations underlying the formation of many schools.

The development of science and mathematics required other factors as well, particularly the idea of co-operation in research. Here both the Pythagoreans and the medical schools (in their very different ways) had important contributions to make. But in neither case was the chief motive for these associations any idea of the value of scientific research for its own sake. Religious and political ties helped to keep the Pythagoreans groups together, and the medical schools were exclusive associations formed from professional motives like a medieval guild or a modern trade union. Moreover the doctors, like the Pythagoreans, were on occasion secretive about their discoveries (*ibid.*, p. 394).

More generally, the production of scientific and philosophical works and the spreading of ideas were greatly hampered by a deeply entrenched cultural tradition practised by many Greek philosophers who, because of their distrust of the written word, confine what they regard as their most important doctrines to oral teachings (*ibid.*, p. 383). A diametrically opposed stance is expressed by al-Jāhiz* (d. 868), one of the famous Arabic prolific authors⁹

Our duty is to do for those who will come after us what our predecessors have done for us. For we found more knowledge¹⁰ than they found, just as those who will come after us will find more knowledge than we did. What is the scientist waiting for to display his knowledge in the open, what prevents the servant of the truth from devoting himself without fear to the task that he was assigned, now that the word has become possible, the times are good, the star of caution and of fear is extinguished, a wind favourable to study is blowing, babble and ignorance are no longer current, eloquence and knowledge are circulating freely in the market? For a man does not find a teacher to train him and an expert to educate him at all times (Al-Jāhiz* 1969, I pp. 86-87).

On the methodological and epistemological levels, we find the already mentioned sharp distinction between mathematics and empirical sciences and mainly physics. In his *Almagest*, Ptolemy further widens the already existing gap between mathematics and physics by subordinating the latter to the former, the implication of this methodological decision and of his overall approach to astronomy will be convincingly refuted by Ibn al-Haytham (d. 1041),. The fourth limitation which is proved to have serious repercussions on the development of science is indicated by Ibn al-Haytham. He makes clear that his *al-Shukūk* is motivated first and foremost by epistemological considerations designed to break the deadlock caused by the Greek synthetic approach of exposing scientific theories which represents more an obstacle than an incentive to the progress of science since it closes the door for further theoretical research (for more details see Tahiri's paper).

What these shortcomings indicate is that Greek science and philosophy were developed in the context of Greek culture to a point that no further progress could be made unless deep changes in the approach to scientific practice happened. Any translation movement of Greek works would not be able to overcome these obstacles if the translation project was to be reduced to the only task of recovering and preserving the Greek heritage. The success of the translation project is due to the growing awareness that the scientific inquiry concerning nature as it was understood and practised by the Greeks was not able respond to the new questions and problems raised by this time. This awareness was actually brought to the

forefront by a major shift of focus from the heritage of the Greek idea of logos to the Arabic concept of knowledge.

5. Knowledge in the Arabic-Islamic culture

The 'Abbasīd dynasty¹¹ (750-1258) certainly gets great credit for making knowledge at the centre of their political strategy by working out and supporting the first ambitious scientific research project in history which gives rise to the surge of an intensive scientific and cultural activity in Bagdad led by the prestigious institution *Bayt al-Ikma*. By learning from the mistakes of the Umayyads' rule¹² (661-750), the 'Abbāsids succeeded where their predecessors failed. Short of full legal legitimacy, the ingenuity of the house of al-'Abbās lies in capturing the imagination of the Arabic-Islamic society by focusing, as we shall see later, on one of the fundamental components of its identity. The 'Abbāsids' strategy was a resounding success because it was a response to the demand of the society since the quest for knowledge had already begun in earnest. This sets a precedent in the Arabic-Islamic history since knowledge proves to be for the first time the only credible alternative by means of which the political body can effectively justify its rule. As a result of the vulnerability of the political power due to the conditional support of the legal authority, the emerging distinctive political and social configuration is that the body politics finds its rule dependent on its unlimited support for knowledge and not knowledge which should rely on the goodwill of the politicians. This outcome of the balance of power indicates that one of the main features of the political and social ideal favoured by the Islamic society is the one where the political power should be at the service of knowledge and not the other way round. By putting knowledge at the top of their political agenda, the 'Abbāsids wanted to show that their accession to power was a force for good and it was to some extent since they succeeded in winning the support of the majority of the Islamic society. This explains the remarkable longevity of their rule which reaches its climax with Hārūn al-Rashīd (786-809) whose legendary name is associated in the West with the famous *Arabian Nights*; but in the Arabic-Islamic conscience, he is remembered as one of the enlightened caliph (al-Rashīd literally means the well-guided) for chairing regular meetings of top intellectuals (jurists and theologians, linguists and grammarians, poets and writers, scientists and philosophers) to discuss hot topical legal, cultural and scientific issues.

But the development of Arabic science was undoubtedly not the work of politicians, it was the result of the unprecedented interactions among the intellectual elite whether they were jurists, grammarians, theologians, poets, scientists or philosophers, and its explanation must ultimately be found in the dynamics of Arabic culture and its specific approach to knowledge underlying the whole translation enterprise summarized by al-Kindī with the following words:

We ought not to be ashamed of appreciating the truth and of acquiring it wherever it comes from *و ينبغي لنا ألا نستحيي من استحسان الحق، و اقتناء الحق، من أين أتى الحق* even if it comes from races distant and nations different from us. For the seeker of the truth nothing takes precedence over the truth *الحق لا شيء أولى بطالب الحق من* and there is no disparagement of the truth, nor belittling either of him who speaks it or of him who conveys it. The status of no one is diminished by the truth; rather does the truth ennoble all (our emphasis, al-Kindī 1974, p. 58).

Al-Kindī's passage contains three crucial points which show the intertwining ethical and epistemological dimensions of the translation movement, namely:

- (i) The trans-national and trans-cultural conception of the unity of science
- (ii) Since each society can have some form of truth, the second step in acquiring knowledge, which is the harder task, is in recognising and appreciating it. The question now is how? The answer relates to the confluence of grammar, logic and Law

in the translation project — this point is not explicit in this paragraph but it links the first and the third point and has been developed by al-Kindī before (recall the passages quoted in section 2 above).

(iii) The supremacy of the truth (not authority), the search for which is the driving force behind the progress of knowledge, is the ultimate goal of the scientific inquiry.

In relation to the first point, it is important to see that the search for the unity of science involves a determined ethical perspective: the humility of learning from others and the ability of acknowledging one's own ignorance, and a social dimension: the need for seeking the interaction with other people. The idea of search for knowledge and its ethical and social implications is deeply entrenched in the Arabic-Islamic culture which goes back to the teaching of Islam i.e. to the seventh century.¹³ Indeed the Arabic people of the seventh century *knew that they knew* little about the external world, a fact eloquently expressed by the Qur'ān (Sūrat 17, verse 85) “و ما أوتيتم من العلم إلا قليلا” (you are given only a little knowledge).” Hence they are not only willing but what is more interesting they are ready to learn from the contributions of previous civilisations. The Arabic-Islamic society has thus no privilege over other societies since the latter can have something that the former does not have: some form of truth, knowledge, wisdom. The Arabic intellectuals of the ninth century such as al-Kindī and Ibn Qutaybah were just following the same Islamic teaching, that was followed by their predecessors, which makes seeking knowledge a duty for every believer. Ibn Qutaybah (d. 889) explains the rationale behind the search for knowledge

Knowledge is the stray camel of the believer العلم ضالة المؤمن; it benefits him regardless from where he takes it: it shall not disparage truth should you hear it from polytheists, nor advice should it be derived from those who harbour hatred; shabby clothes do no injustice to a beautiful woman, nor shells to their pearls, nor its origin from dust to pure gold. Whoever disregards taking the good from its place misses an opportunity, and opportunities are transient as the clouds. ... Ibn 'Abbās [the Prophet's uncle] said: “Take wisdom from whoever you hear it, for the non-wise may utter a wise saying and a bull's eye may be hit by a non-sharpshooter” (Ibn Qutaybah 1986, p. 48).

Since the Arabic-Islamic society cannot have the whole truth, it is urged by Islamic teaching to learn from a wide range of different societies by going as far as China.¹⁴ If knowledge fails to come to the Arabic peninsula, its inhabitants have instead the duty to go after it, this is after all one of the main *raisons d'être* of the existence of the human being according to the Islamic doctrine. This is what led Sabra to speak of the translated Greek works in terms of an “invited guest” which is warmly welcomed by the traditional Arabic culture. Respecting the culture of one's neighbours, no matter how different from the Arabic culture, and getting acquainted with the culture of very distant people appears to be the first step in acquiring knowledge. Acknowledging one's own ignorance amounts in fact to acknowledging the contributions of those people to the formation of the unity of science. Al-Kindī expresses here his deep sense of gratitude to all ancient civilisations on behalf of the newly rising Arabic-Islamic civilisation:

It is proper that our gratitude should be great فينبغي أن يعظم شكرنا to those who have contributed even a little of the truth, let alone to those who have contributed much truth, since they have shared with us the fruits of their thoughts and facilitated for us the true yet hidden inquiries, in that they benefited us by those premises which facilitate our approaches to the truth. If they had not lived, these true principles with which we have been educated towards the conclusions of our hidden inquiries would have not been assembled for us لم يُجتمِعْ لنا even with intense research throughout our time (our emphasis, al-Kindī 1974, p. 57).

In relation to point ii) and iii), it is important to see that the way to acquire knowledge implemented by the translation project is connected with a specific feature of the Arabic notion of knowledge that stems actually from the development of Arabic society before the translation era, namely the role of Law and Grammar. Both disciplines were considered very

early as scientific disciplines. They were and are even today the most important scientific disciplines for the Arabic culture because of the vital role they play in organising their social and cultural life. Moreover, as already mentioned in section 2 above, grammar and logic (including poetics and rhetoric) were conceived as the instruments of the scientific programme inherent to the notion of knowledge underlying the translation project. The link of knowledge with Law had the function of putting the scientific programme of knowledge acquisition into practice. The link of knowledge with logic had the function of designing a grammar of superior order able to render a language with the help of which different kinds of knowledge could be expressed and studied. Actually one might argue that this notion of knowledge stems from the use of the word *'ilm*. Indeed; the Arabic word عِلْم or *'ilm* can mean both science and knowledge and it is remarkably used by the Arabic tradition in a wide sense similar to our usage today and quite different from the Greek meaning of *logos* — if the latter is understood as a theoretical notion of knowledge separated from the notion of practice. It is Franz Rosenthal (1970) who connected the notion of knowledge of classical Islam, designed to introduce a major transformation in scientific and social practice, with Islam. In his researches, Rosenthal describes first the central position occupied by knowledge in the life of the Islamic society to the extent that he identifies knowledge as the distinctive character of the Islamic civilization:

'ilm is one of those concepts that have dominated Islam and given Muslim civilization its distinctive shape and complexion. In fact, there is no other concept that has been operative as a determinant of Muslim civilization in all its aspects to the same extent as *'ilm*. This holds good even for the most powerful among the terms of Muslim religious life such as, for instance, *tawīd* “recognition of the oneness of God”, *ad-dīn* “the true religion”, and many others that are used constantly and emphatically. None of them equals *'ilm* in depth of meaning and wide incidence of use. There is no branch of Muslim intellectual life, of Muslim religious and political life, and of the daily life of the average Muslim that remained untouched by the all-pervasive attitude toward “knowledge” as something of supreme value for Muslim being (Rosenthal 1970, p. 2).

If the Arabic *'ilm* can fairly be rendered by the English word “knowledge”, however Rosenthal finds that “knowledge” falls short of expressing all the factual and emotional contents of *'ilm*. His book is designed to explain how Islam has created a knowledge based-society to the extent that he concludes that “Islam is *'ilm*” (ibid, also chapter V). Rosenthal suggests that the root of *'ilm* has a strong pragmatical feature that seems to derive from the term عَلَّمَ *'alama* which means “way signs”:

For the Bedouin, he elaborates, the knowledge of way signs, the characteristic marks in the desert which guided him on his travels and in the execution of his daily tasks, was the most important and immediate knowledge to be acquired. In fact, it was the kind of knowledge on which his life and well-being principally depended (ibid., p. 10).

From this perspective, knowledge, *'ilm*, is designed to be put to some practical use since it is oriented towards action. More precisely, knowledge can be seen as a mode of action i.e. as a way of acting according to a certain purpose.

Rosenthal’s study of the notion of *'ilm* might also explain the relation between knowledge and الشريعة or *sharī'a*, i.e. Islamic Law, the prevailing understanding of Islam; and *sharī'a* means way since it is designed to show how Muslims should behave according to certain rules or principles. This is how Islam has always been understood by the Islamic society. Furthermore since it was the first scientific discipline to be set up, Law is the knowledge par excellence in two respects: (a) Law is knowledge in itself by establishing the principles and rules which guide the action of the individual and the society; (b) and metatheoretically the knowledge of Law is knowledge by indicating the way for the constitution of future scientific disciplines. Indeed by borrowing some of its central methodological elements such as of analogy, Law served as a model for the constitution of Grammar. Furthermore the notion of Law as a normative metatheory of knowledge becomes logic. Significant is that logic; knowledge of

knowledge, is also called *'ilm*. Logic has in classical Islam an epistemic character and an epistemological role. Logic is epistemic because it is about the relation between an individual and some proposition(s) and has an epistemological role because it enables us to study all kinds of scientific knowledge. Back to Rosenthal again:

Logic for the Muslims the 'organ' or 'instrument' (*ālah*), the instrument for logical speculation (*ālah an-naz*ar*), the instrument for each discipline (*'ilm*) and the means enabling the student to get its real meaning. It explained, and stood for, every one of the disciplines of knowledge. [...] It was the science of scales (*'ilm al-mizān*), weighing the correctness of every statement. It compared to 'an equilibrating standard' (*'iyar al-mu'addil*) by which the objects of knowledge are weighed.' It was 'the leader of the sciences' or 'chief science' (*ra'īs al-'ulūm*) [...]. It was, in a word, 'the science of knowledge' (*'ilm al-'ilm*) or 'the science of the sciences' (*'ilm al-'ulūm*) (ibid., p. 204).

The Andalusian encyclopedic thinker Ibn Zazm (d. 1064) further explains why logic (*manḥiq*, a noun derived from *nuḥq* which literally means speech) is knowledge of second order:

The *nuḥq* mentioned in this discipline is not speech (*kalām*). It is the *discernment* among things and the *thinking about* the sciences and the crafts, business enterprises and the management of affairs (our emphasis, ibid., pp. 203-204).

Logic is knowledge of second order because its subject matter is knowledge of first order i.e. the rest of scientific and social disciplines. Since logic is first and foremost knowledge, though be it of second order, it has a clear normative aspect, i.e. its purpose is

to provide all the rules (*qawānin*) that have the task of setting the intellect straight and of directing man toward what is right and toward the truth regarding any of *intelligibilia* with respect to which man may possibly err, all the rules that can preserve him from errors and mistakes with respect to the *intelligibilia*, and all the rules for checking on the *intelligibilia* with respect to which one cannot be certain that someone did not err in the past (Rosenthal 1970, p. 205).

The application of logic is universal and its purpose as we would say today is to determine valid statements for every domain of objects. The universal aspect of the normativity of logic with its epistemic character and its epistemological role has been summarised by al-Ghazālī's (1058-1111) definition of logic: "Logic is the canon (*qānun*), providing the rules and norms that is applicable to *all* human knowledge and on which *all* human knowledge rests" (our emphasis, ibid. p. 204). The formal nature of logic which consists in making explicit the structure of all scientific and social disciplines is considered by the Arabic tradition as the means by which knowledge could be unified as is rightly stressed by Rosenthal:

The history of logical studies in Islam remains to be written. [...] It is clear, however, that regardless of changes in approach and method, Muslim logicians never lost sight of the fact that the primary function of their labours was to find about "knowledge" and to contribute to a comprehensive epistemology for all aspects of Muslim intellectual endeavor, including theology and jurisprudence (ibid., p. 208).

The spirit of establishing rules and procedures for every scientific discipline which characterises the Arabic tradition explains also why geometry and algebra have come to be conceived by Arabic mathematicians as calculations (see Rashed and Heffer's contributions respectively). It turns out therefore that *ars analytica*, the metamathematical theory which has the task according to Ibn al-Haytham to provide the method of finding mathematical proofs, is nothing other than mathematical logic as Rashed brilliantly explained in section 3 of his paper.

According to the Arabic meaning, knowledge is useful. Its usefulness lies in being an action guide since it comprises some principles of prediction. From this point of view, Islamic Law and Arabic Grammar are scientific disciplines since they fix by means of rules the pattern of the behaviour of both the society and its language, such rules act as way signs which are designed to be followed in the future. Another striking feature worth mentioning is that النحو or *naḥw* which is the Arabic word for Grammar shares the same meaning as that of *'ilm* and

sharī'a since it also means “direction” or “way” and the context in which it is originally used means “follow this way”.

These aspects of the notion of knowledge of the Arabic tradition led to overcome one of the main weaknesses of the inherited tradition of Greek science: the already mentioned underestimation of the notion of experimentation. It is once more the relation between theory and practice which is at stake here and the Arabic scholars noticed that this feature of their notion of knowledge might lead to new advances in relation to the stagnated science of the ancient Greek tradition. Ibn Qutaybah, who is more known as a man of literature and linguistics than as scientist, devotes a whole book to the pre-Islamic astronomy in the introduction of which he declares his main motivation.

My purpose in everything that I reported here has been to confine myself to what the Arabs *know* about these matters and *put to use* (الإقتصار على ما تعرف العرب في ذلك و تستعمله), and to exclude that which is *claimed* (يُدَّعيه) by those non-Arabs who are affiliated with philosophy (المنسوبون إلى الفلسفة) and by mathematicians-astronomers (أصحاب الحساب). The reason is that I consider the *knowledge* of the Arabs (علم العرب) to be *knowledge that (1) is plain to sight* (الظاهر للعيان), (2) *true when put to test* (النافع لنازل البر و راكل البحر), (3) *and useful to the traveller by land and sea* (الصديق عند الإمتحان). God says ‘It is He who has appointed for you the stars, that by them you might be guided in the shadows of land and sea.’ [Qur’ān 6:97] (our emphasis and numeration; Ibn Qutaybah 1956, pp. 1-2).

This is in fact more than a mere provocation, it is a strong challenge to those astronomical works which either were translated or written following the Greek tradition. What is at stake here is the epistemological status of Greek scientific works: how can we know, let alone be sure, that a given discourse, among the various discourses concerning the nature of the physical world, is real knowledge and not simply a mere speculation. These epistemological and related questions concerning the nature of knowledge and its development have become quickly the dominant topic in the Arabic tradition as Gutas explains:

Because of the spirit of research and analysis it inculcated, different fields of scholarly endeavour unrelated to the translations gained in sophistication, a plethora of ideas was available for ready consumption, and the areas covered by the translation literature were no longer the only ones to impress the powerful minds. Intellectual debates of all sorts became the order of the day and patrons became interested not only in the *transmitted* knowledge from the Greeks but in the main problems posed by this knowledge and in the various ideological challenges to it (Gutas 1998, p. 124).

Giving the status of the knowledge in the Islamic society, the Arabic tradition has shifted the focus of research from logos understood as theoretical speculations to the research of a complex notion of knowledge, where philosophy had no privilege status. According to this view, knowledge is not and cannot be dominated by a particular profession and surely not by philosophers since it is usually compared to the depth and magnitude of an ocean the grasp of which goes beyond the capacity of one man or a section of the scientific community. By identifying itself with knowledge, the Islamic civilisation has conceived a distinctive and global project, of which the translations were only an important first step, for its intellectuals, whether they are jurists or theologians, grammarians or linguists, philosophers or mystics, scientists or artists, writers or poets, who are all invited to co-operate to its development. It is in this dynamic and diverse intellectual life that we have to understand Ibn Qutaybah’s intervention. Though being a non-specialist in science and philosophy, Ibn Qutaybah is not just criticising the Greek scientific tradition since he puts some concrete proposals to advance the debate. In the passage mentioned above, Ibn Qutaybah makes three interconnected suggestions to scientists and philosophers designed to help them check any discourse’s claim to knowledge. Namely:

1) A scientific discourse should be first intelligible, but what does it mean for a set of words and inscriptions to be intelligible? Hence the second suggestion:

2) A discourse concerning nature is intelligible if it can be put to the test. According to this view, a claim such as “a table is made of earth, water, air and fire” is an absurdity since it cannot be put to the test. The fact that according to this point of view; intelligibility assumes the possibility of testing suggests that the Arabic tradition would reject the thesis of incommensurability. This applies in particular to Ibn al-Haytham critics to Ptolemy’s *Almagest* discussed by Tahiri’s paper in this volume and in the first chapter of Rashed’s latest volume of *Mathématiques Infinitésimales du IX^e au XI^e siècle*.¹⁵ Relevant to our discussion is that Rashed discusses the semantical changes brought about in the traditional conceptual apparatus by Ibn al-Haytham’s attempt to elaborate an entirely new astronomical theory. We have here a concrete historical case of a scientific discipline going through the first critical transition of its evolution where semantical change goes hand in hand with theory change. More precisely and contrary to what the sociological doctrine wants us to believe, the emergence of the new theory assumed the intelligibility of the old theory — an intelligibility which was tested and subject of scientific controversies. It is, one might claim, within the dialogue triggered by scientific controversies that the semantical changes take place. In our example the point at stake is the notion of *falak* which was used by Arabic astronomers to translate the central concept of Greek astronomy *orb* which refers to the spherical bodies that cause the motion of the planets. In his *Configuration* however, Ibn al-Haytham is led to change its meaning in the sense of “the apparent path of a particular star in the sky ... without referring to the spherical bodies” (Rashed 2006, p. 44). This is the Arabic meaning of *falak* already strongly defended by Ibn Qutaybah. In his *Adab al-kātib* or *Education of the Secretaries*, he explains that *falak* means the “orbit (*madār*) of the stars with which they are associated *و الفلك مدار النجوم الذي يضمها*” (Ibn Qutaybah 1988, p. 69).¹⁶ It seems thus that Ibn al-Haytham reinstates the original meaning of the Arabic word. It can be fairly assumed that Ibn al-Haytham should have been aware of the controversy between the Arabic and the Greek approaches to astronomy since it was widely known (it was explicitly reported for example by one of his predecessors al-|ūfī (903-986) in his *Kitāb |uwar al-kawākib* or the *Book of Constellations*). It remains to be determined whether Ibn al-Haytham was specifically aware of Ibn Qutaybah’s philological arguments. It turns out that before Ibn al-Haytham, Ibn Qutaybah was one of the first leading critics of Greek astronomy by strongly expressing his deep dissatisfaction with the way astronomical research was conducted. Anyway in fact it is Ibn al-Haytham, more than any body else, who seems finally to satisfy Ibn Qutaybah’s requirements. Ibn al-Haytham’s powerful works signal a major breakthrough in scientific practice since they show that the intense theoretical researches undertaken since the beginning of the translation movement have finally begun to bear their fruits. This is one of the first major breakthroughs in the history of science in relation to the influence and heritage of Greek science. We would have liked to call it in fact a revolution in the proper sense of the word since this is what actually happened. The Arabic tradition has indeed turned upside down the Greek scientific practice.¹⁷

The onslaught on Greek scientific claims gathers momentum by spreading to other scientific disciplines like medicine. Ibn Māsaway (d. 857), a personal physician to the caliphal court¹⁸, seems to have learned Ibn Qutaybah’s lesson. He wants to put effectively to the test Galen’s medical claims by dissecting his son had the caliph, as he complains in the following passage, not intervened to prevent him from doing so:

Had it not been for the meddling of the ruler and his interference in what does not concern him, I would have dissected alive this son of mine, just as Galen used to dissect men and monkeys. As a result of dissecting him, I would thus come to *know the reasons* for his stupidity, rid the world of his kind, and *produce knowledge* for people by means of what I would write in a book: the way in which his body is composed, and the course of his arteries, veins, and nerves. But the ruler prohibits this (our emphasis).

Ibn Māsaway's story¹⁹, that recalls Abraham's sacrifice, illustrates how the son was offered up as a sacrifice to scientific knowledge (and held up not by the intervention of the Divinity but of the ruler Law). It seems to me that Ibn Māsaway's statement expresses too the attitude of the whole Arabic scientific practice towards the Greek scientific and philosophical discourse which is held not as truth but as just claims needing to be carefully checked and systematically tested. Dissecting the Greek logos with the aim of producing knowledge is the hallmark of the period of the translation movement which reaches its climax in the eleventh century when Ptolemy's optical theory was overthrown by Ibn al-Haytham's *al-Manāz'ir* (or *Optics*) and his *Almagest* was completely discredited by *al-Shukūk*.

Ibn Qutaybah's second suggestion actually involves two powerful incentives to the progress of science. The first is a heuristic one by directing theoretical research to subjects where testing claims and counterclaims are possible. The second is methodological: scientists are prompted to devise adequate methods and instruments aimed at testing their hypotheses. The underlying idea is that the refutation of the proponent's claims should not be purely rhetorical. Real arguments and counterarguments should be fully substantiated and systematically backed by hard evidence.²⁰ Understanding what is said and to be sure of its truth-value by systematically testing its content are two heuristic suggestions designed to check the claim of a discourse to knowledge. The second suggestion announces in fact the third since the link of knowledge to testing involves some form of a twofold action: the action of testing and the result of a knowledge aimed to improve a given practice

3) Ibn Qutaybah's last point, which he further supports with the verse from the Qur'ān, that the Arabic astronomical knowledge is "useful to the traveller by land and sea" remarkably confirms Rosenthal's insight into the pragmatism of the Arabic understanding of knowledge. The crux of Ibn Qutaybah's point is that the truth of any theory must be reflected in its ability to trigger some practical benefits at some stage of its development. Furthermore by requiring from a physical theory to be of practical benefit, Ibn Qutaybah puts a strong pressure on scientists and philosophers to justify the huge resources devoted to theoretical researches. They have to show in particular that their inquiries are not simply a waste of time and money but they are relevant to the needs of the society by yielding tangible results. It appears thus that Ibn Qutaybah's third suggestion is the ultimate test for any discourse on nature since any acquired knowledge must yield sooner or later some concrete results. The point at stake actually is the relation between theory and praxis which in the Arabic tradition seems to involve a non-vicious circle known nowadays as internal pragmatism: theory should improve practice and practice should improve theory. This explains why the Arabic tradition closely binds theory to experience. We have witnessed indeed in this period an unprecedented surge of interest in all kinds of empirical science. Contrary to the stagnated heritage of the Greek culture which fails to see the role of practice in shaping scientific theories, the Arabic tradition has cultivated the modern way of doing science by developing theoretical scientific branches, like mathematics for example, for their own sake and at the same time putting them at the service of empirical sciences (actually Ardeshir's paper indicates that Ibn Sīnā makes what seems to be the first clear distinction in history between pure and applied mathematics). Geometry was masterfully used in architecture and agriculture; algebraic techniques were conceived to assist Islamic laws and to stimulate trade by facilitating commercial transactions; astronomy was developed to respond to religious and other practical needs giving rise to the emergence of practical astronomy: many observatories were built for more accuracy and lasting observations; hospitals were set up to benefit from and to direct medical researches; etc. to put it shortly, science has never been in action as it was in the Arabic tradition as a result of closely tightening theory and practice. The close combination of علم ('ilm) and عمل ('amal, the Arabic word for action) is effectively and definitely crystallized in the Muslim mind by the Arabic language due to the similarity of the two words in sound and meaning to

the extent that it becomes unthinkable to conceive knowledge without corresponding actions as is articulated by Ibn Qutaybah: “if there were no action, one would not search for knowledge, and if there were no knowledge, one would not search for action *العَمَلُ لَمْ يُطَلَّبِ الْعِلْمُ*” (Ibn Qutaybah 1986, II p. 141). The nature of the relationship between knowledge and action is further studied by al-Ghazālī for whom knowledge is the form of action or as we would say today the construction of a procedure since “action can take form only through knowledge of the manner in which the action can be undertaken” (mīzān, p. 328). It is thus not surprising that the ability to match science with action is the basic skill required from the education of future civil servants of the empire by the influential writer Ibn Qutaybah whose *Education of the Secretaries* was offered to Ibn Khāqān, a senior Secretary of State.

In addition to my works [which provide linguistic, literary, and religious training], it is indispensable for the [secretary] to study geometrical figures for the measurement of land in order that he can recognize a right, an acute, and an obtuse triangle and the heights of triangles, the different sorts of quadrangles, arcs and the other circular figures, and perpendicular lines, and in order that he can *test his knowledge in practice on the ground and not on the survey-registers* *التَّفَاتُرِ الْأَرْضِيِّينَ لَا فِي الْمَخْتَبَرِ لَيْسَ كَالْمُعَايِنِ*. The Persians [i.e. the Sasanians] used to say that he who does not know the following would be deficient in his formation as state secretary: he who does not know the principles of irrigation, opening access-canal to waterways and stopping breaches; [measuring] the varying length of days, the revolution of the sun, the rising-points [on the horizon] of the stars, and the phases of the moon and its influence; [assessing] the standards of measure; surveying in terms of triangles, quadrangles, and polygons of various angles; constructing arched stone bridges, other kinds of bridges, sweeps with buckets, and noria waterwheels on waterways; the nature of the instruments used by artisans and craftsmen; and the details of accounting (our emphasis, Ibn Qutaybah 1988, I p. 15).

The underlying idea is that a purely descriptive theory has less value if its assertions cannot be translated into practice since the aim of science is not to describe nature which is the Greek way of inquiring (through logos) but to produce knowledge by effectively acting upon it. It is this outstanding insight which led the Arabic tradition to ignore the sharp demarcation lines drawn by the Greek imagination that keep the various scientific disciplines quite apart. But the practical benefit goes beyond the material aspect of theoretical researches. The usefulness of a scientific theory should nevertheless be understood in a wider sense including the possible application of its concepts and forms of reasoning to another theoretical, empirical or even social discipline. Logical concepts were fruitfully used in Grammar and the analysis of the Arabic language, logical rules were applied to legal reasoning, Ophthalmology was fully and definitely integrated into Optical studies, Algebra was closely developed in conjunction with Geometry, Arithmetic was effectively applied to Algebra, etc. Was this interdisciplinary approach a happy coincidence or something which was carefully worked out? One of the remarkable features of many Arabic and Islamic intellectuals is the encyclopedic nature of their formation which was sustained throughout the classical Islamic era from al-Kindī to Maimonides, to refer just to those major figures who are known to the western historians (see Rashed's paper). Gutas has rightly emphasised the crucial role played by the encyclopedic formation in al-Kindī's top objective

It is important, first of all, to keep in mind that al-Kindī was not a philosopher in the sense that he was only or primarily a philosopher. He was a polymath in the translated sciences and as such very much a product of his age. He wrote on all the sciences mentioned above: astrology, astronomy, arithmetic, geometry, medicine. This broad and synoptic view of all sciences, along with the spirit of encyclopedism fostered by the translation movement for the half century before his time, led him to develop a *research program* whose aim was *to acquire and complete the sciences* that were transmitted from the ancients (our emphasis, Gutas pp. 119-120).

The underlying idea of encyclopedism is that science is conceived as a whole or unity and not as a mere collection of scientific disciplines which have nothing to do with each other, and the cross-fertilisation of the various scientific branches is the means by which the whole body of knowledge can make further and sustained development. Al-Kindī's answer to the fundamental task that he sets himself inaugurates a new and fruitful approach to science. By seeking the progress of knowledge through the cross-fertilisation of scientific disciplines, the first Arabic philosopher introduces a major shift in the role of the philosopher since his path was closely followed by all his successors. Indeed al-Kindī's successors have further specified that logic, as explained above, is the knowledge which could unify all knowledge. Strikingly, we have to wait until the twentieth century to see the very same idea explicitly expressed by Otto Neurath²¹:

Encyclopedism based on logical empirism was the general historical background which underlay the proposal of an international encyclopedia of unified science. The general purpose of the *International Encyclopedia of the Unified Science* is to bring together material pertaining to the scientific enterprise as whole. [...] The collaborators and organizers of this work are concerned with the analysis of sciences, and with the sense in which science forms a unified encyclopedical whole. The new *Encyclopedia* so aims to integrate the scientific disciplines, so to unify them, so to dovetail them together, that advances in one will bring about advances in the others (Neurath 1938, p. 24).

That is what all the papers of the present volume have in common: they illustrate the idea of the unity of science in the Arabic tradition by exposing the connection, established by Arabic scientists and philosophers, between different scientific disciplines that contributed to the growth of knowledge. Bearing in mind that this is just only a start and a sample of the way in which interdisciplinary scientific exchanges were constantly sought and systematically practised throughout the classical Islamic period, we hope that our volume will inaugurate a new and fruitful approach to the study of the Arabic tradition. Furthermore, according to our view, the aim of this volume coincides with the general aims and motivation of the whole collection *Logic, Epistemology and the Unity of Science*. One can even see the research project of the encyclopedists as a resumption of an old research programme that goes back to the first Arabic philosopher, but this is the start of another story.

The book is divided into two parts, the first on Epistemology and Philosophy of Science and the second on Logic, Philosophy and Grammar. Ibn Sīnā receives the lion's share in both parts. This is hardly surprising given the great interest in Ibn Sīnā's philosophy due to the wide availability of his philosophical and scientific writings and to both the originality of his thought and his encyclopaedic approach to knowledge. Scholars and historians have come to recognise Ibn Sīnā's works as a watershed in the history of Arabic science and philosophy. As more Arabic philosophical and scientific documents become available, it can be expected that in the coming years we will witness new research on other major Arabic-Islamic thinkers with a similarly thorough and in-depth investigation as that on Ibn Sīnā.

II OVERVIEW

As already mentioned, Part I contains papers which focuses on the connection between, epistemology and science. In the first chapter of this part Mohammad Ardeshir discusses the question of the foundation of mathematics underlying Ibn Sīnā's philosophy: what and where are mathematical objects? From the analysis of the role of abstraction in the emergence of some fundamental concepts such as existence, object and unity, Ardeshir concludes that for Ibn Sīnā mathematical objects are those that have mental existence. In relation to epistemology of mathematics Ardeshir discusses Ibn Sīnā's answer to the question: how can we know mathematical objects? Ardeshir explains that for Ibn Sīnā intuition and thinking

involved respectively in the discovery of mathematical propositions and the construction of mathematical proofs are eventually the means by which mathematical knowledge is attained.

Deborah Black's paper goes a step further in investigating Ibn Sīnā's epistemology by tackling the difficult question of self-knowledge. The question now is not how we know mathematical objects for example but how we know that we know mathematical objects? To deal with the complex problem of self-knowledge, Ibn Sīnā adopts a new way of reasoning that we nowadays call thought experiment. Black explains that Ibn Sīnā recognises two distinct levels of self-knowledge. (1) Primitive self-awareness: soul's awareness of itself and (2) reflexive self-awareness, which comes from our awareness of cognizing some object other than ourselves. But for Ibn Sīnā, the latter is a kind of second order knowledge and it presupposes primitive self-awareness which ensures the unity of the soul's operations. Black presents Ibn Sīnā's flying-man-argument - that might be considered one of the earliest uses of a mental experiment – in order to discuss the relation of self-awareness to the other reflexive varieties of self-knowledge. The paper could be seen as Ibn Sīnā's answer to some of the questions that Hans van Ditmarsch searches in his contribution on Ibn Khaldūn.

Albrecht Heffer's contribution challenges the prevailing myth according to which "European mathematics is rooted in Euclidean geometry". This view, cultivated and sustained by modern historians of mathematics, was influenced by the growing epistemological dominance of the Euclidean ideal doctrine from the seventeenth century onwards. "Mathematics consists entirely of calculations" seems to be the conclusion drawn by Wittgenstein following the collapse of Hilbert's aprioristic programme. Heffer ironically finds that this image of mathematics as procedures performed on the abacus fits in very well with pre-seventeenth century conception of mathematical knowledge. He shows how the practice of algebraic problem solving within the abacus tradition, which leads to the emergence of symbolic algebra, grew out of Arabic sources. Indeed early Arabic algebra provides rules and procedures for solving problems and the validity of the rules was accepted on the basis of their performance in problem solving. The prime motivation of Heffer's analysis of the basic concepts of early Arabic algebra is to provide an explicitation of the epistemic foundations of the conception of mathematics-as-calculation developed in the Arab world. Interesting is the fact that the conception of mathematics-as-calculation is not confined to algebra but seems to be a more unifying approach to the practice of mathematics since it is also applied to geometry (see Roshdi Rashed's contribution, third section).

In his *Ibn Sīnā naturalized epistemology*, Jon McGinnis reveals the dynamic aspects of the author of *al-Shifā*'s epistemology when it applies to empirical sciences. McGinnis focuses on the study of *Kitāb al-Burhān* which attracted little attention so far from the scholars and in which Ibn Sīnā exposes what can be called his theory of the logic of scientific discovery. The study is divided into two sections. The first treats Ibn Sīnā's theory of demonstrative knowledge, and how Ibn Sīnā envisions the relation between logic and empirical science, where it is argued that one of the primary functions of *Kitāb al-Burhān* is to provide heuristic aids to the scientist in his causal investigation of the world. The second half concerns Ibn Sīnā's empirical attitude in *Kitāb al-Burhān* towards acquiring the first principles of a science, where such cognitive processes as abstraction, induction and methodic experience are considered. McGinnis discusses Ibn Sīnā's scepticism towards empirical induction and Ibn Sīnā's preference for methodic experience (*tajriba*). Method experience; explains McGinnis, is a type of reasoning that applies to empirical science and purports the need of revision when new empirical data become available. According to McGinnis' paper, it turns out that the kind of logic suitable for the formalisation of empirical sciences intended by Ibn Sīnā is not

deductive logic but something what we would nowadays call some kind of non-monotonic and/or *ceteris paribus* reasoning.

Roshdi Rashed paper tackles the following crucial question: is there a philosophy of mathematics in classical Islam? If so, what are the conditions and the scope of its presence? To answer these questions, it is not sufficient, points out Rashed to present the philosophical views on mathematics, but one should examine the interactions between mathematics and theoretical philosophy. Rashed' paper proposes to tackle the question in a new and unexplored way and that touches the main conceptual target of our volume, namely: the unity of science in the Arabic tradition. Indeed, as remarked by our author, the links between mathematics and philosophy are sometimes tackled in the works of the philosophers of Islam as al-Kindī, al-Fārābī, Ibn Sīnā, etc.; but in a so-to-say totally external way. In fact, there is a remarkable lack of studies aiming to understand the repercussions of the mathematical knowledge of the thinkers of classical Islam on their philosophies, or to discuss the impact on their own philosophical doctrines of their activities as scientists. Rashed's arguments Mathematics has provided to theoretical philosophy some of its central themes, methods of exposition and techniques of argumentation. The aim of Rashed's paper is to study some of the numerous interactions between mathematics and philosophy, in the context of tackling the question of philosophy o mathematics in classical Islam. More precisely, some of the themes discussed in this rich paper are mathematics as a model for the philosophical activity (al-Kindī, Maimonides), mathematics in the philosophical syntheses (Ibn Sīnā, Naṣīr al-Dīn al-Ḥaythamī), and finally the constitution of *ars analytica* (Thābit ibn Qurra, Ibn Sīnā, al-Sijzī, Ibn al-Haytham). From the point of view of logic; this remarkable paper can be also understood as complementing the studies of Ahmed, Schöck and Thom who studied the interactions between logic; grammar and metaphysics but did not tackle the interaction between logic and mathematics as Rashed does.

Hassan Tahiri's paper stresses the epistemological consequences of Ibn al-Haytham's *al-Shukūk*. The author presents Ibn al-Haytham's systematic refutation of Ptolemy's *Almagest* as paradigmatic for the creative attitude of the Arabic Tradition towards the heritage of Greek science. Contrary to his *Optics*, explains Tahiri; *al-Shukūk* is not only a book of science but a book about science since it is motivated by epistemological considerations designed to break the deadlock caused by the Ptolemaic exposition of science. Tahiri's main contribution is that it bridges the assumed historical gap between ancient and modern science by emphasising on the huge impact of *al-Shukūk* on later astronomical researches up to Copernicus. This historical fact, which shakes the basis of the received view's claim according to which Copernicus' *Revolutionibus* was the starting point of the scientific revolution, should, according to Tahiri; prompt the historians to revise the prevailing periodisation of the history of science. One remarkable point suggested in Tahiri's paper is his perspective on controversies that offers a new way to understand the relation between logic, epistemology and the role of the Arabic tradition. The point suggested by Tahiri is that through controversies; particularly in relation to the heritage of Greek science, the Arabic tradition expressed one of their most important achievements: the development of countermodels to the stagnated model of the ancient Greek science and therefore motivated the impulse to unexplored new paths of scientific enquiry.

Part II is composed of papers which exhibit the connection between logic, philosophy and grammar and starts with a paper of Asad Ahmed on the dichotomy *jiha-mādda* in the work of Ibn Sīnā as compared with the Greek version *tropos-hūlē*. The paper begins with the study of the word (*tropos*) in Aristotle, it shows how it became a technical term for the Commentators;

how, as part of *eidos*, it came to be dichotomous with *hūlē*; how the *eidos-hūlē* and *tropos-hūlē* dichotomy was known to al-Fārābī; how Ibn Sīnā inherited this dichotomy; and finally, what role this dichotomy, along with several associated concepts, had to play in Ibn Sīnā's modal logic. According to Ahmed, the dichotomy *jiha-mādda* seems to have become a determining factor for Ibn Sīnā's conversion rules of modal propositions and thus play a central role in his modal syllogistic. Moreover, the author suggests that this distinction is at the base of the distinction between unconditioned and conditioned necessity expressed by the couple *dh#tī/wa{fī*. While this paper explores the purely logical reasons underlying these distinctions. Cornelia's Schöck and Paul Thom's contributions to our volume complete the picture of these and related notions by providing the grammatical and metaphysical background.

Allan Bäck chooses, in his paper, to deal with the epistemological implication of a socio-cultural phenomenon which pervades our modern societies: multiculturalism. According to the author, the fact of the matter is that the emergence of the multiculturalism doctrine or at least its current surge can be seen as symptomatic of the abandonment of the flawed systematic philosophical approach, either to the foundation of science or to the explanation of its development, following the epistemological triumph of the historical-sociological approach to science. The author explains what is wrong with the current understanding of multiculturalism which, according to the view of Bäck, is related to the politically corrected practice reflecting the balance of power of the various conflicting social forces rather than to a philosophical position. The Arabic-Islamic tradition offers another approach to multiculturalism based on the principle of diversity which succeeded in producing a more tolerant society in which different communities lived together side by side according to their own customs and beliefs but without degenerating into a kind of relativism that serves as a justification of the "equal validity of all cultures." The search of certain form of unity or objectivity into the extant diversity seems to be the hallmark of the Arabic-Islamic tradition. *Islamic Logic* is designed to show how this approach is actually implemented in logical studies and more precisely in the investigation conducted by some Arabic-Islamic thinkers into the relationship between Greek logic and the Arabic language. It might be worth mentioning that the author of this contribution begins by discussing a motivation and invitation letter penned by Shahid Rahman. Now, perhaps it might be important to point out that Rahman's aim was to avoid contributions where the main argument is to show that; Arabic author X, wrote the same as the nowadays author Y of the European (and modern) tradition. Interesting is the fact that Bäck's paper brings out what the editors were seeking for: a new alternative concept to our modern notion of multiculturalism based in the study of the Arabic tradition.

Hans van Ditmarsch contribution relates to the work of Ibn Khaldūn who was a 14th century historiographer. From a family originating in Seville, prior to its conquest ("reconquest") by the king of Castille, Ibn Khaldūn lived an itinerant life serving as a magistrate for Spanish and Moroccan Islamic courts. He is very well known in History but his epistemological and logical writings have not yet captured the attention of the specialists in the field. The unfortunate lost of Ibn Khaldūn's book on logic is a major impediment to the study of his thought on those issues. Hans van Ditmarsch, an international expert in dynamic epistemic logic, explores those fragments of Ibn Khaldūn's *Prolegomena*, the *Muqaddimah*. More precisely, the hypothesis van Ditmarsch was trying to confirm or reject was whether Ibn Khaldūn considered the three properties of knowledge as formalized in the logic S5: truthfulness, positive introspection, and negative introspection. In a recent publication of the author — not accidentally — entitled 'Prolegomena' refers to the existence of text fragments

that suggest that the answer to that tripartite question is: yes, yes, no. The two relevant parts in Ibn Khaldūn's *Prolegomena* studied by van Ditmarsch are the chapters 'on reflection', and 'on the nature of human and angelic knowledge' in volume 2 (426-430 and 433-435), and a chapter "logic" in volume 3 (149-160). The author summarises these notions as follows. Reflection is the faculty that distinguishes humans from animals, who only possess the faculty of perception. Reflection provides proof of the existence of the human soul, because it allows us to know things that are not directly observed. Reflection also allows us to interact with the sphere of angels. The power of reflection can be measured as the maximum length of a cause-effect chain: "some people can still follow a series of five or six", and as the ability to avoid actions that result in unpleasant consequences (the remark on the power of reflections suggests for the modern modal logician transitivity of the knowledge operator). It is tempting to see such reflection on acquired knowledge as a form of introspection in the modern epistemic logical sense. It is then comforting for a modal logician that awareness of knowledge provides proof of the existence of the soul. That knowledge of something corresponds to its being true seems also easily read into various phrases. The author did not find a reference to negative introspection. However it worth recalling that in the epigraph on the introduction of our volume it has been stated that the awareness of not knowing is considered to be in the Arabic tradition a condition of learning. Certainly this is a weaker statement than negative introspection that requires that for any proposition p if we do not know p , then we know that we do not know it.

The contribution of Ahmad Hasnawi's is intended to shed new lights on the little known but complex issue of the treatment of the quantification of the predicate by Ibn Sīn# and contains the first translation of the first two chapters of *Al-'Ibāra* — the third book of the logical collection of his philosophical encyclopedia entitled *al-Shifā'* (The Cure). Ahmad Hasnawi's paper can be seen as a response to Wilfrid Hodges's forthcoming "Ibn Sīn#'s *Al-Ib#* on multiple quantification: how East and West saw the issues" (presented at Trinity college's colloquium on the Aristotelian *Peri Hermeneias* 2005). Among the points discussed by Hodges, we mention in particular the reduction of the sixteenth doubly quantified sentences generated by the adjunction of the four quantifiers (every, not any, some and not every) to the subject-predicate sentences. Unlike Ammonius and Tarán, Hodges points out that Ibn Sīn# succeeded in halving the list by "noting that if we replace the subject determiner in one of these sentences by its contradictory, then we get a sentence that is true if and only if the original sentence was false." For Hodges then, this is not a rule because Ibn Sīn# fails to further halve the resulting list. According to Hodges, the real rule applied by Ibn Sīn# and that prevented him from conducting the second reduction is stated much later. Hodges formulated it as follows: "In a sentence with a determined predicate, take the predicate as a whole, including the determiner, and regard it as a single universal". A claim challenged by Ahmad Hasnawi's paper. First of all, he reminds the reader that Ibn Sīn# broadened the study of the quantification of the predicate by systematically discussing the significance and the logical status of singular and indefinite sentences. On the question of double quantification, Hasnawi argues that what Hodges considers as a mere observation, which allows Ibn Sīn# to halve the list of sixteen doubly quantified sentences, is in fact a rule since it follows a systematic procedure. And contrary to Hodges' claim, Hasnawi mentions a passage where Ibn Sīn# states the equivalence of two sentences of the reduced list indicating that he was aware of the possibility of reducing further the remaining eight sentences. This evidence suggests, according to Hasnawi, that Ibn Sīn# seems to be more interested in the systematic explanation of the quantification of the predicate, designed to interpret the logic of doubly quantified sentences on the model of the sentences with an indefinite predicate (S is not-P), than with the systematic reduction of the doubly quantified sentences. More significantly, Ibn Sīn# calls

“deviating” propositions such propositions where the predicate is quantified because, according to Ibn Sīn#, they do not correspond to the common use of language. That is why Ibn Sīn# declares that “there is no great utility in studying them in depth” since they have little application. This, according to Hasnawis’s Appendix II; explains also why Ibn Sīn#’s successors seem to follow his advice by generally dismissing deviating propositions from their logical studies. An important point missed out by Hodges since his paper gives the misleading impression that Ibn Sīn#’s treatment of the quantification of the predicate is representative of the entire eastern tradition.

Cornelia Schöck tackles the issue of the relationship between Neoplatonic and Peripatetic metaphysics and logic on one hand and Arabic grammar on the other hand. She first reminds the reader of the little known fact that this relationship has its roots in a much older dispute between the grammarians and the theologians (*mutakallimūn*) in relation to the meaning of the “derived name” (*ism mushtaqq*). By broadening the perspective of her investigation, Schöck seeks to explain the origin of the distinction between the understanding of predications ‘with regard to essence/essentially’ (*dh#ī*) and ‘with regard to description/descriptionally’ (*wa{fī*). The first is derived from a technical term of Aristotelian logic, namely the logical term “essence”, and the second comes from Arabic grammar. On the basis of the grammatical distinction of the Arabic notion of “derivation” (*ishtiq#q*), Schöck shows how Ibn Sīn#’s logico-linguistic analysis arrived at his famous two types of use of the ‘derived’ (*mushtaqq*) in language. Schöck explains that according to Ibn Sīn# “the derived” (*al-mushtaqq*) — namely “[the name of] the agent” (*[ism] al-f#il*) and “the description/attribute which is similar to [the name of] the agent” (*al-ifa al-mushabbaha bi-l-f#il*) (cf. above § 4) — can be used in language to indicate five different meanings, namely:

[1.] It can stand ‘with regard to essence/essentially’ (*dh#ī*) to indicate:

[1.a] an essence and a quiddity to which is attributed an essential potency and quality, as for example ‘rational’ (*n#iq*) in the statement ‘All rational have the power of volition’;

[1.b] an essence and a quiddity to which is attributed a passive-potency (*quwwa*) to be in a state (*1#l*) of being and to be in a contrary state of being, as for example ‘moving’ (*muta1arrik*) in the statement ‘All moving are resting’;

[1.c] an essence and a quiddity to which is attributed an active-potency (*quwwa/qudra*) for an action (*fi’l’amal*) and for a contrary action, as for example ‘speaking’ (*n#iq*) in the statement ‘all speaking are keeping quiet’ or as for example ‘standing’ (*q#im*) in the statement ‘all standing are sitting’.

[2.] It can stand ‘with regard to description/descriptionally’ (*wa{fī*) to indicate:

[2.a] an essence and a quiddity to which is attributed a quality (*kayfiyya*) by which the substance is in a state (*1#l*) of being, as for example ‘moving’ (*muta1arrik*) in the statement ‘All moving are changing [when moving]’;

[2.b] an essence and a quiddity to which is attributed a quality (*kayfiyya*) by which the substance is connected (*muqtarin*) (cf. above § 6) and related (*mu*#f*) (cf. below § 8) to an acting/doing (*fi’l/fa’l’amal*), as for example ‘walking’ (*m#shin*) in the statement ‘All walking are changing [when walking]’.

One of the most significant products of this process of mutual rapprochement between grammar and logic, the author points out, is the synthesis of the Aristotelian accidental predication with the Arabic ‘description’ (*wa{fī*). This explains why statements of empirical sciences belongs to the *wa{fī*-reading in which the necessary relation between the two terms is restricted to the time of the duration of the attachment of an accident to the essence and substance denoted by the subject-term. This is the time *when* the essence and substance is described as either being in a certain state (*1#l*) or as performing an action (*fi’l’amal*).

Schöck further examines the metaphysical implications of the *dhātī/waʿfī* distinction. She argues that the latter is not only basic for Ibn Sīnā's modal syllogistic and epistemology, but also for al-Ghazālī's semantical-logical explanation of the names of God. The modern philosopher of logic might learn from Ahmed's, Schöck's and Thom's contributions that the distinction between definite descriptions and proper names might have a long and fascinating history.

Paul Thom's contribution starts where Schöck's contribution ends, namely with the investigation of the relationship between logic and metaphysics in Ibn Sīnā's modal syllogistic and therefore completes the logical and grammatical researches of Ahmed and Schöck.

Thom points out that Ibn Sīnā, unlike Aristotle, states truth-conditions for the propositions that constitute his modal syllogistic. Ibn Sīnā's characterisation of the subject of an absolute or modal proposition as standing for whatever it applies to, "be it so qualified in a mental assumption or in external existence, and be it so qualified always or not always, in just any manner", leaves open two ways to construe the propositions, namely *de re* and *de dicto* – the author remarks that his formulation self-consciously rejects the idea that the subject-term of an absolute or modal proposition applies just to what actually exists. Recent discussions of Ibn Sīnā's modal syllogistic have adopted a simple *de re* reading of Ibn Sīnā's *dhātī* propositions, and therefore either ignored or rejected the possibility of metaphysical applications for his modal theory. Thom contests this interpretation and identifies a class of metaphysical propositions (such as absolute propositions, statements of final causality and those of which the predicate is constitutive of the subject) which do not exhibit a simple *de re* form but involves both *de dicto* and *de re* elements. Interestingly, his attempt of interpreting Ibn Sīnā's *dhātī* propositions that incorporates *de dicto* element shows that the combined *de dicto/de re* analysis gives just as accurate a formal representation of Ibn Sīnā's modal syllogistic as does the simple *de re* analysis. Besides its application in metaphysics, Thom provides theoretical reasons for preferring it over the simple *de re* analysis.

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Lille October 9th, 2007

Hassan Tahiri
 Shahid Rahman
 Tony Street

Notes

¹ Although this fundamental feature of scientific practice in the Arabic tradition has yet to attract the attention of many scholars and historians, we nevertheless find that the spirit of continuous research and the close cooperation between Arabic and Islamic intellectuals are best illustrated by the work of a series of astronomers and mathematicians of the thirteenth and the fourteenth centuries. They are called the Marāgha School because they worked in close collaboration at the observatory of Marāgha (situated in north-western Iran) on a specific astronomical research project which was clearly defined in the eleventh century by Ibn al-Haytham in his landmark *al-Shukūk* (for more details, see Tahiri's paper).

² One of the fruitful direct contacts between the Arabs and the Chinese was the introduction of paper-making technology into the Islamic world in the early eighth century, making obsolete all other writing materials. The quick use of paper in writing leads to the unprecedented spreading knowledge. Jonathan Bloom devotes a whole book to this worldwide phenomenon created by the emergence of the paper industry. By discussing the social and scientific impact of paper, he shows "how its use in the islamic lands during the Middle Ages influenced almost every aspect of medieval life, [...] how paper utterly transformed the passing of knowledge and served as a bridge between cultures" (Bloom 2001).

³ Gutas is referring here to a number of scientists (mainly algebraists and astronomers) such as al-Khwārizmī, Ya'qūb ibn Abī-Manṣūr and Banū-Mūsā.

⁴ Gutas' claim seems to be challenged by Rashed's recent study of al-Kindī's works, see fn. 5.

⁵ P. 150.

⁶ One could assume that al-Kindī was speaking here as if he was, at least implicitly, the director of the programme of *Bayt al-Ikma* for three reasons: (1) according to Rashed, "le calife al-Ma'mūn se l'attacha et l'intégra à la "Maison de la sagesse", *Bayt al-Ikma*, qu'il avait fondée; [...] il avait d'ailleurs été chargé par al-Ma'mūn de contrôler les traductions faites au *Bayt al-Ikma* et d'en améliorer l'arabe." (Rashed 1998, p. v); (2) he had the strong support of the ruling power because of his close ties with the caliph al-Ma'mūn and his successor al-Mu'ta'im (833-842). The latter had appointed him as tutor of his son Ahmad and was the addressee of a number of his epistles including *On the First Philosophy*. (Gutas 1998, p. 123, Rashed 1998, p. v); (3) he tries to implement effectively his programme by gathering around him a circle of scientists and collaborators (Gutas, p. 119; Rashed 1998, p. v). Rashed adds: "de nombreux thèmes et concepts élaborés chez les Grecs ont été choisis et repensés par al-Kindī, intégrés à l'œuvre originale qu'il lui-même construite."

⁷ On the distinctive relation that binds the Arabic people to their language, the arabicist Nadia Angheliescu writes in the first chapter of her *Language et culture dans la civilisation arabe*:

Le seul prodige que [the prophet] Muhammad revendique comme signe de son investiture divine c'est le Qur'ān, dont la perfection sur le plan de l'expression défie toute imitation. "Pour l'Islam — note Louis Massignon — le miracle est verbal, c'est l'*i'jāz* coranique ; la chose essentielle en Islam, c'est la langue arabe du Qur'ān, miracle linguistique". Personne ne réussira à atteindre la perfection de ce monument de langue — ce que proclama Muhammad lui-même, en lançant à ses coreligionnaires un défi par delà les siècles —, mais tous s'efforcent d'imiter son vocabulaire, son style, ses procédés rhétoriques. La culture arabe tout entière est marquée par cette démarche de la 'beauté' de l'expression, par le haut prix que l'on attache à la forme, à la sonorité, à l'emphase, décelables même chez ceux qui ne maniaient pas la langue arabe littéraire. Jusqu'à nos jours, cette magie du verbe continue à s'exercer même sur un public moins cultivé ou illettré. Le célèbre historien des Arabes, P. K. Hitti, fait mention lui aussi de l'extraordinaire force de la parole dans l'espace arabe : "Aucun peuple du monde, probablement, n'est tellement saisi d'admiration devant l'expression littéraire et n'est tellement impressionné par le mot prononcé ou écrit comme le sont les Arabes. Il est presque inconcevable qu'une langue puisse exercer sur les esprits de ses détenteurs une influence aussi irrésistible que l'arabe. L'auditoire moderne de Bagdad, de Damas ou du Caire peut s'enflammer au plus haut point rien qu'à entendre réciter un poème ou prononcer un discours dans l'arabe classique, même s'ils ne les comprennent que vaguement ou partiellement. Le rythme et la rime, la musicalité produisent sur les auditeurs l'effet de ce qu'ils nomment la 'magie permise' (*si'r lalāl*)."

Cette ‘magie’, il faut bien le dire, c’est l’arabe littéraire qui l’exerce — cette langue qui fut depuis toujours placée sur le piédestal de la plus haute estime. Ce n’est là sans doute qu’une forme particulière de manifestation de ce prestige dont jouit toute langue littéraire ou ‘langue standard’ : il s’agit d’une attitude qui se manifeste par la ‘loyauté linguistique’, par la ‘fierté’, par la ‘fidélité aux normes’, attitude que Paul Garvin considère comme relevant de la *fonction symbolique* prêtée à la langue standard, en général, et qui, dans le cas de l’arabe, acquiert une résonance toute particulière (Angelescu 1998, pp. 13-14).

This fascination of the arabic language gives rise to the developpment of linguistic studies which begun as early as the eight century and whose effect will be felt in Europe eight centuries later:

On ne peut examiner l’attitude des Arabes à l’égard de la langue durant les siècles de leur épanouissement culturel, sans révéler l’importance de la science linguistique, avec ses diverses ramifications. Reflets du ‘logocentrisme’ de la société arabo-islamique, les études linguistiques arabes connurent une vogue que l’on ne retrouve guère dans d’autres espaces culturels. Il existe des milliers d’ouvrages consacrés aux différentes disciplines que l’on désigne aujourd’hui sous le nom de grammaire, lexicologie, lexicographie, sémantique, rhétorique, et qui constituent l’un des composants de base du fonds d’or de la culture arabe médiévale. Les résultats de cette laborieuse activité d’analyse et de réflexion linguistiques sont relativement peu connus en dehors du monde arabe, si l’on fait abstraction des travaux des orientalistes européens, qui surtout à partir du XVI^e siècle prirent les grammaires et les dictionnaires arabes pour modèle” (ibid., p. 67).

⁸ The Arabic-Islamic society proudly calls itself the nation of *iqra*’ (أمة اقرأ) in reminiscence of the first verse, or rather the first word, to be revealed to the prophet. The symbolic significance of this lies in the meaning of *iqra*’ which has to do with reading, learning and lecturing.

⁹ Angelescu explains how al-Jāhiz* personage became to symbolise the book-based nature of Arabic culture:

Al-Jāhiz* lisait tant, qu’il impressionnait ses contemporains : on raconte qu’il était rarement vu sans livre à la main, qu’il passait ses nuits chez quelque libraire pour finir un livre qui l’intéressait, ou qu’il faisait de longs voyages pour se procurer des livres dont il avait entendu parler. Il avait collectionné un nombre impressionnant de livres et la légende dit que, vieux et paralysé, il aurait trouvé la mort sous les piles de livres qui s’étaient effondrés sur son corps : une mort on ne peut plus symbolique pour un personnage symbolique. Dans le Bagdad de l’époque, il existait de nombreuses librairies (à en croire certains auteurs, une seule rue en possédait une centaine), mais surtout des bibliothèques publiques et privées : toute personne marquante se faisait un point d’honneur de sacrifier sa fortune pour acheter des livres. Toutes les disciplines sont cultivées et, à ce que’on affirme, en dehors des mathématiques et de la philosophie qui restent néanmoins l’apanage des spécialistes, aucune branche de la science n’échappe à cet esprit encyclopédique accapareur du siècle (Angelescu 1998, pp. 55-56).

¹⁰ Al-Jāhiz* actually uses the more general term *عبرة* or *’ibra* which can be translated as lesson. It is rendered by knowledge here since this is the topic discussed in the passage. *’ibra* is one of the key words in Arabic culture since it indicates not only the necessity of change but also seems to describe how change is brought about. In its general sense, it conveys the idea that the good development of a society as well as of an individual depends on their ability to draw the right lessons from their own experiences and the experiences of other people (past and present). It should be reminded here that Ibn Khaldūn’s *al-Muqaddima* (see Ditmarsch’s paper) is also known by its shortened subtitle *Kitāb al-’ibar* (the book of lessons), reflecting the Arabic-Islamic approach to History.

¹¹ After successfully ousting the Umayyads, the ‘Abbasīds moved the capital to Bagdad, their new founded city. Their rule lasted for more than five centuries until it was brought down by the invasion of the Mongol.

¹² The Umayyads established Damascus as the capital of the Islamic state. Their rule did not last long mainly because not only of their inability to broaden the basis of their power as Gutas explains (pp. 17-19), but also of their failure to win the hearts and the minds of the masses due to their lack of vision for the long term development of the society.

¹³ For more details on the impact of Islamic teaching, whose exhortation goes back to the seventh century, on the permanent establishment of the ‘search after knowledge’ tradition, see Rosenthal chapter V, section 1 “On Knowledge”, p. 70.

¹⁴ “Seeking knowledge is a duty for every believer” and “Seek knowledge, even if it be in China” are among the most famous sayings — about knowledge — attributed by the tradition to the prophet.

¹⁵ In this chapter Rashed presents a recently discovered astronomical material entitled the *Configuration of the Movements of each of the Seven Wandering Stars* which was written by Ibn al-Haytham after his famous *al-Shukūk*. The historical significance of this monumental work can hardly be overemphasised since it demonstrates

that Ibn al-Haytham has finally come to the conclusion that astronomy cannot be founded as a physical theory by just reforming Ptolemy's *Almagest*.

¹⁶ The same explanation could be found in his *Kitāb al-Anwā'* where he criticises the way astronomers use the Arabic word *falak*. Referring to the *Almagest* in which Ptolemy assumes that the heavenly bodies are moved by spherical bodies, Ibn Qutaybah admits that he cannot comprehend Ptolemy's statement speaking of something that it can hardly be seen "I have heard *فقد سمعت* and some who say that *aflak* (the Arabic plural of *falak*) are circles (*a}wāq* the plural of *jawq*) around which move the stars and the sun and the moon, and that the sky is above them [all]"; and he continues his strong attack by expressing his puzzlement as to how *falak* has become to refer in their astronomical works to supposedly large physical bodies that can only be heard of but can never be seen: "I have no way to find out how is that and I do not find it corroborated (*شاهداً shāhid*)" by the Arabic tradition (our emphasis, Ibn Qutaybah 1956, § 139, p. 124).

¹⁷ A point which is not missed by the eminent historian of science Gérard Simon when he describes Ibn al-Haytham's approach to optics as a scientific revolution, making it de facto the first scientific revolution in the history of science. For, he remarks, that the Greek conception of sight finds itself transformed by his work. Indeed, Ibn al-Haytham establishes experimentally that the phenomenon of sight is the result of light coming in and not out from the eye as is assumed by Ptolemy:

La révolution opérée par le très grand savant arabe Ibn al-Haytham, connu en occident sous le nom d'Alhazen, qui a substitué à une théorie de la vision faisant sortir de l'œil des rayons lumineux une théorie antagoniste faisant entrer dans l'œil des rayons lumineux ; ce qui l'a obligé à se demander sur de nouvelles bases comment la vision pouvait être un sens à distance, faisant percevoir le monde extérieur, alors que c'est dans le corps que se produit la sensation (Simon 2003, p.7).

Simon explains that the *timing* of this revolution is rather an evidence once again of the fact that scientific change is brought about by a change of approach in the conduct of the scientific inquiry. This is particularly true in relation to connection of sight with optics where the empirical turn triggered by Ibn al-Haytham has its roots in the role given to sight by the Arabic science and culture. Indeed as emphasised by Ibn Qutaybah in his objections to Greek astronomy, sight has been always considered and used in the Arabic tradition not as the platonic apprehension of ideas but as the instrument with the help of which the validity of uttered, reported or written statements could be systematically checked and tested:

Culturellement, la possibilité de géométriser la vision n'est pas surprenante pour des théoriciens qui conçoivent le flux visuel comme une émanation de l'âme, et pour des astronomes pensant que la vue nous livre ce qu'il y a de plus noble et de plus divin dans le monde, l'harmonie des mouvements célestes. La vision, pour un Ptolémée, peut échapper partiellement à la contingence et au désordre du monde sublunaire, car elle est le sens qui nous met en contact avec les régions éthérées, à la manière dont l'ouïe est un sens intellectuel parce qu'elle donne à percevoir les rapports mathématiques des harmonies musicales. L'optique, là encore, s'insère dans la culture de l'antiquité, et plus particulièrement ici dans une tradition pythagoricienne et platonicienne. Avec Ibn al-Haytham, et en particulier son *Traité d'optique*, l'insertion culturelle de l'optique change. Elle reste certes science de la vision et science des géomètres, mais, en tant que désormais elle se donne la lumière pour objet et l'œil pour champ d'étude, elle devient science de la matière et tisse des liens très neufs avec la médecine. En bref, elle s'autonomise et se complexifie, tout en gagnant en rigueur expérimentale (ibid., pp. 87-88).

¹⁸ Though being a personal physician to al-Ma'mūn and his successors, Gutas points out that it seems that he "conducted his research in the course of his practice as chief physician in the hospital in Bagdad" (p. 118).

¹⁹ For further details see al-Qifī 1903, pp. 390-392 and Gutas 1998, pp. 118-119.

²⁰ Al-Ghazālī formulates Ibn Qutaybah's first two criteria in the following way: "knowledge is the perception (*ta}awwur*) of things through thorough understanding (*ta}laqquq*) of quiddity and definition, and assent (*ta}dīq*) with regard to them through pure, verified (*mu}laqqaq*) certainty" (*Maqā}id*, II, 86, in Rosenthal p. 62).

²¹ Interesting is the fact that Rahman and Symons (2004, pp. 3-16) show that Neurath's Encyclopedism is linked to a conception of the relation between theory and practice strikingly close to that of Arabic tradition as discussed above.

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PART I
EPISTEMOLOGY AND PHILOSOPHY OF SCIENCE

Ibn Sīnā's Philosophy of Mathematics

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Abstract. We try to find the answers to two main questions of philosophy of mathematics in Ibn Sīnā's philosophy, i.e. what and where are mathematical objects? And how can we know mathematical objects? Ibn Sīnā's ontology implies that mathematical objects are *mental objects*. In his epistemology, Ibn Sīnā emphasises on *intuition* and *thinking* as two main ways of attaining mathematical knowledge. Moreover, Ibn Sīnā's analysis of mathematical propositions *implies* that they are synthetic a priori judgements in the sense of Kant.

1. Introduction

In this paper we try to find the answers of Ibn Sīnā to two main questions of philosophy of mathematics. These two questions are: (1) what and where are mathematical objects? and (2) how can we know mathematical objects?

Ibn Sīnā elaborated his philosophy in many of his writings without any remarkable change or modification. It is well known that his most detailed book is *al-Shifā'* (*The Book of Healing*). In "theology" or "metaphysics" (*al-Ilāhiyyāt*) of *al-Shifā'*, he discusses mathematics in at least three books (*maqālat*). In book 1, when he tries to characterize *theoretical sciences* by their subject matter, he defines and studies the subject matter of mathematics. In book 3, he puts forward his views on the natures of unity and number. Finally, in book 7, where he criticizes Platonism, he comes back to the subject of mathematical objects, and he criticizes Pythagoras as well.

A natural question related to the arrangement and the order of the topics of *al-Shifā'* may be the following:

Why Ibn Sīnā discusses mathematics in general and, mathematical objects in particular, in metaphysics? And why that is so crucial to metaphysics?

The same question, of course, may be raised for Aristotle's *Metaphysics*. There also we find that discussions about the concept of number are distributed almost everywhere in the book. In the edition that I have now on my desk (Aristotle 1958), the book ends with the section entitled *critique of the other theories of numbers*. Is it not surprising that *Metaphysics* ends with numbers?

Back to the question, we distinguish three plausible answers:

(1) The concept of number originates from the concept of unity, and this latter is a central notion of metaphysics. So it seems natural a discussion of the concept of number to be included in any book on metaphysics like *al-Ilāhiyyāt* of *al-Shifā'*.

(2) The Pythagorean metaphysics was a dominant philosophy in Aristotle's time and he should have defended his philosophy against Pythagoras. Ibn Sīnā is more or less a follower of Aristotle. So he does the same as Aristotle, even if it is not clear whether Pythagorean philosophy were still active in his time.

(3) There is a third reason that makes it more plausible to me why numbers are discussed in metaphysics, and that comes from a well-known postulate of Aristotle, according to which *the concepts of existence and unity are two universal concepts which have the same extension*.

Existent and unit are the same and have the same nature, in the sense that they accompany each other... since if we say “one human” and “human”, both refer to one thing ... “one human” and “existent human” does not show anything else (Aristotle 1958, book of *Gamma*, chapter 1).

This postulate of Aristotle is accepted by almost all Islamic philosophers. For example, Ibn Sīnā admits it when he discusses about the subjects of philosophy

And since unit and existence have the same extension, it is necessary that we study unit as well¹. (Ibn Sīnā 1997, book 1, chapter 4, p. 36)

And Mullā |adrā says:

Existence and unity are really the same or it [existence] is a necessary corollary of unity (Mullā |adrā 1981, chapter 4, vol. 3, page 298).

In other words, whatever really exists is really one, and vice versa, i.e., whatever is really one is really existent. Now, since “existence” is clearly the most crucial notion of metaphysics, this would imply that the notion of “one” is as important as the notion of “existence” in metaphysics.

2. Ontology of mathematical objects

In this section we want to find out the place where Ibn Sīnā believes that mathematical objects exist. We believe the ontology of a particular science, like mathematics, is highly related not only to the subject matter of that science, but also to the philosophical tradition in which the latter is characterized. Let us see then how Ibn Sīnā classifies the subject matter of different *theoretical* sciences.

2.1. The subject matter of mathematics

According to Ibn Sīnā (Ibn Sīnā 1997, book 1, chapter 1), philosophical sciences are divided into two categories, *theoretical* sciences and *practical* sciences. Theoretical sciences themselves are subdivided into three parts: physics (*al-ʿabīʿiyyāt*), mathematics (*al-Taʿlīmīyyāt*)² and metaphysics (*al-Ilāhiyyāt*):

As we said [in other places], theoretical sciences are of three kinds, physics, mathematics and metaphysics.

And also it was said: the subject matter of physics is bodies, in so far as it is in motion and at rest, and its problems are about accidents that occur to bodies with respects to motion and rest.

The subject matter of mathematics is quantity abstracted from matter or from whatever has quantity. The problems of mathematics are the ones that bear upon the quantity, as quantity. And in the definition of [science of] mathematics, there will not be any reference to a particular kind of matter or power of motion.

Metaphysics is about things that are apart from matter, in both aspects, existence and definition (Ibn Sīnā 1997, book 1, chapter 1, pp. 11-12.).

In his classification of philosophical science, Ibn Sīnā follows Aristotle (see also Wisnovsky 2001). He argues that the subject matter of sciences is “existent” (*mawjūd*) and an existent (a) may be found combined with matter in both existence and definition (or term) (*ʾadd*), or (b) is combined with matter neither in existence nor in definition, and finally (c) is not combined with matter in definition but with matter in existence. The first kind of existent is the subject matter of physics, the second one is that of metaphysics and the last one is that of mathematics. In the same place, Ibn Sīnā describes the subject matter of mathematics in detail.

But the subject matter of mathematical sciences is magnitude, either abstracted (*mujarrad*) from matter in the mind, or accompanied by matter in the mind, and is number, either abstracted from matter or accompanied by matter. And mathematics does not discuss the existence of abstract

magnitude or abstract number or number with matter, but after accepting the existence of quantity, it is concerned with the accidents of quantity (Ibn Sīnā 1997, book 1, chapter 2, p. 18).

These four kinds of mathematical objects described above correspond to four branches of mathematics, namely (a) geometry, (b) astronomy, (c) arithmetic and (d) music.

Magnitude, which means *quantum continuum* or *continuous quantity* in the above passage, is the subject matter of geometry and astronomy. Geometry studies magnitude and quantity abstracted from matter in the mind, even if they are accompanied by matter in the external world. It means that quantum continuum is accompanied by matter in the real world, but mind can separate it from matter and considers its properties. In astronomy, magnitude is accompanied by matter, both in mind and in the real world. A similar distinction holds between arithmetic and music. In arithmetic, the abstract number is studied, and in music the number and relations between them are discussed when accompanied by sounds.

It is clear that if the subject matter of mathematics is magnitude (quantum continuum) or number (quantum discretum), there is no place in mathematics for questions like “what is the nature of magnitude?” or “is magnitude a substance or an accident?” or “whether quantum discretum must be realized in matter or outside of matter?” etc. Since in mathematics, *the accidents* of magnitude and numbers are studied, not their *essence* and states of existence. These questions lie in the domain of philosophy or metaphysics.

There is a minor point in the above citation that is worth mentioning. The concept of “abstract” used in the above passage is *not* the same as the one Ibn Sīnā uses often elsewhere in his philosophy. In his philosophical research, when Ibn Sīnā uses this concept, it is in opposition with “material”. What he means here by “abstraction” is the *possibility* for the estimation (*wahm*) to seize magnitude and number apart from matter. When we discuss the epistemology of Ibn Sīnā, we will come back to this again.

Coming back to the subject matter of mathematics³, we can say that according to Ibn Sīnā, it consists of *things that are accompanied by matter in the external world and are abstracted from matter in mind*. He continues:

But number can be applied to both sensible objects and non-sensible objects, so number, in so far as it is number, does not belong to sensible objects (Ibn Sīnā 1997, book 1, chapter 2, pp. 19-20).

His main point here is that discussion about number and its relations should be understood as abstracted from sensible objects, not when it may belong to sensible objects. So discussion about numbers is not about sensible objects. About “magnitude”, the question whether it is a “substance” or an “accident” is less clear:

But magnitude, [then] is a common name. Some times it is referred to dimension, which is the substratum of natural body, and sometimes, what is meant by it is the quantum continuum, which is referred to line, surface and solid. You have already learned the difference between these two meanings (Ibn Sīnā 1997, book 1, chapter 2, p. 20).

Here Ibn Sīnā recalls his previous discussion of the difference between these two meanings of “magnitude”. I believe he refers the logic of *al-Shifā'* (third section, chapter 4), where he says *two bodies that are different with respect to size, are not different in receiving three dimensions*; this is exactly the first meaning of magnitude. What is the source of difference in any two bodies and is subject of change is the quantum continuum susceptible of admitting three directions, that is, length, width and depth.

Then Ibn Sīnā continues his discussion on the subject matter of metaphysics. He says:

From what is said until now, it became clear that existent, as existent, is the basis of all these subjects, and it must be the subject matter of this science [philosophy], for the reason we mentioned (Ibn Sīnā 1997, book 1, chapter 2, p. 21).

It is a well-known fact in the tradition of Islamic philosophy that the “problems” of every science are “the essential accidents of the subject matter” of that science (see, e.g., Ibn Sīnā

1997, book 1, chapter 2). The essential accidents of “existence” includes, in the first place, among other things, “unit and plural”, “potential and actual”, “universal and particular” and “necessary and possible” (see Ibn Sīnā 1997, book 1, chapter 1).

It is very interesting to know that when Ibn Sīnā tries to explain the reason for which metaphysics is called *māba'd-a}-}abī'iyi* (*whatever is after physics*), he encounters to some difficulties with regard to mathematics.

And the meaning of [“meta” in] “metaphysics” is relative with respect to our perception, since when we observe the world for the first time, we perceive the natural existence. But if we consider this knowledge in itself [not in relation to us], it is better to be named as “prephysics” (*māqabl-a}-}abī'iyi*), since it discusses matter that is prior to physics, in both its substantial (*bi-dhāt*) and conceptual (*bi-al-'umum*) aspects (Ibn Sīnā 1997, book 1, chapter 3, p. 31).

Ibn Sīnā argues that the prefix “meta” in “metaphysics” is related to the stages of our perception. As we will see later in his epistemology, sense perception is the first level in human understanding of the world. This level of perception and some other levels that are closer to sense perception than to intellection, are ways of knowing *physics* or nature. On the other hand, if we consider philosophy in itself, it is prior to physics, since its questions are about matters that have priority relative to natural objects. For example, philosophy discusses the Separate, or abstracts that are the cause of nature, and *every cause is prior to its effect*. Moreover, in philosophy we discuss subjects that are more general than natural objects and *every general matter is prior to a particular one*. He then says:

But perhaps somebody may claim: the subjects of *pure mathematics* (*riyā'iyāt-al-ma'l*ā*) which are discussed in arithmetic and geometry, are also before physics and in particular, number, whose existence is not related to physics, since it sometimes exists even in non-physical objects, so the sciences of arithmetic and geometry might be counted as “prephysics” (Ibn Sīnā 1997, book 1, chapter 3, p. 31).

It is worthwhile to note that Ibn Sīnā distinguishes *pure* mathematics from the other parts of mathematics, which nowadays are known as *applied* mathematics. We don't know if that is the first time in the history of science that this distinction is made. However, what interests us here is that he does not count astronomy and music as belonging to pure mathematics. According to Ibn Sīnā, in arithmetic “number” is discussed exactly as “the pure magnitude” is in geometry. In astronomy and music, on the contrary, the subject is quantities and numerical proportions between stars in astronomy, or numerical proportions between sounds in music. The main point of the above critique is that arithmetic and geometry discuss their objects without any relation to external objects exactly as subjects discussed in metaphysics. We count things in the Separate as well.

In answering to the above critique concerning the subject matter of metaphysics, Ibn Sīnā first considers geometry:

What can be said as answer to this critic is this: the subject of that part of geometry in which lines, surfaces and solids are studied is clearly not separated from physics as regards existence, so the predicates of such subjects are not separated from physics, *a fortiori* (Ibn Sīnā 1997, book 1, chapter 3, p. 32).

In this part of geometry, he says, line is divided, for example, into straight, curved and other types of lines and surface is divided into affine surface or non-affine surface and also non-affine surface is divided into convex and concave. It is clear that the subject matter of this part of geometry is based on matter, since in the universe of abstracts there does not exist line, surface or solid. We can only have lines, surfaces and solids in physical objects. Line without surface, surface without solid and solid without body is *unrealizable*⁴. For the other part of geometry, whose subject matter is “absolute magnitude”, Ibn Sīnā argues:

In parts of geometry where the subject matter is absolute magnitude [not line, or surface or solid], the absolute magnitude is the subject with respect to its potentiality for every proportion, and this

potentiality for proportions is realized not for magnitude that is a form for body or a principle for physics, but for magnitude that is an accident (Ibn Sīnā 1997, book 1, chapter 3, p. 32).

Ibn Sīnā says that we sometimes study “absolute magnitude” in geometry, and not special properties of lines, surfaces or solids. If we consider this part of geometry, the above critique becomes more serious. We did not accept geometry as part of metaphysics, since lines, surfaces and solids are realized only in nature whereas “absolute magnitude” does not depend on physical world and is something abstracted from matter. Does this mean that we should consider this part of geometry as a part of metaphysics? Ibn Sīnā's answer is negative. He argues that when we discuss absolute magnitude in geometry, we mean magnitude in so far as it accepts different relations. That finds determination in lines, surfaces and solids. The other meaning of absolute magnitude, namely, the form of body, is not related to geometry. This meaning of absolute magnitude as the form of body is the principle of natural objects and prior to them, and so discussion about it belongs to metaphysics.

Mullā ḡadrā does not accept Ibn Sīnā's argument and argues

Magnitude, as it is, does not exist unless in one of these three species; as every genus is related to its species. How then he [Ibn Sīnā] believes that the absolute magnitude [as a genus] is possible to be apart from physics but apartness of line, surface and solid [as species] from physics is not allowed in physics? Moreover, each of these species is realizable in non-physical world - as it will become clear later -, so the right answer is this: each one of these three species of magnitude is the subject of geometer exactly when it accepts proportions and divisions, like squaring, cubing and other attributions and it accepts these proportions when it belongs to and depends on physical objects (Mullā ḡadrā 1925, p. 20).

We believe that the above discussion on the meaning of “absolute magnitude” is debatable, and is outside of the scope of this paper.

Let us see Ibn Sīnā's argument against including arithmetic in metaphysics.

And as regards number, the critique is more serious and it seems that the science of number [arithmetic] may be counted as [part of] metaphysics... But the reason why arithmetic is not a part of metaphysics will be clear to you soon. The subject matter of arithmetic is not number in all its aspects, since number, sometimes is found in the Separate (*mufḡriq*) [like intellect (*'aql*)], sometimes in physical objects and sometimes in estimation, in which it is abstracted from every accident [whether physical or abstracted], although it is impossible for number to exist in the external world except in the state of an accident. The number in the Separate is impossible to be the subject of increase or decrease, but it remains only constant. Aye, number should be in such a way that has potentiality of every increase and decrease or every proportion, [and this] is possible only if number be realized in bodies, which has the potentiality of being counted, or number be realized in estimation, and in both cases [realization of number in bodies or in faculty of estimation], number is not out of physics (Ibn Sīnā 1997, book 1, chapter 3, p. 32).

The answer of Ibn Sīnā to the critics that the science of arithmetic is metaphysical may be explained as follows: we may consider number in different ways, asking questions like “has number a real existence?” or “is the existence of number essential or accidental?” etc; and seeking answers for these questions is outside of the science of arithmetic and lies within the sphere of metaphysics.

Number as it is considered in arithmetic is such that it accepts variations and changes. In arithmetic, when we add, subtract or divide two numbers, in fact we look at it as a decrease or an increase of number. Such concept of number is not the same as the one discussed in metaphysics, where it is considered to be *constant*. The reason that such numbers are constant is based on the fact that their *referents*, i.e., separated matters, are *not* subject to any change. So the natural question is this: what kind of number is subject to change, i.e., to be increased or decreased? The answer is: in two cases number has the potency of different proportions. In one case where number is related to physical objects, and since the physical universe is subject to changes, then the number, i.e., the quantity of those objects, will necessarily change. The other case is when number has potency of different proportions in estimation

(*wahm*). That means the estimative faculty is able to abstract a number from every numbered object, and to add, subtract and multiply it by other numbers. Even in this case, Ibn Sīnā believes that the source of these changes in the estimation faculty is states and proportions of the physical universe, and if there were no changes in the physical universe, the estimative faculty would not be able to imagine the abstracted numbers with different proportions⁵.

2.2. Ontology of mathematical objects

According to the tradition of Islamic philosophy, the objects of mathematics are *quantities* and quantities are *not* substances. In his *al-Shifā'* (*al-Ilāhiyāt*, chapters 3 and 4), Ibn Sīnā argues that both *number*, as a discrete quantity and *magnitude* as a continuous quantity, are *accidents*. That means quantities do not have an *independent existence* in the external world, and so they need some *substrata* to exist.

It is worth knowing that in chapter 2 of the same book, he explains the different meanings of the concept of “unit”, which is the basis of his notion of quantity. Despite different meanings, the concept of “unit” shares the property of “not being divisible actually” (see Ibn Sīnā 1997, book 3, chapter 3), and “not accepting plurality” (*ibid*). He divides the notion of “unit” into “essential unit” and “accidental unit”. The notion of “numerical unit” is defined as a kind of “essential unit”, which is sometimes called “particular unit”.

And as regards plurality, it is evident that it must be defined in terms of unity, since the unit, is the principle of plurality and existence, and the essence of plurality derives from that [unit]. Besides, in order to give any definition of plurality, we use the term “unit”. And it is for this reason that in the definition of plurality, you may say: “plurality is exactly the set (*mujtama'*) of units”. You note that in this definition of plurality, “unit” is used, and something else, which is the notion of set (Ibn Sīnā 1997, book 3, chapter 3, p. 112).

Ibn Sīnā then criticizes the people who believe that the above explanation is an *essential* (or real) *definition* of “plurality”. By an essential definition, he means, like other Islamic philosophers and logicians, a *definition by terms* (*1add*), which refers to the essence of the defined object (*definiendum*). A common example for an essential definition is the definition of “human” as “rational animal”. Ibn Sīnā argues that the concept of “set”, which is used in the above definition, conceptually includes “counting” and “repetition”. In his argument against any possible essential definition of the concept of “unit”, he uses some *epistemological* premises.

But it seems that plurality is more known in the imagination (*khayāl*) than unity, as unity is more known in the intellect than plurality, and both are universal matters and are immediately imagined, nevertheless we first imagine “plurality” through sensible things, and we understand unity without any “intellectual” principle, and even if we believe some “intellectual” principle for that, it is an imaginative one. So our definition of plurality in terms of unity is an “intellectual” definition, in the sense that the term “unity” in the definition is immediately imagined (Ibn Sīnā 1997, book 3, chapter 3, p. 113).

Ibn Sīnā admits that our perception of “plurality” happens before “unity”, and that is because “plurality” comes directly from sensible objects. The first notion of “plurality” is made when we see something that is not “this”. Contrary to “plurality”, “unity” takes place in imagination as a *negation* of “plurality”, so it will be formed in perception in the next step. Since our perception of “unity” is immediate and non-theoretical, we may define “plurality” in terms of “unity” only through intellection. When it is said that “unity” is something that does not have “plurality”, it means that the meaning of “unity”, which is self-evident for us, is against the meaning of “plurality”. So in this setting, it hints that “unity” is the negation of “plurality”.

He finally argues that the above explanation is just a clarification of the concepts “unit” and “plural”, and that “plurality” is just another *name* for “a set of units”.

He then says:

Now, we investigate the nature of numbers and its properties... Number exists in the external existent objects, and it also exists in our soul (mind), and this saying: “number has no existence, except in our soul” is not noteworthy. Aye, if he [who holds this view] means that number has no existence apart from any existent object, except in our soul, then he is right, since we have already proved that it is impossible for the unit to have external existence, and naturally, number which its existence is based on unit, is also the like (Ibn Sīnā 1997, book 3, chapter 5, p. 126).

So, according to Ibn Sīnā, we may say that number has two levels of existence. On the first level, it exists only in the mind, that is, abstracted from any existent matter, on the second level, it exists with the external objects. His argument for the existence of the second level is the following: in non-unit objective things, i.e., a collection including more than one object, there are clearly some⁶ units. So number necessarily exists, since number is nothing except units that possesses some place in the order of numbers, and each place, itself, is a *unique* species. In this way, each number, as a *species*, is itself a unit, and so it has some special properties attributed to its unity. It is clearly impossible to prove some properties for something that has no reality in the outside world.

So each number has a particular nature and a form, and the imagination of that in the soul comes from that nature, and that nature is the unity, which constitutes the essence of that number. And the meaning of a number is not a plurality without a form of unity. So it is not true to say: “number is the sum of units, not unit”, since a number, as it is a sum, is a unit that carries some properties such that a number with other sum and order does not have. And it is not surprising for something to be unit with respect to its kind, like 10 or 3, and also to be plural, with respect to other aspects (Ibn Sīnā 1997, book 3, chapter 5, p. 127).

A number like 10, as a species-unit, has a special form and so possesses properties of being 10, but as a plurality, it may not be the subject of those properties. Instead it has some properties of plurality in opposition to that unity. That is exactly the meaning of “plurality in unity” and “unity in plurality”, and there is no contradiction here. We *view* a number in two *different* ways.

Ibn Sīnā believes that the definition of, for example, 10 as the sum of 9 and 1 is *wrong*. His argument is:

It is not true to say that: the number 10 is 9 and 1, or that is 5 and 5, or that is 1 and 1 and ... until it ends up to 10; since in the statement “ten is nine and one”, nine is predicated of ten, and then you conjoin to it the one. This is similar to say “ten is black and sweet”, which in this sense, the meaning of the original claim will be: “ten, is both nine and one”, and if what you mean by that conjunction is not a definition, but it [has the meaning] like the statement “human is animal and rational”, namely, human is such an animal that is rational. In this sense, the meaning of your claim will be: “the number ten, is nine, and it is also one”, and this is impossible (Ibn Sīnā 1997, book 3, chapter 5, pp. 127-8).

It is clear that Ibn Sīnā is considering here the case where *conjunctions* play a *logical* role. A statement like “A is B *and* C”, logically means that “A is B” *and* “A is C”. By this interpretation for “and”, it is impossible that both statements “10 is 9” and “10 is 1” be true. But there is a mathematical interpretation for “and” which simply means “addition”. That is what Ibn Sīnā ignores. We will explain the origin of his ignorance after the next passage. Ibn Sīnā then considers other possible meanings that the above definition may have. He concludes that,

And the number ten is the sum of nine and one, when both are present and the conclusion is something different from each one of them (Ibn Sīnā 1997, book 3, chapter 5, p. 128).

Contrary to modern axiomatic definition of natural numbers, where for example, 10 is defined by $9 + 1$, namely, ten is the *successor* of nine, Ibn Sīnā will not accept it as an essential definition. The notion of *addition* for natural numbers as is defined in set theory, is *finally* in term of two primitive (*undefined*) concepts, i.e., *set* and *membership*, with familiar symbol “ \in ”. It may be suggestive to interpret “set” in Ibn Sīnā’s philosophy, by “plurality”. This

interpretation, of course, is debatable. Even if we have some support for the mentioned interpretation, it is too hard to find a similar interpretation for the set theoretical notion of *membership*, which is a *two-place relation*. It is a well-known fact that Ibn Sīnā's logic does not *permit* two-place relations⁷.

He finally goes to the conclusion that we are *not* able to present an *essential definition* for number, i.e., it is *undefinable*.

And since it is hard to imagine a definition for number in terms of units, its definition necessarily is nothing more than a description (*rasm*) (Ibn Sīnā 1997, book 3, chapter 5, p. 128).

Recall that a *description (rasm)* is a definition in terms of *accidental* properties of *definiendum*. Ibn Sīnā then concludes that

As the great ancient philosopher, the First Teacher [Aristotle] said: "Do not suppose that the number six is three and three, but it is six, at once and immediately" (Ibn Sīnā 1997, book 3, chapter 5, p. 129).

2.3. Existent and Object

In book 1, chapter 5 of *al-Shifā'*, Ibn Sīnā begins a long discussion on the relation between "existent" and "object". His objective is, among other things, to establish the following two facts:

- (1) "Existent" and "Object" are self-evident and immediately imagined concepts.
- (2) "Existent" and "Object" are conceptually different, but extensionally the same.

On the first fact, Ibn Sīnā emphasizes the *epistemological* value of the two concepts, in the sense that they are the most general concepts.

So we say that the meanings of existent, object and necessity are immediately pictured in soul, namely, their imagination do not need any more known objects to be imagined (Ibn Sīnā 1997, book 1, chapter 5, p.39).

In this section we are more interested in the second fact. He asserts:

We say: the meanings of "existent" and "object", are both imagined in soul and they have two different meanings. So "existent" and "affirmed" (*positive*), (*al-muthbat*) and "realized" (*al-muḥal*), are [different] names with the same meaning, and we are confident that the meanings of these words are present to the soul of anyone who reads this book. Sometimes "object" and all its synonyms in all other languages refer to another meaning [other than "existent"], since everything has a reality [an essence], which is due to that reality [essence], that the object is what it should be. So a triangle has the reality [essence] of being a triangle, and whiteness has the reality [essence] of being white, and this is a meaning of "object" which sometimes we call as "specific existent", and by "specific existent", we do not mean an affirmative existent. The word "existent" also refers to several meanings, one of which is the reality [essence] an object has, as if the specific existent of a thing is exactly the reality [essence] that the thing were based on it (Ibn Sīnā 1997, book 1, chapter 5, pp. 41-2).

We may summarize Ibn Sīnā's view in this way: "object" is the same as "specific existent", and they both refer to the essence of things. So, in some sense, "object" or "specific existent" refer to the essence of things, and "existent" refers to the things themselves.

Ibn Sīnā continues his argument to establish that "object" and "existent" are conceptually different. He then provides an argument to show that these two concepts have the same extension, i.e., we can consider something as the extension of the concept of "object", if and only if it is an extension of the concept of "existent", i.e., "*object*" and "*existent*" are *extensionally equivalent*.

And the necessity of the meaning of existence cannot be separated from "object", namely, the meaning of existent is always necessary for an object, since an object either exists in the external world, or in estimation or in the intellect (Ibn Sīnā 1997, book 1, chapter 5, p. 43).

As Ibn Sīnā says, if an object is in the external world, it satisfies the predicate “existent”, and if it is not in the external world, it can be considered as existent only when we imagine it in the mind, and if something exists neither in the external world nor in the mind, it is *not* an object at all. So the necessary condition for something, which does not exist in the external world, to be an object is to exist in the mind (estimation or intellect).

Before closing this section, we would like to consider another ontology of mathematical objects that we believe is *wrongly* attributed to Ibn Sīnā. In Rashed 1984, R. Rashed argued that Ibn Sīnā codified a comprehensive doctrine of philosophy such that embraces al-Fārābī's admission of irrational numbers as mathematical objects. In Rashed 1984, it is argued that since algebra is the intersection of arithmetic and geometry, its tool, i.e., “an algebraic unknown”, can be read as an “object”, which represents a number or a geometric magnitude. Something more can be said, since a number may be irrational, so an object can represent a quantity that can only be known by approximations. Although this algebraists' tool must be universal enough to cover different contents, it also must exist independent of what is determined so that getting better approximations be possible.

R. Rashed rightly believes that an Aristotelian theory is not able to have such ontology, so it is necessary to suggest a new ontology by which we are able to speak of an object without any specific property, an ontology that permits us to know an object without being able to represent it in an exact way. He finally claims:

Ainsi, tout existent est une chose, mais la réciproque n'est pas exacte, bien qu'il soit impossible qu'une chose n'existe ni comme sujet concret, ni dans l'esprit (Rashed 1984, page 35).

But Ibn Sīnā argues against people who claim: “the concept of object is more general than of existent” (Ibn Sīnā 1997, book 1, chapter 5). His argument is based on this fact that in predication in statements and in our knowledge of objects, knowledge is about concepts that are *in our mind*, so they have *mental existence*, although they may not exist in the external world. In fact, as already mentioned, a necessary condition for something to be an object is that it should exist *mentally*.

And this happened to them, because of their ignorance to this truth that [the subject of] predication is something that has existence in soul, although they may be nothing in the external world, and the meaning of predication of such things is that they have some relations with the [external] existent (Ibn Sīnā 1997, Article 1, book 5, p. 45).

3. Epistemology of mathematical sciences

In this section we briefly explain Ibn Sīnā's epistemology to find the answer to our second question, i.e., “how can we know mathematical objects”? As D. Gutas pointed out (Gutas 2001), Ibn Sīnā's epistemology was under “inevitable shifts of emphasis and terminology over the years”. These modifications culminated in the writings of the final period of Ibn Sīnā's philosophical activity, especially in *Pointers and Reminders (al-Ishārāt wat-tanbīhāt)*. In description of his epistemological theory, we essentially make use of this book.

In this book, as elsewhere, Ibn Sīnā identifies the mental faculties of the soul in terms of their epistemological function. According to Ibn Sīnā, knowledge begins with *abstraction*. The concept of “abstraction” (*tajrīd*) in Islamic Philosophy, and in epistemology in particular, is a significant notion. In fact what distinguishes the levels of perception boils down to the degree of “abstraction” (*tajrīd*). We briefly mention that contrary to Arabic word “*tajrīd*”, the word “abstraction” loses the sense of the intensification of existence and reality that takes place as the degree of *tajrīd* increases. So a better translation for the Arabic word “*tajrīd*”, may be the English word “disengagement”, or “detachment”. Nevertheless, in this paper we use the common word “abstraction” and its derivatives for “*tajrīd*” and its corresponding derivatives. There are vast investigations on different possible meanings of “abstraction” in the Islamic

philosophy, and in Ibn Sīnā's philosophy in particular. We refer the reader to a survey of this topic in Hasse 2001.

In the purest sense, "abstract" (*mujarrad*) is an attribute of God, the Necessary Existence in itself, since the Necessary Existence has no attachment to or dependence upon anything other than itself.

More specifically, "abstract" is the attribute of the intellect that is able to see things as they actually are, that is, without their entanglement in the obscurities of imagination and sense perception. It is also the essential attribute of the forms or quiddities that the intellect perceives (In this final use, it comes close to the term "abstracted").

Perception (*idrāk*) of an object consists in attaining a true image (idea) (*mithāl*) of the object by the one who perceives (*mudrik*) [subject], and the *mudrik* observes that. So either when that is perceived object is exactly the same object outside of the *mudrik*, which possibility is incorrect; since then something which does not exist outside [of the *mudrik*], should necessarily exist [outside]. The examples are many geometrical figures, or many impossible hypotheses - things that do not exist. Or the perception [of an object] is a true image of the object pictured on the *mudrik* himself in such a way that it has no difference in quiddity (*māhiyya*) with that [the object], and it is the form, which remains (Ibn Sīnā 1960, physics, chapter 3, p. 33).

What Ibn Sīnā intends here is an *explanation* of the concept of perception (*idrāk*), not presenting a (real) *definition*. He explains that perception of an object may be described in two ways. He argues against the possibility that perception consists in *transferring* of things outside of the *mudrik* to *mudrik*. His argument is based on some evident *counterexamples*, e.g., *assumptions* in geometry, which we *know* that they do *not* exist in the outside world, but we *know* them. He then accepts the other possibility, according to which perception is to attain the form and the quiddity of the object. An immediate consequence of his description for perception is that we may perceive objects that may not exist outside us, and only the form of the objects are perceived by *mudrik*.

Now let us see how Ibn Sīnā stratifies the levels of perception:

Sometimes an object is sensible, and that is when it is seen; [and] sometimes it is imagined, and that is when the object itself is absent and its form is present in *mudrik*; as when you have seen Zayd; and then when he is absent from you, you imagine him. Sometimes an object is intelligible (*ma'qul*), and that is [like] when you understand the meaning of human from Zayd, a meaning that holds for other things as well. When Zayd is sensible, it is with appearances [which are different from his quiddity] which do not affect his quiddity, whenever they [appearances] have been disappeared; like to have place, position, quality and determined quantity, such that if they are replaced by something else, the reality of human's quiddity would not have been changed. Sense will perceive Zayd in a state that has these appearances, namely, appearances which are interconnected with him, because of the matter from which he was created. Sense will not remove those appearances from Zayd and will not perceive him unless by the connection that exists between sense and its matter; for this reason, whenever this connection is lost, the form of that [Zayd] will not be present to the sense. But imaginative faculty imagines Zayd with all these appearances and cannot abstract him absolutely from these appearances. But it can abstract him from the positional relation upon which sense was dependent; so Zayd is present in imagination even when his positional relation is absent. But intellect can abstract a quiddity from all its personal appearances and establish it in such a way as if it [intellect] manipulates the sensible to a form of intelligible. But an object without these appearances - appearances not necessary for its quiddity - is intelligible in itself and does not need any manipulation to be prepared to be intelligible; but it may need to be abstracted by a faculty that is responsible for intellection (Ibn Sīnā 1960, physics, chapter 3, p. 34).

In this passage, Ibn Sīnā is going to describe different stages of perception. In his description, perception is classified into four stages based essentially on a hierarchy of abstractions of objects. These four stages are: (1) sense perception (*ḥiss*), (2) imagination (*takhayyul*), (3) estimation (*wahm*) and (4) intellection (*ta'aqqul*). A natural objection may be the absence of the third type, i.e., estimation, in the passage quoted from *al-Ishārāt wat-tanbīhāt*. The answer

is this: in the above passage, Ibn Sīnā, as a definite example, considers “Zayd”, that cannot be perceived by estimation, as we will see the reason when we explain the meaning of the term “estimation” according to Ibn Sīnā. In his other books, like *al-Shifā'*, he does not take any definite example, and so he is able to distinguish all four types of perception. Now we explain these four types of perception in more details:

(1) Sense-perception (*Iiss*)

Sense perception responds to the particular with its given form and material accidents, such as place, time, position, quality, etc. As a mental event, being a perception of an object rather than the object itself, perception occurs in the particular. So a sensible object has three conditions: (i) presence of the object, (ii) with material accidents, and (iii) particularity. All these conditions hold for a definite object like Zayd, when we see him. Then such a concept is definite and does not hold for more than a person. Now let us analyze this activity of the soul in details. To classify the formal features in abstraction from material accidents, we must retain the images given by sensation and also manipulate them by disconnecting parts and aligning them according to their formal and other properties. However, retention and manipulation are distinct epistemological functions, and cannot depend on the same psychological faculty; therefore Ibn Sīnā distinguishes faculties of relation and manipulation as appropriate to those diverse epistemological functions.

(2) Imagination (*khayāl*)

In imagination, among three above conditions, the first condition does not hold, i.e., the same Zayd is absent. Ibn Sīnā identifies the retentive faculty as ‘representation’ and charges the imagination with the task of reproducing and manipulating images. To conceptualize our sense perception and to order it according to its quiddities, we must have and be able to re-invoke images of what we experienced but is now absent. For this we need sensation and imagination; in addition, to order and classify the content of representation, we must be able to discriminate, separate out and re-combine parts of images, and therefore must possess imagination and reason. By carrying out this manipulation, imagination allows us to produce images of objects we have not already seen out of the images of things we have experienced. So imagination can also generate images of intelligibles.

(3) Estimation (*wahm*)

That is to perceive the particular meaning of non-sensible, like perception of kindness a father has for his child. So estimation is also of particular concepts, which have not been perceived by any senses. Among the conditions necessary for sense perception, only the condition (iii) holds for estimation. This is a faculty for perceiving non-sensible “intentions that exist in the individual sensible objects”. A sheep fears a wolf because it estimates that the animal may do it harm; this estimation is more than representation and imagination, since it includes an intention that is additional to the perceived and abstracted form and concept of the animal.

(4) Intellection (*ta'aqqul*)

Intellection is the final stage of perception in which none of the three conditions hold. It is to perceive the universal concepts, e.g., “human” by abstraction of Zayd, removing all material appearances that will not change the quiddity of “human”.

According to Ibn Sīnā, intelligibles divide into two kinds, material intelligibles and immaterial intelligibles. So we have two kinds of intellection, depending on the corresponding objects. In material intellection, we first perceive a particular human, like Zayd, by sense perception in the presence of the material object, as described in the stage of sense perception. Then it is understood by common sense while the object is absent; and then the object will be abstracted from all material features such that it will be prepared to be understood by intellection. In immaterial intellection, the object itself is intelligible such that it does not need to be abstracted from material accidents, like abstract realities, souls, etc.

Now our main question reduces itself to “*on what stage of perception we perceive mathematical objects, in particular, number and magnitude?*”

Corresponding to the classification of philosophical sciences in terms of their subject matter, intellection is divided into two kinds, *theoretical* intellection and *practical* intellection. Theoretical intellection is responsible for knowledge and perception of intelligibles, and practical intellection is like a ladder to attain moral values, Ibn Sīnā distinguish four levels or layers for theoretical intellection, (a) *potential* intellection (*al-'aql al-hayoulāni*), a stage where no intelligible is perceived yet, but it has the capacity or potentiality to accept *primary* intelligibles, (b) *dispositional* intellection (*al-'aql bi-al-malaka*), a stage where intellection has passed from the pure potentiality and has perceived the primary intelligibles, and is prepared to acquire the *secondary* intelligibles⁸, either by *thinking* or by *intuition* (*1ads*), (c) *actual* intellection (*al-'aql bi-al-fi'l*), a stage where the intellection *knows* that he has acquired the secondary intelligibles, and finally, (d) *active* intellection (*'aql mu}laq*), where the intellection *observes* the secondary intelligibles (see Ibn Sīnā 1960, physics, chapter 3, section 10).

So in the levels (a) and (b), intellection has only the potentiality to acquire the secondary intelligibles whereas in the levels (c) and (d), it can present, observe and study them. For acquiring the secondary intelligibles in the stage (b), Ibn Sīnā mentions *two ways*, first *thinking*, and the second *intuition*. In the next chapter of the same book, he describes four differences between “thinking” and “intuition”. These characteristics may be summarized as the following: (i) contrary to “thinking”, there is no *search* in “intuition”, and that is when, without enough background or premises, we sometimes acquire the middle term [of a syllogism], intentionally, or unintentionally, (and in both cases, without any movement of the mind), (ii) in contrast to “thinking” which may be unsuccessful in its search, “intuition” hits spontaneously the middle term and comes to the point immediately, (iii) “thinking” is often about particulars, since it searches by the assistance of the imaginative faculty, and (iv) “thinking” takes place in time but “intuition” is immediate and spontaneous.

According to Ibn Sīnā, *knowledge* is acquired in the second level of theoretical intellection, which is in *contact* with the third level, i.e., the active intellect through thinking *and* intuition. For the mathematical sciences, the meaning of thinking is not much debatable. Ibn Sīnā himself was a well-known logician of his time and also knew the *Elements* of Euclid. So it is clear that, for him, “thinking” in mathematical sciences is nothing else than *deductions* or *proofs* of mathematical propositions by means of axioms and rules. This *simple* or *formal* picture of mathematical thinking does *not* explain what really is going on in the mathematician’s mind. The process of catching the *middle terms*, as the medium or means, to prove the main claim or proposition, is not explained by this simple picture of the mathematical thinking. In the mathematical science, *lemmata* play the same role as *middle terms* in a syllogism. Here Ibn Sīnā introduces a new way or method to fill the gap. That is called “intuition”. His description of the concept of intuition establishes a crucial element in the process of mathematical discovery. It is worth mentioning that, as D. Gutas interprets in Gutas 2001, Ibn Sīnā probably came up to his theory of intuition by his own experience as a mathematician. His example to explain different ways where intuition occurs is the states of problems solving in geometry. As D. Gutas explained in Gutas 2001, in standard version of Ibn Sīnā’s theory of intuition, *all* intelligible knowledge is acquired *only* through intuition. In his “revised” version, which is met with in the writings of the later period of Ibn Sīnā’s philosophical activity, “a second way of acquiring the middle terms and the intelligibles is introduced. This is thinking, which is now defined as a movement of the soul in search of the middle terms, thus taking over a large part of the former definition of intuition.” (See Gutas 2001, for details)

The theory of intuition in epistemology of mathematical sciences is very involved and complicated. In modern epistemology of mathematical sciences, there are different and various interpretations and explanations for the concept of "intuition". The most attractive ones are Gödel's and Brouwer's concepts of intuition. Each one of these concepts of intuition has been interpreted in different ways. For the Gödel's notion of intuition, see, e.g., Maddy 1996 and Parsons 1996, and for the Brouwer's concept of intuition, see, e.g., van Stigt 1990. To locate the place of Ibn Sīnā's theory of intuition in this complicated geography of theories of intuitions needs a separate paper.

Before closing this paper, we will briefly investigate the status of mathematical propositions in Ibn Sīnā's philosophy. According to his logic, universals, as *predicates* in propositions, are either essences or accidentals. Note that the concept of "accidental" here is different from the concept of "accident" (*'ara**), which is against the concept of "substance" (*jawhar*). The concept of "accidental" (*'ara*ī*) is in opposition to the concept of "essential" (*dhātī*). *Accidentals* are divided into two types, *necessary* and *unnecessary* accidentals. A necessary accidental is defined as an accidental which is impossible to be separated from the essence. In fact, every science discusses the necessary accidentals of its subject matter. A necessary accidental is necessary either for *existence* or for *quiddity*. For example, "heat" is a necessary accidental for the *existence* of the "fire". On the other hand, the necessary accidentals for quiddity are divided again into two types, *self-evident* necessary and *non-self-evident* necessary. A self-evident necessary accidental itself is divided into two smaller types, it may be *strictly* self-evident or *non-strictly* self-evident. Instead of giving definitions of these nested terms, let us look at some examples (Ibn Sīnā 1960, logic, chapter 2):

1. In the proposition "A triangle has angles", the predicate "angle" is a strictly self-evident necessary accidental for the subject "triangle".
2. In the proposition "The number four is even", the predicate "even" is a non-strictly self-evident necessary accidental for the subject "the number four".
3. In the proposition "The sum of angles of a triangle is equal to two right angles", the predicate "the sum of angles being equal to two right angles" is a non-self-evident necessary accidental for the subject "triangle".

According to Ibn Sīnā, a demonstration transfers *truth, certainty and necessity* from the premises to the conclusions. Premises or *first principles* are generally divided into two parts, the first principles for *all sciences* are called *common principles* (*al-u{ūl al-muta'ārafa*), and the first principles for every special science called *postulates* (*al-u{ūl al-maw*ū'a*). For example, "whole is bigger than [its] part" or "contradiction is impossible", etc are common principles, and "the shortest line between two points is a straight line" is a postulate for the science of geometry. Ibn Sīnā has a vast investigation in his different writings on the *ways* the common principles are acquired by the mind. A class of these common principles called as *awwaliyyāt*, are acquired *only* through the intellective faculty. These are propositions that are *obvious* for the intellective faculty and accepting them is *necessary*. The above two examples of the common principles are of this category. Contrary to the common principles, which are certain, the postulates are *susceptible of doubt* (*mashkūk*).

Mathematical science is one of the main parts of the demonstrative sciences, which is based on *the certain premises* and demonstrations or proofs which transfer certainty from premises to conclusions. Mathematical premises are either the common principles (*awwaliyyāt*), like the proposition "the whole is bigger than [its] part", or innates (*fi}rī*), like the proposition "the number four is even" (see Ibn Sīnā 1960, logic, chapter 9). According to Ibn Sīnā, mathematical propositions are *certain, necessary* and have *essential truth*.

A natural question for a philosopher of mathematics is:

What relations may exist between Ibn Sīnā's characterization of mathematical propositions and mathematical knowledge, on the one hand, and Kant's classification of propositions into

analytic and synthetic propositions and mathematical knowledge into a priori and a posteriori knowledge, on the other hand?

The following quotation gives a *partial* answer to the above question:

It is not the case that every science uses postulates, but in some sciences only definitions and *awwaliyyāt* are used, for example in arithmetic. But in geometry, all kinds of principles [definitions, common principles and postulates] are used (Ibn Sīnā 1956, chapter 12).

The immediate conclusion is, according to Ibn Sīnā, that *arithmetical knowledge is a priori* and *geometrical propositions are synthetic* in the sense of Kant. Moreover we can conclude that, by Ibn Sīnā's analysis, arithmetical propositions are *not* analytic, since the negations of arithmetical propositions are *not* self-contradictory. So according to Ibn Sīnā, *arithmetical propositions are synthetic* in the sense of Kant as well. We admit that our conclusion about non-analyticity of arithmetical propositions is debatable. One may argue that Ibn Sīnā's concept of the common principles (*awwaliyyāt*) is wider than the *usual set of the logical* axioms. That would imply that arithmetical propositions are analytic. We leave *open* this problem⁹.

We have not found any *explicit* claim of Ibn Sīnā on priority or posteriority of geometrical postulates. However, based on his writings, in particular his discussion on the difference between common principles and postulates, and an example from geometry in Ibn Sīnā 1956, chapter 12, we believe that, most probably, he will admit geometrical knowledge as *a priori knowledge*.¹⁰

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Notes

1. All translations from Arabic into Farsi are my translations, and all terms and expressions inside [] are my *interpretations*.
2. Mathematics is translated into "*al-Ta'līmiyyāt*", which literally means "what is related to *ta'līm*", and "*ta'līm*" itself means "teaching and learning". This translation of "mathematics" into Arabic is very close to the original meaning of the word "*mathēma*".
3. It should be mentioned that after having presented his definitions of the subject matter of three branches of philosophical sciences, i.e., physics, mathematics and metaphysics, Ibn Sīnā immediately discusses the subject matter of logic. Apart from how he describes that, the point is that he includes "Logic" in theoretical philosophy, at least as far as the description of its subject matter is concerned. That is, somehow implausible in his doctrine. Note that metaphysics of *al-Shifā'* is as called "thirteenth art" (*fann*). The best way to justify counting the different parts of *al-Shifā'*, is to start with physics including 8 arts, and then mathematics including 4 arts, and finally metaphysics starts from thirteenth art. In this way, metaphysics matches with the overall plan of the book. There is no place for logic in metaphysics of *al-Shifā'*. That means, according to Ibn Sīnā, logic is *not* a branch of theoretical *sciences* (See also Sabra 1980 for more details.).
4. According to Mullā |adrā the estimative faculty abstracts line, surface and solid from matter, but these are not separated from matter in external existence (see Mullā |adrā 1925, page 20). He then concludes that, at least, this part of geometry cannot be counted as a part of metaphysics.
5. Here Mullā |adrā has a third reason for not considering arithmetic as part of metaphysics. He says that number, which is the subject of arithmetic, and the unity, which is the principle of arithmetical numbers, is different from the unity that exists in the Separate and, moreover, the Separate does have numbers constructed of units. A number, which is a quantity, may have proportions and such a number can be only found in matter, since such a

number is an accident of physics, not something as a principle of the physical objects (see Mullā |adrā 1925, page 20).

6. Here “some” means “at least two”. It is worth knowing that according to Ibn Sīnā, “number” is another name for “plurality” and this concept is applied only for sets with *at least two elements*, so “numbers” starts from 2, i.e., zero and one are *not* numbers. Unit is the building block of all numbers, but it is *not* a number itself (see Ibn Sīnā 1997, book 3, chapter 3). So *empty set* and *singletons* do *not* exist even in the mind.

7. See, for example, the *admissible* syllogisms in Ibn Sīnā 1956.

8. A natural question may arise here: Is there any relation between “secondary intelligibles” and “immaterial intellection”? It is plausible to assume that the objects of immaterial intellection that can be perceived *through* “forms” of objects are *necessarily* secondary intelligibles. However, there are also objects of immaterial intellection that are perceived without having “forms”, like ego (See also Sabra 1980).

9. Kant’s notion of intuition is *interpretable* in the concept of construction, and his conclusion on the synthetic (a priori) property of mathematical statements is based on his notion of intuition. The term “construction” in Kant’s time had an established use in at least one part of mathematics, i.e., in geometry. It is natural to assume that what Kant primarily has in mind are constructions in geometry (see Hintikka 1992 for more details). As is mentioned before, Ibn Sīnā came up to his notion of intuition mainly through his experiences in geometry. So Ibn Sīnā’s notion of intuition *may* have relation to what is called *construction of middle terms*.

10. There are many other important questions in philosophy of mathematics that are not considered in this paper. One of the most controversial is the concept of “infinity”. Ibn Sīnā’s theory of infinity is very similar to Aristotle’s, in the sense that he does *not* believe in *actual infinity*, and he believes in potential infinity as a procedural character, see, e.g., Ibn Sīnā 1960, a }-[]abī’iyyāt.

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Avicenna on Self-Awareness and Knowing that One Knows

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Abstract. One of the most well-known elements of Avicenna's philosophy is the famous thought experiment known as the "Flying Man." The Flying Man argument attempts to show that the soul possesses innate awareness of itself, and it has often been viewed as forerunner to the Cartesian *cogito*. But Avicenna's reflections on the nature of self-awareness and self-consciousness are by no means confined to the various versions of the Flying Man. Two of Avicenna's latest works, the *Investigations* and the *Notes*, contain numerous discussions of the soul's awareness of itself. From an examination of these works I show that Avicenna recognizes two distinct levels of self-knowledge: (1) primitive self-awareness, which is illustrated by the Flying Man; and (2) reflexive self-awareness, which comes from our awareness of cognizing some object other than ourselves. While Avicenna assigns primitive self-awareness a central role in ensuring the unity of the soul's operations, he encounters a number of difficulties in his efforts to explicate the relation of primitive self-awareness to the reflexive varieties of self-knowledge that he inherits from the Aristotelian tradition.

It is a commonplace in the history of philosophy that issues surrounding self-awareness, consciousness, and self-knowledge do not become prominent until the early modern period. For medieval philosophers, particularly those in the Aristotelian tradition, the nature of self-knowledge plays only an ancillary role in psychology and epistemology. This is a natural consequence of Aristotle's characterization of the intellect as a pure capacity that has no nature of its own: "Thus that in the soul which is called mind ... is, before it thinks, not actually any real thing."¹ Until the intellect has been actualized by some object, there is nothing for it to reflect upon; hence self-knowledge for Aristotle — at least in the case of human knowers — is derivative upon knowledge of other things: "Thought is itself thinkable in exactly the same way as its objects are."²

Like all historical generalizations, of course, this truism admits of striking individual exceptions. The most obvious and well-known exception in the medieval Islamic tradition is Avicenna (Ibn Sīnā, 980-1037), whose famous thought experiment known as the "Flying Man" centres on the human soul's awareness of itself. But Avicenna's reflections on the problems of awareness and consciousness are by no means confined to the various versions of the Flying Man.³ In particular, two of Avicenna's latest works, the *Investigations* and the *Notes* — both of which are in the form of remarks compiled by Avicenna's students⁴ — contain a wealth of tantalizing and often problematic reflections on the soul's awareness of itself (*shu'ūr bi-al-dhāt*).⁵ The purpose of the present study is to consider the account of self-awareness that emerges from these works against the backdrop of Avicenna's Flying Man. I will show that Avicenna recognizes two distinct levels of self-knowledge, the most basic of which is exemplified in the experience of the Flying Man, which I will label "primitive self-awareness."⁶ Primitive self-awareness violates many of the strictures placed on self-knowledge by the Aristotelian principles rehearsed above, and Avicenna differentiates it from the reflexive awareness of oneself via one's awareness of an object that is characteristic of Aristotelianism. He also distinguishes primitive self-awareness from our knowledge of our bodies and psychological faculties and from our scientific understanding of our essential

natures as humans; and he explicitly recognizes the capacity for “knowing that we know” as a distinctive form of self-knowledge. Primitive self-awareness plays a central role in ensuring the unity of the soul’s operations, especially its cognitive ones, and Avicenna appears to have seen the absence of such a unifying centre of awareness as a major lacuna within Aristotelian psychology. But in the end it remains unclear whether Avicenna is able to provide a coherent account of the relations among primitive self-awareness and the other varieties of self-knowledge that he inherits from the Aristotelian tradition.

1. The Flying Man: A Sketch

The broad contours of the Flying Man are generally well-known, so I will merely summarize the salient points here. To set up the thought experiment, Avicenna admonishes the reader to imagine herself in a state in which all forms of sensible perception are impossible, and he identifies two fundamental sources of sense knowledge to be bracketed: (1) everything previously acquired from experience, that is, all knowledge anchored in memory and imagination; and (2) any occurrent sensations. In order to accomplish this, she is supposed to imagine herself: (1') in a pristine, newly-created state, but fully mature (*kāmilan*);⁷ this allows her to disregard all empirical knowledge, while presupposing an intellect with full rational capacities; and (2') suspended in a void so that her limbs do not touch one another and she can neither see, hear, touch, smell, nor taste anything. This prevents her both from feeling her own body and from sensing external objects.⁸ Avicenna then asks whether self-awareness would be absent in such a state. Would a person, while deprived of all sensory experience, be entirely lacking in self-awareness? Avicenna believes that no one “endowed with insight” would deny that her awareness of herself would remain stable even in these conditions.⁹ He is confident that even under these extreme conditions, the subject would continue to affirm “the existence of his self” (*wujūd dhāti-hi*).¹⁰ Assuming that we share his intuition on this point, Avicenna points out that this affirmation takes place despite the fact that all sense perception, both internal and external, is cut off. We remain aware of the existence of our selves, but under the state hypothesized in the Flying Man we are entirely oblivious to the existence of our bodies; hence this affirmation of our existence cannot be dependent upon the experience of having a body. Avicenna thus concludes that since “it is not possible for the thing of which one is aware and not aware to be one in any respect,” it follows that the self cannot be either the whole body nor any one of its parts.¹¹

This last move in the Flying Man, which is repeated in all of its versions, is of course problematic, since it seems to contain the obviously fallacious inference pattern, “If I know x but I do not know y , then x cannot be the same as y .” The question of whether Avicenna explicitly or implicitly commits this fallacy — a charge often laid against the Cartesian *cogito* as well — has been much discussed. It is not a question that I plan to take up here for its own sake, however, since it is primarily of relevance to the question of Avicennian dualism. It is noteworthy, however, that while the Flying Man argument focuses primarily on the impossibility that self-awareness is a mode of sense perception, the primitive character of the experience exemplified in the Flying Man poses parallel and equal difficulties for the claim that it could be a mode of intellectual understanding as well, as we will see below.¹²

2. Primitive Self-Awareness

The scenario imagined in the Flying Man is designed to show that self-awareness is always present in the human soul, independently of our awareness of other objects, in particular the objects of sense faculties. In the *Notes* and *Discussions*, Avicenna attempts to provide a more systematic account of the epistemic primitiveness of self-awareness over all other forms of

knowledge by employing the fundamental epistemological distinction between innate and acquired knowledge.¹³ Self-awareness is placed in the realm of innate knowledge, and comparisons are drawn between self-awareness and other paradigmatic cases of innate knowledge:

Self-awareness is essential to the soul (*al-shu'ūr bi-al-dhāt dhātī li-l-nafs*), it is not acquired from outside. It is as if, when the self comes to be, awareness comes to be along with it. Nor are we aware of [the self] through an instrument, but rather, we are aware of it through itself and from itself. And our awareness is an awareness without qualification, that is, there is no condition for it in any way; and it is always aware, not at one time and not another.¹⁴

A bit later in this passage, he makes this same assertion in even more striking terms, identifying self-awareness with the soul's very existence:

Our awareness of ourselves is our very existence (*shu'ūr-nā bi-dhāt-nā huwa nafs wujūd-nā*). ... Self-awareness is natural (*gharīzah*) to the self, for it is its existence itself, so there is no need of anything external by which we perceive the self. Rather, the self is that by which we perceive the self.¹⁵

We can isolate a number of claims made in these passages regarding the nature of primitive self-awareness and what it means to say that it is “innate” or “natural”:

1. It is essential to the soul; nothing could be a (human) soul if it did not possess self-awareness;
2. There is no cause outside the soul from which it acquires awareness of itself;
3. No instrument or medium is required in order to become self-aware; we perceive the self “through itself”;
4. Self-awareness is direct and unconditioned;
5. It is present in the soul from the beginning of its existence;
6. It is continual, not intermittent and episodic; and
7. The self *just is* awareness: for the self to *exist* at all is for it to be aware of itself.

These points are closely interrelated and can be further reduced to two groups: 1, 5, 6, and 7 all articulate the basic thesis that the self-awareness is an essential attribute of human existence, constitutive of the very fabric of our being; 2, 3, and 4 express the principal consequence of this basic thesis, namely, that self-awareness cannot be causally dependent upon anything at all outside the soul. Self-awareness is direct and unmediated in any way.

It seems obvious that such a view is entirely at odds with the Aristotelian thesis that the human soul can only have knowledge of itself concomitant with its awareness of an object. Indeed, the points that Avicenna emphasizes in these passages seem deliberately formulated so as to invoke and at the same time to reject the Aristotelian claim that self-awareness is a derivative psychological state. But what are the grounds which entitle Avicenna to make this claim? If Avicenna is correct that self-awareness is indeed innate, not acquired, then it will have the epistemic status of a self-evident principle or axiom which need not and cannot be demonstrated on the basis of prior principles. Yet even self-evident principles can become subject to doubt, and in such cases they will require something in the way of argumentative support. Thought experiments are one technique that can be called upon in such circumstances, so we might expect Avicenna to appeal to the experience of the Flying Man to confirm the primitiveness of self-awareness. Yet the Flying Man, colourful though it may be, does not go far enough towards establishing the primitiveness thesis, since it merely prescind from all sensory awareness. The claim made here is a stronger one epistemologically, since it asserts that self-awareness is not merely prior to and independent of corporeality and sensibility, but of all forms of cognitive awareness of other objects. Hence, Avicenna still needs to show that self-awareness is absolutely primitive in every respect, in the sense that it is *presupposed* by our capacity to understand anything at all. As evidence for this claim,

Avicenna offers the following analysis of the conditions under which awareness of other objects is possible:

My apprehension (*idrāk-ī*) of myself is something which subsists in me, it does not arise in me from the consideration of something else. For if I say: "I did this," I express my apprehension of myself even if I am heedless of my awareness of it. But from where could I know that I did this, unless I had first considered my self? Therefore I first considered my self, not its activity, nor did I consider anything by which I apprehended myself.¹⁶

A bit later, Avicenna repeats the same point:

Whenever we know something, there is in our knowledge of our apprehension of it an awareness of ourselves, though we do not know that our selves apprehended it. For we are aware primarily of ourselves. Otherwise when would we know that we had apprehended it if we had not first been aware of ourselves? This is as it were evidence (*bayyinah*), not a demonstration (*burhān*), that the soul is aware of itself.¹⁷

Self-awareness is innate to the soul and cognitively primary because only if I first know my self can I: (1) know anything else about myself; and (2) become aware of other things. Self-awareness is presupposed by any attribution of properties or actions to myself, since such attributions presume the existence of a subject for those attributes; and self-awareness is equally implicit in all the soul's acts of knowing other things, since it is a condition for the recognition of these objects *as* objects distinct from ourselves. Though Avicenna does not explicitly say so here, his position seems to allow that one can be aware of oneself without being concomitantly aware of any object. Self-awareness seems to be an exception to the general rule that all thinking is in some way *intentional* and directed toward an object. In contrast to the Aristotelian orthodoxy, then, the primary object of self-awareness is the *self* as a bare subject, not its *activity* of thinking.

3. Awareness and Consciousness

If primitive self-awareness is absolutely primary, as Avicenna urges, indeed even identical with the soul's existence, why would we ever need to be alerted to such a basic datum of experience? Avicenna himself admits that despite its primitive status, self-awareness is often something of which, paradoxically, we remain ignorant. Thus in the *Notes* he remarks: "A human being may be inattentive to his self-awareness, and [thereafter] be alerted to it"; and again, "But the soul may be oblivious to [itself] (*dhāhilah*), and need to be alerted, just as it may be oblivious to the primaries, and need to be alerted to them."¹⁸ The implication, then, is that consciousness is *not* the same thing as self-awareness, and that we often fail to be conscious of our own selves.

The most striking illustration of the distinction between consciousness and self-awareness is Avicenna's assertion that even in sleep or drunkenness no one would fail to affirm his own existence. This declaration occurs in the version of the Flying Man found in the *Directives*,¹⁹ and a similar point is made in the *Investigations*. In the latter work, Avicenna appeals to the existence of imaginative activity in sleep (i.e., dreaming), and he argues that self-awareness must necessarily be present in a person in whom there is cognitive activity of any kind. The fact that we are not fully conscious of that activity, and that we may fail to recall it when we awaken, is irrelevant. Thus understood, consciousness is not awareness, but rather, a second-order, reflexive operation for which primitive self-awareness is a necessary but insufficient condition:

A doubt was raised to him that someone who is asleep is not aware of himself. So he said: the person who is asleep acts upon his images just as he acts upon his sensibles while awake. And oftentimes he acts upon cogitative intellectual matters just as he does in waking. And in this state of his acting he is aware that he is the one acting, just as he is in the waking state. For if he awakens and remembers his acting, he remembers his awareness of himself, and if he awakens and

he does not remember this, he will not remember his self-awareness. And this is not a proof that he was not aware of himself, for the memory of self-awareness is different from self-awareness, or rather, the awareness of self-awareness is different from self-awareness.²⁰

The claim that we can be *unconsciously* aware of ourselves at first glance seems an oxymoron. Yet the property of being an object of awareness even in the absence of conscious thought is a basic feature of all innate or primary knowledge for Avicenna, and primitive self-awareness too possesses this property in virtue of being innate. Thus the primary concepts and propositions on which all our thought depends are likewise absolutely basic, and we often take them for granted because of their pervasive role in all our cognitive operations.²¹ We are seldom consciously aware of our employment of the principle of contradiction, for example, even though we cannot entertain any proposition unless it conforms to that principle. By the same token, we cannot think of any object unless we are at the same time aware of our selves as the underlying subject of the thought. But in neither of these cases need we be conscious of the role played by our innate knowledge in our knowledge of other things. Indeed, Avicenna seems to imply that it is unusual for innate knowledge of any sort to rise to the level of full consciousness.

Still, the separation of consciousness from awareness is problematic in an Avicennian context, since Avicenna does not have open to him the obvious appeal to memory as a means of explaining how I can be aware of objects of knowledge which I am not consciously entertaining.²² For it is a key tenet of Avicenna's cognitive psychology that the concept of memory applied to the intellect is meaningless. Avicenna argues for this controversial conclusion on the grounds that "it is impossible that [an intelligible] form should be existent in complete actuality in the soul but [the soul] not understand it in complete actuality, since 'it understands it' means nothing other than that the form is existent in it."²³ What, then, can it mean to claim that I am aware of any object — including my self — and yet not actually, that is, consciously, understanding it?

In the case of other examples of innate knowledge, this problem is fairly easily resolved. For primary intelligibles are not fully *innate* for Avicenna in the way we ordinarily understand innateness. In this respect, the legacy of the Aristotelian identification of the human intellect as in pure potency to its intelligibles retains its hold on Avicenna.²⁴ There are two principal characteristics of innate knowledge as it is manifested in the primary intelligibles: (1) we never actively seek to learn them and we are not conscious of when they are acquired; and (2) under normal circumstances we do not consciously differentiate these intelligibles from the derivative intelligibles in which they are implicitly contained. The second of these two characteristics is what allows Avicenna to make sense of the claim that we are *aware* of innate intelligibles — in the sense that they are actually present *in* our minds — even though we are not consciously thinking of *them*. Their innate presence in us is in virtue of their containment in *other* concepts, and hence they do not violate Avicenna's rejection of intellectual memory. If our minds were totally empty of all other thoughts, we would not possess these ideas either.

This solution is open to Avicenna to a limited extent in the case of primitive self-awareness, since self-awareness is a precondition for thinking about any object other than the self. But Avicenna has made the stronger claim that self-awareness *is* the soul's very subsistence and existence. At no point can the soul *exist* unless it is aware of itself, even if it is not consciously or actively thinking of itself. This is not true even of the most fundamental of primary intelligibles. Self-awareness, then, cannot be the soul's implicit consideration of itself as the subject of *other* thoughts, since that would, in effect, reduce primitive self-awareness to Aristotelian reflexive awareness. In primitive self-awareness the self is not present to itself as an intelligible object in the way that other objects are present in its thought. Of what then, is the soul aware when it is aware of nothing but the existence of itself?

4. Awareness and Identity: What Self-Awareness is Not

In my overview of the Flying Man argument, I noted that Avicenna identifies the object to which we are alerted by the thought experiment as the existence (*wujūd*) or individual existence (*annīyah*) of the self or soul (*dhāt; nafs*). While the same terminology is also found in the *Notes* and *Investigations*, in these works Avicenna prefers to speak of our awareness of our *huwīyah* or “individual identity.” Like the various terms for “existence,” “identity” serves to convey the primitiveness of self-awareness, the fact that it is empty of any specific cognitive content. But the term “identity” also captures two additional properties that are distinctive of primitive self-awareness. First and most fundamentally, self-awareness is the only form of knowledge in which cognitive identification — the identity of knower and known — is on Avicenna’s view completely realized in human thought.²⁵

When you are aware of yourself, it is necessary that there is identity (*huwīyah*) here between the one aware and the thing of which there is awareness. ... And if you are aware of something other than yourself, in this case there will be an otherness between the one who is aware the object of awareness. ... As for awareness of the self, the one who is aware of that which he is, is his very self, so here there is identity and no otherness in any respect.²⁶

The second property follows as a corollary of the complete identity between knower and known: self-awareness must be direct and cannot be mediated in any way at all. While the denial of intermediaries in self-awareness is usually linked with attempts to show that self-awareness cannot be a form of sense perception, this is nonetheless a basic feature of primitive self-awareness whose consequences extend to the intellectual as well as the sensible sphere.²⁷

In the course of elaborating upon the claim that we are primitively aware only of our individual identity and existence, Avicenna eliminates three distinct but closely related theses regarding the nature of self-awareness and in particular the sort of knowledge of the self that can be gained in this primitive act. According to Avicenna, primitive self-awareness is neither: (1) an activity of any discrete *part* or *faculty* within the soul; hence it does not have any particular part of the soul as its object; nor (2) is it awareness of the soul’s essential nature or quiddity; nor (3) is it awareness of the aggregate or totality of the soul’s collected parts.

Parts and Faculties. That self-awareness cannot pertain to a part of the soul in the sense of a particular faculty within the soul follows directly from the claim that the sole object of primitive self-awareness is one’s individual identity. Since the self is not identical with any one of its parts or faculties, self-awareness cannot be reducible to any limited form of reflexive understanding by one cognitive faculty to the exclusion of the others, even though the individual faculties of the soul are all capable, at least in a limited way, of reflexive awareness of their own activities. When such reflexive awareness occurs, it is not primitive, but a form of second-order awareness or knowing that one knows:

And as for awareness, you are aware of your identity (*huwīyah-ka*), but yet you are not aware of any one of your faculties such that it is the object of awareness. For then you would not be aware of yourself but of some part of yourself. And if you were aware of yourself not through your self, but rather through a faculty such as sensation or imagination, then the object of awareness would not be [the same as] that which is aware, and along with your awareness of yourself you would be aware that you are aware of your soul (*bi-naḥsi-ka*) and that you are the one who is aware of your soul.²⁸

In this passage and remarks elsewhere, Avicenna tends to focus on the impossibility of the corporeal faculties of sensation and imagination being the powers by which the soul is aware of itself, in the same way that he tends to associate the unmediated character of self-awareness with the denial that self-awareness is a sensory act. Nonetheless, the analysis on which Avicenna’s point is based does not depend in any special way upon the corporeal basis of

sensation — the senses simply provide the most vivid examples of mediated and partial knowledge of the self. Thus, even in one passage where he is responding to a specific question about the soul's ability to understand itself intellectually, Avicenna quickly reverts to counter-examples based upon the limitations of the senses. The response here adds another dimension to the denial that self-awareness can be attributed to the activity of any particular faculty within the soul, for Avicenna eliminates not only reflexive awareness by a faculty of its own acts, but also the grasp of any one part of the soul by another. In such cases the identity criterion for self-awareness is doubly violated, since neither the subject nor the object of awareness is identical with the soul in its totality:

And if this power is subsistent through a body, and your soul is not subsistent in this body, then that which is aware of this body through that faculty would belong to something separate through another form. So there is no awareness of yourself in this case in any way, and no apprehension of yourself through what is proper to it (*bi-khuṣūṣiyati-hā*).²⁹ Rather, some body would sense with something other than itself, in the way that you sense your leg with your hand.³⁰

While the example here centres on the limitations of the senses, the conclusion would seem to be universally applicable to all parts of the soul. To the extent that any cognitive faculty functions as an instrument by which the soul performs a determinate range of activities directed towards a determinate class of objects, its operations will violate the identity criterion for self-awareness, regardless of whether or not the faculty in question uses bodily organs in the performance of those acts.

Universal and Quidditative Knowledge. Despite his tendency to focus on examples drawn from the senses, Avicenna does admit that primitive self-awareness cannot be an act of the intellect in any standard sense. He denies, for example, that self-awareness is implicit in the act of understanding the general concept “soul” or “humanity” which I exemplify as a particular instance, on the grounds that one cannot simultaneously be aware of a whole as well as one of parts. In this case the “whole” is not the self, however, but the universal, and the “part” is not a faculty of the soul, but rather, my self as a particular instance falling under a universal class:

Next he was asked, “And how do I perceive the general intention of the soul; and am I at the same time also aware of my individual soul?” He answered, “No, it is not possible to be aware of something as well as one of its divisions (*wa-tajzi 'ah-hu*).”³¹

While the denial that self-awareness can be accomplished by any isolated part or faculty of the soul thus applies as much to the intellect as to the senses, it is more common to find Avicenna arguing against the identification of self-awareness as an act of intellection on the grounds that self-awareness neither consists in nor supervenes upon universal knowledge of the soul's essential nature:

After this he was asked: “And if I understand the soul through the general intention, am I in that case a soul absolutely, not a particularized, individuated soul; so am I therefore every soul?” The reply: “There is a difference between the absolute considered in itself and universality. For universality is what is said of every soul which has another consideration; and one of these two is a part of my soul, the other is not.”³²

In this passage Avicenna appeals to the distinction between quiddity and universality articulated in Book 5 of the *Metaphysics* of the *Healing*. On this account of universals, any object that I know *exists* in my intellect, and in virtue of that mental existence its quiddity acquires the additional property of universality. An intelligible universal is thus an instance of some quiddity — in this case “humanity” — enjoying a form of conceptual existence in which it is combined with the properties peculiar to that realm of existence.³³ This entails, as Avicenna here indicates, that when any absolute quiddity is instantiated in mental existence it is but one *part* or constituent of the resultant universal. By the same token, when the quiddity “humanity” is combined with a set of properties peculiar to concrete, extramental existence to

form an individual human, it once again is but a part or constituent of an entirely distinct entity. Thus, while my own proper self and my universal concept of “human being” *share* the same essence or quiddity, “humanity,” “humanity” itself is not completely *identical* with either my self nor that concept. While there is *partial* identity between my universal concept of “human” or “soul” and my self, then, the identity is not complete. So on these grounds too intellectual knowledge even of my own nature fails to meet the identity criterion for primitive self-awareness.

The understanding of the universal under which my own nature falls is thus neither necessary nor sufficient for self-awareness. Indeed, as Avicenna notes in the first of the two passages cited above,³⁴ to the extent that the universal and the particular are two different sorts of cognitive objects, when I am actively contemplating the universal “human,” any explicit awareness of my individual self will be precluded by another axiom of Avicenna’s cognitive psychology, namely, that the soul can only consciously think of one intelligible at a time: “For it is not in the capacity of our souls to understand intelligible things together in a single instant.”³⁵ With this we have yet another explanation for Avicenna’s claim that primitive self-awareness must in most instances be differentiated from conscious attention. For by and large my everyday conscious thoughts are focused on objects *other* than my own individual identity and existence, and I cannot, on Avicennian principles, actively and consciously attend to my individual existence while at the same time actively thinking other thoughts. That is why, one presumes, thought experiments like the Flying Man are needed.

Collections of Parts. Thus far I have considered Avicenna’s grounds for rejecting two of the three candidates that might be put forward as sources of self-awareness — one of the soul’s particular cognitive faculties, or its intellectual understanding of its own essential nature. But Avicenna also rejects the claim that self-awareness might be nothing more than our perception of the total aggregate or collection of our various parts. One question posed in the *Investigations* wonders whether a human being just is the collection of his parts (*jumlah-hu*), and if so, whether the totality of that collection constitutes the object of his awareness. In response Avicenna argues that self-awareness cannot be equated with awareness of the sum total of one’s parts, since it is possible to be aware of one’s individual existence while lacking awareness of the collection in its entirety. This follows from Avicenna’s claim that self-awareness is the very existence of the self and thus something that is always present at every moment in which the self subsists. But the totality of one’s parts does not display any stability and continuity, for those parts change over time, and many of them are hidden from us under ordinary circumstances. Avicenna casts the “hidden parts” argument as an inference based on the mutability and hiddenness of our internal organs, an emphasis that might once again lead us to suppose that the main impediment to self-awareness here derives from the bodily side of our selves.³⁶

For many a person who is aware of the being of his existence (*bi-wujūdi ānīyati-hi*) is not aware of the collection, and were it not for autopsy there would have been no knowledge of a heart, nor a brain, nor any principal nor subordinate organ. Whereas before all this he was aware of his existence. Moreover, if the object of awareness remains an object of awareness while, for example, something of the collection is separated in such a way that there is no sensing of it, in the way that a limb is cut off from an anaesthetized amputee, then it is conceivable that this could happen to him and he would not sense it, nor be aware that the collection has been altered, whereas he would be aware of his self, that it is his self, as if he had not been altered. And as for the thing from the collection which is other than the collection, it is either the case that it is an internal organ or an external organ. And it may be that none of the internal organs is an object of awareness at all, but existence (*al-ānīyah*) is an object of awareness prior to autopsy. And that of which there is awareness is different from that of which there is no awareness. And the external organs may be missing or changed, whereas the existence of which we are aware is one thing in its being an object of awareness as an individual unity (*wa ldatan shakh ḥīyatan*).³⁷

In its appeal to the constancy of my awareness of the individual unity that is my self, even in the absence of complete awareness of my bodily members, this line of reasoning appears to commit the same suspect fallacy of which the Flying Man argument is often accused: I am aware of my self; I am not aware of the totality of my parts; therefore my self is distinct from the totality. But Avicenna's distinction between primitive self-awareness and conscious thought lessens the sophistical appearance of the argument in the present context, and it allows us to give the argument a purely epistemological interpretation. On the basis of that distinction, the "ignorance" of our brains or hearts to which Avicenna refers cannot be understood as a simple failure to be *conscious* of them. So the argument merely illustrates the epistemological conclusion that primitive self-awareness is not the *same kind* of knowledge as bodily consciousness: it tells us nothing about the underlying nature of the self nor its distinction from the collection.

Yet if we follow this line of interpretation, we will also be prohibited from identifying primitive self-awareness as identical with any conscious state of an immaterial mind or soul. For it can surely be claimed that non-philosophers and materialists lack *consciousness* of their non-material parts as well, that is, of their immaterial minds and rational souls, despite the continuity of their self-awareness. That is, after all, what allows them to be materialists. So if Avicenna's argument here is meant to apply to bodily parts in particular, and not equally to the immaterial faculties of the soul, it is inadequate. What it does establish is that if self-awareness is indeed a necessary concomitant of our existence underlying all our derivative conscious states, it must be an entirely different mode of knowing from any of those states, be they sensible or intellectual.

5. Individuation and Self-Awareness

We have seen, then, that despite a few indications to the contrary, Avicenna generally appears to recognize that he cannot draw any determinate conclusions regarding the nature of the self based on his analysis of self-awareness alone. Given the very primitiveness of that state, the most one can do is to establish what self-awareness is not. But there is one suspect presupposition that continues to inform Avicenna's discussions of primitive self-awareness, and that is the assumption that there *is* an underlying self of some sort which is, at a bare minimum, a single, individual unity to which all the soul's manifold activities are somehow ultimately referred.

The problem that is lurking here is one which brings Avicenna up against the anomalies in his dualistic account of human nature. Avicenna claims that human souls are subsistent entities in their own right, and yet, since there are multiple individuals in the species "human," those individuals can only be distinguished from one another by the diversity of their matter.³⁸ If the self is indeed a unity, as Avicenna's account of self-awareness implies, and if its unifying function is incompatible with corporeality, then self-awareness would seem to be a function of the soul itself.³⁹ But Avicenna has admitted, perhaps reluctantly, that self-awareness cannot be a function of the intellect, since the self is not a universal. So we are faced with the question, what mode of cognition corresponds to a self that is at once subsistent and individual, but not entirely immaterial, and not the sole exemplar of its own nature or quiddity? The dilemma that Avicenna faces here is nicely captured in the *Investigations*:

He was asked: By what faculty do we perceive our particular selves? For the soul's apprehension of intentions is either through the intellective faculty — but the awareness of the particular self (*al-dhāt al-juz'īy*) is not intellected; or through the estimative faculty — but the estimative faculty apprehends intentions conjoined to images. And it has been shown that I am aware of my essence even if I am not aware of my limbs and do not imagine my body.⁴⁰

Avicenna's immediate response to the problem is simply to note that the impediment to the intellectual understanding of an individual is matter, which is intrinsically unintelligible, not individuality *per se*. Hence, if there is some aspect of the human soul's individuation that is not simply reducible to matter and material accidents, the individual self may in some way be intelligible. Still, Avicenna remains non-committal as to the exact faculty to which primitive self-awareness should be traced:

He answered: It has been shown that the universal intention is not apprehended through a body, and that the individual intention which is individuated through material accidents to a determinate magnitude and a determinate place is not perceived without a body; but it has not been shown that the particular cannot be apprehended at all without a body, nor that the particular cannot be converted into the judgement of the universal. Rather, when the individuation of the particular is not by means of magnitude, place, and the like, then there is no hindrance to the one's being aware of it—so I suppose it would be the intellect. The impossibility of this has not been shown anywhere. And there is no harm in there being a material cause of this individual, and of its being a material thing in some respect, so long as the concomitant individuating form is not itself a material form, but is instead one of the forms characteristic of that whose individuation is not through a body. The intellect or the intellective soul cannot, however, perceive an individual particular by means of material forms with magnitude.⁴¹

Even if we grant that the material aspects of human nature in and of themselves do not rule out the possibility of an intellectual grasp of ourselves as individuals, it is difficult to see how such knowledge would fit the account that Avicenna has given of primitive self-awareness. When Avicenna does attempt to describe more precisely how such intellectual self-awareness might be accomplished, the explanation turns on the possibility of singling out an individual by means of its accidents through a process whereby I understand myself by combining my grasp of “humanity” with my understanding of properties that are peculiar to me.⁴²

So he replied: If this self-awareness is not called an “intellection” (*'aqlan*), but rather, the term “intellection” is proper to what belongs to the awareness of the abstract universal, then one could say that my awareness of myself is not an intellection and that I do not understand my self. But if every perception of what subsists abstractly is called an “intellection,” it need not be granted that every intelligible of everything is a universal intention subsisting through its definition. Though perhaps if it is to be granted, it is only granted in the case of external intelligibles; nonetheless it is certain this is not to be granted absolutely. For not everything has a definition, nor is every intelligible just a simple concept, but rather, the thing may be understood through its states, so that its definition is perceived mixed with its accidents. In this way, when I understand my self I understand a definition to which is conjoined an inseparable accident (*'ārid lāzim*).⁴³

Avicenna's point, then, seems to be we can conceptualize complex intelligibles such as “laughing human” or “political human,” and that these concepts can provide a model for intellectual self-awareness of our individual identities. My understanding of my self on that model would consist of the definition of “human” plus a series of necessary accidents conjoined to that definition, which in concert would contract that definition to pick out me alone.⁴⁴ But there are obvious difficulties with this solution. From a metaphysical perspective, it is not clear what property or set of properties could count as a necessary accident singling out my individual self, since Avicenna generally rejects bundle theories of individuation.⁴⁵ More importantly in the present context, however, this model seems to lack entirely the immediacy which is the characteristic feature of primitive self-awareness. Even if it is indeed possible for me to grasp my own individuality intellectually through a process such as the one just described, such an intellection could in no sense be counted as one in which I am simply aware of my individual existence and identity prior to any conscious awareness I have of either my essence or my attributes.

Avicenna's account of primitive self-awareness thus seems to require a different paradigm of intelligibility which would allow for direct acquaintance with an immaterial particular. In a few places Avicenna indicates that such an account might be developed on the basis of

parallels between self-awareness and sensible observation. This, at least, is implied by Avicenna's inclusion of propositions expressing self-awareness under the category of "observational" (*al-mushāhadāt*) premises in the *Directives*, a category which is principally comprised of sensible propositions such as "the sun is shining," and "fire is hot." In this context, however, Avicenna does not distinguish sharply between primitive self-awareness and our awareness of our mental states, since the task at hand is to classify propositions based upon their reliability, rather than to explore the cognitive processes that underlie them.⁴⁶ So these propositions have already been filtered by the intellect and no longer display the immediacy of the perceptual acts on which they are based. In the *Notes* too Avicenna compares self-awareness to the knowledge we gain of an individual by direct acquaintance (*al-ma'rifah*) and through observation (*al-mushāhadah*).⁴⁷ But ultimately Avicenna fails to develop these suggestions in any comprehensive way, so that the exact nature of primitive self-awareness remains somewhat mysterious.

6. Second-Order Awareness and Knowing that One Knows

Thus far I have focused solely on Avicenna's account of primitive self-awareness, since that is the form of self-knowledge to which Avicenna devotes the most attention. But Avicenna does not entirely neglect other forms of self-knowledge, and in the course of his accounts of self-awareness he often invokes the distinction between primitive self-awareness on the one hand, and awareness *that* we are aware on the other hand. Whereas primitive self-awareness is a form of innate knowledge and thus is of a piece with the soul's very existence, awareness *of* awareness is something which we must acquire through conscious effort:

As for its awareness that it is aware of itself, this it has through acquisition. And for this reason it does not know that it is aware of itself, and likewise for the rest of the things for which it acquires the power to become aware. And this is something which is not existent in it, which it needs to procure for itself.⁴⁸

Unlike primitive self-awareness, whose exact character remains obscure despite its pervasiveness, awareness that we are aware is an intellectual act, and hence it is always at the level of actual conscious thought:

But our being aware that we are aware is an activity of the intellect. Self-awareness belongs to the soul in actuality, for it is always aware of itself. And as for the awareness of the awareness, it is potential. And if the awareness of the awareness were actual, it would always be [so], and there would be no need for the consideration of the intellect.⁴⁹

At first glance it might appear that this acquired form of awareness is the Avicennian counterpart to the traditional Aristotelian conception of self-awareness as an act concomitant with the understanding of other things. Yet there are reasons to think that such a comparison is not entirely apt. Avicenna's model is clearly a propositional one, whereas the Aristotelian notion of an awareness that is concomitant with our knowledge of an object seems prior to any propositional judgment. I suspect, however, that Avicenna would claim that there really is no such thing as reflexive self-awareness in the Aristotelian sense, since he rejects both of the principles upon which the Aristotelian account is based.⁵⁰ So Avicenna would probably agree that Aristotelian reflexive knowledge is either nothing but awareness that we are aware, and hence it is indeed propositional; or that it offers a flawed account of primitive self-awareness and is to be rejected outright. Similarly, it is not clear whether our intellectual grasp of our own natures or quiddities — i.e., our simple understanding of the intelligibles "human" and "soul" — would count as instances of awareness *that* we are aware in Avicenna's eyes. Here too it seems unlikely that Avicenna would consider such knowledge to be a form of second-order *awareness*. For in order to count as "awareness," it would seem necessary for the

knower to apply the concept “human” to her understanding of herself. Failing that, her knowledge of humanity would seem to constitute *self*-knowledge only incidentally.

Despite his relative silence on the exact scope and nature of second-order awareness, there are a couple of short and provocative passages in which Avicenna attempts to offer some account the role it plays within human knowledge. Two functions seem paramount: (1) second-order awareness is necessary for conscious thought to occur; and (2) second-order awareness plays a role in the attainment of certitude (*al-yaqīn*).

With respect to (1), Avicenna argues that the complete identity that characterizes primitive self-awareness necessitates that a different sort of cognitive act must occur in order to acquire knowledge *that* one is aware: “For so long as you know (*ta’rifu*) yourself, you do not know that this awareness of it from yourself is yourself.”⁵¹ This is a direct consequence of Avicenna’s distinction between awareness and consciousness. Since self-awareness under normal circumstances is something that we are not attentive to, it must be made the subject of conscious reflection by the intellect in order to play an active role in our cognitive pursuits. And the role that second-order awareness plays in those pursuits seems to be in its own way a central and foundational one, especially for the philosopher. For certitude, the epistemic goal at which philosophy is supposed to aim, is defined as an act of second-order knowledge.

Hence, with respect to (2), Avicenna argues, in a very compact statement prefaced to one of his accounts of primitive self-awareness, that insofar as certitude entails knowing that one knows, it is akin to and perhaps dependent on second-order awareness:

Certitude is to know that you know, and to know that you know that you know, *ad infinitum*. And the apprehension of one’s self is like this. For you apprehend your self, and you know that you apprehend it, and you know that you know that you apprehend it — *ad infinitum*.⁵²

Avicenna does not make it entirely clear here whether “knowing that one knows” and “being aware that one is aware” are synonymous. Does Avicenna believe that knowing that one knows is simply a special case of second-order awareness focused on one’s awareness of a particular object, or does he intend to make the stronger claim that certitude is ultimately *dependent* upon our capacity to bring primitive self-awareness to the level of conscious attention? Some remarks on the nature of our *feeling* of certitude in the *Psychology* give us reason to think that Avicenna would indeed assign self-awareness a foundational role in all certain knowledge.

In the passage in question, Avicenna presents the phenomenon of a person who feels certain that she knows the answer to some question as soon as it is posed to her, even when she has never actually worked out the point at issue before. In effect, she teaches herself as well as her audience during the course of her articulation of the reply. Avicenna’s account of what is going on in such cases is somewhat problematic, although it coheres well with the general principles that are laid out in this part of the *Psychology*. What Avicenna argues is that in cases such as these the knower is actually certain of the reply she is about to give, and that actual certitude is only possible if one’s belief is indeed true and one’s knowledge actual. Given the respondent’s actual certitude, the knowledge in question cannot be potential, even proximately so, “because it is impossible to be certain that something actually unknown is known by him but stored away. For how could you be certain of the state of something unless the thing (*al-amr*) itself in relation to which you were certain were known?”⁵³

Now in the case of our knowledge of things other than ourselves, the inference from the strength of our psychological certitude to the reality of that about which we are certain is clearly suspicious. But the point does shed light on the role that Avicenna envisages for self-awareness in the attainment of certitude. For as we’ve seen, primitive self-awareness is the only form of knowledge that is, from the first moment of our existence, always actually present in us. And certitude, as here described, rests on an *actual* relation between the knower and that of which she is certain. Primitive self-awareness, then, is the only form of knowledge

in which the actual relation between the knower and the object known is guaranteed. Moreover, since the person who is actually certain of anything must grasp the relation between herself and the other objects of which she is certain, primitive self-awareness would also seem to be an ingredient within any additional claims we have to be certain of the nature of things other than ourselves. Certitude thus consists in the awareness that we are aware; it is not a distinct form of second-order knowledge in which primitive self-awareness plays no central role.

7. Knowing that We Know and the Problem of Infinite Regress

Avicenna's identification of certitude as a form of knowing that one knows is not unprecedented in the Islamic philosophical tradition. Al-Fārābī (ca. 870-950) had already stipulated this as one of the conditions of certitude in his discussions of the nature of demonstrative science:

Certitude is for us to believe concerning the truth to which assent has been given that it is not at all possible for the existence of what we believe of this thing to be different from what we believe; and in addition to this, we believe concerning this belief that another [belief] than it is not possible, even to the extent that whenever there is formed some belief concerning the first belief, it is not possible in one's view for it to be otherwise, and so on *ad infinitum*.⁵⁴

The principal function that this claim plays in al-Fārābī's epistemology is to differentiate knowledge from true opinion: while true opinions may indeed correspond with reality, al-Fārābī argues that only when we *know* that our belief in their correspondence is necessary does our opinion rise to the level of certitude. Al-Fārābī himself often uses the term "awareness" (*shu'ūr*) to explicate this second order-knowledge, and what he appears to have in mind is a criterion that involves the subject's direct acquaintance with the evidence upon which her belief is based, the fact that it rests on the subject's "own vision."⁵⁵ This in turn entails concomitant self-awareness, al-Fārābī suggests, since I must also recognize that it is *my* knowledge that is the guarantor of my belief. If a subject is certain of his belief, his cognitive state must be that of "someone who considers the thing at the time when he is considering it and is aware that he is considering it."⁵⁶

One striking feature of al-Fārābī's account of knowing that one knows is the claim that certitude entails an infinite regress of second-order acts of awareness. It is this feature of al-Fārābī's criterion that Avicenna himself echoes in the *Notes*, and it is also a point of contention in a debate over second-order knowledge between al-Ghazālī (1058-1111) and Averroes (Ibn Rushd, 1126-1198). Al-Fārābī himself does not comment much on the infinity condition: he does not state whether the infinity is potential or actual, for example. Given the Aristotelian prohibition against actual infinities, we might presume that the regress here is necessarily potential. If I am certain of something, then I will be able, if challenged, to assert second-order, third-order, etc. claims as required, but I need not and perhaps cannot actually accept an infinity of meta-propositions. The second-order claim is sufficient to establish certain knowledge, since it secures my grasp on the evidentiary basis for my belief. Hence, there is no danger that a sophisticated challenger might disturb my certitude by charging that while I may know that I know *p*, I may not *really* know that my knowledge won't falter when I reach a tenth-order or hundredth-order claim, for example.

Yet some version of the possibility of an infinite regress of self-awareness claims does seem to worry Avicenna. It is not, however, the infinite regress of second-order awareness that concerns him, but rather, the view that holds that our becoming alerted to our primitive self-awareness (as, for example, by performing the Flying Man), constitutes a *repetition* of the act of primitive self-awareness itself. This Avicenna denies: "A human being may be inattentive to his self-awareness, and be alerted to it; but he is not aware of himself twice."⁵⁷ Here, the

core of Avicenna's concern seems to be the preservation of the privileged character of self-awareness amongst the soul's cognitive acts. But the prohibition against the "repetition" of our selves in ourselves does not prevent an infinite regress of acts of knowing *that* we know. Rather, second-order awareness must necessarily be of a different kind from primitive self-awareness and have a distinct object from it: there must be some form of epistemic ascent here.

The problem posed by the infinite regress of awareness resurfaces in an exchange between Avicenna's critics, al-Ghazālī and Averroes. This debate is especially instructive for our purposes since many of al-Ghazālī's claims presuppose the Avicennian paradigm of self-awareness, in which second-order awareness is a distinct act of understanding from primitive self-awareness, whereas Averroes's responses are more faithful to the traditional Aristotelian picture. Unlike Avicenna and al-Fārābī, however, al-Ghazālī, explicitly rejects the possibility of an infinite regress of second-order acts:⁵⁸

Rather, he knows his being a knower by another knowledge, [and so on] until this terminates in a knowledge of which he is oblivious and does not know. We do not say that this regresses *ad infinitum* but that it stops [at a point] with a knowledge relating to its object, where [the individual] is oblivious to the existence of the knowledge but not [to that] of the object known. This is similar to a person who knows blackness, being, in his state of knowing, psychologically absorbed with the object of his knowledge — namely, blackness — but unaware of his [act of] knowing blackness, paying no heed to it. If he pays heed to it, it will require another knowledge [and so on] until his heeding ceases.⁵⁹

Al-Ghazālī's perspective here is ultimately far removed from Avicenna's. Al-Ghazālī seems to make self-awareness entirely dispensable to human knowledge, and its incidental character is even more pronounced than in the classical Aristotelian picture, where reflexive self-awareness, while not a necessary condition presupposed by all other knowledge, is nonetheless an inevitable by-product of it. Certainly al-Ghazālī's remarks are incompatible with the claim that certitude — that is, demonstrated, scientific knowledge — depends upon second-order acts of awareness. On al-Ghazālī's view, second-order awareness actually seems to be an *impediment* to complete awareness of the object of one's thought. For according to the above passage, in order to thwart the objectionable infinite regress of reflexive acts, we eventually posit a stage in which our absorption in the object known and our attention to it is so all-embracing that we lose ourselves entirely in the object and fail to note the otherness between it and ourselves.

Averroes's response to al-Ghazālī's remarks in the *Incoherence of the "Incoherence"* staunchly defends the Aristotelian view that self-knowledge is indistinguishable from our concomitant awareness of other things. Averroes does allow for an exception to this claim in cases where we are talking about my knowledge of my *individual* soul (*'ilm bi-naḥsi-hi al-shakhḥīyah*), by which Averroes means nothing but my ability to perceive my own individuating states and actions.⁶⁰ But on Averroes's view this sort of individual self-knowledge is clearly inferior to the self-knowledge that *is* identical with what is known, since in the latter case the knower has universal, essential knowledge of "the quiddity which is proper to him." Averroes's point here is not simply that we only truly know ourselves when we have attained a scientific understanding of human nature. Rather, Averroes makes the following assertion based upon the identification of rationality as the essential difference of humanity:

The essence of a human being (*dhāt-hu*) is nothing but his knowledge of things (*'ilm al-ashyā*). ... The quiddity of a human is knowledge, and knowledge is the thing known in one respect and something different in another. And if he is ignorant of a certain object of knowledge (*ma'lūm mā*), he is ignorant of a part of his essence (*juz'an min-dhāti-hi*), and if he is ignorant of all knowables, he is ignorant of his essence.⁶¹

Despite its reliance on the identity of knower and known, Averroes's claim here is stronger than the Aristotelian position that the soul knows itself in the same way that it knows other things. The Aristotelian claim is simply that self-awareness can only occur reflexively, once another object is known. Aristotelian self-knowledge in this sense is episodic. Avicennian self-awareness, by contrast, is continuous and uninterrupted. Averroist self-knowledge, unlike either of these models, is progressive and cumulative: the acquisition of knowledge is a form of self-realization for Averroes, and hence my self-knowledge increases in proportion to the increase in my overall store of knowledge.⁶²

On the basis of this stronger understanding of the identity of knower and known, Averroes denies that there could be any problem in positing an infinite regress of meta-levels of awareness. There is no need to cut off an infinite regress by positing some mysterious stage at which the knower fails entirely to be conscious of herself, because there is nothing problematic about the sort of infinity that is implied by a series of claims that a subject knows *that* she knows:

Now al-Ghazālī's answer, that this knowledge is a second knowledge (*'ilm thānī*) and that there is no infinite series here, is devoid of sense, for it is self-evident that this implies such a series, and it does not follow from the fact that when a man knows a thing but is not conscious that he knows the fact that he knows, that in the case when he knows that he knows, this second knowledge is an additional knowledge to the first; no, the second knowledge is one of the conditions of the first knowledge, and its infinite regress is, therefore, not impossible; if, however, it were a knowledge existing by itself and additional to the first knowledge, an infinite series could not occur.⁶³

I take Averroes's point here to be the following: since my knowledge of an object is one and the same act of knowledge as my knowledge of myself, there is implicitly contained in that single knowledge a potentially infinite series of propositions asserting my knowledge *that* I know, *that* I know *that* I know, and so on. Self-knowledge is an *ingredient* within our knowledge of other things to the extent that certitude requires us to know that we know. Knowing that we know does not, then, generate an infinite series of distinct acts of knowing as al-Ghazālī maintains, and hence there is no need to terminate the series by positing some act of awareness in which self-knowledge is entirely absent. Such a move is absurd in Averroes's eyes, not the least because it places a form of ignorance at the core of the explanation of knowledge. There is no little irony in the fact that much the same objection could be made against the function that Avicenna assigns to primitive self-awareness: both primitive self-awareness and self-absorption into the object known rest our knowledge on modes of awareness that lie below the threshold of consciousness and that, as such, remain actually unknown.

8. Conclusion

It is clear from the many attempts that Avicenna makes to clarify the nature of primitive self-awareness that he considered it to be a fundamental principle in his own philosophy and a necessary and important corrective of the prevailing philosophical view that made self-awareness of secondary importance in the explication of human knowledge. It appears from his various characterizations of primitive self-awareness that emphasize its utter basicity and complete self-identity that Avicenna believed that some such state of pre-conscious awareness was necessary to ground the *unity* of the human being as the single knowing subject to which her diverse cognitions, grounded in various faculties, are referred. It is this concern with the unity of awareness, rather than the desire to establish the immateriality of that unifying subject, that is of paramount importance to Avicenna, even in the Flying Man experiment — a point which is attested to by Avicenna's decision to incorporate two of the three versions of the Flying Man into arguments for the unity of the soul.⁶⁴

Nonetheless, it is impossible to deny that Avicenna is strongly attracted by the possibility of moving from an analysis of the primitiveness and simplicity of self-awareness to the conclusion that a being possessed of this capacity cannot be essentially corporeal. Thus in both the contexts in which the Flying Man is used to support the unity of the soul Avicenna eventually makes the additional claim that no body could act as the unifying or binding entity that he has discovered. And while Avicenna is in general careful to differentiate primitive self-awareness from simple intellectual understanding, his focus is in most instances fixed on establishing its non-sensory character.

More fundamentally, it seems reasonable to suppose that Avicenna's insistence on the necessity of positing some unifying principle of awareness is itself rooted in his commitment to the subsistence of the human soul and the merely relational character of its link to the body. It can hardly be an empirical inference, after all, for by Avicenna's own admission primitive self-awareness, as such, is prior to all conscious thought. Yet Avicenna's claim that self-awareness is indistinguishable from the very existence of the human soul follows quite naturally on the assumption that the fundamental attribute of the separate intellects — that of being always actually engaged in a "thinking of thinking" — must also be manifested in human intellects if they are to be intellects at all.⁶⁵ While Avicenna may agree with Aristotle that the human soul is indeed in mere potency to objects of knowledge *other* than itself, if the soul is essentially immaterial and rational, then there can be no point in its existence at which it is not in some sense actually cognitive. To the extent that the human soul is truly an intellectual soul, it must have the characteristic property of all subsistent intellects, that of being actually intelligible to itself. No intellect can ever be empty of this bare minimum of self-awareness. The Aristotelian view of self-knowledge, then, can be accommodated into Avicennian psychology to a limited extent. But that view, like the more basic characterization of the soul as the form or perfection of the body, captures only those limited aspects of human knowledge that pertain to its temporal — and temporary — physical state.

Notes

¹ *De anima* 3.4, 429a23-24.

² Aristotle, *De anima* 3.4, 430a1-2, and more generally to 430a9. Cf. 429b5-9. All translations of Aristotle are from Barnes 1984. For parallel remarks regarding sensible self-awareness, see *De anima* 3.2, 425b12-13, and more generally to 426a26. The claim that the intellect can only think itself after it has thought some other object is in turn a consequence of the principle of cognitive identification according to which the knower in some way becomes the object known in the act of perceiving or thinking. See *De anima* 2.5, 417a18-20; 418a3-6; 3.4, 429b29-30a1; 3.7, 431a1-6; 3.8, 431b20-432a1.

³ The Flying Man was popular amongst medieval readers of the Latin Avicenna, and modern commentators have often compared it to the *cogito* of Descartes. It occurs three times in Avicenna's major philosophical writings: twice in the *Psychology of the Healing* (1.1, 13 and 5.7, p. 225), and once in *Directives* p. 119. There is a vast literature on the Flying Man. Some important recent articles are Marmura 1986; Druart 1988; Hasnaoui 1997. For the influence on the Latin West, see Gilson 1929-30, pp. 39-42; Hasse 2000, pp. 80-92. The label "Flying Man" is not Avicenna's; as far as I can tell, it originates with Gilson 1929-30, p. 41 n. 1.

⁴ For the nature of these works and their place in Avicenna's philosophical development, see Gutas 1988, pp. 141-44, and Reisman 2002. Many relevant passages from the *Investigations* have been discussed and translated into French in Pines 1954.

⁵ I translate *shu'ūr* throughout as "awareness," which is the most natural English equivalent. While the term usually denotes self-awareness, it is occasionally used more broadly for awareness of other objects. See *Notes* pp. 30, 148, 162. In such cases it is close in meaning to *idrāk*, "apprehension" or "perception" (taken broadly without restriction to sensation).

⁶ The *Notes* and *Discussions* also consider the relation between animal and human self-awareness, where the former includes a human being's awareness of the activities taking place within the animal powers of her soul. On this see Black 1993, especially pp. 236-39.

⁷ *Kāmil* is a technical term in Islamic philosophy, and in Avicenna's psychology the cognate term *kamāl* is equivalent to the Greek *entelecheia* — "perfection" or "actuality" — used by Aristotle in the definition of the soul as the "first perfection of a natural body" (ἡ ψυχή ἡ πρώτη τελεία; *De anima* 2.1,

412a27-29; 412b5-6). Given that one version of the Flying Man occurs at the end of Avicenna's discussion of soul as entelechy (*Psychology* 1.1), one might suppose that Avicenna intends us to take *kāmil* here in its technical sense. But I am inclined to read it more colloquially as meaning something like "mature." The purpose of this portion of the thought experiment is to force us to bracket any knowledge we have gained from experience, while still presupposing we have the full intellectual capacities of an adult. But if *kāmil* refers to the soul as a "first perfection," then the state of a newly born *infant* would also be included; and if it refers to the soul as a "second perfection," then the soul would no longer seem to be in a pristine state, and this would render the experiment unable to alert us to the *primitiveness* of self-awareness. For a comprehensive study of Avicenna's account of the soul as perfection, and of his teleology in general, see Wisnovsky 2003, especially pp. 113-41.

⁸ Anscombe 1975, pp. 152, 156 proposes a similar thought experiment involving sensory deprivation. One interesting difference between the Flying Man and accounts of self-awareness and personal identity in modern philosophy is Avicenna's claim that *memories* as well as occurrent sensations can be bracketed without threatening personal identity.

⁹ *Directives* 119.

¹⁰ This is the language of *Psychology* 1.1, p. 13. Avicenna uses the phrase *wujūd dhāti-ka* as well as *wujūd annīyati-hi* in 5.7, p. 225; at *Directives* p. 119, *annīyati-hā* is used. *Annīyah* is a technical neologism within classical Islamic philosophy commonly rendered as "existence" or "individual existence." For its origins see Frank 1956; d'Alverny 1959.

¹¹ *Psychology* 5.7, p. 226.

¹² See below at nn. 28-31.

¹³ This distinction is a variation on the distinction between necessary or innate (**arūrī*) and acquired (*muktasab*) knowledge common among the *mutakallimūn*. On this see Marmura 1975, 104-5; Dhanani 1994, pp. 22-38. For the role of the Flying Man argument in Avicenna's attempts to refute the Mu'tazilite view of the soul and its self-awareness, see Marmura 1986, pp. 383-84.

¹⁴ *Notes* 160; cf. *Notes* pp. 30 and 79.

¹⁵ *Ibid.*, p. 161.

¹⁶ *Ibid.*, p. 161. Avicenna goes on to draw an analogy with our need to know who Zayd is prior to identifying any properties as belonging to him. See n. 47 below.

¹⁷ *Ibid.*, p. 161.

¹⁸ *Notes* pp. 147 and 79-80.

¹⁹ *Directives* p. 119: "The self of the sleeper in his sleep and the drunkard in his drunkenness will not slip away from himself, even if its representation to himself is not fixed in his memory."

²⁰ *Investigations*, §380, 210.

²¹ For the parallel between primary intelligibles and the Flying Man, cf. Marmura 1986, p. 394 n. 6.

²² Compare Avicenna's distinction between awareness and conscious thought with a similar distinction later drawn by Leibniz, in which memory plays a key role: "Mais je suis étonné comment il ne vous est pas venu dans la pensée que nous avons une infinité de connoissances, dont nous ne nous appercevons pas toujours, pas même lorsque nous en avons besoin, c'est à la memoire de les garder, et à la reminiscence de nous les représenter" (*New Essays*, pp. 76-77). For the comparison with Leibniz, cf. Pines, 1954, p. 31.

²³ *Psychology* 5.6, p. 217. As far as sense memory is concerned, we should recall that the Flying Man explicitly brackets sense memories as well as occurrent sensations.

²⁴ See *Psychology* 5.5, pp. 208-9, for example.

²⁵ For Avicenna's refutation of cognitive identification as a general feature of human cognition, see *Psychology* 5.6, pp. 212-213, and *Directives* p. 180. Avicenna does not recognize the identity of knower and known as an Aristotelian principle — which it obviously is — and he claims instead that it is an innovation of Porphyry. For discussion of this point see Black 1999a, pp. 58-60.

²⁶ *Notes*, pp. 147-48. While this passage uses *huwīyah* to describe relation between the subject and object of self-awareness, other texts also use *huwīyah* to designate the object itself. See *Investigations* §55, p. 134; §370, p. 207; and §424, pp. 221-222.

²⁷ Sensible awareness is by definition mediated, since both the external and internal senses require bodily organs. On this point see *Investigations* §349, p. 196; §358, p. 199; §367, p. 204; §375, p. 209; *Notes* p. 80; *Directives* p. 119. The related claim that dependence on bodily organs entails that the senses cannot be fully reflexive or aware of themselves is made in *Psychology* 5.2, pp. 191-94. For the Neoplatonic background to this claim, see Gerson 1997. Rahman 1952, pp. 103-104, pp. 111-114 discusses the parallels in the Greek commentators.

²⁸ *Investigations* §55, p. 134. Cf. §424, p. 221.

²⁹ This expression is not common in the texts on self-awareness that I have examined, but it appears to be more or less synonymous with *huwīyah*. Cf. the use of *mutakha* {*ah* at *Investigations* §427, p. 223.

³⁰ *Investigations* §424, 221-222.

³¹ *Ibid.*, §332, p. 192.

³² *Ibid.*, §331, p. 192. Similar allusions to Avicenna's accounts of quiddity and universality are found in several other passages on the nature of self-awareness, for example, *Investigations* §372, pp. 208-9 (cited at n. 42 below); §422, p. 221; and §426, pp. 222-223.

³³ *Metaphysics* 5.1-2. For a general overview of this aspect of Avicenna's metaphysics, see Marmura 1992; for the theory of mental existence implied by this account, see Black 1999a, pp. 48-62.

³⁴ At n. 31 above.

³⁵ *Psychology* 5.6, p. 214.

³⁶ Likewise, in *Psychology* 5.5, 209, the term *jumlah* — “aggregate” or “collection” — is employed to explain the limited capacity of the senses to grasp true unity.

³⁷ *Investigations*, §370, p. 207; the question posed here refers explicitly to the Flying Man “hypothesis” (*al-far**) in the *Shifā'*. Cf. *Investigations*, §§357-358, p. 199; *Directives* pp. 119-121; *Psychology* 5.7, pp. 225-26.

³⁸ For a recent discussion of the philosophical issues facing Avicenna on this point, see Druart 2000.

³⁹ Avicenna argues at length for the unity of the soul in *Psychology* 5.7, and both this version of the Flying Man and the version in the *Directives* are intended to focus attention on the unity of the self as much as on its incorporeality.

⁴⁰ *Investigations* §371, p. 208.

⁴¹ *Ibid.*, §371, 208.

⁴² At *Investigations* §427, p. 223, Avicenna suggests that this is also the model whereby we should understand how the separated soul would be aware of itself.

⁴³ *Investigations* §372, p. 208.

⁴⁴ Cf. *Investigations* §426, pp. 222-23, in which Avicenna makes a similar point in the course of comparing self-awareness with our knowledge of other humans.

⁴⁵ See especially *Isagoge* 1.12, pp. 70-71; translation in Marmura 1979, pp. 50-52; for an overview of Avicenna's account of individuation, see Bäck 1994, pp. 39-53.

⁴⁶ *Directives* 56.

⁴⁷ *Notes* 161. “Acquaintance” (*ma'rifah*) is usually identified by Avicenna as a perceptual act performed by the senses and differentiated from intellectual knowledge. See especially *Demonstration* 1.3, p. 58: “The perception of particulars is not knowledge, but rather, acquaintance (*laysa 'ilman bal ma'rifatan*).” But as is noted in Marmura 1986, p. 387, Avicenna also uses the cognate term *'arif*, common in discussions of mystical knowledge, to describe the act of self-awareness one experiences in the Flying Man.

⁴⁸ *Notes* 30; cf. *Notes* p. 147.

⁴⁹ *Notes*, p. 161. Cf. *Investigations* §380, p. 210 (cited at n. 20 above), where Avicenna treats the *memory* of self-awareness as a form of awareness that we are aware.

⁵⁰ That is: (1) cognitive identification; and (2) the claim that the rational soul has no nature of its own prior to thinking of other objects. Cf. above at nn. 1, 2, and 25.

⁵¹ *Notes* p. 161.

⁵² *Ibid.*, p. 79.

⁵³ *Psychology* 5.6, pp. 214-15.

⁵⁴ Al-Fārābī, *Demonstration*, p. 20. Cf. *Conditions* p. 97.

⁵⁵ For al-Fārābī's use of *shu'ūr* and cognates, see *Conditions* pp. 98-99.

⁵⁶ *Conditions*, pp. 100-101.

⁵⁷ *Notes* p. 147; cf. *Investigations* §425-26, pp. 222-223; and §422, : “And attention [to the reality known] is not existent for it three times, but rather, its abstraction itself is in us; otherwise it would proceed to infinity.”

⁵⁸ It is worth noting, however, that in *Niche* c.1, §18, p. 8. Al-Ghazālī paints the possibility of such a regress in more positive terms: “Finally, it perceives its own knowledge of something, the knowledge of its knowledge of that thing, and its knowledge of its knowledge of its knowledge. Hence, in this single instance the intellect's capacity is infinite.”

⁵⁹ Al-Ghazālī, *Incoherence*, Discussion 6, §37, p. 106.

⁶⁰ Averroes, *Incoherence*, Discussion 6, §51, pp. 335-336; Van Den Bergh 1954, pp. 200-201.

⁶¹ *Ibid.*, p. 336; Van Den Bergh 1954, p. 201, slightly modified.

⁶² This is not surprising, of course, since the cumulative view of self-knowledge forms the core of the traditional doctrines of the acquired intellect (*al-'aql al-mustafād*) and conjunction (*ittiḥāl*) with the Agent Intellect; on this see Black 1999.

⁶³ Averroes, *Incoherence* §81, p. 351; Van Den Bergh 1954, p. 211.

⁶⁴ Those in *Psychology* 5.7 and *Directives* p. 121; cf. n. 39 above.

⁶⁵ So in *Psychology* 5.6, Avicenna's most sustained discussion of human knowledge, he consistently evokes the cognition of the separate intellects as his model of what understanding is, and then modifies this model where necessary in order to fit the exigencies of “ensouled knowledge” (*'ilm nafsānīyah*, p. 215).

Abbreviations

Primary texts are cited by the following abbreviated titles:

- Averroes: 1930 (1954), *Incoherence: Tahāfut al-Tahāfut (The Incoherence of "The Incoherence")*, ed. by M. Bouyges, Beirut, Imprimerie Catholique, Trans. in Van Den Bergh 1954.
- Avicenna: 1892, *Directives: Al-Ishārāt wa-t-tanbīhāt (Directives and Reminders)*, ed. by J. Forget, Leiden.
- Avicenna: 1956, *Demonstration: Al-Shifā': Al-Burhān*, ed. by A. E. Affifi and I. Madkour, Cairo, General Egyptian Book Organization.
- Avicenna: 1947, *Investigations: Al-Mubā'athāt* in A. R. Badawi (ed.), *Aris}ū'inda al-'Arab*, Cairo, pp. 122-239; I have not had access to the more recent and more complete edition of the *Mubā'athāt* by M. Bidarfar, Qum, 1992.
- Avicenna: 1952, *Isagoge: Al-Shifā', Al-Madkhal*, ed. by G. Anawati, M. El-Khodeiri, F. al-Ahwani, and I. Madkour, Cairo, General Egyptian Book Organization.
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A Conceptual Analysis of Early Arabic Algebra

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Abstract. Arabic algebra derives its epistemic value not from proofs but from correctly performing calculations using coequal polynomials. This idea of ‘mathematics as calculation’ had an important influence on the epistemological status of European mathematics until the seventeenth century. We analyze the basic concepts of early Arabic algebra such as the unknown and the equation and their subsequent changes within the Italian abacus tradition. We demonstrate that the use of these concepts has been problematic in several aspects. Early Arabic algebra reveals anomalies which can be attributed to the diversity of influences in which the *al-jabr* practice flourished. We argue that the concept of a symbolic equation as it emerges in algebra textbooks around 1550 is fundamentally different from the ‘equation’ as known in Arabic algebra.

1. Introduction

The most common epistemology account of mathematics is based on the idea of apriorism. Mathematical knowledge is considered to be independent of experience. The fundamental argument for an apriorist assessment of mathematics is founded on the concept of a formal proof. Truth in mathematics can be demonstrated by deductive reasoning within an axiomatic system. All theorems derivable from the axioms have to be accepted solely on basis of the formal structure. The great mathematician Hardy cogently formulates it as follows (Hardy 1929):

It seems to me that no philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner or another, the immutable and unconditional validity of mathematical truth. Mathematical theorems are true or false; their truth or falsity is absolute and independent of our knowledge of them. In some sense, mathematical truth is part of objective reality.

When some years later, Gödel proved that there are true statements in any consistent formal system that cannot be proved within that system, truth became peremptory decoupled of provability. Despite the fact that Gödel’s proof undermined the fundament of apriorism it had little impact on the mainstream epistemological view on mathematics. Only during the past decades the apriorist account was challenged by mathematical empiricism, through influential works from Lakatos (1976), Kitcher (1984) and Mancosu (1996). These authors share a strong believe in the relevance of the history of mathematics for an epistemology of mathematics.

The apriorist view on mathematics has not always been predominant in western thinking. It only became so by the growing influence of the Euclidean axiomatic method from the seventeenth century onwards. With respect to algebra, John Wallis was the first to introduce the axioms in an early work, called *Mathesis Universalis*, included in his *Operum mathematicorum* (1657, 85). With specific reference to Euclid’s *Elements*, he gives nine *Axiomata*, also called *communes notationes*. From then on, the epistemological status of algebra was transformed into one deriving its truth from proof based on the axiomatic method. Before the seventeenth century, truth and validity of an algebraic derivation depended on correctly performing the calculations using an unknown quantity. While Wittgenstein was heavily criticized for his statement that “Die Mathematik besteht ganz aus

Rechnung” (Mathematics consists entirely of calculations), (1978, 924; 468), his image of mathematics as procedures performed on the abacus, fits in very well with pre-seventeenth-century conceptions of mathematical knowledge. Algebraical problem solving consisted of formulating the problem in terms of the unknown and reducing the form to one of the known cases. Early Arabic algebra had rules for each of six known cases. While geometrical demonstrations exist for three quadratic types of problems, the validity of the rules was accepted on basis of their performance in problem solving.

The idea that European mathematics has always been rooted in Euclidean geometry is a myth cultivated by humanist writings on the history of mathematics. In fact, the very idea that Greek mathematics is our (Western) mathematics is based on the same myth, as argued by Jens Høyrup (Høyrup 1996, 103):

According to conventional wisdom, European mathematics originated among the Greeks between the epochs of Thales and Euclid, was borrowed and well preserved by the Arabs in the early Middle Ages, and brought back to its authentic homeland by Europeans in the twelfth and thirteenth century. Since then, it has pursued its career triumphantly.

Høyrup shows that “Medieval scholastic university *did* produce an unprecedented, and hence specifically European kind of mathematics” (ibid.). But also outside the universities, in the abacus schools of Florence, Siena and other Italian cities, a new kind of mathematics flourished supporting the practical needs of merchants, craftsman, surveyors and even the military man.

Symbolic algebra, the Western mathematics *par excellence*, emerged from algebraic practice within this abacus tradition, situated broadly between Fibonacci’s *Liber Abbaci* (1202) and Pacioli’s *Summa* (1494). Practice of algebraic problem solving within this tradition grew out of Arabic sources. The epistemic foundations of a mathematics-as-calculation was formed in the Arab world. An explicitation of these foundations is the prime motivation of our analysis of the basic concepts of early Arabic algebra.

2. Starting point

While the original meaning of the Arabic concepts of algebra will be an important guideline for this study, we relinquish the search for the “exact meaning”. Several scholars have published studies on the origin of the term algebra, the meaning of *al-jabr* and *al-muchābala* and the Arabic terms for an unknown. Some have done so with the aim of establishing the correct meaning with the aid of Arabic etymology and linguistics (e.g. Gandz 1926, Saliba 1972, Oaks and Alkhateeb 2005). Strictly taken, the precise meaning of these Arabic terms and concepts is irrelevant for our study. Even if there would be one exact meaning to be established, it was not available for practitioners of early algebra in Europe. With a few exceptions, such as Fibonacci,¹ the flourishing of algebraic practice within the abacus tradition depended on a handful of Latin translations and vernacular interpretations or rephrasing of these translations. Unquestionably, certain shifts in meaning took place within the process of interpretation and diffusion during the twelfth and thirteenth centuries. Rather than the Arabic terms and concepts, the concepts conveyed by the first Latin translations will be our starting point.

2.1. Latin translations of al-Khwārizmī’s Algebra

Three Latin translations of al-Khwārizmī’s *Algebra* are extant in sixteen manuscripts (Hughes 1982). These translations have been identified as from Robert of Chester (c. 1145), Gerard of Cremona (c. 1150) and Guglielmo de Lunis (c. 1215), although there is still discussion whether the latter translation was Latin or Italian. What became available to the West was only the first part of al-Khwārizmī’s treatise. The second part on surveying and the third on

the calculation of legacies were not included in these Latin translations. The full text of the *Algebra* became first available with the edition of Frederic Rosen (1831) including an English translation. Rosen used a single Arabic manuscript, the Oxford, Bodleian CMXVIII Hunt. 214, dated 1342. The value of his translation has been questioned by Ruska (1917), Gandz (1932, 61-3) and Høystrup (1998, note 5). Some years later Guillaume Libri (1838, Note XII, 253-299) published a transcription of Gerard's translation from the Paris, BNF, Lat. 7377A, an edition that has been qualified as 'faulty' and corrected on eighty accounts by Hughes (1986, 211, 231). Later during the century, Boncompagni (1850) also edited a Latin translation from Gerard, but it was later found that this manuscript was not Gerard's but Guglielmo de Lunis' (Hughes 1986). Robert of Chester's translation was first published with an English translation by Karpinski (1915). However, Karpinski used a manuscript copy by Scheubel, which should be seen more as a revision of the original.

It is only during the past decades that critical editions of the three Latin translations have become available. The translation by Gerard of Cremona was edited by Hughes (1986), based on seven manuscript copies. Hughes (1989) also published a critical edition of the second translation from Robert of Chester based on the three extant manuscripts. A third translation has been edited by Wolfgang Kaunzner (1986). Although this text (Oxford, Bodleian, Lyell 52) was originally attributed to Gerard, it is now considered to be a translation from Guglielmo de Lunis (Hughes 1982, 1989). An Italian translation of 1313 from the Latin is recently published by Franci (2003). It has been argued by several scholars that Gerard of Cremona's translation is the best extant witness of the first Arabic algebra (Høystrup 1998).

2.2. Latin translations of other Arabic works

Apart from al-Khwārizmī's *Algebra* there have been Latin translations of other works which contributed to the diffusion of Arabic algebra. The *Liber algorismi de pratica arismetrice* by John of Seville (Johannes Hispalensis) precedes the first Latin translations and briefly mentions algebra (Boncompagni 1857, 112-3). Also of importance is Abū Bakr's *Liber mensurationum*, translated by Gerard of Cremona in the twelfth century (Busard, 1968). Although this work deals primarily with surveying problems it uses the methods as well as the terminology of the early Arabic *jabr* tradition. Jens Høystrup, who named the method "naive geometry" or "the tradition of lay surveyors", has pointed out the relation between this work and Babylonian algebra (Høystrup, 1986, 1990, 1998, 2002). Following Busard, he has convincingly demonstrated that the operations used to solve these problems are concretely geometrical. Therefore this work can help us with the interpretation of operations in early Arabic algebra.

The *Algebra* of Abū Kāmil was written some decades after that of al-Khwārizmī and bears the same title *Kitāb fī al-Jabr wa al-muqābalah*. Several versions of the manuscript are extant. An Arabic version MS Kara Mustafa Kütübhanesi 379 in Istanbul; a fourteenth-century copy of a Latin translation at the BNF at Paris, Lat. 7377A, discussed with partial translations by Karpinski (1914) and published in a critical edition by Sesiano (1993) who attributes the Latin translation to Guglielmo de Lunis (1993, 322-3). A fifteenth-century Hebrew version with a commentary by Mordecai Finzi, is translated in German by Weinberg (1935) and in English by Levey (1966). Levey also provides an English translation of some parts of the Arabic text. Other texts include Ibn Badr's *Ikhtīṣār al-Jabr wa al-muqābala* which was translated into Spanish (Sánchez Pérez, 1916) and al-Karajī's *Fakhrī fī al-Jabr wa al-muqābalah* with a partial French translation (Woepcke, 1853).

3. The evolution of the concept of an unknown

3.1. The unknown in early Arabic algebra

The unknown is used to solve arithmetical or geometrical problems. The solution commences with posing an unknown quantity of the problem as the abstract unknown. By analytical reasoning using the unknown, one arrives at a value for it. In algebraic problem solving before Arabic algebra, the abstract unknown is not always the symbolic entity as we now understand. As an essential part of the analytical reasoning, it is an entity related to the context of the problem and the model used for problem solving. For Babylonian algebra, it is shown by Høyrup (2002) that the model was a geometrical one. The unknown thus refers to geometrical elements such as the sides of a rectangle or a surface. In Indian algebra we find the unknown (or unknowns) used for monetary values or possessions as in the rule of *gulikāntara* (Colebrooke 1817, 344). The terms used in Arabic algebra reflect both the geometrical interpretation of the unknown as well as the one of a possession. We will argue that the difficulties and confusions in the understanding of the concept of the Arabic unknown are induced by diverse influences from Babylonian and Indian traditions.

3.1.1. Arabic terminology

The central terms in Arabic algebra are *māl*, *shay'* and *jidhr*. In addition, the monetary unit *dirham* is also used in problems and in their algebraic solutions. It is generally accepted that the term *māl* refers to possession, or wealth or even a specific sum of money. The *shay'* is translated as 'thing' ever since the first commentators wrote about it (Cossali 1797-9). From the beginning, *shay'* was considered the unknown (Colebrooke 1817, xiii).

The difficulties of interpretation arise when we translate *māl* by 'square' and *shay'* by 'root'. Rosen (1831) and Karpinski (1915) both use 'square' for *māl* on most occasions. Karpinski even uses the symbolic x^2 . However, when the problem can be stated without the use of a square term, they both change the interpretation of the *māl*. For example in problem III.11, Rosen uses 'number' and Karpinski employs x instead of x^2 as used for the other problems.² This already contributes to the confusion as the Latin translation uses the same word in both cases. Moreover the choice of the word 'square' is misleading. Neither the geometrical meaning of 'square', nor the algebraical one, e.g. x^2 , are adequate to convey the meaning of *māl*.³ For the geometrical problems, al-Khwārizmī elaborates on the use of *māl* for the algebraic representation of the area of a geometrical square. If the meaning of *māl* would be a square, why going through the argumentation of posing *māl* for the area?⁴ The algebraic interpretation of a square is equally problematic. If *māl* would be the same as the square of the unknown then *jidhr* or root would be the unknown. However, this is in contradiction with the original texts in which *māl*, if not the original unknown by itself, is at least transformed into the unknown. Høyrup (1998, 8) justly uses the argument that *māl* is used in linear problems in al-Karajī's *Kāfī* (Hochheim 1878, iii, 14). This corresponds with the use of a possession in Hindu algebra, in formulating algebraic rules for linear problems, such as the *gulikāntara*.

Arab	<i>māl</i>	<i>shay'</i>	<i>jidhr</i>	<i>dirham</i>	' <i>adad mufrad</i>
	□ □ □	□ □ □	□ □ □	□ □ □ □	□ □ □ □ □ □ □
Hispalensis	(none)	res	radix	(none)	numerus
Robert	substantia	res	radix	dragma	numerus
Gerard	census	res	radix	dragma	numerus simplex
Guglielmo	census	res	radix	dragma	numerus
Abū Kāmil (latin)	census	res	radix	dragma	numerus simplex

Table 1: The terms used in early Latin translations of Arabic texts.

3.1.2. The ambiguity of *māl*

The interpretation of *māl* as the unknown, pure and simple, is not as straightforward as often presented. While *māl* (in Robert's translation *substancia* and in Gerard's *census*) is used to describe the problem, the algebraic derivation depends on operations on other terms than the original 'possession'. Also Hughes points out the problem in his commentary of Robert of Chester's edition:⁵

Terminology also must have jolted Robert's readers. In problems four and six of Chapter I and in five, ten, and thirteen of Chapter II, *substancia* in the statement of the initial equation becomes *res* or *radix* in its solution. Excursions such as these must have challenged the reader.

Let us look more closely at problem III.13, as it is instructive to point out what constitutes a transformation in the original concept of *māl*:

Karpinski 1930, 118

I multiply a square by two-thirds of itself and have five as a product.

Explanation. I multiply x by two-thirds x , giving $\frac{2}{3}x^2$, which equals five. Complete $\frac{2}{3}x^2$ by adding to it one-half of itself, and one x^2 is obtained. Likewise add to five one-half of itself, and you have $7\frac{1}{2}$, which equals x^2 . The root of this, then, is the number which when multiplied by two-thirds of itself gives five.

Hughes 1989, 61

Substanciam in eius duabus terciis sic multiplico, ut fiant 5. Expositio est, ut rem in duabus terciis rei multiplicem, et erunt $\frac{2}{3}$ unius substancie 5 coequancia. Comple ergo $\frac{2}{3}$ substancie cum similitudine earum medii, et erit substancia. Et similiter comple 5 cum sua medietate, et erit habebis substanciam vii et medium coequantem. Eius ergo radix est res que quando in suis duabus terciis multiplicata feurit, ad quinarium excrescet numerum.

Substancia here is used in the problem text as well as the solution. But clearly it must have a different meaning in these two contexts. In the beginning of the derivation *substancia* is replaced by *res*. In the English translation, Karpinski switches from 'square' to x . By multiplying the two *res* terms, x and $\frac{2}{3}x$, two thirds of a new *substancia* is created. This second *substancia* is an algebraic concept where the first one, in the problem text, is a possession and may refer to a sum of money. While Gerard of Cremona uses *census* instead of *substancia*, his translation has the same ambiguity with regard to *census*.⁶

3.1.2.1. The root of real money

This anomaly of Arabic algebra is discussed now for almost two centuries. Libri (1838), Chasles (1841, 509), and others have noticed the problem. Some have chosen to ignore it while others pointed out the inconsistency, but did not provide any satisfactory answer. Very recently, two analyses have reopened the discussion. In the yet to be published Høystrup (2006) and Oaks and Alkhateeb (2005) the double meaning of the *māl* is prominently present in their interpretation of early Arabic algebra.⁷ Høystrup (2006) adequately describes the anomaly as "the square root of real money". As *māl* or *census* originally is understood as a possession, and the unknown is designated by *shay'* or *res*, which is the root of the *census*, problems looking for the value of a possession thus deal with the root of real money when they use the *shay'* in their solution. According to Høystrup the difference between the two was already a formality for al-Khwārizmī.

3.1.2.2. Abū Kāmil towards a resolution of the ambiguity

We find the anomaly also in the algebra of Abū Kāmil, almost a century later. But Abū Kāmil is the first to point out that the transformation of a value or possession into an algebraic quantity is an arbitrary choice. His double solution to problem 52 is very instructive in this

respect. The problem commences as follows (translation from the Arabic text, f. 48^v; Levey 1966, 164, note 167):

If one says to you that there is an amount [*māl*] to which is added the root of its $\frac{1}{2}$. Then the sum is multiplied by itself to give 4 times the first amount. Put the amount you have equal to a thing and to it is added the root of its $\frac{1}{2}$ which is a thing plus the root of $\frac{1}{2}$ a thing, (then multiply it by itself) [sic]. It gives a thing plus the root of $\frac{1}{2}$ a thing. Then one multiplies it by itself to give a square plus $\frac{1}{2}$ a thing plus the root of 2 cubes [*ka'bin*, a dual of *ka'b*] equal to 4 things.

The Latin translation makes the anomaly apparent (Sesiano 1993, 398, 2678-2683):

Et si dicemus tibi: Censui adde radicem medietatis eius; deinde duc additum in se, et provenie[n]t quadruplum census. Exemplum. Fac censum tuum rem, et adde ei radicem medietatis eius, et [prov-] erunt res et radix $\frac{1}{2}$ rei. Que duc in se, et provenie[n]t census et $\frac{1}{2}$ rei et radix 2 cuborum, equales 4 rebus.

In symbolic representation the solution depends on:

$$\left(x + \sqrt{\frac{1}{2}x}\right)^2 = 4x$$

As is common, the translator uses *census* for the possession or amount of money in the problem formulation. The solution starts by stating literally ‘make from the *census* your *res*’ (“Fac censum tuum rem”) which could easily be misinterpreted as “make x from x^2 ”. In the rest of the solution, *res* is used as the unknown.

Abū Kāmil adds a second solution: “You might as well use *census* for the possession”, he reassures the reader (Sesiano 1993, 399, 2701-2705),

Et, si voleris, fac censum tuum censum, et adde ei radicem medietatis ipsius, et erunt census et radix medietatis census, equales radici 4 cens[ibus]uum, [et] quia di[x]cis: “Quando ducimus e[um]Ja in se, [erit] proveniet quadruplum census”. Est ergo census et radix $\frac{1}{2}$ census, equales radici 4th censu[um]s. Et hoc est 2 res.

Here, the symbolic translation would be:

$$\left(x^2 + \sqrt{\frac{1}{2}x^2}\right)^2 = 4x^2$$

The *census* is now used for the possession. But there is still a difference between the *census* of the problem formulation and the *census* of the problem solution. “Fac censum tuum censum” should here be understood as “put the amount you have equal to the square of a thing”. What Abū Kāmil seems to imply by providing alternative solutions to a single problem, is that there are several ways to ‘translate’ a problem into algebraic form. The possession in the problem text is not necessarily the unknown. You can use the unknown for the possession, but you might as well use the square of the unknown. In the abacus tradition from the thirteenth to the sixteenth century, this freedom of choice was highly convenient for devising clever solutions to problems of growing complexity. The ambiguity in the concept of *māl*, by many understood as a nuisance of Arabic algebra, could have facilitated the conceptual advance to the more abstract concept of an algebraic quantity.

3.1.3. Conclusion

There is definitely an anomaly with the original concept of an unknown in early Arabic algebra. On the one hand, *māl* is used as the square term in quadratic problems of the type ‘*māl* and roots equal number’ such as the prototypical case four from al-Khwārizmī

$$x^2 + 10x = 39$$

Early Arabic algebra provides procedures for problems which can be reduced to one of the six standard types. On the other hand, *māl* is also used for describing the quantity of a problem, mostly a sum of money or a possession. Possibly, at some time before al-Khwārizmī's treatise, these two meanings were contained in a single word and concept. As problems dealing with possessions were approached by algebraic method from the *al-jabr* tradition, a transformation of the concept *māl* became a necessity. We notice in al-Khwārizmī's *Algebra* and all the more in that of Abū Kāmil, a shift towards *māl* as an algebraic concept different from a possession or a geometrical square. The confusion and discontent expressed by several twentieth-century scholars with terminology in early Arabic algebra stems from a failure to see the conceptual change of the *māl*.

We do not know much about the origin of the *al-jabr* tradition, preoccupied with quadratic problems and their 'naive' geometric demonstrations. Jens Høyrup (1994, 100-2) speculates on a merger of two traditions. The first is the class of calculators employing the *lisāb* for arithmetical problem solving. The second stems from the tradition of surveyors and practical geometers, going back to Old Babylonian algebra. We would like to add the possible influence from Hindu algebra. While the *al-jabr* tradition is definitely different from the Indian one in methods and conceptualization, the type of problems dealing with possessions are likely to have been imported from the Far East. The ambiguities within the concept of *māl* reflects the variety of influences.

3.2. Multiple solutions to quadratic problems

A second particularity of Arabic algebra is the acceptance of double solutions for one type of quadratic problems. The recognition that every quadratic equation has two roots is generally considered as an important conceptual advance in symbolic algebra. We find this insight in the mostly unpublished works of Thomas Harriot of the early seventeenth century. More influential in this respect, is Girard's *Invention Nouvelle en Algebre*, published in 1629. However, it is less known that early Arabic algebra fully accepted two positive solutions to certain types of quadratic problems. It is significant that this achievement of Arabic algebra has largely been neglected during the abacus tradition, while it might have functioned as a stepping stone to an earlier structural approach to equations. We believe there is an explanation for this, which is related to the concept of an unknown of the abacus masters. Let us first look at the first occurrence of double solutions in early Arabic algebra.

3.2.1. Two positive roots in Arabic algebra

Two positive solutions to quadratic problems are presented in al-Khwārizmī's fifth case of the quadratic problems of "possession and number equal to roots". This problem, in symbolic form, corresponds with the normalized equation

.....

al-Khwārizmī talks about addition and subtraction leading to two solutions in the following rule for solving the problem:

From Robert's translation
(Hughes 1989, 34):

Primum ergo radices per medium dividas et fient 5. Eas ergo in se multiplica et erunt 25. Ex hiis ergo 21 diminuas quem cum substancia iam pretaxauimus, et remanebunt 4. Horum ergo radicem accipias id est 2, que ex medietate radicum id est 5 diminuas et remanebunt tria, vnam radicem huius substancie constituen- cia, quam scilicet substanciam novenus complet numerus. Et si volueris ipsa duo que a medietate radicum iam diminuisti, ipsi medietati id est 5 ad 20 dicas, et fient 7.

The rule from the Arabic manuscript
(Rosen 1931, 42):

When you meet with an instance which refers you to this case, try its solution by addition, and if that do not serve, then subtraction certainly will. For in this case both addition and subtraction may be employed, which will not answer in any other of the three cases in which the number of the roots must be halved. And know, that, when in a question belonging to this case you have halved the number of the roots and multiplied the moiety by itself, if the product be less than the number of dirhems connected with the square, then the instance is impossible; but if the product be equal to the dirhems by themselves, then the root of the square is equal to the moiety of the roots alone, without either addition or subtraction.

The procedure thus corresponds with the following formula:

$$x_{1,2} = \frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

al-Khwārizmī states that the problem becomes unsolvable when the discriminant becomes negative. When the square of $b/2$ equals the number (of dinars) there is only one solution which is half the number of roots. The gloss in Gerard's translation of problem VII.1 gives a geometric demonstration with the two solutions. This problem from al-Khwārizmī is also treated by Abū Kāmil (Karpinski 1914, 42-3; Sesiano 1993, 330-6). A lesser known Arabic manuscript, which most likely predates al-Khwārizmī, also has the geometric demonstration with double solutions (Sayili 1985, 163-5).

Chasles (1841, 504) mentions a Latin translation of Gerard (Paris, BNF, anciens fonds 7266) from a treatise on the measurement of surfaces, by an Arab called Sayd. A problem of the same type, corresponding with the symbolic equation

$$x^2 + 3 = 4x$$

is solved by addition and subtraction ("Hoc namque est secundum augmentum et diminutionem"), referring to the values $x = 2 + 1$ and $x = 2 - 1$, resulting in the double solution $x = 3$ and $x = 1$.

In conclusion: double positive solutions to one type of quadratic problems were fully accepted in the earliest extant sources of Arabic algebra.

3.2.2. Speculation on the origin of double solutions

Dealing with quadratic problems, Diophantus never arrives at double solutions. If the problem has two positive solutions, he always finds the larger one (Nesselmann 1842, 319-21; Tropfke 1933-4, 45). So, where do the double solutions of Arabic type V problems originate from? If not from Greek descent, the most likely origin would be Hindu algebra. However, Rodet (1878) was the first to critically investigate the possible influence of Hindu sources on Arabic algebra. One of his four arguments against such lineage is the difference in approach to double solutions of the quadratic equation. As the Hindus accepted negative values for roots and numbers they had one single format for complete quadratic equations, namely

$$ax^2 \pm bx = \pm c$$

whereas the Arabs had three types. The Hindu procedure for solving complete quadratic problems accounts for double solutions as stated by Bhāskara (and his predecessors):⁸

If the root of the absolute side of the equation be less than the number, having the negative sign, comprised in the root of the side involving the unknown, then putting it negative or positive, a two-fold value is to be found of the unknown quantity: this [holds] in some cases.

The “root of the absolute side of the equation” refers to the $\pm c$. The Hindu procedure to find the roots of a quadratic equation can be illustrated by the following example (Bhāskara stanza 139; Colebrooke 1817, 215-6):

The eighth part of a troop of monkeys, squared, was skipping in a grove and delighted with their sport. Twelve remaining were seen on the hill, amused with chattering to each other. How many were they in all?

Using the unknown *ya* 1 for the number of monkeys, Bhāskara solves the problem as follows:

<i>ya</i> v $\frac{1}{64}$ <i>ya</i> 0 <i>ru</i> 12	literally transcribed:
<i>ya</i> v 0 <i>ya</i> 1 <i>ru</i> 0	$\frac{1x^2}{64} + 0x + 12 = 0x^2 + x + 0$
<i>ya</i> v 1 <i>ya</i> 64 4 <i>ru</i> 0	bringing to the same denominator:
<i>ya</i> v 0 <i>ya</i> 0 <i>ru</i> 768	$x^2 - 64x = -768$
	making the left side a perfect square:
	$(x - 32)^2 = 256$
<i>ya</i> 1 <i>ru</i> 32 2	extracting the root results in:
<i>ya</i> 0 <i>ru</i> 16	$x - 32 = \pm 16$

The two solutions thus become $x = 48$ and $x = 16$.

In the next problem the acceptance of two solutions is more challenging (Bhāskara stanza 140; Colebrooke 1817, 216):

The fifth part of the troop [of monkeys] less three, squared, had gone to a cave; and one monkey was in sight, having climbed on a branch. Say how many they were.

This leads to the equation:

<i>ya</i> v 1 <i>ya</i> 5 5 <i>ru</i> 0	literally transcribed:
<i>ya</i> v 0 <i>ya</i> 0 <i>ru</i> 250	$1x^2 - 55x + 0 = 0x^2 + 0x - 250$

with solutions $x = 50$ and $x = 5$. Bhāskara has some reservations about the second solution because one fifth of five minus three becomes negative.

The very different approach towards quadratic problems and the acceptance of negative roots in Hindu algebra makes it an improbable source for the double solutions of type V problems in Arabic algebra.

If not from Greek or Indian origin, there is only one candidate left. Solomon Gandz, in an extensive, and for that time, exhaustive comparison of solutions to quadratic problems from Babylonian, Greek and Arabic origin concluded (Gandz 1937, 543):

Greek and Arabic algebra are built upon the rock of the old Babylonian science and wisdom. It is the legacy of the old Babylonian schools which remain the very foundation and cornerstone of

both the Greek and Arabic systems of algebra. The origin and early development of the science cannot be understood without the knowledge of this old Babylonian legacy.

Although the relation should now be qualified and differentiated more cautiously, recent studies, such as the groundbreaking and novel interpretation by Høyrup (2002) endorse some line of influence. If we look again at the type V problem from al-Khwārizmī, the resulting equation

$$x^2 + 21 = 10x$$

which is given in its direct form, corresponds remarkably well with a standard type of problem from Babylonian algebra:

$$a + b = 10$$

$$ab = 21$$

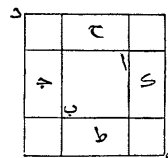
The important difference between the two is that Babylonian algebra uses a geometrical model for solving problems. The two parts a and b are represented as the sides of a rectangle ab and they function as two unknowns in the meaning we have defined elsewhere.⁹ Arabic algebra uses geometry only as a demonstration of the validity of the rules and its analytic part is limited to reducing a problem to one of the standard forms using a single unknown. al-Khwārizmī systematically uses the unknown for the smaller part. Thus in problem VII he proceeds as follows (de Lunis; Kaunzner 1989, 78):

Ex quarum unius multiplicatione per alteram 21 proveniant. Sit una illarum res, altera 10 minus re, ex quarum multiplicatione proveniunt 10 res minus censu, que data sunt equalia 21. Per restaurationem igitur diminuti fiunt 10 res censui ac 21 equales ecce quintus modus, resolve per eum et invenies partes 3 et 7.

al-Khwārizmī multiplies x with $10 - x$, with value 21. After “restoration” this leads to the standard form of the equation above. While the rule for type V prescribes trying addition first and then subtraction (in the Robert translation), the solutions arrived at here are 3 first and then 7. We believe that the recognition of two solutions to this type of quadratic problem is a direct relic of the Babylonian solution method. Although the geometric proof for this problem, present in the Arabic texts and the three Latin translations, does not correspond with any known Babylonian tablets, some of al-Khwārizmī’s geometric demonstrations ought to be placed within the surveyor’s tradition which descends in all probability from Old Babylonian algebra.

Høyrup (2002, 412-4) points out that al-Khwārizmī’s provides two rather different geometrical demonstrations to the case “possessions and roots equal number”. Only one corresponds with the procedure described in the text. The other, shown in Figure 1, corresponds remarkably well with the Babylonian table BM 13901, nr. 23.

ليتم لنا بناء السطح الاعظم بما نقص من زواياه الربع لان كل عدد يضرب ربعه في مثله ثم في اربعة يكون مثل ضرب نصفه في مثله فاستغينا بضرب نصف الاجذار في مثلها عن الربع في مثله ثم في اربعة وهذا صورته



وله ايضا صورة اخري تودي الي هذا وهي سطح اب وهو المال فاردنا ان نزيد عليه مثل عشرة اجذاره فنصفنا العشرة فصارت خمسة نصيرناها سطحين علي جنبتي سطح اب وهما سطحاً ج د فنصار طول كل سطح منهما خمسة اذرع وهو نصف العشرة الاجذار وعرضه مثل ضلع سطح اب فبقيت لنا مربعة من زوايا سطح اب وهي خمسة في خمسة وهي نصف العشرة الاجذار التي زدناها علي جنبتي السطح الاول فعلمنا ان السطح الاول هو المال وان السطحين الذين علي جنبتيه هما عشرة اجذار فذلك كله تسعة وثلاثون وبقي الي تمام السطح الاعظم مربعة خمسة في خمسة فذلك خمسة وعشرون زدناها

Figure 1: A geometrical demonstration by al-Khwārizmī (from Rosen 1841, 10).

According to Høyrup, al-Khwārizmī’s proof must have been derived from this tradition. This way of demonstrating may then have been more familiar than the *al-jabr* itself.

3.2.3. Double solutions in the abacus tradition

We continue to find double solutions in the early abacus tradition. The first vernacular algebra by Jacopa da Firenze (1307) mentions double solutions to the fifth type, both in the rules and in the corresponding examples. Maestro Dardi (1344, van Egmond 1983), in an extensive manuscript some decades later, continues to account for double solutions (Franci 2001, 83-4). Significantly, he leaves out the second positive solution for the geometrical demonstration of $x^2 + 21 = 10x$ which is copied from the Arabic texts. Later treatises gradually drop the second solution for this type of problem. For example, the anonymous Florence Fond. Prin. II.V.152, later in the fourteenth century, has an intermediate approach. The author writes that:

In some cases you have to add half the number of cosa, in others you have to subtract from half the number of cosa and there are cases in which you have to do both.

However, when applying the rule to an example with two positive solutions, he proceeds to perform only the addition.¹⁰ For the equation

$$x^2 + 9 = 10x$$

he gives the solution $x = 9$ and does not mention the second root $x = 1$. Also Maestro Biagio mentions addition and subtraction in his sixth rule but only applies the addition operation, as in problem 3 where two positive solutions are possible (Pieraccini 1983, 3).

Later abacus masters abandon the second solution altogether. For example the Riccar. 2263 gives only one solution to the problem $a + b = 10, ab = 22$ (Simi 1994, 33). Pacioli only uses addition for the fifth case of the quadratic problems (Pacioli 1494, 145). Maestro Gori, in the early sixteenth century, generalizes his rules to a form where the powers of the unknown are relative to each other. The Arabic rule V corresponds with his rule 4 in which “one finds three terms in continuous proportion of which the major and the minor together equal the middle one” (Siena L.IV.22, f. 75^r; Toti Rigatteli 1984, 16). This corresponds with the equation type

$$ax^{2n} + c = bx^n$$

Here Gori is in complete silence about a second possible solution, in the explanation of the rule, as well as in the examples given.

3.2.4. Double solutions disappearing from abacus algebra

Why do we see these double solutions for quadratic problems fading away during algebraic practice in the abacus tradition? It could be interpreted as an achievement of Arabic algebra which becomes obscure in vernacular writings. In our understanding, the abandonment of double solutions has to be explained through the rhetorical structure employed by abacus writers. The strict, repetitive and almost formalized structure of the problem solution text is a striking feature of many of the algebraic manuscripts in Italian libraries. The solution always starts with a hypothetical reformulation of the problem text by use of an unknown. For example, Gori, as an illustration of the rule cited above, selects a division problem of ten into two parts with certain conditions given. The solution commences in the typical way “suppose that the smaller part equals one cosa” (“pongho la minor parte sia 1 co.”, *ibid.* p. 17). One particular value of the problem is thus represented by the unknown. The unknown here is no indeterminate as in later algebra; it is an abstract representation for one specific quantity of the problem. Given that this recurring rhetoric structure, which is so important for the abacus tradition, commences by posing one specific value, it makes no sense to end up with two values for the unknown. For the type of division problems which have descended from Babylonian algebra the quadratic expression leads to the two parts of the division. However, if one starts an argumentation stating that the *cosa* represents the smaller part, one does not expect to end up with the value of the larger part. The concept of an unknown in the abacus tradition is closely connected with this rhetorical structure in which the choice of the unknown excludes double solutions by definition.

3.3. The unknown in the abacus tradition

With al-Khwārizmī’s treatise and more so with Abū Kāmil’s *Algebra*, the unknown became a more abstract concept, independent of a geometrical interpretation. While the unknown in one type of quadratic problems allowed for double solutions, this was gradually reduced to a single value through the rhetorical structure of abacus treatises. Let us now summarize the development of the concept unknown within the abacus tradition.

The ambiguity of the *māl* was carried over, to some degree, from the Arabic texts to the abacus tradition by Fibonacci. Høyrup (2000, 22-3) has pointed out the inconsistent use of Latin words for *shay’* and *māl* by Fibonacci.¹¹ For most of the algebra part, Fibonacci uses the *res* and *census* terminology of Gerard of Cremona. However, in the middle of chapter 15 he switches from *census* to *avere* for *māl* (Sigler 2002, 578-601). For Høyrup this is an indication that vernacular treatises may have been circulating around 1228, the time of the second edition of the *Liber abbaci*. The Milan Ambrosiano P 81 sup, (fols. 1^r-22^r) is a later revision of Gerard’s translation. Here the author uses *cosa* for *res* (Hughes 1986, 229). While this manuscript is probably of later origin, the use of the vernacular *cosa* rather than *census* or

res is characteristic for the abacus tradition. With the first vernacular algebra extant, by Jacopa da Firenze in 1308, the use of *cosa* removed most of the original ambiguities. Where the conversion from the *māl* as a possession to the *māl* as an algebraic entity will have defied the student of Arabic algebra, the vernacular tradition eliminated these difficulties. When Jacopa provides the solution to a problem on loan interest calculation he commences as follows (Høyrup 2000, 30):

Fa così: pone che fusse prestata a una cosa el mese de denaro, si che vene a valere l'anno la libra
12 cose de denaro, che 12 cose de denaro sonno el vigesimo de una libra, si che la libra vale
l'anno 1/20 [de cosa] de una libra.

By posing that the loan was lent at one *cosa* in *denaro* a month, the calculation can be done in *libra* leading to a quadratic equation with a standard solution. The rhetorical structure of the solution text starts from a conversion of a quantity of the problem, in this case the number *denari* lent, to an unambiguous unknown *cosa*. Reformulating the problem in terms of the unknown, the problem can be solved by reducing the formulation to a known structure. In this case to *censi* and *cosa* equal to numbers.

This was basically the function and meaning of the *cosa* for the next two centuries within the abacus tradition. In all, the notion of the unknown in the abacus tradition was fairly constant and unproblematic.

4. Operations on polynomials

Most current textbooks on the history of algebra consider operations on polynomial expressions as natural to a degree that they do not question the circumstances in which these operations emerged. This is rather peculiar as most algebra textbooks, from the late abacus tradition onwards, explain these operations at length in their introduction. Focusing on the operations which have led to the formation of new concepts, we consider operations on polynomials crucial in the understanding of the equation as a mathematical concept. A possible reason for this neglect of conceptual innovations is the structural equivalence of algebraic operations with arithmetical or geometrical ones.

al-Khwārizmī (c. 850) introduces operations on polynomials in the Arabic version of his *Algebra* after the geometrical proofs and before the solution to problems. Strangely, he treats multiplication first, to be followed by a section on addition and subtraction, and he ends with division. Algebraic and irrational binomials are discussed interchangeably. Geometrical demonstrations are provided for the irrational cases. This order is followed in the three Latin translations. Abū Kāmil in his *Algebra* (c. 910) extends the formal treatment of operations on polynomials from al-Khwārizmī with some geometrical demonstrations and some extra examples, and moves division of surds to the first part. Al-Karkhī (c. 1000) improves on the systematization, but still follows the order of multiplication, division, root extraction, addition and subtraction. He treats surds after algebraic polynomials.

Hindu and Arabic treatments of operations on polynomials differ too widely to suspect any influence from either side. The order of operations and the way negative terms are treated are systematically dissimilar in both traditions. Nonetheless, there is the historical coincidence in the introduction of operations on polynomials in two dispersed traditions.

4.1. The abacus and cossic tradition

Although Fibonacci's algebraic solutions to problems use operations on polynomials throughout chapter 15, he does not formally discuss the subject as known in Arabic algebra. Typically, such preliminaries are skipped in early abacus writings and the authors tend to move directly to their core business: problem solving. A formal treatment of operations on polynomials is found gradually from the fourteenth century onwards.

Maestro Dardi in his *Aliabraa argibra* commences his treatise with an extensive section dealing with operations on surds (1344, Siena I.VII.17, fols. 3^v-14^r; Franci 2001). A short paragraph deals with the multiplication of algebraic binomials in between the geometrical demonstrations and the problems (ibid. f. 19^v). This is the location where we found the subject in al-Khwārizmī's *Algebra*. As far as we know, the anonymous Florence Fond. prin. II.V.152 dated 1390, is the first abacus text which has a comprehensive treatment on the multiplication of polynomials (ff. 145^r-152^r; Franci and Pancanti 1988, 3-44). It provides numerous examples with binomials and trinomials, including roots and higher powers of the unknown. Some curious examples are

$$(8x+0)(9x+5)$$

a form including a zero term we are familiar with from Hindu algebra, and the complex form

$$(6x^5 + 6x^4 + 6x^3 + 6x^2 + 6x + 6)(6x^5 + 6x^4 + 6x^3 + 6x^2 + 6x + 6)$$

Still, the examples are limited to multiplying polynomials.

During the fourteenth century, such introduction becomes more common and with the anonymous Modena 578 (1485, van Egmond 1986) we find a more systematic treatment of the addition, subtraction and multiplication of unknowns and polynomials. Finally, Pacioli (1494) raises the subject to the level of an algebra textbook.

5. The symbolic equation as a novel concept

5.1. The concept of an equation in Arabic algebra

Because the following paragraphs will deal with operations on equations, we have to make clear what the meaning is of an equation in early Arabic algebra. In fact, *there are no equations in Arabic algebra* as we currently know them. However, some structures in Arabic algebra can be compared with our prevailing notion of equations. Many textbooks dealing with the history of algebraic equations go back to Babylonian algebra. So, if there are no equations in Arabic algebra, what are they talking about? Let us therefore try and interpret the concept within Arabic algebraic treatises more positively.

Some basic observations on early Arabic algebra should not be ignored:

- The Latin translations do not talk about equations but about *rules for solving certain types of quadratic problems*. This terminology is used throughout: “the first rule”, “demonstration of the rules”, “examples illustrating the rules”, “applying the fourth rule”, etc. Apparently, these rules can be transformed directly into symbolic equations, but this is true for many other rules which cannot even be considered algebraic, such as medieval arithmetical solution recipes.¹²
- There is no separate algebraic entity in al-Khwārizmī's treatise which corresponds with an equation. The closest we get to an entity are “modes of equating” or “the act of equating”, referring to actions, not to a mathematical entity. The best way to characterize a mathematical entity is by the operations which are allowed on it. In early Arabic algebra there are no operations on equations. On the other hand, there are operations on polynomials. al-Khwārizmī has separate chapters on these operations.
- Early Arabic algebra is preoccupied with quadratic problems. Although linear problems are later approached algebraically by al-Karkhī, no rules are formulated for solving linear problems, as common in Hindu algebra. Therefore, if we consider the

rules for solving quadratic problems equations, then there is no analogous case for linear problems.

The correct characterization of the Arabic concept of an equation is *the act of keeping related polynomials equal*. Guglielmo de Lunis and Robert of Chester have a special term for this: *coaequare*. In the geometrical demonstration of the fifth case, de Lunis proves the validity of the solution for the “equation”

$$x^2 + 21 = 10x$$

The binomial $x^2 + 21$ is coequal with the monomial $10x$, as both are represented by the surface of a rectangle (Kaunzner 1989, 60):

Ponam censum tetragonum abgd, cuius radicem ab multiplicabo in 10 dragmas, quae sunt latus be, unde proveniat superficies ae; ex quo igitur 10 radices censui, una cum dragmis 21, coequantur.

Once two polynomials are connected because it is found that their arithmetical value is equal, or, in this case, because they have the same geometrical interpretation, the continuation of the derivation requires them to be kept equal. Every operation that is performed on one of them should be followed by a corresponding operation to keep the coequal polynomial arithmetical equivalent. Instead of operating on equations, Arabic algebra and the abacus tradition operate on the coequal polynomials, always keeping in mind their relation and arithmetical equivalence. At some point in the history of algebra, coequal polynomials will transform into an equation. Only by drawing the distinction, we will be able to discern and understand this important conceptual transformation. We will now investigate how and when this transformation took place.

5.2. Operations on ‘equations’ in early Arabic algebra

Much has been written about the origin of the names *al-jabr* and *al-muchābala*, and the etymological discussion is as old as the introduction of algebra into Western Europe itself. We are not interested in the etymology as such (as does for example Gandz 1926) but in the concepts designated by the terms. The older writings wrongly refer to the author or inventor of algebra by the name Geber. Several humanist writers, such as Ramus, chose to neglect or reject the Arabic roots of Renaissance algebra altogether (Høyrup 1998). Regiomontanus’s Padua lecture of 1464 was probably the most damaging for a true history of algebra. John Wallis, who was well-informed on Arabic writings through Vossius, attributes the name algebra to *al-jabr w’al-muchābala* in his *Treatise on Algebra* and points at the mistaken origin of Geber’s name as common before the seventeenth century (Wallis 1685, 5). He interprets the two words as operations and clearly not as Arabic names:¹³

The Arabic verb *Gjābara*, or, as we should write that found in English letters, *jābara* (from whence comes the noun *al-gjābr*), signifies, to restore ... The Arabic verb *Kābala* (from whence comes the noun *al-mulābala*) signifies, to oppose, compare, or set one thing against another.

Montucla (1799, I, 382) repeats Wallis’ comments on Geber by Wallis but seems to interpret *al-muchābala* as the act of equating itself:

Suivant Golius, le mot arabe, gebera ou giabera, s’explique par religavit, consolidavit; et mocabalat signifie comporatio, oppositio. Le dernier de ces mots se rapporte assez bien à ce qu’on fait en algèbre, dont une des principales opérations consiste à former une opposition ou comparaison à laquelle nous avons donné le nom d’équation.

We want to understand the concept of the ‘equation’ within the context of the dissemination of early Arabic algebra in Western Europe. We will approach this conceptual reconstruction from the operations that were performed on the structures we now call equations. Changes in the operations on these structures will allow us to understand the changes in the concept of an

equation. In a fairly recent publication, Saliba (1972) analyzed the possible meanings of *al-jabr* and other operations in the Arabic text of the *Kitāb al-mukhtaṣar fī ʿIsāb al-Jabr wa al-Muqābalaḥ* (c. 860) by al-Khwārizmī, but also the lesser-known works *Kitāb al-Badīʿ fī al-ʿIsāb* (Anbouba, 1964), *Kitāb al-Kāfī fī al-ʿIsāb* by al-Karajī (c. 1025), and its commentaries, the *Kitāb al-Bāhir fī ʿilm al-ʿIsāb* by Ibn ʿAbbās (12th century) and the *Kitāb fī al-Jabr wa al-Muqābalaḥ* from Ibn ʿAmr al-Tannūkhī al-Maʿarrī. Concerning the use of operations, Saliba concludes (1972, 190-1):

We deduce from them the most common definitions of the algebraic operations commonly denoted in those texts by the words *jabr*, *muqābala*, *radd* and *ikmal*.

The understanding of the precise meaning of these operations is an ongoing debate since the last century and earlier. There are basically two possible explanations. Either the Arabic authors of algebra treatises terms used the term inconsistently, or there are fundamental difficulties in understanding their meaning. Saliba is clearly convinced of the former, and seizes every opportunity to point at differences in interpretation and double uses of some terms. Others believe that there are no inconsistent uses at all and attempt to give an interpretation of their own. A recent discussion, on the *Historia Mathematica* mailing list, has raised the issue of interpretation once again.¹⁴ Jeffrey Oaks writes that:

the words used to describe the steps of algebraic simplification, *ikmāl* (completion), *radd* (returning), *jabr* (restoration) and *muqābala* (confrontation), are not technical terms for specific operations, but are non-technical words used to name the immediate goals of particular steps. It then follows, contrary to what was previously thought, that al-Khwārizmī and other medieval algebraists were *not* confusing and inconsistent in their uses of these words.

We do not want to be unsporting by claiming that a middle position is here more appropriate. We tend to defend the latter position. While there may be some inconsistent uses of the terms between authors and possibly even within a single treatise, the proper meaning of the operations can be well established within the context in which they occur. We will show that some confusions can be explained by translating or scribal errors and that a symbolic interpretation of the operations as Saliba's is highly problematic. We found out that while our interpretation of the *al-jabr* operation is new with respect to most twentieth-century discussions, it is not divergent from nineteenth-century studies, as Chasles' (1841) and Rodet's (1878).

5.2.1. Al-jabr

5.2.1.1. Early occurrences

The *jabr* operation is commonly interpreted as “adding equal terms to both sides of an equation in order to eliminate negative terms”.¹⁵ It appears first in al-Khwārizmī's book in the first problem for the ‘equation’ $x^2 = 40x - 4x^2$. In this interpretation the *al-jabr* is understood as the addition of $4x^2$ to both parts of the equation in order to eliminate the negative term in the right-hand part. As a typical symbolical interpretation we give the description from Saliba (1972, 192):

If $f(x) - h(x) = g(x)$, then $f(x) = g(x) + h(x)$; which is effected by adding $h(x)$ to both sides of the equation and where $f(x)$, $h(x)$, $g(x)$ are monomials. E.g. if $x^2 - 10x = 19$ then $x^2 = 19 + 10x$

Saliba (1972) points out that the Arabic root *jabara* has a double meaning. On the one hand ‘to reduce a fracture’, on the other ‘to force, to compel’. He believes the second interpretation is justified as it corresponds with his mathematical understanding. We will argue the contrary. Surprisingly, the symbolic interpretation such as van der Waerden's and Saliba's has, until very recently, never been challenged. The rule corresponds with one of the later axioms of

algebra: you may add the same term to both sides of an equation.¹⁶ As such, the rule seems to be in perfect correspondence with our current understanding of algebra. However, we will show this is not the case.

Let us follow the available translations of the original text. The first of al-Khwārizmī's illustrative problems is formulated as the division of 10 into two parts such that one part multiplied by itself becomes four times as much as the two parts multiplied together. Using the unknown for one of the parts, the other is 10 minus the unknown. al-Khwārizmī proceeds as follows (Rosen 1831, 35-6):

Then multiply it by four, because the instance states “four times as much”. The result will be four times the product of one of the parts multiplied by the other. This is forty things minus four squares. After this you multiply thing by thing, that is to say one of the portions by itself. This is a square, which is equal to forty things minus four squares. Reduce it now by the four squares; and add them to the one square. Then the equation is: forty things are equal to five squares; and one square will be equal to eight roots, that is, sixty-four; the root of this is eight, and this is one of the two portions, namely, that which is to multiplied by itself.

The *jabr* operation is thus described by “reduce it now by the four squares, and add them to the one square”. Remark that this description is somewhat odd. The operation here seems to consist of two steps, first reducing the four squares from it and secondly, adding them to the one square. For the second problem, Rosen (1831, 37) also uses the term *reduce* in the context “Reduce it to one square, through division by nine twenty-fifths”, which is clearly a different type of operation of division by a given factor. On most other occasions Rosen translates the *jabr* operation as “separate the <negative part> from the <positive part>”.¹⁷ Karpinski's translation gives a different interpretation. He used Scheubel's copy of the Latin translation by Robert of Chester and translates the passage as “Therefore restore or complete the number, i.e. add four squares to one square, and you obtain five squares equal to $40x$ ” (Karpinski 1915, 105). Karpinski does not use ‘restore’ in the second sense. In his view, *restoring* describes a one-step operation. The addition of the four squares to the one square explains the act of restoration. Can we find this interpretation confirmed by the first Latin translations?

Although we find in Hispalensis (Boncompagni 1857, 112-3) a corrupted version of the title of al-Khwārizmī's book, “Exceptiones de libro qui dicitur *gleba mutabilia*”, *al-jabr* is not further discussed.¹⁸ The *jabr* operation is most commonly translated into Latin by the verb *restaurare* and appears only once in Robert of Chester's translation for this problem (Hughes 1989, 53): “Restaura ergo numerum et super substanciam 4 substancias adicias” which literally means “Therefore restore the number and to the square term add 4 square terms”. The other occurrence is in the title *Liber Algebrae et Almuchabolae de Questionibus Arithmeticis et Geometricis. In nomine dei pii et misericordis incipit Liber Restauracionis et Opposicionis Numeri quem edidit Mahumed filius Moysi Algaurizmi*. Robert also uses the verb *complere* twice as an alternative translation for *al-jabr* (Hughes 1989, 56:1, 57:21).

The second Latin translation by Gerard of Cremona (c. 1150) uses *restaurare* eleven times. For the first problem Gerard formulates the *jabr* operation as “deinde restaurabis quadraginta per quatuor census. Post hoc addes census censui, et erit quod quadraginta res erunt equales quinque censibus”.¹⁹ Thus, the two Latin translations agree. Translated in symbolic terms, when given $40x - 4x^2 = x^2$, the $40x$ is restored by the $4x^2$ and only then, *post hoc*, the $4x^2$ is added to the x^2 . If we look at the actual text used by Karpinski (published by Hughes 1989, 53) “Restaura ergo numerum et super substancia, 40 rebus absque 4 substancias adicias, fientque 40 res 5 substancias coequentes”, the same interpretation can be justified. The *al-jabr* or restoration operation consists of completing the original term $40x$. It is considered to be incomplete by the missing four *censi*. The addition of the four *censi* to the *census* is a second step in the process, basically different from the *al-jabr* operation. The other occurrences of the operations within the problem sections are listed in the Table 2.

With this exhaustive list of all occurrences of the *jabr* operation in al-Khwārizmī's *Algebra* we can now draw an interpretation for the meaning of the operation:²⁰

- The restoration is an operation which reinstates a polynomial to its original form. We use polynomial as a generalization of the several cases. In VI.4 it is a simple number which is being restored. Also cases VII.5 and VII.6 refer to the single number 100, instead of $100 + x^2$. However in problem VI.5 is the binomial $100 + 2x^2$ which is restored. This is consistent with the other Latin translations.
- The restoration consists of adding (back) the part which has been diminished (“que fuerunt diminute”) to the polynomial. The restoring part can itself be a polynomial, as in problem VII.4 with $2x^2 - 1/6$ as the restoring part.
- The restoration operation is always followed by the addition of the restoring part to the other (coequal) polynomial.

Prob	Meta-description	Actual text	pp.
VI.1	$x^2 = 40x - 4x^2$	Deinde restaurabis quadraginta per quattuor census. Post hoc addes census censui	247-248
VI.3	$4x = 10 - x$	Restaura itaque decem per rem, et adde ipsam quattuor.	248
VI.5	$100 + 2x^2 - 20x = 55$	Restaura ergo centum et duos census per res que fuerunt diminute, et adde eas quinquaginta octo.	249
VII.1	$10x - x^2 = 21$	Restaura igitur decem excepta re per censum, et adde censum viginti uno.	250
VII.4	$21x + \frac{2}{3}x - 2x^2 - \frac{1}{6} = 100 + 2x^2 - 2x$	Restaura ergo illud, et adde duos census et sextam centum et duobus censibus exceptis viginti rebus	251
VII.5	$100 + x^2 - 20x = \frac{1}{2}x$	Restaura igitur centum et adde viginti res medietati rei.	252
VII.6	$100 + x^2 - 20x = 81x$	Restaura ergo centum, et adde viginti radices octoginta uni.	252
VII.8	$52\frac{1}{2} - 10\frac{1}{2}x = 10x - x^2$	Restaura ergo quinquaginta duo et semis per decem radices et semis, et adde eas decem radicibus excepto censu.	253
	$52\frac{1}{2} = 20\frac{1}{2}x - x^2$	Deinde restaura eas per censum et adde censum quinquaginta duobus et semis.	253
VIII.1	$100 + x^2 - 20x = 81$	Restaura ergo centum et adde viginti radices octoginta uni et erunt centum et census,	257

Table 2: All the occurrences of the restoration operation in al-Khwārizmī's *Algebra* in the Latin translation by Robert of Chester.

In such interpretation of Arabic algebra, the basic operation of *al-jabr*, from which the name of algebra is derived, does not consist of adding a negative term to the two parts of an equation. Instead, it refers to the completion of a polynomial which is considered incomplete by the presence of what we now would call, a negative term. An understanding of *al-jabr* in early Arabic algebra is inextricably bound with a geometric interpretation. We conjecture the *al-jabr* operation to be a generalization of the basic geometrical acts like cutting and pasting as we know them from Babylonian algebra. The original use of the restoration may refer to the restoration of a geometrical square. As we have discussed above, the *māl* as the Arabic concept of the unknown is a mixture of the meaning of possession, known from Hindi sources and from the geometrical square. While the original form of the *jabr* operation may have been purely geometrical, the operation can easily be generalized to simple numbers or polynomials. The demonstration of the solution to the quadratic problems in chapter 7 of al-Khwārizmī's *Algebra* gives us the most likely context of interpretation. Given that $x^2 + 10x = 39$, the demonstration depends on the completion of the polynomial $(x + 5)^2 - 25$ with value 39 (see Figure 2). The *jabr* operation restores the *māl*, the square term, in the polynomial. Hence, the value of the completed square $(x + 5)^2$ can be determined through a separate operation of adding 25 to 39. Also the third translation, by Guglielmo de Lunis (c. 1215), uses *restauracio*. In eight problems the operation is applied in the same meaning as the two other translations. We will therefore not discuss these further.²¹

However, in two similar problems, 4 and 6, *restaurare* is also used for a different kind of

$5x$	<i>māl</i>
x^2	$5x$

Figure 2□: completing the square (from al-Khwārizmī)

operation. This happens in situations where an expression involves a fraction of the *māl* as in

$$\frac{1}{12}x^2 + 1 + \frac{1}{3}x + \frac{1}{4}x = 20 \quad \text{and} \quad \frac{1}{12}x^2 = x + 24$$

In these two cases *restaurare* consists of multiplying the polynomials by 12. This operation is called *al-ikmāl* in Arabic and will be discussed below.

5.2.1.2. Al-jabr in later Arabic sources

Let us verify if this new interpretation of Arabic algebra can be sustained in later texts.

Abū Kāmil uses the term *restaurare* (as the Latin translation for *jabare*) forty times in his *Algebra*. On other occasions he uses *reintegrare* or *ikmāl* as a synonym for *restaurare*. All occurrences have the same meaning as with al-Khwārizmī and can be reconciled with our new interpretation. For the third problem Abū Kāmil constructs the ‘equation’ $4x = 10 - x$ and proceeds “Restaura ergo 10 per rem cum re, et appone adde rem 4 rebus; et erunt 5 res, equals 10 dragmis” (Sesiano 1993, 361:1117).²² Also here the restoration consists of completing the 10 and the following step is adding x to the $4x$. As with one case of al-Khwārizmī, the *jabr* operation with Abū Kāmil frequently refers to the restoration of a polynomial. For example the coequal polynomials

$$42\frac{1}{2}x - 4\frac{1}{4}x^2 = 100 + 2x^2 - 20x$$

are restored as follows (Sesiano 1993, 365:1285-91):

Restaura ergo 100 dragmas et 2 census cum 20 radicibus, et adde illas ad 42 res et $\frac{1}{2}$ rei diminutis 4 censibus et $\frac{1}{4}$; et erunt 62 res et $\frac{1}{2}$ rei diminutis 4 censibus et $\frac{1}{4}$ census, equals 100 dragmis et 2 censibus. Restaura item 62 res et $\frac{1}{2}$ rei cum 4 censibus et $\frac{1}{4}$, et adde illos 100 dragmis et 2 et censibus; et erunt 100 dragme et 6 census et $\frac{1}{4}$ census, que quantur 62 rebus et $\frac{1}{2}$ rei.

The first restoration refers to the $100 + 2x^2$, the second to $62\frac{1}{2}x$.

Interestingly, the critical edition adds some omissions in the Latin translation which are present in an Arabic copy of the original. In this case the original had “Restaura ergo 100 dragmas et 2 census diminutis 20 rebus cum 20 radicibus”. This reaffirms our interpretation of restoration as “restore <the defected polynomial> with <the part that was diminished>”.

Jeffrey Oaks and Haitham Alkhateeb defend the position on the *Historia Mathematica* forum, that the *al-jabr* operation for $10x - x^2 = 21$ should be interpreted as follows:

Think of $10x - x^2$ as a diminished $10x$. Its identity as $10x$ is retained even though x^2 has been taken away from it. Its restoration to its former self is accomplished by adding x^2 to the other side of the equation.

This was answered by Luis Puig, who apparently raised the issue in a publication previously.²³ In Puig’s reconstruction of the *al-jabr* operation for the same problem, it is the $10x$ which is restored: “Restaura luego las diez cosas del tesoro [substraído] y añádelo a veintiuno. Resulta entonces diez cosas, que igualan veintiún dirhams y un tesoro” (Puig 1998). On the discussion forum, Puig refers to the distinction made by al-Karkhī between *nombres simples* and *nombres compósés*. This distinction is indeed quite relevant for an interpretation of the *al-jabr* operation. In the *Al-Fakhrī*, partially translated by Woepcke, al-Karkhī gives an introduction to algebra treating the multiplication of polynomials. A marginal comment on the distinction of the two types of ‘numbers’ is as follows (Woepcke 1853, 50):

Il y a des personnes qui sont d’avis que ce nombre ($10 - a$) est composé, puisqu’il est formé par deux expressions d’un ordre différent. Mais il n’est pas ainsi, parce que en disant : dix moins chose, vous indiquez un seul nombre de l’ordre des unités ; si, au lieu de cela, il y avait eu : dix plus chose, cela aurait été composé. Cependant, placez les expressions de ce genre dans quelle catégorie vous voudrez, cela ne change rien aux principes du calcul.

The special status of ‘incomplete’ or ‘defected’ simple numbers can further explain the nature of the *al-jabr* operation. As the bone surgeon, *algebrista* in old Spanish, splints a broken leg, so does the *al-jabr* operation restore an incomplete number.²⁴ While a negative term is considered a defect, the addition of a positive term is considered a constructive step for a

composed number. It also explains that we should not consider the $-x^2$ in $10 - x^2$ as a negative term, but as the defect of the incomplete number 10. While al-Karkhī's distinction between simple and composed numbers is essential in contextualizing the *al-jabr* operation, it cannot be stated that *al-jabr* refers to the completion of simple numbers only. In a problem of Abū Kāmil's *Algebra*, we find an interesting case in which the 'defected polynomial' consists of four terms (Sesiano 1993, 390-1):

Et si dicemus tibi: Divisi 10 in duas partes, et multiplicavi unam [in aliam] duarum partium in se et aliam in radicem 8; deinde proieci quod [agregatum] productum fuit ex multiplicatione unius duarum partium in radicem 8 ex eo quod provenit ex multiplicatione (alterius) in se, et remanserunt 40 dragme. Exemplum. Faciamus unam duarum partium rem, reliquam vero 10 diminuta re. Et ducamus 10 diminuta re in se, et erunt 100 dragme et census diminutis 20 rebus. Deinde multiplica rem in radicem de 8, et proveniet radix 8 censuum. Quam prohice ex 100 dragmis et censu diminutis 20 rebus, et remanebunt 100 dragme et census 20 [radicibus] rebus diminutis et diminuta radice 8 censuum, que equantur 40 dragmis. Restaura ergo 100 et censum cum 20 [radicibus] rebus et radice 8 censuum, et adde (eas) ad 40 dragmas. Et habebis 100 dragmas et censum, que equantur 40 dragmis et 20 rebus et [rei] radici 8 censuum.

This solution of a division problem can be described symbolically as follows. Consider the two parts to be x and $10 - x$. Multiplying the second by itself and the first by the root of 8, the difference equals 40. Thus:

$$(10 - x)(10 - x) - \sqrt{8}x = 40$$

Expanding the square of the second part and bringing the x within the square root, this leads to

$$100 - 20x + x^2 - \sqrt{8}x = 40$$

So, now the question is, in al-Karkhī's terminology: what is restored here, the composed number or the simple number? The text of Abū Kāmil leaves no doubt: "Restaura ergo 100 et censum cum 20 [radicibus] rebus et radice 8 censuum". Thus the polynomial

$100 + x^2$ is restored by $\sqrt{\quad}$.

After that, the two terms are added to 40.

So, if close reading of the original text provides us with this divergent interpretation of the basic operation of Arabic algebra, why did scholars, proficient in Islam sciences and algebra fail to see it? Take for example Solomon Gandz, the leading expert on Arabic and Babylonian algebra in the early days of *Isis* and *Osiris*. Devoting an article on "The origin of the term 'Algebra'", Gandz (1926, 440) concludes that the *al-jabr wa al-muqābalaḥ* "ought to be rendered simply as *Science of equations*". Arguing against the older interpretation of restoration, he raises an intriguing question: "Why should we use an artificial surgical term for a mathematical operation, when there are such good plain words as *zāda* and *tamma* for the operation of addition and completion?" (ibid., 439). This should indeed ring a bell. Maybe *al-jabr* is not just "a mathematical operation" as we tend to see it. Maybe the operation is something very different from addition. The specific choice of the term *al-jabr* instead of other "good plain words" deserves an explanation within the context of early Arabic algebra and is no argument against an interpretation as restoration.

5.2.1.3. Older interpretations

Troubled by the question why the interpretation of *al-jabr*, as the restoration of a defected polynomial, is virtually absent in the twentieth century, we looked at some earlier studies. In Chasles (1841, 605-616) we recognize several important aspects of our interpretation:

Quand, dans un membre d'une équation, une quantité positive est suivie ou affectée d'une quantité négative, on *restaure* la quantité positive, c'est-à-dire qu'on la *rétablit dans son intégralité*. Pour cela on ajoute aux deux membres de l'équation une quantité égale, au signe près, à la quantité négative. Dans le langage de notre algèbre actuelle, nous dirions qu'on fait passer la quantité négative, du membre où elle se trouve, dans l'autre membre. Mais les Arabes ne pouvaient s'exprimer ainsi, parce qu'ils ne considéraient pas de quantités *négatives* isolément. Quoi qu'il en soit, c'est, à mon sens, cette opération de *restauration*, telle que je viens de la définir, que les Arabes ont appelée *jebr*, et les traducteurs *algebra*.

He considers *al-jebr* as a restoration of a positive quantity to its original integrity. In doing so, one must “add an equal quantity to the two members of the equation”. Chasles rightly adds that isolated negative quantities are not recognized in Arabic algebra.

Woepcke (1854, 365) is less concerned with the aspect of restoration and considers *al-jebr* as “the action of removing a negative particle and consequently replacing it at the other member to conserve the equality”.²⁵ Rodet (1878, 38), based on the authority of Freytag (1830) for a translation of *jabara* as “post paupertalum ditivait”, uses *enrichissant*. Thus he interprets the restoration of $100 - 20x = 40$ by al-Khwārizmī as:

Il commence par faire disparaître le terme négatif $- 20x$, en enrichissant, comme il dit, les 100 unités de déficit que leur a causé la soustraction des $20x$. Pour compenser cet enrichissant, il doit naturellement ajouter $20x$ dans le second membre de l'équation.

Carra de Vaux (1897) wrote a short note on the meaning of *al-jebr* in *Bibliotheca Mathematica* after inspecting a manuscript of Ibn El-Hāim in the Ambrosiano Library in Milan (&, 64, sup. f. 28^r). In that text the term is also applied to the restoration of a quantity with a missing fraction: “Thus to make $5/6$ equal to one whole, you divide 1 by $5/6$ which leads to $1 + 1/5$ and then multiply it with $5/6$. Otherwise, you can take the difference of $1 - 5/6$ and $5/6$ which is $1/5$ and this you add to $5/6$ to obtain one”. There is one occasion in al-Khwārizmī's problems in which the same operation is performed. In problem III.13, discussed above in §3.1 *complere* was used in the same way. By using the same term for the operation, al-Khwārizmī shows that adding

— to —

is basically the same act as restoration

— back to the form x^2 .²⁶

Carra de Vaux's note also includes a reference to the encyclopedia of the Turkish historian Hādīj Khalīfa (c. 1650). Here a definition of *djebr* is given strong support for our favored interpretation: “le djebr c'est ajouter ce qui manque à l'une des deux quantités mises en équation pour qu'elle devienne égale à l'autre”.²⁷

It is with some surprise that we have to admit the relevance of the nineteenth-century analyses in the current discussions on the interpretation of Arabic algebra. It seems that with Hankel and Cantor the interpretation as adding the term to both parts of an equation, was generally accepted.²⁸ Many twentieth-century authors have neglected to look up the studies of nineteenth-century scholars and missed their valuable comments.²⁹

In summary, we believe that the *al-jebr* operation in early Arabic algebra can be characterized as follows:

- An operation aiming at the restoration of a defected quantity to its original completeness.
- The restored quantity could initially have been a simple number in the sense of al-Karkhī, but for Abū Kāmil it also applies to polynomials.

- The operation is probably derived from or to be interpreted in a geometrical sense.
- The operation is not performed on an equation but on the affected part of one of two coequal polynomials.
- The addition of the defected part to the coequal polynomial is not a part of but a consequence of the operation.

5.2.2. Al-muqābala

The second operation, *al-muqābala*, is generally understood as the addition of homogeneous terms in a polynomial. So the operation allows to rewrite $x^2 + 20x + 100 = 20x + 100 + 21$ as $x^2 + 20x + 100 = 20x + 121$ (from al-Khwārizmī's third problem, Hughes 1989, 58). The Latin word for this is simply *summa* and derived from its geometrical interpretation of adding areas together. A second, equally important meaning of *al-muqābala* is the elimination of a term by subtracting it from the coequal polynomial. The latin term for this is *opponere* and is used in problem III.5 of al-Khwārizmī's *Algebra* (Hughes 1989, 56:3):

habebis 100 et duas substancias absque 20 radicibus 58 coequantes. Comple igitur 100 et 2 substancias cum re quam diximus et adde eam super 58, et fiet 100 et due substancie, 58 et 20 res coequancia. Hoc igitur oppone id est ex numero 29 proicias et remanebunt 21 et substancia 10 res coequancia.

Thus al-Khwārizmī applies *al-jabr* to $x^2 + 20x + 100 = 20x + 100 + 21$ in order to restore $100 + x^2$, translated on this occasion by *complere*. Omitted here by the scribe is a step which divides both polynomials by two to arrive at the coequal $x^2 + 20x + 100 = 20x + 100 + 21$. Then he applies *al-muqābala* to eliminate the number 29 from the second polynomial by subtracting it from the first, resulting in $21 + x^2 = 10x$. Hughes (1989, 20) understands the division by two as *complere*, but we believe this to be mistaken, as *complere* is also used, in the meaning described here, in problem two of the second chapter “habebis 40 et 20 res 100 coequantes. Hec ergo centeno opponas numero et 40 ex 100 auferas et remanebunt 60, 20 res coequancia” (Hughes 1989, 57/23). Rosen (1838, 40), who used the Arabic manuscript, does include the missing step as “Reduce this to one square, by taking the moiety of all you have. It is then: fifty dirhems and a square, which are equal to twenty-nine dirhems and ten things”. The Latin translation of Abū Kāmil's *Algebra* paraphrases *muqābala* as *mukabala* or *mucabele* and explains it as *oppositio* (Sesiano 1993, lines 527 and 532), but does not use the term within the problems. The verb *complere* only appears in its strict geometrical sense. Saliba (1972, 199) finds only one occasion in which al-Karkhī uses *muqābala* in the same sense as al-Khwārizmī. He believes that al-Karkhī also uses *muqābala* for the two operations discussed below.

While our interpretation of *al-jabr* considers the operation of completion as distinct from the subsequent step of adding the completed part to the coequal polynomial, *al-muqābala* appears to operate on the coequal polynomials within the same operation.

5.2.3. Al-radd and al-ikmāl

The last two operations called *al-radd* and *al-ikmāl* are less controversial. They normally refer respectively the division or to the multiplication of coequal polynomials by a constant. However, in some cases *ikmāl* is used synonymously with *jabr* by Abū Kāmil and *tamma* (to complete) for the *ikmāl* operation.

Arab	al-jabr	al-muqābala	al-radd	al-ikmāl
	الجبر	المقابلة	الرد	الإكمال
Rosen (from Arab)	reduce separate	reduce	reduce	complete
Robert of Chester	restaurare complere	opponere	converte	complere
Karpinski (from Robert)	restore complete	by opposition	reduce	complete
Gerard	restaurare	opponere	reducere	reintegrare
Guglielmo	restaurare	eicere	reducere	restaurare reliquitur integer
Abū Kāmil	restaurare reintegrare	opponere	reducere	complere (geometrical)

Table 3: Terms for the basic operations of Arabic algebra in the main Latin translations.

The best reference problem is problem III.5, as it combines the first three operations in a single problem solution. While Robert leaves out the *al-radd* step, he uses the verb *converte* for reducing the square term in problems III.3 and III.12 (“ergo ad unam converte substanciam”). The completion of the square term appears in problems III.4 and III.6.

5.3. Operations on equations in the abacus tradition

In the course of the fourteenth century, the original context of *al-jabr* as restoring a defected or incomplete quantity was almost entirely abandoned. The initial *al-jabr* operation, acting on a single quantity was extended by Abū Kāmil to be applied on polynomials. While the Arabic understanding of the operation continues to be present in some Latin treatises, we witness a clear shift in meaning of the operation.

With Fibonacci’s *Liber Abbaci* and the early vernacular algebra texts, the operation acts simultaneously on two coequal polynomials. The relation between the words used for restoration and its etymological root becomes disconnected. In the beginning of the fourteenth century, restoration involves both the addition and the subtraction of a term to coequal polynomials, sometimes within the same derivation. With maestro Biagio, from the fourteenth century onwards, the terminology discards all references to the restoring aspect and simply operates on both parts in order ‘to level out’ the positives as well as the negatives. The simultaneous operation on coequal polynomials is the beginning of what constitutes an algebraic equation. We cannot yet consider *ragguagliare* as an operation on an equation, but the simultaneous addition, subtraction, division and multiplication of coequal polynomials by some quantity contributes to the further transformation of this structure into a symbolic equation.

6. Conclusion

The symbolic equation has resulted from a series of developments in algebraic practice spanning a period of three centuries. The concept of a symbolic equation as it emerges in algebra textbooks around 1550 is fundamentally different from the ‘equation’ as known before the sixteenth century. This transformation of the equation concept was completed through the practice of algebraic problem solving. We can distinguish several phases of development which were necessary to realize the modern concept of an equation. We will now summarize these developments as discussed here, and place them within a broader

framework. We will present them in logical order which does not perforce coincide with consecutive historical events. Several of these developments overlap and have reinforced each other.

6.1. The expansion of arithmetical operators to polynomials

A process of expansion and generalization has allowed applying the operations of addition, subtraction, division and multiplication to other entities than natural numbers. This expansion process can be looked at from the viewpoint of the objects as well as of the operators. Operations on polynomial terms emerged as an expansion of the operators. These were introduced in Hindu texts around 600 and in Arabic algebra before 800. Essential differences in approach suggest an independent development in these two traditions. The presentation of operations on polynomials together with or following the operations on irrational binomials provides strong support for a historic process of generalization from irrationals to algebraic polynomials. We have written evidence that operations on polynomials were introduced in Europe through the Latin translations of Arabic works on algebra. Possibly there has been some influence too from Hindu algebra through sub-scientific traditions. The abacus tradition paid little attention to a formal treatment of operations on polynomials. Only from the end of the fourteenth century some abacus treatises devote a section to the multiplication of binomials or trinomials. Early German cossist texts of the fifteenth century were the first to formally introduce these operations. They reflect the structure of an *algorism* applied to terms involving unknowns. By the beginning of the sixteenth century every serious work on algebra has an introduction explaining at least addition, subtraction and multiplication of algebraic polynomials.

6.2. The expansion of the number concept

The process of applying arithmetical operations on terms with unknowns invoked an expansion of the number concept. The cossist tradition forwards the idea, which later becomes omnipresent in algebra textbooks, that cossic numbers are some kind of number, next to whole numbers, fractions and surds. Systematic treatments of arithmetic and algebra typically include binomials in the exposition of the numeration, the types of numbers in arithmetic. This evolution culminates in the *Arithmetica* of Cardano (1539). Cardano departs from the prevailing structure and treats the operators one by one. For each operation he discusses its application to whole numbers, fractions, irrationals and polynomial expressions. Polynomials, which he calls *de numeratione denominationem*, are thus presented as part of the number concept. The idea of polynomials as numbers is abandoned by the end of the sixteenth century. Later interpretations of higher-order polynomials with multiple roots and the unknown as a variable are in direct contradiction with a cossic number having one determinate arithmetical value.

6.3. Equating polynomial expressions

The very idea of an equation is based on the act of equating polynomial expressions. In fact, the Latin terms *aequatio* and *aequationis* refer to this action. Also the Sanskrit words *samīkarana*, *samīkarā*, or *samīkriyā*, used in Hindu algebra can be interpreted in this way. The word *sama* means ‘equal’ and *kri* stands for ‘to do’. The meaning of an equation in the first Latin texts is most correctly conveyed by the terminology used by Guglielmo de Lunis and Robert of Chester. The term *coaequare* denotes *the act of keeping related polynomials equal*. The whole rhetoric of abacus texts is based on the reformulation of a problem using the unknown and the manipulation of coequal polynomials to arrive at a reducible expression in the unknown. One looks in vain for equations in abacus texts. Every reference to an equation is purely rhetorical, meaning that the only equation discussed is that <coequal polynomial 1> equals <coequal polynomial 2>. If the manuscript contains illustrations or marginal comments

then these are always polynomials or operations on polynomials. Only by the end of the fifteenth century do we find equations in the non-rhetorical meaning. They first appear in German texts such as the Dresden C 80. Apparently Italian algebra was too dependent on a rigid rhetorical structure to view an equation as a separate entity. Pacioli's *Summa* (1494), full of marginal illustrations, does not give a single equation.³⁰ In Rudolff (1525) and Cardano (1539) we find the first illustrations of an equation in print. Both in the literal and the historical sense, we find the *construction* of an equation by equating polynomials (see Figure 3, from Cardano 1539, 82).

$$\begin{array}{r} 196.\bar{p}.336\text{ co.}\bar{p}.144\text{ ce.} \\ 4356.\bar{m}.2504\text{ co.}\bar{m}.1452\text{ ce.} \\ \hline 4160.\text{xqualia } 3240\text{ co.}\bar{p}.1596\text{ ce.} \end{array}$$

Figure 3: Cardano's construction of an equation by equating polynomial expressions.

6.4. Operations on coequal polynomials

The concept of an equation is shaped by the operations on coequal polynomials. The early development of the equation concept is determined by the first Arabic texts on algebra. Arabic algebra emerged from several competing traditions which are reflected in the meaning of the unknown and the operations allowed on coequal polynomials. These influences are most likely the 'high' tradition of calculators and the 'low' tradition of practical surveyors. A third influence of solving recreational problems concerning possessions may stem from Indian practice. The conceptual ambiguity of the *māl*, the unknown in Arabic algebra, can be explained through this diversity of influences. Also the *al-jabr*, the basic operation of Arabic algebra is challenging for a modern interpretation. Early Arabic texts interpret *al-jabr* as the restoration of a defected polynomial. The restoration of such polynomial to its integral (positive) form requires the subsequent step of adding the restored term to the coequal polynomial. This operation has transformed into the more general addition of terms to coequal polynomials. The characterization of the *al-jabr* as the restoration of one defected polynomial depends on the distinction made between co-equal polynomials and equations. When viewing Arabic algebra as operating on equations, such an interpretation would be meaningless.

Other operations such as bringing together homogeneous terms and dividing or multiplying coequal polynomials by a common factor can be related directly to their Arabic archetypes. These operations have been applied and discussed only implicitly in abacus problem solving. An explicit or formal exposition of the possible operations on coequal polynomials is first seen by the end of the fifteenth century in Germany. The formulation of rules and making these operations explicit contributed to the idea of operating on a single algebraic entity. It will take two more centuries to formulate these rules as axioms of algebra.

6.5. Expansion of arithmetical operators to equations

The transformation of operations on coequal polynomials to operations on equations is a subtle one. Only by making the distinction between the two can we understand and discern the changes in the concept of an equation.

The first explicit use of a multiplication of an equation is found in Cardano (1539, f. HH1^r) where he uses two unknowns to solve a linear problem. Eliminating one unknown, he arrives at an equation, expressed in the second unknown, which he multiplies with 35, as shown in Figure 4. Operating on equations here is closely connected with the use of the second unknown.

$$\begin{array}{r} 60 \frac{2}{3} \text{ aequalia } 2 \frac{2}{3} \text{ quā.} \\ \hline 35 \\ \hline 2008 \text{ aequalia } 72 \text{ quā.} \\ 39 \text{ Valor quā.} \end{array}$$

Figure 4: First operation on an equation in Cardano’s *Arithmetica Practicae*

6.6. Operations between equations

The second unknown has been the driving force behind the introduction of operations between equations. Cardano (1545) not only performs operations on equations but also he was the first two subtract equations in order to eliminate one of the unknowns (*Opera Omnia*, III, 241).

Using Cardano’s method of eliminating a second unknown from the *Ars Magna* and Stifel’s extension of algebraic symbolism for multiple unknowns, Jacques Peletier (1554) operates on an aggregate of linear equations.

$$\left\{ \begin{array}{r} 7 \text{ . pol. } \bar{p} \text{ . } 3 \text{ . quan. } \quad 132 \text{ .} \\ 7 \text{ . pol. } \bar{p} \text{ . } 25 \frac{2}{3} \text{ quan. } \quad 494 \frac{2}{3} \\ \hline 22 \frac{2}{3} \text{ quan. } \text{ aequales } 362 \frac{2}{3} \end{array} \right.$$

Figure 5: Cardano (1545) subtracts the first equation from the second to result in the third

He adds and subtracts pairs of equations in a systematic way to solve a set of linear equations. Buteo’s text (1559) corresponds closely with our meta-description in modern symbolism. The concept of a symbolic equation can thus be regarded as completed. The method was further refined by Gosselin (1577) from which we know that he had some influence on Viète (Cifoletti 1993).

7. Epistemological consequences

We have presented a detailed analysis of the basic concepts of algebra since the first extant texts in the Arab world and their subsequent introduction in Western Europe. The basic concepts of algebra are the unknown and the equation. We have demonstrated that the use of these concepts has been problematic in several aspects. Arabic algebra texts reveal anomalies which can be attributed to the diversity of influences from which the *al-jabr* practice emerged. We have characterized a symbolic equation as a later development which builds upon the basic Arabic operations on coequal polynomials. The concept of an equation can be considered as a solidification of the possible operations on coequal polynomials. In this way, the equation sign, as it was introduced by Robert Recorde (1557), represents not only the arithmetical equivalence of both parts, but at the same time symbolizes the possible operations on that equation. The equation, the basis of symbolic algebra, emerged from the basic operations on pre-symbolic structures, as we have studied them within Arabic algebra. The equation became epistemological acceptable by the confidence in the basic operations it represented. Knowledge depending on this new concept, such as later algebraic theorems or problems solved by algebra, derived their credibility from the operations accepted as valid for the concept. This new mathematics-as-calculation, derived from Arabic algebra, became *the* interpretation of mathematical knowledge in the sixteenth and seventeenth centuries. The

introduction of symbolism allowed for a further abstraction from the arithmetical content of the algebraic terms. Operating on and between equations became such powerful tool that it stood as a model for a *mathesis universalis*, a normative discipline of arriving at certain knowledge. This is the function Descartes describes in Rule IV of his *Regulae*. Later, Wallis (1657) uses *Mathesis Universalis* as the title for his treatise on algebra. As a consequence, the study of algebra delivered natural philosophers of the seventeenth century a tool for correct reasoning in general. In the early modern period, algebra functioned as a model for analysis, much more than Euclidean geometry did.

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Notes

¹ Although it has been argued that Fibonacci used a Latin translation of al-Khwārizmī's *Algebra*, particularly Gerard of Cremona's translation (Miura 1981, 60; Allard 1996, 566), one has to account for the fact that he had direct access to Arabic sources. Leonardo was educated in Bugia, at the north of Africa, now Bejae in Algeria, and travelled to several Arabic countries. He writes in his prologue of the *Liber Abbaci* that he "learnt from them, whoever was learned in it, from nearby Egypt, Syria, Greece, Sicily and Provence, and their various methods, to which locations of business I travelled considerably afterwards for much study" (Sigler 2002, 15-6).

² al-Khwārizmī's *Algebra* contains several problems which have been numbered in some translations. We will use the part numbers of the treatise as Roman numerals, followed by the sequence number and refer to the Latin translation if the problem numbering differs. Problem III.11 in Robert's translation is as follows: 'Terciam substancie in eius quartam sic multiplico, ut tota multiplicacionis summa ipsi coequetur substancie' (Hughes 1989, 61). The problem is given by Karpinski in modern symbolism as

$$\left(\frac{x}{3}\right) \cdot \left(\frac{x}{4}\right) = x, \text{ while the form } \left(\frac{x^2}{3}\right) \cdot \left(\frac{x^2}{4}\right) = x^2$$

would be more consistent with his interpretation of the *māl*.

³ Also argued by Høyrup (1998, note 11).

⁴ For a representation of *māl* as a geometrical square see Figure 2 in the discussion on *al-jabr* below.

⁵ Hughes (1989, 18-9). Apparently Hughes mixes up the chapter numbering. Read instead "problems four and six of part II and in five, ten, and thirteen of part III".

⁶ This problem is numbered 14 in chapter VIII of the Gerard's translation (Hughes 1986, 260).

⁷ A preliminary version of both these articles came to our attention when most of this chapter was already written. The analysis of Oaks and Alkhateeb (2005) and especially their section on 'the deliberate shift from the original *māl* to the algebraic *māl*' agrees with our observation. In fact, they discern three different meanings for *māl*. For the third meaning, they refer to the "division rule". If the result of the division of *a* by *b* is *c*, then the value of the *māl* *a* can be "recovered" by multiplying *b* and *c*.

⁸ We will follow the analysis of Rodet (1878, 84-8). The English translation is from Colebrooke (1817, 208).

⁹ Heffer, A.: "The Regula Quantitatis: From the Second Unknown to the Symbolic Equation", forthcoming.

¹⁰ f. 155^v; Franci and Pancanti, 1988, 54: "Quando le chose sono uguali a censi ed al numero prima si parta ne' censi e poi si dimezi le chose e l'una metà si moltiplica per se medesimo e di quella moltiplicazione si tralgha il numero, la radice del rimanente aggiunto ovvero tratto dall'altra metà delle chose, chotanto varà la chosa e tieni a mente che sono quistioni dove di bisogno agiugnere la metà delle chose e sono di quelle che àno bisogno di

trarre del la metà delle cose e sono di quelle che per l'uno e per l'altro si solvono. Esenpro al'agiugnere, prima dirò chosi”.

¹¹ Fibonacci, *Liber Abbaci*, second edition of 1228, on which Boncompagni's transcription is based. Høyrup (2002) suggests that the inconsistencies stem from the later additions and believes there must have existed an Italian vernacular text from before 1228 in which the term *avere* was used.

¹² Although Hughes (1986, 1989) consistently talks about equations, he implicitly agrees with this position when he writes that Gerard “uses the word *questio* to signify our term *equation*” (Hughes 1986, 214).

¹³ From the English edition, Wallis 1685, 2. Chasles (1841, 612) criticizes Wallis for the algebraic interpretation of the terms *al-jabr* and *al-muchābala* as synthesis and analysis. However, Chasles has been very selective in his reading of the *Treatise on Algebra*.

¹⁴ The discussion has been archived at <http://mathforum.org/kb/forum.jspa?forumID=149&start=0>

¹⁵ From van der Waerden (1980, 4). Compare with “Addition gleicher Terme zu beiden Seiten einer Gleichung, um subtraktive Glieder zu eliminieren”, Alten e.a. (2003, 162) and “to add the absolute value of a negative term from one side of an equation to itself and to the other side”, Hughes 1986, 218. Hughes (1989, 20) defines the synonymous Latin term *complere* as “to transfer a term from one side of the equation to another”.

¹⁶ Axioms play a role in the formulation of algebraic theory only from the seventeenth century. See chapter 8 for a further discussion on this.

¹⁷ Rosen 1831, 42, 43, 47, 48, 52, 52, for the problems discussed below. Problems of section VIII (in Gerard's translation) do not appear in the Arabic manuscript.

¹⁸ There exist two copies of an Arabic manuscript by Abd al Hamīd ibn Wāsi^c ibn Turk, called *Logical Necessities in Mixed Equations*, studied by Sayili (1985). There are good reasons to believe that this work on algebra predates the one of al-Khwārizmī's. Interestingly, except for the title, there is no reference to *al-jabr*.

¹⁹ This is the same formulation as the version of Libri (1938, I, 275), from the Paris Latin 7377A.

²⁰ Some clarifications may be necessary. The solution to VII.1 possibly contains a scribal error. Before the restoration step, $(10 - x)$ is multiplied with x . Consistent with the other cases, the restoration thus refers to $10x$, instead of $10 - x$ as in the text. Problem VI.5 refers to “the roots that have been diminished”, thus $20x$

²¹ Problems 1, 3, 5, 7, 8, 9, 10 and 11 in the numbering by Kaunzner (1986).

²² The line numbers from the Sesiano transcription are given after the column. Some other examples from Sesiano (1993): “Restaura ergo eas cum 9 rebus” (1132), “Restaura 10 radices per censum” (1174), “Restaura igitur 100 dragmas cum 20 rebus” (1243).

²³ Puig 1998, 16, discussed in the *Historia Mathematica* mailing list.

²⁴ For the meaning of *algebrista* see Smith (1958, II, 389). For a quotation from Don Quixote see Cantor 1907, I, 679, note 3, and Kline 1964, 95).

²⁵ Woepcke 1854, 365 : “*Algèbre* signifie dans la langue technique l'action, d'ôter la particule de la négation et ce qui la suit, et de reporter, en conservant l'égalité dans l'autre membre”.

²⁶ Hughes (1989, 18-9) misses the point when he writes in his commentary that al-Khwārizmī “does not use the multiplicative inverse to obtain $x^2 = 7\frac{1}{2}$ ” and that this “must have jolted Robert's readers”. However, the

performed operation is perfectly comprehensible given our interpretation of *al-jabr*.

²⁷ Translation by Carra de Vaux, from Flügel 1835-58, II, 582.

²⁸ Cantor (1907, I, 676) uses *Wiederherstellung* as the German translation of *al-jabr* and defines it as follows: “Wiederherstellung ist genannt, wenn eine Gleichung der Art geordnet wird, dass auf beiden Seiten des Gleichheitszeichens nur positive Glieder sich finden”. This is a curious definition as the equation sign appeared only in *Recordes* (1557). Hankel even cites the *Arithmetica* of Diophantus as a source for the *al-jabr* of the Arabs: “Wenn aber auf der einen oder auf beiden Seiten negative Grössen vorkommen, so muss man diese auf beiden Seiten addiren, bis man auf beiden Seiten positive Grössen erhält und das ist al gebr”. The quotation is taken from the Bachet (1621), *Diophanti Alexandrini Arithmeticonum*, p. 11.

²⁹ A notable exception is TROPFKE (1933, II, 66): “In dem Beispiele $13x - 5 = 7x + 4$ ist die linke Seite unvollständig, da ein *fehlendes* Glied vorkommt; si muß also mit 5 *ergänzt* werden, die dann auch rechts *hinzuzufügen* ist”. This interpretation is not respected by the editors of the 1980 edition.

³⁰ Except for the standard rules of algebra, the six Arabic types and two impossible cases (Pacioli 1494, f. 149^f).

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Avicenna's Naturalized Epistemology and Scientific Method*

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Abstract. This study provides a survey of Avicenna's theoretical or abstract discussions of the methods of science and the psychological processes laying behind them as they appear in his *Kitāb al-Burhān*. Since that text has not been studied in-depth, the paper is primarily exegetical, focusing what might be termed Avicenna's 'naturalized epistemology'. The study is divided into two sections. The first treats Avicenna's theory of demonstrative knowledge, and how Avicenna envisions the relation between logic and science, where it is argued that one of the primary functions of *Kitāb al-Burhān* is to provide heuristic aids to the scientist in his investigation of the world. The second half concerns Avicenna's empirical attitude in *Kitāb al-Burhān* towards acquiring the first principles of a science, where such cognitive processes as abstraction, induction and methodic experience are considered.

No treatise by Avicenna, at least not among his major philosophical encyclopedias, is exclusively dedicated to what might be called 'traditional epistemology'; rather, Avicenna's theory of knowledge is found in his psychological works and his work on scientific method, namely, *Kitāb al-Burhān*. By 'traditional epistemology' I mean the investigation into how knowledge or science is possible in the light of skeptical challenges. The traditional epistemological answer involves identifying a set of foundational criteria — whether *a priori* truths, sense data or a combination of both — by which one can justify or verify certain beliefs, and so can be said to have justified, true beliefs, that is, knowledge or science. In contrast with traditional epistemological approaches a naturalist approach to epistemology has re-emerged among contemporary philosophers. Paul Roth describes this naturalized epistemology thus:

Naturalism in epistemology can be characterized negatively by its eschewal of any notions of analytic or *a priori* truths. Positively, naturalism asserts a normative and methodological continuity between epistemological and scientific inquiry. The techniques endemic to the former are only a subset of the historically received and contingently held norms and methods of the latter (Roth (1999, 88)).

Bearing in mind these two opposing approaches to the theory of knowledge, it is worth noting that in Avicenna's works on psychology and scientific method he does not obsess over how to respond to the skeptic, or how to provide an *a priori* foundation for knowledge or even how to justify the knowledge one claims to have.¹ His concern is with describing the psychological processes involved in knowledge acquisition as well as the proper methods employed by successful scientists within the various sciences. In short, for Avicenna the traditional epistemological question, "How *ought* we acquire our beliefs?" is replaced, or at the very least is answered in part by, the question "How *do* we acquire our beliefs?", where the normativity of reason is in fact grounded in the practices of good science.

It is Avicenna's emphasis on this latter descriptive question as opposed to the former normative question, as well as his appeal to the *a posteriori* as opposed to the *a priori* that I am calling 'Avicenna's naturalized epistemology'.² In this respect the type of foundationalism I am denying of Avicenna is a rather strong one, namely, an epistemological theory that

asserts that the justification or verification of a body of beliefs must ultimately be based on what contemporary philosophers have variously termed ‘*a priori* truths’, ‘self-evident truths’, ‘self-presenting truths’, and ‘the given’. Foundationalism in this sense should not be confused with the thesis that certain sciences may be subordinate to other sciences, as for example physics might be thought to be more basic than chemistry. In the case of subordinate sciences the higher science frequently provides the explanations of various principles simply assumed in the lower science. This latter position more properly belongs to projects of unifying the sciences rather than epistemic foundationalism, and one can happily endorse one, while not endorsing the other, as in fact Quine did.

As already noted those interested in Avicenna’s theory of knowledge must look predominately to either his psychological works or his work on demonstration. Since most current research has focused on Avicenna’s psychological treatises, I want to augment our understanding of Avicenna’s theory of knowledge by considering his far less studied *Kitāb al-Burhān* of the *Shifā’*. Since this work has not been studied in-depth, my intent in this paper is primarily exegetical, namely, to present a number of the more salient features of *Kitāb al-Burhān*.³ In addition, however, I shall argue for what I have called Avicenna’s ‘naturalized epistemology’. This involves two stages. First, I treat Avicenna’s theory of demonstrative knowledge, and how Avicenna envisions the relation between logic and science, where I contend that *Kitāb al-Burhān*, far from endorsing any foundational project in epistemology, is primarily concerned with providing heuristic aids to the scientist in his investigation of the world. The second stage concerns Avicenna’s empirical attitude in *Kitāb al-Burhān* towards acquiring the first principles of a science, where I consider the cognitive processes of abstraction and to a lesser extent induction and methodic experience.⁴

1. Demonstrative Knowledge

Avicenna’s *Kitāb al-Burhān* roughly follows Aristotle’s *Posterior Analytics*, although Avicenna’s organization and development of Aristotelian themes are often uniquely his own. It is worth noting that among contemporary Aristotelian scholars it is an open question whether Aristotle intended the *Posterior Analytics* to be a discussion of science in general, or of some specific sciences and not others, or indeed whether it merely presents an account of how to formalize for pedagogical reasons a science already obtained.⁵ The situation is not the same for Avicenna’s *Kitāb al-Burhān*. Avicenna clearly saw this work as providing a completely general philosophy of science applicable to all sciences. “The goal of this book is to provide a means for acquiring the assent that is certain and the true and real concepts, and so the benefit of the book is obvious, namely, to arrive at the sciences occasioning certainty and the true and real concepts beneficial to us” (I.1, 7.12-14; 53.15-14). Moreover, this conception of the goal of *Kitāb al-Burhān* is witnessed by Avicenna’s regular use of examples drawn from all the sciences, such as medicine, physics, mathematics and metaphysics.

For Avicenna knowledge or scientific understanding (Arabic علم; Greek ἐπιστήμη) is roughly divided into two kinds: knowledge of the first principles of a given science and knowledge acquired through demonstration. Avicenna notes that both an account and description of how one acquires the first principles of a science properly fall under the purview of psychology (III.5, 160.17-18; 222.12-13), whereas a discussion of the methods and tools used by the scientist in acquiring demonstrative knowledge belongs to the subject of *Kitāb al-Burhān*; nevertheless, Avicenna does make comments in *Kitāb al-Burhān* relevant to how the scientist acquires the first principles of a science, which I shall turn to in the second half of this paper. For now, however, I begin with his discussion of demonstrative knowledge and the demonstrations leading to it.

Unlike Aristotle, who at *Posterior Analytics* I 2 offered a list of the conditions that the premises in a demonstration must meet — namely that they are true (ἀληθές), primitive (πρώτον), immediate (ἀμέσον) (that is, not themselves derived demonstratively), better known than (γνωριώτερον), prior to (πρότερον) and explanatory of (αἴτιον) the conclusion— Avicenna offers no such succinct list. Instead Avicenna's discussions of the conditions required of scientific first principles are interspersed throughout book I of *Kitāb al-Burhān*, sometimes treated explicitly, but more frequently implicitly. Thus Aristotle's 'truth condition' appears to be subsumed under Avicenna's 'certainty condition' (يقين), which includes both being true or real (الحق) and necessary (الضروري) (I.7 30.17-31.10; 76.4-14). Avicenna's use of 'certainty', a condition conspicuously absent from Aristotle's list, is significant. Throughout *Kitāb al-Burhān* Avicenna uses 'certainty' in two conceptually distinct ways.⁶ Thus, sometimes 'certainty' refers to one's assurance or knowledge of some natural necessity, and in this sense 'certainty' seems to be relative to the knower and the justification and warrant one has for a belief. More frequently, however, 'certainty' refers to the necessity or inevitableness of some causal relation in the world, which, though captured in the premises and conclusions of a demonstration, nonetheless is independent of any knower and his syllogizing, and in fact provides the very basis for knowledge and syllogisms. For Avicenna, as we shall see, one has the former type of certainty, that is, psychological assurance, only when one is aware of the latter type of certainty, that is, one recognizes that a necessary or inevitable causal relation obtains between two things. Here we should also note that though Aristotle himself does not include necessity in his initial list of conditions for the premises of a demonstration, based upon what he does say at *Posterior Analytics* I 4 and 6, it is natural enough to think that he thought necessity was a hallmark of such principles. Avicenna just makes this condition explicit in his notion of certainty.

Concerning the remainder of Aristotle's conditions, Avicenna, as far as I can see, never explicitly discusses the 'primitiveness' of principles, but this may be because أول ('primitive') is often taken as a synonym for 'principle', and so it might have been thought that this condition must obviously hold of a principle. As for being 'immediate', Avicenna mentions in passing at I.6 (30.10-12; 77.3-5) that some knowledge is بلا واسطة ('without middle'), but he probably does not intend this condition to be an absolute requirement of a scientific principle, but only relative to a given science; for he clearly believes that some of the principles in a subaltern science might be demonstrated in a higher science (I.12, 58.14-17; 110.13-15). At *Kitāb al-Burhān* I.11 Avicenna has a detailed discussion of the conditions 'prior to' (أقدم) and 'better known' (أعرف) than the conclusion, in which, like Aristotle, he distinguishes between 'prior and better known to us' and 'prior and better known by nature'. Unfortunately, his extremely rich and nuanced discussion is worthy of a study in its own right and would take us well beyond the scope of this paper. Concerning Aristotle's final condition, 'causally explanatory of the conclusion', this condition too seems to be subsumed under Avicenna's certainty condition and will be discussed more thoroughly below.

A demonstration according to Avicenna is "a syllogism constituting certainty," (I.7, 31.11; 78.15). In other words, it is a deduction beginning with premises that are certain or necessary that concludes that not only such and such is the case, but that such and such cannot not be the case (I.7, 31.7-8; 78.11-12).⁷ Thus, demonstrative knowledge involves possessing a syllogism that makes clear the necessity or inevitableness obtaining between the subject and predicate terms of its conclusion. In addition, Avicenna divides demonstrative knowledge itself into two categories depending upon the type of demonstration employed. Thus there is the demonstration *propter quid*, or demonstration giving 'the reason why' (برهان لِم) and the demonstration *quia*, or demonstration giving 'the fact that' (برهان لأن).⁸ Avicenna further divides the demonstration *quia* into two sub-species: a demonstration that leads from one correlative effect to another correlative effect, called an "absolute demonstration *quia*" (الإطلاق)

(برهان لأن على دليل), and a demonstration that leads from an effect to the cause, called an ‘indication’ (دليل).

Concerning the two types of demonstration *quia*, Avicenna suffices himself with providing definitions and examples of both kinds. Thus the absolute demonstration *quia* “accords with the existing middle term’s neither being a cause nor an effect of the major’s existing in the minor; rather, [the middle term] is something related to or coextensive with [the major term] in relation to its cause, where [the middle term] accidentally accompanies it or something else simultaneous with it in the nature” (I.7, 32.7-10; 79.17-19). He gives the following syllogism as an example: whoever exhibits a cloudy viscous urine is feared to have encephalitis; this individual (who is suffering from a fever) has exhibited such symptoms; thus this individual is feared to have encephalitis. In this case, notes Avicenna, neither the symptoms nor having encephalitis is the cause or effect of the other; rather, they are both effects of some unstated cause, which Avicenna identifies with the heated humors’ motion towards the head and their evacuation from it. What is important to note about the absolute demonstration *quia* is that even though the syllogism neither proceeds from nor leads to a cause, there nonetheless is a necessary, natural causal relation between the two terms, namely, they both are effects of some common cause, even if that cause is not made explicitly clear in the syllogism. Had there been no such causal relation, and the two terms had been merely coincidental accidents, then there would have been no demonstration. We shall return to this point shortly.

The second of the two demonstrations *quia*, namely, an indication, “accords with [the middle term’s] existing as the effect of the major’s existing in the minor” (I.7, 32.10; 79.19-20), and here Avicenna provides several examples. For instance, every recurring tertian fever is a result of the putrefaction of bile; the individual (who is suffering from a fever) has a recurring tertian fever; therefore, his fever is a result of the putrefaction of bile. Similar examples are given concerning the Moon’s relative position in relation to the Sun and the Moon’s various phases; the Moon’s being eclipsed when it passes between the Earth and the Sun; and a piece of wood’s burning when put into contact with fire. What is common in these examples is that one starts from some effect and concludes to the effect’s cause.

Demonstration in the most proper sense is the demonstration *propter quid*. The demonstration *propter quid* is a syllogism “that gives the cause with respect to both issues [namely, *that* such and such is the case as well as *why* such and such is the case], such that [the syllogism’s] middle term is like the cause for granting assent to the major’s existence belonging to the minor (or its denial), and so it is a cause of the major’s existence belonging to the minor (or its denial)” (I.7, 32.5-7; 79.13-16). In his examples of the demonstration *propter quid*, Avicenna returns to the examples used in clarifying an indication, but now he converts the examples such that the middle term is the cause of the effect. Thus, he again gives the example of tertian fever: whoever suffers from a putrefaction of bile owing to the bile’s congestion and the pores being obstructed is suffering from a recurring tertian fever; this individual is suffering from these symptoms; therefore, this individual is suffering from a recurring tertian fever. In short, the demonstration *propter quid*, like the demonstrations *quia*, inherently involves necessary, natural, causal relations. Unlike the demonstrations *quia*, however, the demonstration *propter quid* makes clear exactly what that causal relation is.

As Avicenna’s examples suggest, he believes that there is an inherent relation between demonstrations and causes. At *Kitāb al-Burhān* I.8 he develops this line of thought and argues in two steps that there is demonstrative knowledge if and only if one has necessary, perpetual certainty concerning the relation between two terms, where this certainty only occurs when one recognizes that a causal relation holds between the two terms. Avicenna’s first step is to indicate that knowledge of causal relations provides this necessary, perpetual certainty. His second step is to show that other kinds of relations that purport to provide this type of certainty in fact do not do so.

In *Kitāb al-Burhān* Avicenna's first stage — namely, indicating that the knowledge of causes ensures necessary, perpetual certainty — is example driven and he defers a full account of the underlying metaphysics of causality to first philosophy.⁹ For our purposes it would be beneficial to consider one of Avicenna's metaphysical arguments for this thesis. The argument that I shall consider, though by no means Avicenna's most well known or even preferred argument for causal necessity, does have the advantages of being concise as well as highlighting a point that will be of interest later, namely, how one comes to know that something has a causal power.¹⁰

In the *Najāt* (XI.2.i, 546.3-547.5), Avicenna begins with the claim that any proposition is necessary whose opposite entails an absurdity (محال) or a contradiction in the sense defined in Aristotle's *Metaphysics*, namely that something cannot both be and not be at the same time and in the same respect (Γ 4, 1005b19-20). Now grant, for example, that fire, which has the actual active power to burn, is put into contact with cotton, which has the actual passive power to be burned. Next assume that the expected effect, namely, the burning of the cotton, does not occur. Under these conditions one of two things would explain the cotton's not burning. Either the fire, which was assumed to have the actual active power to burn, does not in fact have the active power to burn, and thus there is a contradiction; or the cotton, which was assumed to have the actual passive power to be burned, does not in fact have the passive power to be burned, which again is a contradiction. In either case, then, the assumption that the expected effect does not occur, given the actual presence of its causes, entails an absurdity or contradiction. Thus, the opposite of the assumption must be necessary, namely, the expected effect necessarily occurs given the actual presence of its causes.

The previous argument might appear to be a piece of *a priori* reasoning, opposed to the sort of naturalism that I want to ascribe to Avicenna. On closer examination, however, one sees that the content of the argument requires that one already knows that things such as fire and cotton have their respective active and passive causal powers. This knowledge, as we shall see in the second half of this paper, is not known *a priori*, but is acquired either through a process of abstraction (التجريد) or methodic experience (التجربة), both of which, as I shall argue, involve a strong empirical element. In this respect, then, Avicenna's argument is clearly not intended to show that there are causal relations by some piece of *a priori* reasoning. In a very real sense Avicenna just takes the reality of causal relations for granted as part of his naturalism; for to deny causal relations would make the events in the world matters of mere happenstance and so would leave unexplained the manifest regular and orderly occurrence of events. In effect, to deny causal relations would undermine the very possibility of science understood as an investigation and explanation of the world's order, a position that Avicenna simply will not countenance. Instead, Avicenna's argument shows that to deny that causal relations are necessary is in effect to deny causal relations outright and so give up on the project of science. Avicenna's second stage in arguing that demonstrative knowledge is only acquired through knowing causes is to show that other kinds of relations that purport to provide necessary certainty in fact do not do so, or in the very least do not provide scientifically informative knowledge. Avicenna does not provide a global argument for this thesis, but proceeds on a case by case basis, where the two most prominent cases are the so called 'relative syllogism'¹¹ and cases of repetition or exclusion (الإستثناء).

Concerning the relative syllogism, one might argue as follows: Zayd is a sibling; all siblings have a sibling; therefore Zayd has a sibling. In this case one has argued from the relation of being a sibling to the existence of Zayd's sibling, where being a sibling is not the cause of the existence of the other individual, and yet one knows with certainty that the other individual exists given that Zayd is a sibling. Although Avicenna undertakes "a close examination and analysis" of the logic underlying this case, his concluding remarks suffice for our purposes.

Know that the intermediacy of the relative is something of little profit with respect to the sciences. That is because your knowledge that Zayd is a brother is your very knowledge that he has a brother or it is something included in your knowledge of that. Thus the conclusion is no better known than the minor premise. If that is not the case, and instead one is ignorant of [the conclusion] until it is proven that [Zayd] had a brother, then the individual simply does not understand (تصوّرت) “Zayd is a brother.” Cases such as these should not be called syllogisms let alone demonstrations (I.8, 41.18-42.1; 90.3-7).

Inasmuch as science and demonstrative knowledge are intended to provide one with a deeper understanding of the workings of the world, relative syllogisms simply fail; for, as Avicenna observes in his detailed analysis of the relative syllogism, to recognize a relation is simply for “the two relata to be simultaneously present in the mind” (I.8, 41.10-11; 89.15-16). In other words, it is not the relation that makes clear the existence of the two relata, but the existence of the two relata that makes clear the existence of the relation.¹²

Avicenna next considers the case of الإستثناء, which we shall leave un-translated for the moment. He gives the following example where one seems able to draw a conclusion with necessary certainty and yet the conclusion is not causally related to the premises.

When we know that this number is not one of two, we know with absolute unchanging certainty through the intermediary of [‘its not being one of two’] that [this number] is singular. Now that does not result from a cause; for it is not the case that its not being one of two is a cause of its being singular; rather, it is more appropriate that its being singular is something that in itself is a cause of its not being one of two and is something external to the essence of [not being one of two], since it is through a consideration of something else [41.1-4; 89.6-10].

The purported counterexample involves a hypothetical syllogism of the form ‘if not p , then q ; not p ; therefore q ’. Here one infers the necessary and certain existence of q from the non-existence of p , but the non-existence of something can hardly be called a ‘cause’, at least not in any rich sense of cause as some real ontological feature of the world, which of course is what Avicenna intends.

Although Avicenna’s resolution of this objection is relatively clear, the target or scope of his solution is not as clear. As for his solution (I.8, 42.1-7; 90.?-14), he argues that the middle term is either (1) some characteristic or sign (علامة) that does not essentially require that the number not be one of two or (2) one knows that the number is not one of two owing to some cause. In the first case, where there is nothing belonging to the essence of the number that requires that the number not be one of two, one does not know the premise with necessary, unchanging certainty. If the initial premise is not known with certainty, however, then neither can the conclusion be known with certainty; and so one cannot be said to have scientific understanding of the conclusion. In the second case, where there is something belonging to the essence of the number that explains its not being one of two, the cause would be that the number is singular and so by knowing that it is singular, one knows that it is not one of two. In that case, however, one already knows the conclusion before one knows the premise, and as such the conclusion of the purported example is uninformative and so not scientifically interesting.

The difficulty is determining the scope of Avicenna’s conclusion, that is to say, what does Avicenna precisely mean by الإستثناء. He clearly does not mean القياس الإستثنائي, or the ‘repetitive syllogism’ understood as an entire class, since at the end of his discussion he contrasts the counterexample with the informative repetitive syllogism, which conclude to some new knowledge acquired only after the ‘repetition’ (الإستثناء).¹³ In this case, Avicenna may be critiquing any syllogism that uses a conditional premise, where the antecedent and consequent of the conditional are not causally linked. Alternatively, Avicenna may be using الإستثناء in a non-technical sense, and so may mean simply ‘exemption’ or ‘exclusion’. Thus, Avicenna may be concerned with proofs that purport to provide necessary certainty about some class of things on the basis that a given class of things is exempt or excluded from some other class of

things. In this case the exemption or exclusion may be treated as a type negation, where a negation is hardly a cause in the sense of some real ontological feature of the world.

Perhaps we do not need to choose between these two alternatives; for it would seem that Avicenna has the philosophical wherewithal to exclude from the purview of scientific knowledge both types of proofs, again, namely, those involving no causal link between the elements of a conditional proposition and those involving negations. In the first case, it must be shown that Avicenna's original argument can be generalized to exclude from scientific discourse all hypothetical syllogisms in which there is absolutely no link between the antecedent and consequent of the conditional premise or premises. Clearly the first horn of the argument can be generalized, since if one of the premises is not known with certainty, then the conclusion cannot be known with certainty either. The difficulty is with the second horn, since perhaps there is some third thing that essentially explains the correlation, and yet the conclusion is not explanatory of the premise, as appears in Avicenna's own version of the argument. To give a hackneyed example: if something does not have a heart, it does not have a kidney; x does not have a heart; therefore, x does not have a kidney. Structurally, this example is identical with Avicenna's own; however, not having a kidney certainly is not the cause of not having a heart, or vice versa, but it was precisely that the conclusion was causally explanatory of one of the premises that Avicenna found objectionable in his initial argument. Still, if one knows with necessary, unchanging certainty that there is a necessary correlation between the two, even though not necessarily a direct cause-effect relation between them, that knowledge will presumably be on the basis of possessing an absolute demonstration *quia*, but in that case knowing that x does not have a heart is to know that x does not have a kidney. Thus one would have concluded to something already known, and so the syllogism is uninformative and not suitable for providing scientific knowledge. In short, the argument of Avicenna's second horn might be generalized thus: if two things are not merely related by happenstance, then to know that they are essentially related requires possessing an absolute demonstration *quia*; however, if one already possesses an absolute demonstration *quia* that two effects are essentially dependent upon a single cause, then given the existence of one effect one already knows that the other effect must exist. Simply put, such cases of الإستثناء will be scientifically uninformative.

Alternatively, if Avicenna intended الإستثناء to indicate a type of negation rather than a subclass of repetitive syllogisms, he could draw on earlier arguments he presented in the *Introduction* (المدخل) of the *Shifā'*. There he argued that though negations have a place in logic, they should be avoided in scientific discourse precisely because a negation inasmuch as it is a negation does not refer to any positive feature in the world, and yet science is concerned about finding out the way the world *is*. For Avicenna, negations are rather "entailments that belong to things relative to a consideration of certain (positive) accounts (معان) that do not belong to [the things]" (Avicenna (1952, I.13, 79.3-4)). In other words, when a proposition involves a negation, such as x is *not* one of two, the negation is relative to or follows upon certain positive accounts or factors that do belong to the thing, such as being singular, where the negative attribute is interpreted in terms of its failing to be among the positive accounts that do belong to the thing. As such negations are parasitic on what is. Thus insofar as الإستثناء might be understood as a type of negation, it provides one with information about the thing only to the extent that one already knows the causes or positive factors that constitute the thing, and so again negations are scientifically uninformative.

The relational syllogism and الإستثناء (however it might be understood) were the two main contenders for purported modes of necessary and certain reasoning that do not involve causal relations.¹⁴ Both either failed to provide the requisite knowledge or were scientifically uninformative. Thus, demonstrative knowledge must concern causal relations; for only causal

relations provide the necessary certainty that Avicenna takes to be the hallmark of good science.

To this point I have primarily focused on presenting and explaining the content of Avicenna's theory of demonstrative knowledge found in *Kitāb al-Burhān*. What should be clear from these comments is that for Avicenna there is an intimate link between logic and the scientific enterprise. I now would like to speculate about how I believe Avicenna envisions this relation. In the demonstration *propter quid*, as well as to a lesser extent the demonstrations *quia*, knowledge or scientific understanding is not for Avicenna about justifying one's beliefs or verifying science. Instead it is about laying bare the underlying causal structure of the world, which is done primarily through a logical analysis of empirical data, where this analysis involves identifying the middle term ultimately required for rational thought. Here we are led to a fundamental epistemological insight — first articulated by Aristotle and then wholeheartedly embraced and developed by Avicenna — namely that the causal explanations sought in the various sciences are the middle terms used in logic.¹⁵

Whereas Aristotle appears simply to assert this identification in his *Posterior Analytics*, Avicenna, in other works, suggests what the underlying metaphysics might be that explains this relationship between the objects of science and the objects of logic. Thus in Avicenna's *Introduction to the Shifā'* and the *Metaphysics* of the *Najāt* he claims that there is something common to both the intelligibles, which are the objects of rational thought, and their concrete instances and the causal interactions among them, which are the objects of scientific inquiry. Thus, Avicenna writes in his introduction:

The essences of things may be either in concrete particulars or in the conceptualization [of those things] (التصوّر), and so [essences] are considered from three [different] aspects. [One] aspect of essence indicates what it is to be that essence, not relative to one of the two existences [that is, concrete particulars or their conceptualization], and what follows upon them, [but only] insofar as it thus, [that is to say an essence considered in itself]. [A second] aspect belongs to [essence] insofar as it is in concrete particulars, so that at that time accidents, which individualize its existing as that, follow upon it. [A third] aspect belongs to [essence] insofar as it is conceptualized, so that at that time accidents, which individualize its existing as that, follow upon it (Avicenna (1952, I.2, 15)).

Avicenna identifies the essence considered in itself—that is the common link between the particulars, or the objects of science, and the forms existing in the intellect, or the objects of logic and rational thought—with a certain 'thingness' (الشيئية).¹⁶ Thus in the *Najāt*, he writes: “There is a difference between the thingness and the existence in concrete particulars; for the intrinsic essential account [of what something is] (المعنى) has an existence in concrete particulars and in the soul and is something common [to both]. That common thing, then, is the thingness” (XI.1.xii, 519.17-520.2).

For Avicenna, then, there is an inherent link between the objects of science and rational thought *via* the concept of thingness or the essences considered in itself. Although the two share a common link, they are, however, not absolutely identical for Avicenna; rather, as Avicenna will strenuously argue throughout the entirety of *Kitāb al-Burhān* I.10, the objects of science are in one sense prior to the objects of logic. Consequently, scientific analysis and examination are likewise in a sense prior to logical formulation, and as such logical notions are dependent upon and indeed mirror what is discovered as a result of good scientific methods. Hence, if an Aristotelian or Avicennan syllogistic provides humans with a universal logic, that is, a logic that sets the norms for rational thought (and there are good reasons for thinking that the *falāsifa*, including Avicenna, held this) and yet logical notions are dependent upon and reflect what is discovered through good scientific practices, then the way good science *in fact* proceeds is precisely the way one *ought* to acquire knowledge. In short, for Avicenna, epistemological questions concerning the normativity of reason should be replaced, or at least informed by, descriptions of what science does.

The relationship between logic and science is central to Avicenna's naturalized epistemology, and thus we should be careful to state both what he intends and does not intend by this relation. Avicenna does not mean that by using logic one can rationally reconstruct the external world from sense data (or perhaps sense data and purportedly *a priori* truths) in the way Russell attempted in his *Our Knowledge of the External World*, and perhaps Carnap as well under one natural interpretation of his *Der logische Aufbau der Welt*.¹⁷ For Avicenna such a foundationalist project would add nothing to one's understanding of how the world works, and thus in very real sense such a project would be vacuous for Avicenna. Moreover, such a project runs the risk of imposing some logical structure or constraints upon the world, which may not in fact be in the world, whereas for Avicenna the relation is just the reverse. Logic maps onto the way the world is, not because one has imposed some logical reconstruction on the world, but because the world structures and constrains the way one reasons about it.

For similar reasons Avicenna does not envision the relation between logic and science as how we might today see mathematics' standing to science, namely, as an idealization of the way the world would behave if it were composed of perfectly elastic bodies, lacking friction and the like.¹⁸ Human cognitive faculties, for Avicenna, are such as to discover the causal structure inherent in the world itself, and even if humans can invent logically and mathematically idealized models of the world, this is at best derivative of first understanding the causal structures in the world.

For Avicenna, I contend, the significance of the relation between middle terms and causes is that it allows all the advancements made in logic (or at least Aristotelian and Avicennan logic) to be used to further one's scientific investigations and inquiries concerning the nature of the world. Here let me use an overly simplistic instance to make the point. For Avicenna one can express all inferences using a finite set of paradigm syllogisms. Moreover, the syllogism allows one to infer a relationship between two terms by means of a middle term; for example this individual's suffering from tertian fever follows from his suffering from a putrefaction of bile. Consequently, when the scientist seeks the causal explanation of some phenomenon (that is to say, he asks why a given relationship holds between two terms), he is assured that when there is a causal explanation that links the two terms, that relationship can be expressed as a syllogism. Furthermore, the causal explanation of this relationship serves as the syllogism's middle term. Thus, since all scientific demonstrations or discoveries are expressible syllogistically, and since the syllogism has a specific structure, the scientist can use his knowledge of the syllogism to guide his initial inquiries; for only premises of a certain form and arranged in a certain way constitute a valid syllogism. In short, since there is an inherent relation between causes, that is, the objects of scientific inquiry, and the middle term, that is, the fundamental notion of Aristotelian and Avicennan logic, the scientist can be assured that the logical features that belong to the syllogism likewise hold of scientific explanations. In short, the scientist can use his knowledge of logic to facilitate scientific investigation.

A concrete, even if overly simplistic, example may help clarify.¹⁹ Imagine that a scientist wants to discover the cause of or reason why all dogs have incisors. For the Avicennan scientist, his knowledge of the syllogism immediately begins directing his search. The causal explanation must be of a form such that the conclusion "all dogs have incisors" follows logically. The only syllogism that renders such a conclusion is Barbara, namely, one that is in the first figure and has all universal affirmative premises. Hence the scientist knows before he begins his investigation that the answer (at least in its simplest form) has the following logical structure:

1. all x have incisors;
2. all dogs are x ;
3. therefore, all dogs have incisors.

The scientist's inquiry, then, is for x , that is, the middle term that causally links dog and having incisors. Granted the syllogism has not provided the scientist with an answer to the inquiry, and thus the scientist must still undertake an empirical investigation. Still that one should investigate the world fits well with Avicenna's empiricist leanings, which I shall discuss more fully below. Furthermore, the scientist is steered clear of certain false avenues of pursuit. For instance, he can neglect any observations that hold only of some dogs or some of the things that have incisors.²⁰ Similarly, he can set aside those observations that hold of no dogs or no things that have incisors.²¹ The reason he need not consider such premises is that one can never validly infer a positive, universal conclusion from them. Thus here is one way that logic's relation to science can facilitate scientific discovery, namely that a knowledge of the syllogism both allows the scientist to break down complex scientific questions into more manageable ones and also saves him from false steps in his investigation.

To summarize this section, demonstrative knowledge must concern causal relations; for only causal relations guarantee the necessary certainty that Avicenna takes to be the hallmark of science and knowledge. Moreover by linking the causal relations sought by scientists with the notion of the middle term, Avicenna could avail himself of the machinery presented in his logical works for the purpose of scientific investigations. Although there is much more to say about Avicenna's views of knowledge acquired through demonstration, the above at least gives one a sense of Avicenna's theory of demonstration and its relation to epistemology. In the last half of this paper I want to consider Avicenna's second kind of knowledge, namely, the knowledge and acquisition of first principles and the role of sensory perception in acquiring these principles.

2. Acquiring First Principles

Like Aristotle before him, Avicenna claims that all demonstrative knowledge, that is, knowledge that involves intellectual (الذهني) teaching and learning, must proceed from prior knowledge (*Posterior Analytics* I 1; *Kitāb al-Burhān* I.3), namely, knowledge that is not itself a product of a demonstration. The prior knowledge Avicenna has in mind is the existence claims and definitions of a science (I.12). For example in the science of physics, the physicist begins with the knowledge that motion exists as well as a definition of motion. In addition, the physicist will initially have some operational definitions such as accounts of what is meant by 'place', 'time', 'the continuum', 'void' and the like, that is to say, those things either purportedly required if there is to be motion or the necessary accidents that follow upon there being motion. The physicist subsequently investigates and sees if anything in the world corresponds with these initial nominal definitions. This initial knowledge insofar as it makes up the first principles of a given science is not demonstrated within the science itself — though in some cases it may be demonstrated in a 'higher science' (في علم قبله أو معه) — but either must be accepted if any science is to proceed at all or if the special science is to proceed, in the latter case it is one of the science's posits (وضع) (I.12, 58.14-17; 110.13-15).

Avicenna frequently states in *Kitāb al-Burhān* that a discussion of how the first principles of a science are acquired belongs to the subject of psychology (علم النفس); for an account of how we acquire first principles for Avicenna ultimately involves describing the various psychological and cognitive processes involved in human thought as well as any natural posits required to explain what we as human cognizers in fact do. Indeed, scholars working on Avicenna's psychology, such as Dimitri Gutas, Dag Hasse and Peter Adamson, to mention just three, have greatly advanced our understanding of such Avicennan cognitive processes as intuition or intellectual insight (الحدس),²² abstraction (التجريد) and discursive thought (الفكر).²³ It is not my intent here to delve into Avicenna's psychological works, but hopefully to augment what he says in those works with comments he makes in *Kitāb al-Burhān*, particularly with respect to

his empiricism and the roles of abstraction, induction (الاستقراء) and methodic experience (التجربة).²⁴

In *Kitāb al-Burhān*, Avicenna exhibits a strong empiricist leaning in his account of how one acquires the first principles of a special science or of science in general, which is radically opposed to any theory of *a priori* or innate knowledge. This empirical element, especially with respect to the natural sciences, is seen most clearly in the comments that he makes at III.5, where he discusses Aristotle's claim that "if a certain sense is wanting, then necessarily a certain knowledge is also wanting" (*Posterior Analytics* I 18, 81a38-39). In basic agreement, Avicenna comments Aristotle:

It is said, 'Whoever loses a certain sense, necessarily loses a certain knowledge,' which is to say that one cannot arrive at the knowledge to which that sense leads the soul. That is because the starting points from which one arrives at certain knowledge are demonstration and induction, that is, essential induction. Inevitably induction relies on sensory perception, while the universal premises of demonstration and their principles are obtained only through sensory perception, by acquiring the *phantasmata* (خيالات) of the singular terms through the intermediacy of [sensory perception] in order that the intellectual faculty freely acts on them in such a way that it leads to acquiring the universals as singular terms and combining them into a well-formed statement. If one wants to explain these [principles] to someone who is heedless of them (and there is no more suitable way to draw attention to them), then it can only be through an induction that relies on sensory perception. This follows because [the principles] are primitive and cannot be demonstrated, as for instance, the mathematical premises taken in proving that the Earth is at the center [of the universe], and the natural premises taken in proving that earth is heavy and fire light. That is why the principles of the essential accidents of every subject are learned first through sensory perception. Then from the sensibles some other intelligible is acquired, for example, the triangle, plane and the like in geometry, regardless of whether they are separable or inseparable. Indeed, then, the ways to arrive at them are initially through sensory perception (III.5, 158.11-159.3; 220.5-15).

Avicenna freely admits that the above is merely a concise statement and that the details will need to be worked out in the science of psychology. Fortunately, Avicenna also quickly sketches out those details in the remainder of III.5.

Thus Avicenna begins, "Something of the intelligible is not sensible, and something of the sensible inasmuch as it is what presents itself to sensory perception is not intelligible, namely, what presents itself for the apprehension of the intellect, even if sensory perception is a given starting point for acquiring much of the intelligible." Avicenna claims here that the objects of science, though starting from sensory perception, cannot be reduced simply to the perceptibles; rather, the objects of science are the intelligibles, which, though derived from the sensibles, are not identical with them.

To make his point, he has one consider a perceptible human, for example, Zayd or Omar, and the intelligible human, namely, what is common to Zayd and Omar that makes them both fall under the kind human. The perceptible human only presents itself to the senses as having a determinate magnitude, qualities, position, place and the like, all features that in some sense are unique to the individual at the time he is being perceived. In contrast, the intelligible human is something common to all humans, and as such is related to Zayd in the exact same way it is related to Omar as well as any other human. Indeed, Avicenna claims that the intelligible human is related to all instances of human "by way of absolute univocity" (المطلق بالتواطؤ). Thus, since what is sensibly perceived to belong to Zayd, Omar and other humans is not what is understood to belong to the form of humanity as it is found in the mind, Avicenna concludes that "the intelligible human is not what is conceived in the *phantasm* of the perceptible human" (III.5, 159.14-15; 121.8).

Since it is the intelligibles, or more exactly their definitions, that most frequently play the role of first principles in a science, it is necessary to see how the perceptibles are converted into intelligibles. Avicenna's answer is that this conversion takes place in part through the

cognitive process of abstraction (التجريد).²⁵ Fortunately, Avicenna again outlines the most salient features of this psychological process.

[T]he essences perceptible in existence are not in themselves intelligible, but perceptible; however, the intellect makes them so as to be intelligible, because it abstracts their true nature (حقيقتها) from the concomitants of matter. Still, conceptualizing the intelligibles is acquired only through the intermediacy of sensory perception in one way, namely that sensory perception takes the perceptible forms and presents them to the imaginative faculty, and so those forms become subjects of our speculative intellect's activity, and thus there are numerous forms there taken from the perceptible humans. The intellect, then, finds them varying in accidents such as it finds Zayd particularized by a certain color, external appearance, ordering of the limbs and the like, while it finds Omar particularized by other [accidents] different from those. Thus [the speculative intellect] receives these accidents, but then it extracts them, as if it is peeling away these accidents and setting them to one side, until it arrives at the account in which [humans] are common and in which there is no variation and so acquires knowledge of them and conceptualizes them. The first thing that [the intellect] inquires into is the confused mixture in the *phantasm*; for it finds accidental and essential features, and among the accidents those which are necessary and those which are not. It then isolates one account after another of the numerous ones mixed together in the *phantasm*, following them along to the essence [of human] (III.5, 160.7-17; 222.1-11).

This, then, is Avicenna's theory of abstraction in a nut shell.

Avicenna's language of 'extracting' (ينزع) and 'peeling away' (يقشر) may give the appearance that the intellect undertakes some mysterious process of 'dematerializing' or 'eliminating' certain features in the *phantasm* when it abstracts the intelligible. I believe that what Avicenna has in mind is actually simpler and more commonplace; for one can augment Avicenna's account here with comments that he makes about abstraction in his *Physics*, where one sees that far from being anything mysterious, much of the abstractive process is simply a matter of selective attention.

Analysis (التحليل)²⁶ is to mark a distinction owing to things whose existence truly is in the composite; however, they are mixed in the view of the intellect. Thus some of them are separated from others through their potency and definition, or some of them indicate the existence of something. So, when [the intellect] closely attends to (تأمل) the state of some of them, it moves from it to another (Avicenna (1983, II.9, 142.4-6)).

'Analysis', which Avicenna is treating very much like abstraction in the present passage, at least in part simply involves the far from mysterious process of selectively attending to certain features of the *phantasm*, that is, the sensible object as it appears in the intellect, to the exclusion of other features.

Clearly, this is not Avicenna's whole story concerning abstraction and acquiring first principles; for as he says later, acquisition of the first principles also involves "a conjunction of the intellect with a light emanated upon the soul and nature from the agent that is called the 'Active Intellect', that is, something leading the soul in potency to actuality. Be that as it may, sensory perception is a starting point, beginning with the accident, not the essence, of what [the intellect] has" (III.5, 161.6-8; 223.3-5). Admittedly, talk of 'emanation' and a separate 'Active Intellect' may sound peculiar, even mysterious, to modern ears. In fact, however, Avicenna's appeal to the Active Intellect is part and parcel of his naturalism and is well-integrated into both his physics and psychology; for in physics Avicenna would appeal to the Active Intellect to explain in part the acquisition of a new material form during substantial and accidental change, and analogously in psychology the acquisition of an intelligible form.²⁷ Avicenna's appeal to the Active Intellect in both cases, then, might be seen as an inference to the best explanation. He simply puts forth a natural posit needed to explain certain physical phenomena. In this respect Avicenna's positing the Active Intellect is loosely on par with Newton's initially positing his three laws and the concept of universal gravitation.²⁸ Although Newton could not demonstrate these aspects of his physics, if one granted them to him, he could explain a whole range of natural phenomena. The case is

similar for Avicenna, and though we today do not accept Avicenna's explanation, before we congratulate ourselves for having more advanced views than Avicenna, it should be noted that psychologists and cognitive scientists are still far from explaining the phenomena that Avicenna was addressing, namely, how mental states are generated from physical states and how thinking actually takes place. One can hardly fault Avicenna for not adequately explaining in terms that we today would prefer what we ourselves have not yet fully explained.

Let me be clear: I am not belittling the role that Avicenna finds for the Active Intellect in human cognition, but merely emphasizing another aspect of this phenomenon, which until recently has not been given its proper due. Abstraction, which begins with sensory perception, strips away one set of accidents, namely, those that follow on matter, and so prepares the way for the application of a new set of accidents, namely, the intelligible accidents, such as universality, that are acquired from the Active Intellect and are required if there is to be understanding. Both the roles of sensory perception and the Active Intellect are essential for a full account of Avicenna's view vis-à-vis human cognition.

In addition to abstraction, Avicenna lists three other ways that sensation is involved in acquiring the first principles of a science, or as Avicenna himself describes it, how "granting assent to the intelligibles is acquired through the senses" (III.5, 161.1-162.9; 222.17-224.8). These include (1) the particular syllogism (القياس الجزئي), (2) induction (الاستقراء) and (3) methodic experience (التجربة). Avicenna's comments concerning the particular syllogism are brief, consisting of two sentences.

[T]he particular syllogism [involves] the intellect's having a certain universal generic judgment, and then the individuals of a species belonging to that genus are sensibly perceived. So the species form is conceptualized together with [the genus], and that judgment is then predicated of the species. In that case, then, an intelligible that was not [possessed] is acquired (III.5, 161.11-13; 223.8-10).

Since, this method requires one of the other three methods to explain the generic judgment presupposed by the particular syllogism, I shall keep my comments short. Imagine that one possesses some generic judgment, for example, all animals are mortal, or any other universal claim that can be predicated of the genus animal. Next, if the argument is not to be jejune, imagine that a biologist comes across something that he has never experienced before, and so has no knowledge about it, yet from sensory perception he recognizes that it is an animal. From this perception and his prior generic judgment concerning all animals, he can conclude that this newly discovered species of animal is also mortal and has whatever other properties follow upon being an animal in general.

The latter two empirical methods of acquiring knowledge of first principles, namely, induction and methodic experience, are far more interesting, and show Avicenna's unique development of Aristotelian themes as well as his departure from Aristotle.²⁹ Avicenna parts company with Aristotle in his overall attitude towards induction (or least how later Aristotelians understood induction) and is skeptical of the merit of induction as an adequate tool of science. At *Kitāb al-Burhān* III.5 he describes induction in the following lackluster terms:

When the particular instances [of the first principle] are considered inductively, they call the intellect's attention to the belief of the universal; however, the induction that proceeds from sensory perception and the particulars in no way makes belief of a universal necessary, but only draws attention to it. For example, [when] two things both touch a third thing, but not each another, they require that that [third] thing is divisible. This aforementioned claim, however, may not be something established in the soul as well as it is sensibly perceived in its particular instances, which the intellect does notice and believes (III.5, 161.14-18; 223.11-15).

At most induction is merely a pointer (منبه) that draws one's attention to the pertinent facts surrounding some state of affairs. Induction, then, does not make clear what the cause of that

state of affairs is or even that there must be a cause. Although Avicenna's reservations towards induction might incline one to think that he is being anti-empirical, and so retarding science, such an assessment is far from the truth.

Earlier at *Kitāb al-Burhān* I.9 as well as in *Kitāb al-Qiyās* IX.22, Avicenna lays out what he finds problematic about induction. Induction has two elements: one involves the sensible content of induction and the other the rational structure of induction, namely, the syllogism associated with induction. If induction is to provide one with the necessary and certain first principles of a science, then the necessity and certainty of the conclusion of an inductive syllogism must be due either to induction's sensory element or its rational element or some combination of both. On the one hand, the purported necessity and certainty of induction cannot be known solely through induction's sensory element; for in good empirical fashion Avicenna recognizes that necessity and certainty are not direct objects of sensation. On the other hand, if the necessity and certainty are due to induction's rational component, then the syllogism associated with induction should not be question begging. Yet, complains Avicenna, in the scientifically interesting cases one of the premises of an induction will be better known than its conclusion, and so the induction is neither informative nor capable of making clear a first principle of a science.

At *Kitāb al-Qiyās* IX.22, Avicenna claims that induction in fact is successful in those cases where its divisions are exhaustive, as for example when animal is divided into mortal and immortal, or rational and irrational. The difficulty arises when one uses some other type of division that does not involve contradictory pairs. Unfortunately, Avicenna's discussion both in *Kitāb al-Qiyās* and *Kitāb al-Burhān* about the problematic type of division used in induction remains predominately in the abstract and the one concrete example he does provide — subsuming body and white under color — is singularly unhelpful. The following example, taken from Aristotle's *Prior Analytics* II 23, however, appears to be what he has in mind. Assume one divides long lived animals into horses, oxen, humans and the like, and then one wants to use this premise to make clear inductively the cause of their longevity. Thus one might reason as follows:

1. all horses, oxen, humans and the like are gall-less (major premise);
2. long-lived animals are horses, oxen, humans and the like (minor premise);
3. therefore, long-lived animals are gall-less.

Avicenna's earlier point was that the induction works only if one can be certain that one has correctly identified all and only long lived animals in the minor premise. One could be certain of this identification only if one knew what it is about this set of animals that guarantees that they and only they are the long-lived ones, but this knowledge would simply be to know the cause of these animals' longevity, the very premise one wanted to make clear. Thus it is not induction's rational element, at least in the scientifically interesting cases of induction, that explains the purported necessity and certainty of its conclusion.

Since necessity and certainty cannot be found in either induction's sensory or rational elements, it would be difficult to explain how it could emerge from the two taken jointly. Again, Avicenna is not dismissing induction out-right; it certainly has its place in science as a means of drawing one's attention to pertinent facts. Still, if induction is intended to establish the facts about some causal relation and so provide the first principle of a science, Avicenna contends that it simply fails.

Avicenna instead wants to replace induction with methodic experience, which like induction has both a sensory and rational, or syllogistic, component. Unlike induction, methodic experience does not purport to explain *what* the causal relation is between two terms of a first principle, but only to identify *that* there is a causal relation.

[Methodic experience] is not like induction; for induction, in chancing upon the particulars, does not occasion universal certain knowledge, even if it might be something drawing attention [to it],

whereas methodic experience does. Indeed, methodic experience is like the observer and perceiver seeing and sensing that certain things belong to a single kind upon which follows the occurrence of a given action or affection. So when that is repeated numerous times, the intellect judges that this is an essential feature belonging to this thing that is not some mere chance occurrence, since that which is by chance does not occur always. An example of this is our judgment that a magnet attracts iron, and that scammony purges bile (III.5, 161.20-162.3; 223.16-224.2).

In methodic experience, there is the regular observation that two things always occur together without any falsifying evidence to the contrary. Thus the scientist reasons that whenever two things always occur together without any falsifying instance there must be a cause relating those two things. One always observes a magnet's attracting iron, for example; therefore, there must be some causal relation between the magnet's attraction and the iron, otherwise it would not always occur. Methodic experience has not explained what this causal relation is, only that there is such a relation; nonetheless, the conclusions arrived at by methodic experience can still be used as first principles of a science in order to explain other phenomena.

It should be further noted that at *Kitāb al-Burhān* I.9, where Avicenna fully discusses methodic experience, he is quite insistent that the necessary knowledge obtained through it is only conditional (بشرط) and applies only to the domain under which the examination was made. "[Methodic experience] does not provide *absolute* universal syllogistic knowledge, but only *conditional* universal [knowledge], that is, this thing which is repeated to the senses adheres to its nature as an ongoing thing with respect to the domain in which it is repeated to the senses, unless there is an obstacle. Thus [the knowledge] is universal with this condition, but not absolutely universal" (I.9, 46.20-23; 96.5-7). It is because knowledge of first principles acquired through methodic experience is limited to the domain under which the examination took place that Avicenna further warns the scientist that in light of new empirical data one may need to revise one's claims.

Thus he considers the case of the scientist who has repeatedly observed that on administering scammony there is always an accompanying purging of bile. The only thing that the observer can legitimately conclude, warns Avicenna, is that those varieties of scammony that he has tested always lead to this result; however, should new varieties of scammony become available that do not conform to the earlier findings, the initial hypothesis must be revised. Avicenna makes this point clearly:

We also do not preclude that in some country a disposition (مزاج) and special attribute (خاصية) are associated with scammony not to purge (or there is absent in it a disposition and special attribute); however, it is necessary that our judgment based upon methodic experience is that the scammony commonplace to us and perceived [before us], either from its essence or from the nature in it, purges bile, unless it is opposed by an obstacle (I.9, 48.4-7; 97.12-14).

Here in Avicenna's account of methodic experience one sees perhaps the strongest piece of evidence for Avicenna's naturalism and empirical stance towards science, namely that scientific hypotheses in principle must be revisable in light of new empirical data.

To conclude by way of summary, Avicenna's naturalized epistemology involves two separate, but closely related aspects: (1) identifying the methods and tools of good science in the case of demonstrative knowledge and (2) describing the psychological processes by which one becomes aware of causal relations in the case of first principles. With respect to the first aspect we have seen that the scientific tools and methods are predominately logical tools; however, Avicenna does not envision logic as providing some means for rationally or logically reconstructing the world beginning solely with *a priori* knowledge perhaps mixed with sense data. Far from endorsing such a foundationalist project, Avicenna sees logic as providing an aid to discovering the rational, causal structure inherent in the world itself. As for the second aspect, I believe Avicenna would happily endorse W. V. O. Quine's position, "Epistemology, or something like it, simply falls into place as a chapter of psychology and

hence of natural science. It studies a natural phenomenon, viz., a physical human subject” (Quine, 25).

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Notes

* I have consulted both Badawī’s and ‘Afīfī’s editions of *Kitāb al-Burhān* [Avicenna (1966) and (1956) respectively]. References to *Kitāb al-Burhān* are to book and chapter, then page and line number of Badawī’s edition. In both cases line numbers have been introduced by myself for ease of reference. In those cases where I have preferred Afīfī’s edition, I have marked the reference with a ‘*’.

¹ For an alternative interpretation of Avicenna’s theory of knowledge, which is more closely along traditional epistemological lines, see S. Nuseibeh (1989; 1996, 836-838). Nuseibeh argues that for Avicenna real knowledge is had only if it is verified. He then proceeds to argue that Avicenna held that there could neither be an empirical nor conceptual verification of any purported piece of knowledge, at least not prior to death, and thus Avicenna should rightly be described as a ‘skeptical’. Nuseibeh’s argument only holds if in fact Avicenna believed that science needed to be in some way verified or justified. In this paper, I shall argue that Avicenna did not hold such a position.

² My understanding of naturalized epistemology comes primarily from the following sources: W. V. O. Quine (1994), P. Kitcher (1992), H. Kornblith (1994) and P. Roth (1999) as well as through numerous discussions with Professor Roth.

³ M. E. Marmura (1990) provides a summary of some of the points in Avicenna’s *Kitāb al-Burhān*.

⁴ I do not consider here the important cognitive process of *حَدَس*, since in *Kitāb al-Burhān* Avicenna has very little to say about it. Moreover, in this work *حَدَس* appears to be exclusively a means for acquiring demonstrative knowledge from already possessed prior knowledge; see *Kitāb al-Burhān* I.3, 13, 6-9; 59.11-13 and III.3, 192, 2-4. Admittedly, in Avicenna’s psychological works *حَدَس* plays a more prominent role in acquiring first principles; see D. Gutas (1988, 159-176; 2001).

⁵ See J. Barnes (1975), P. Byrne (1997), M. Ferejohn (1991), R. McKirahan (1992) and W. Wians (1989).

⁶ This distinction is clearly implicit in Avicenna’s writing (especially at I.8) and explicitly made by al-Fārābī (1987, 98-99), where he speaks of the certainty of a belief as being a ‘congruence’ or ‘adequation’ (المطابق) with the state of affairs in the world.

⁷ It is interesting to note that Avicenna is quite insistent that the certainty, and thus the necessity, in question in a demonstration is not merely the certainty or necessity of the conclusion; for that the conclusion follows of necessity or certainly is true of every valid syllogism. For Avicenna, then, the relevant certainty or necessity concerns the premises, and the certainty or necessity of the conclusion is in turn derived from the premises’ certainty or necessity. See I.7, 31.11-18; 78.15-79.4.

⁸ Aristotle suggests this distinction at *Posterior Analytics* I 13, where he discusses the difference between understanding ‘the fact that’ (τὸ ὄν) and ‘the reason why’ (τὸ δι’ ὅτι).

⁹ Studies on Avicenna’s theory of causation include: M. E. Marmura (1984), R. Wisnovsky (2002) and A. Bertolacci (2002). For a discussion of causalities’ role in relation to medieval Arabic metaphysics in general see T.-A. Druart (2005).

¹⁰ Admittedly the argument I present is only implicit in Avicenna’s text. Still, that the interpretation that I suggest is the way certain later thinkers understood Avicenna’s argument is witnessed by al-Ghazālī’s treatment of causation in his celebrated 18th Discussion of his *Tahāfut al-falāsifa*. There al-Ghazālī treats only the argument for necessary causal relation that I present, and says nothing about Avicenna’s more well-known argument for this thesis from *Najāt* XI.2.iii.

¹¹ It is possible that Galen introduced the relational syllogism as one of the possible demonstrations used in science in his now lost *De demonstratione*, of which large parts, though not the whole, were available in Arabic translation; see N. Rescher (1966, 4-6). Concerning Galen’s theory of the relational syllogism see Galen (1964, ch. XVI).

¹² For a discussion of Avicenna’s metaphysics of relation see M.E. Marmura (1975, 83-99).

¹³ For an excellent survey of the term الإستثناء in Arabic logic see K. Gyekye (1972). For primary Avicennan sources concerning الإستثناء one may consult Avicenna (1964, VIII.1 and 2; 1971, 374) and the English translation of the former text by N. Shehaby (1973, 183-199).

¹⁴ Avicenna also considers the *reductio ad absurdum* (قياس الخلف), but his comments are brief, since he believes that this mode of argument can be converted into a demonstration *quia* (III.8, 42.7-8; 90.15-17).

¹⁵ See Aristotle, *Posterior Analytics* II 2 and Avicenna, *Kitāb al-Burhān* I.8.

¹⁶ For a discussion of Avicenna's conception of 'thingness' see R. Wisnovsky (2000; 2003, ch. 8). For a more general discussion of Avicenna's conception of the 'essence considered in itself' see M. E. Marmura (1979; 1992); and for a more specific discussion of the relation of essences considered in themselves to logic and science see J. McGinnis (forthcoming).

¹⁷ For an alternative interpretation of Carnap's *Der logische Aufbau der Welt*, and I believe a more philosophically satisfying one, see M. Friedman (1992).

¹⁸ Avicenna makes this point explicitly at the end of his *Physics*, where he argue against what we might call a 'mathematized physics'; see Avicenna (1983, IV.15, 331.7-333.9).

¹⁹ For a more complex example that is actually taken from Avicenna's *Physics* see J. McGinnis (forthcoming, section IV).

²⁰ The logical reason is that the distribution of either the minor or middle term will not extend far enough.

²¹ The logical explanation is that the middle term will not connect the two terms.

²² Neither 'intuition' nor 'insight' properly captures the sense of حدس, which more correctly is a quick, though clean, heuristic by means of which one correctly identifies the middle term of a syllogism.

²³ See D. Gutas (1988, 159-176; 2001), D. Hasse (2001) and P. Adamson (2004).

²⁴ For a discussion of Avicenna's empirical methodology, and, more specifically, medieval Arabic physicians' empirical attitude in relation to medicine see D. Gutas (2003). Similar ground is covered, albeit with the intent of showing that Avicenna was a skeptic, in S. Nuseibeh (1981). Both Gutas and Nuseibeh—Nuseibeh explicitly and Gutas only implicitly and with certain qualifications—suggest that for Avicenna the empirical findings of the physician cannot be used to discover, formulate or correct the first principles of medicine, since these principles are given in the higher science of physics. There is a sense in which this claim is true, namely, insofar as Avicenna is banning the majority of the physicians from undertaking this task; however, this proscription is due to the fact that most of these physicians lack a thorough knowledge of physics, which is required for such a task. In principle, however, it seems that Avicenna need not preclude one well-versed in both medicine and physics from using the empirical data acquired in medicine to inform one's understanding of medicine's first principles, provided that the physician-physicist is approaching that data *qua* physicist.

²⁵ For discussions of abstraction that emphasize the role of the Active Intellect as opposed to the role of the human intellect and sensory perception see the following: H. Davidson (1992, ch. 4), F. Jabre (1984) and S. Nuseibeh (1989). Nuseibeh reduces حدس to inspiration and revelation that is emanated by the Active Intellect and in fact he seems to eliminate abstraction altogether from Avicenna's theory of concept formation. For a more recent account of abstraction that emphasizes the role of the human intellect in abstraction and is overall consonant with Avicenna's comments in *Kitāb al-Burhān* see D. Hasse (2001).

²⁶ Although the term used in the context of the *Physics* is not التجريد or التجرد, but التحليل, this in part seems to be a concession to the text upon which Avicenna is commenting, namely, John Philoponus' *Physics* commentary. In its proper technical usage التحليل means 'analysis', that is, a breaking down of a thing into its constitutive parts for the purposes of investigation or definition. Still, Avicenna's context makes it clear that he is considering التحليل as at least closely akin to abstraction; for he addressing the issue of how one ultimately acquires the concepts of 'matter' and 'form', which indeed are first principles in physics. Moreover, even in Avicenna's psychological works he does not use التجريد exclusively for 'abstraction'; rather, he uses a whole complex of terms, such as أفرز, انتزع, and of course جرد. This list, I thus suggest, might also in certain cases include حلل.

²⁷ In Avicenna's psychology the Active Intellect also plays the further role of providing the storehouse for the intelligibles when they are not being thought by humans, and so allows Avicenna to avoid positing that the intelligibles subsist on their own in some Platonic realm of the Forms.

²⁸ Indeed when Newton's *Principia* first appeared he was criticized for his concept of universal gravitation by no less than Huygens for backsliding and introducing scholastic occult qualities; see R. Westfall (1971, 155-159).

²⁹ For a detailed discussion of Avicenna on induction and methodic experience see J. McGinnis (2003) (it should be noted that there I translated التجربة as 'experimentation', whereas I now believe that 'methodic experience' more properly captures the sense of the Arabic); also see J. L. Janssens (2004), which in important ways supplements and corrects my earlier work.

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The Philosophy of Mathematics

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Abstract. Is there a philosophy of mathematics in classical Islam? If so, what are the conditions and the scope of its presence? To answer these questions, hitherto left unnoticed, it is not sufficient to present the philosophical views on mathematics, but one should examine the interactions between mathematics and theoretical philosophy. These interactions are numerous, and mainly foundational. Mathematics has provided to theoretical philosophy some of its central themes, methods of exposition and techniques of argumentation. The aim of this paper is to study some of these interactions, in an effort to give some answers to the questions raised above. The themes which will be successively discussed are mathematics as a model for the philosophical activity (al-Kindī, Maimonides), mathematics in the philosophical syntheses (Ibn Sīnā, Naṣīr al-Dīn al-Ḥāzīnī), and finally the constitution of *ars analytica* (Thābit ibn Qurra, Ibn Sinān, al-Sijzī, Ibn al-Haytham).

The historians of Islamic philosophy take a particular interest in what some, at times, like to call *falsafa* (فلسفة). As they see it, it comprises the doctrines of the Being and the Soul developed by the authors of Islamic culture, indifferent to other kinds of knowledge and independent of all determination other than the link they have with religion. These philosophers would, then, be working in the Aristotelian tradition of Neo-Platonism, heirs of late antiquity under the colours of Islam. This historical bias ensures, superficially at least, a smooth passage from Aristotle, Plotinus and Proclus, among others, to the philosophers of Islam from the 9th century on. But the price is high: it often, but not always, results in a pale and impoverished image of philosophical activity and transforms the historian into an archaeologist, although one deprived of the latter's resources. Indeed, it is not uncommon for the historian to take on as his main task an excavation of the domain of Islamic philosophy, looking for the remnants of Greek works lost in their original but preserved in Arabic translation; or, for want of such a translation, to declare himself satisfied with the fragments of the ancient philosophers often studied with talent and competence by historians of Greek philosophy.

It is true that recently, some historians have turned to doctrines elaborated in other fields beyond the wake of the Greek inheritance: the philosophy of law, developed in magisterial manner by the jurists; the philosophy of *Kalām* (علم الكلام), that is, of the philosophical theologian, refined and subtle; the Sufism of the great masters as al-Zallāj and Ibn 'Arabī and others. Such studies enrich and correct the picture and reflect more faithfully the philosophical activity of the time. They also allow for a better understanding the place of the Greek inheritance in Islamic philosophy.

But the sciences and mathematics have not yet received the same attention as law, the *Kalām*, linguistics or Sufism and, even today, the links — in our opinion essential — between sciences and philosophy, and notably between mathematics and philosophy are disregarded. The links between mathematics and philosophy in the works of the philosophers of Islam as al-Kindī, al-Fārābī, Ibn Sīnā, and others are sometimes tackled, but in what must be termed a totally superficial way. Their views on the links between the two domains are described in an attempt to find a connection between these views and the Platonic or Aristotelian doctrines, or sometimes the possible influence of the Neo-Pythagoreans is examined. This means that there

is no attempt to understand the repercussions of the philosophers' mathematical knowledge on their philosophies, and not even the impact on their own philosophical doctrines of their activities as scientists, which of course most of them were. The historians of philosophy are not alone accountable for this deficiency; the responsibility is also that of the historians of sciences. It is true that, to examine the links between the sciences and philosophy, it is necessary to have a particularly wide scope of competence, a much finer linguistic knowledge than what suffices in geometry, syntactically elementary and lexically poor; and a knowledge of the history of philosophy itself. If to these demands we add a conception of the links between science and philosophy that is itself inherited from the present positivism, it is easier to understand the deep indifference of the historians of science in this domain. Yet — we must remind ourselves — the links between sciences and philosophy are an integral part of the history of sciences.

To be sure, the situation is a little paradoxical: for seven centuries, a scientific and mathematical research of the most advanced was elaborated in Arabic in the urban centres of Islam. Is it likely that philosophers who were sometimes themselves mathematicians, physicians, and so on, should have carried out their philosophical activity as recluses, indifferent to the changes that were taking place under their eyes, blind to a succession of scientific results that were following one another? How is this imaginable in the face of an unprecedented profusion of disciplines and successes: astronomy critical of Ptolemaic models, reformed and renewed optics, the creation of algebra, the invention of algebraic geometry, the transformation of Diophantine analysis, the discussion of the theory of parallels, the development of projective methods, and so forth — the philosophers should have been so insensitive as to remain within the relatively narrow frame of the Aristotelian tradition of Neo-Platonism? The apparent poverty of the philosophy of classical Islam is undoubtedly due to its historians rather than to history.

Nevertheless, to we examine the links between philosophy and science or philosophy and mathematics — to which we will limit ourselves here —, only as they appear in the philosophers' works, is to make only one third of the journey. It is also necessary to question mathematician-philosophers and mathematicians. But to consider mathematics alone demands an explanation at the outset, all the more so as this means of proceeding is in no way the norm in the study of Islamic philosophy.

No scientific discipline has contributed as much to the genesis of theoretical philosophy as mathematics; none has had such ancient and numerous links with philosophy. From antiquity, mathematics has constantly provided central themes for philosophical reflection; it has supplied methods of exposition, argument techniques, and even implements appropriate to its analyses. And finally, it offers itself to the philosopher as an object of study: he sets about clarifying mathematical knowledge itself by studying its object, its methods, by probing its apodictic characters. From start to finish in the history of philosophy, questions have kept recurring on the conditions of mathematical knowledge, its capacity to be extended, the nature of the certainty it reaches, and its place at the heart other kinds of knowledge. The philosophers of Islam are no exception to this rule: al-Kindī, al-Fārābī, Ibn Bājjā, Maimonides among many others.

Other less obvious links have appeared between mathematics and theoretical philosophy. It is common for them to collaborate in order to elaborate a method, a logic even, as the encounter between Aristotle and Euclid over the axiomatic method, or al-Āūsī's appeal to combinatorial analysis to solve the philosophical problem of emanation from the One. But whatever form this link may take, there is one which is particularly noticeable and which, in this case, was created by a mathematician, not a philosopher: we mean the doctrines developed by the mathematicians to justify their own practice. The conditions most propitious for these theoretical constructions are present when a mathematician, ahead of contemporary research,

is confronted with an insurmountable obstacle, as a result of the unsuitability of available mathematical techniques for the new objects that are beginning to emerge. Just think of the different variants of the theory of parallels, notably from the time of Thābit ibn Qurra (d. 901) of a kind of *analysis situs* conceived by Ibn al-Haytham, or of the doctrine of the indivisibles in the 17th century.

The links between theoretical philosophy and mathematics are to be found mainly in four types of works: the works of philosophers; those of the mathematician-philosophers as al-Kindī, Muḥammad ibn al-Haytham (not to be mistaken for al-Zāsan ibn al-Haytham [see Rashed, 1933c, II, pp. 8-19; 2000, III, pp. 937-941]); those of the philosopher-mathematicians as Naḥṣr al-Dīn al-Ḥūsī, and others; and those of mathematicians as Thābit ibn Qurra, his grandson Ibrāhīm ibn Sinān, al-Qūhī, Ibn al-Haytham, and others. Therefore to limit oneself to one group or another when examining the links between philosophy and mathematics is to condemn oneself to the loss of an essential dimension of the field of study.

We have tried on several occasions now to provide an exposition of some of the themes of the philosophy of mathematics; these are but a few soundings intended to reveal the riches of a domain rather more soundings, in fact, than a systematic examination of the domain. Such a project deserves a substantial volume, a volume which has yet to be written. The fact remains that the way that seems best suited to the task differs from merely setting out the views the philosophers may have expressed on mathematics and its importance; rather, it considers which themes were tackled, the intimate links between mathematics and philosophy and their role in the elaboration of doctrines and systems — that is to say the organisational role of mathematics. Notably, we will show how mathematician-philosophers set about solving philosophical problems mathematically, a fruitful approach generating new doctrines, new disciplines even. We will bring out the attempts of mathematicians to resolve mathematical problems philosophically and we shall see it constitutes an investigation which is profound and necessary. I will deal with the following topics:

1° Mathematics as the condition and source of models for philosophical activity. From the numerous philosophers who may illustrate this theme, we have selected just two: a mathematician philosopher and a philosopher who without being a mathematician was yet knowledgeable in mathematics: al-Kindī and Maimonides.

2° Mathematics in philosophical synthesis. It is with the first known synthesis, that of Ibn Sīnā, that mathematics as such intervenes in philosophical works. One of the results — and by no means the least — is the “formal” turn in ontology; which permitted the mathematical treatment of a philosophical problem. Naturally, we will consider here the contribution of Ibn Sīnā, a philosopher well-read in mathematics, which was continued by the mathematician Naḥṣr al-Dīn al-Ḥūsī.

3° The third topic, mainly cultivated by mathematicians dealing with the problem of mathematical invention, is *ars inveniendi* and *ars analytica* with Thābit ibn Qurra, Ibrāhīm ibn Sinān, al-Sijzī and Ibn al-Haytham.

1. Mathematics as conditions and models of philosophical activity: al-Kindī, Maimonides

The links between philosophy and mathematics are essential to the reconstitution of al-Kindī’s system (the 9th century); it is indeed such a dependence that the philosopher advertises when he writes a book entitled *Philosophy can only be acquired through mathematical discipline* (al-Nadīm, ed. 1971, p. 316), and when in his epistle on *The quantity of Aristotle’s books* (□□□□□□□□ (i.e. *Rasā’il*), ed. Abū Rīda, I, pp. 363-384), he presents mathematics as a propaedeutic to philosophical teaching. He even goes as far as calling out to the student in philosophy, warning him that he is facing the following alternative: to begin with the study of

mathematics before tackling Aristotle's books, according to the order given by al-Kindī — and then he can hope to become a true philosopher; or to do without mathematics and come merely to parrot philosophy, if he is capable of memorising by heart. Having mentioned Aristotle's different groups of books, al-Kindī writes:

This is the number of his books, that we have already mentioned, and which a perfect philosopher needs to know, after mathematics, that is to say, the mathematics I have defined by name. For if somebody is lacking in mathematical knowledge, that is, arithmetic, geometry, astronomy and music, and thereafter uses these books throughout his life, he will not be able to complete his knowledge of them, and all his efforts will allow him only to master the 'ability' to repeat if he can remember by heart. As for their deep knowledge and the way to acquire it, these are absolutely non-existent if he has no knowledge of mathematics (*ibid.*, I, pp. 369-370).

For al-Kindī, then, mathematics is at the base of the philosophical programme. By going deeper into its role in al-Kindī's philosophy — which is not our purpose here — one will be able to understand more rigorously the specificity of his work, which indeed historians often approach in two different ways. According to the first interpretation, al-Kindī presents himself as a Muslim representative of the Aristotelian tradition of Neo-Platonism, a philosopher of a doubly late antiquity. The second interpretation sees in him a follower of philosophical theology (*Kalām*), a theologian who would have liked to change its language for that of Greek philosophy. But if we give back to mathematics the role which has been devolved on it in the elaboration of his philosophy, al-Kindī's fundamental options will open up before our eyes. One of them comes from his Islamic convictions, as they were explained and set out in the tradition of philosophical theology, notably that of *al-Tawīd*, (the doctrine of God's unicity) that Revelation delivers us the truth, which is unique and rational. The second one refers us back to Euclid's elements as method and model: what is rational can be reached in a concise, very condensed and almost instantaneous way by Revelation, and can equally be derived through collective and cumulative work — that of philosophers — from truths of reason, independent of Revelation, which should satisfy the criteria of geometric proof. These truths of reason, which are used as primitive notions and postulates, were provided at the time of al-Kindī by the Aristotelian tradition of Neo-Platonism. They were chosen to replace the truths that Revelation offers in philosophical theology since they could fulfil the requirements of geometric thought and make possible an axiomatic style of exposition. The "mathematical examination (الفحص الرياضي)" became then the instrument of metaphysics.

That is in fact the case for the epistles in theoretical philosophy, such as for example *First Philosophy*, and the *Epistle for Explaining the Finitude of the Body of the World*, (Rashed and Jolivet, 1988). To take the latter text as an example, al-Kindī proceeds methodically to prove the inconsistency of the concept of an infinite body. He begins by defining primitive terms: *magnitude* and *homogenous magnitudes*. He then introduces what he calls "a certain proposition (قضية حق)" (*ibid.*, p. 161, l. 16), or, as he explains elsewhere, "the first true and immediately intelligible premises (المقدمات الأولى الحقيقية المعقولة بلا توسط)" (*First Philosophy, ibid.*, p. 29, l. 8), or else "the first obvious true and immediately intelligible premises" (*On the Unicity of God and the Finitude of the Body of the World, ibid.*, p. 139, l. 1), i.e. tautological propositions. These are expressed in terms of primitive notions, of order relations on them, of union and separation operations on them, of predications: finite and infinite. The following statements illustrate such propositions: homogenous magnitudes which are no bigger than each other are equal; or, if one of equal homogenous magnitudes is added to a magnitude which is homogenous to it, then they become unequal (*ibid.*, p. 160). Finally, al-Kindī uses a process of proof, *reductio ad absurdum*, by adopting a hypothesis: the part of an infinite magnitude is necessarily finite.

This is the path al-Kindī follows in his other writings. As in his *First Philosophy*, he proceeds *more geometrico* in his epistle *On the Quiddity of What Cannot Be Infinite and of What is*

called *Infinite*, this is how al-Kindī wants to prove the impossibility that the world and time are infinite. Al-Kindī begins here once again by stating four premises: 1° “Of anything from which some thing is taken away, what remains is smaller than what was before the subtraction was carried out”; 2° “Anything from which some thing is taken away, if what is taken away is given back to the former, it goes back to the original quantity”; 3° “For all finite things, if they are put together, a finite thing is obtained”; 4° “If there are two things such that one is smaller than the other, then the smaller measures the bigger or measures a part of it, and if it entirely measures it, then it measures a part of it” (Rashed and Jolivet, 1998, p. 150). From these premises, inspired directly by Euclid’s *Elements*, al-Kindī intends to establish his philosophical assertion. He then assumes an infinite body from which some finite thing is taken away, and the question is whether what remains is finite or infinite. He then shows that both hypotheses lead to contradictions, and concludes that no infinite body can exist. He goes on, showing that it is the same for the body’s accidents, notably time. And time, movement and the body are reciprocally involved. He then shows that there is no infinite time *a parte ante* and that neither the body, movement, nor time are eternal. There is therefore no eternal thing, and the infinite is only potential, as in the case of numbers. These examples, briefly mentioned, show how al-Kindī articulated simultaneously mathematical principles and methods, and philosophy according to the Aristotelian tradition of Neo-Platonism. It should be noted that al-Kindī the philosopher was also a mathematician as his works in optics (Rashed, 1996) and mathematics (Rashed, 1993a) testify. In philosophy, he was also familiar not only with Aristotle’s accounts and those of the Aristotelian and Neo-Platonist tradition, but also with Aristotelian commentators such as Alexander.

Maimonides (1135-1204), while not productive in mathematics like al-Kindī, was informed about the subject. He obviously has enough knowledge of mathematics to try to read, pen in hand, perhaps even to teach and to comment on, mathematical works as Apollonius’ *Conics*, which is to say, works of the highest level at the time. But his commentary never bears on the fundamental ideas, on the properties really studied in the work; he is interested only in the elementary proof techniques taught, for the most part, in the first six books of Euclid’s *Elements*. Put bluntly, his commentary is nowhere near the level of the works commented upon. But why did Maimonides spend so much time and energy for so meagre an outcome? We can certainly invoke — in Maimonides’ own words — the role of mathematics in training the mind (ترويض الذهن) to reach human perfection (*The Guide for the Perplexed*, ed. Atay, p. 80). But there is more: it has to do with the other connections between mathematics and philosophy. We will confine ourselves to the most important of these.

One must bear in mind that the starting point of Maimonides is dogma and not philosophy: “to elucidate (as he says) the difficulties of dogma (مُشكلات الشريعة), and to make plain its hidden truths, which are far above the comprehension of the multitude.” (*ibid.*, p. 282). This has been one of the major tasks of philosophy since al-Kindī (see his epistle *On the Quantity of Aristotle’s Books*), and consists in reaching the truth passed on by the Scriptures through reason, that is to say, philosophical speculation. To accomplish this task, even simply to initiate it, a perfect concordance had to be assumed between the two kinds of truth, that of the Scriptures and that of reason and philosophy. This “concordance” lies on a principle formulated by Ibn Rushd as follows (1126-1198): “a truth does not contradict a truth but accords with it and testifies for it” (*Faḥl al-Maqāl*, pp. 31-32). In this respect, the means for which Maimonides opted is the same as that with which his predecessors were equipped: “the method based on indubitable proof (الطريق الذي لا ريب فيه)” (*The Guide for the Perplexed*, ed. Atay, p. 187), i.e. to establish by the “true proof (البرهان الحقيقي)” the truth of dogma: the existence of God, His unity and His incorporeality. For these philosophers, this proof can only be conceived of as a mathematical model. And to do so, a language other than that of the

Revelation had to be used, a language whose concepts, defined by reason alone, are endowed with a certain ontological neutrality.

The “true proof”, that is, according to the mathematical model, is the way necessary for the truths of Revelation to obtain further the status of truths of reason, which is in no way peculiar to a particular religion, revealed or not. Such is the first connection between mathematics and philosophy. But these connections, as we shall see, occur at different levels. First of all, Maimonides’ general approach consists in borrowing notions from the Aristotelian philosophy of his predecessors, and proof and exposition techniques from mathematics; it is this approach which has been effectively used, for example, in the major part of the second book of the *Guide*. The method follows that of geometers, to whom he owes certain proof techniques — mainly *reductio ad absurdum* — to establish each element of his exposition. In the *Guide*, there are twenty-five such elements, twenty-five lemmas most of which are quoted, but all of which are taken by Maimonides to have been rigorously proved by his predecessors. To these lemmas, he adds one postulate, and from these twenty-six propositions he infers his “principal theorem”: GOD EXISTS, HE IS UNIQUE, AND HE IS NEITHER A BODY NOR IN A BODY. The importance of this passage is due not so much to the strength of the proof as to the deliberate metaphysical arrangement of a *more geometrico* exposition. The first lemmas were the potential subject of a logical and mathematical treatment since Aristotle, revived by al-Kindī, then picked up by several metaphysicians like Ibn Zakariyā al-Rāzī, Abū al-Barakāt al-Baghdādī (11th-12th), Fakhr al-Dīn al-Rāzī (1150-1210), Naṣīr al-Dīn al-Ḥusī (1201-1274), among others; finally, they are put together in the commentary of the *Guide* by al-Tabrīzī and later, in that of Hasdai Crescas (1340-ca 1414). They concern the impossibility of the existence of an infinite magnitude, and the impossibility of the coexistence of an infinite number of finite magnitudes. The third lemma states the impossibility of the existence of an infinite chain of causes and effects, material or not — thus condemning in advance the infinite regression of causes. Three propositions follow the three lemmas. The first deals with change; four categories are subject to change: substance, quantity, quality, and place. The second concerns motion: motion implies change and transition from potentiality to actuality. The third proposition enumerates the different kinds of motion. The seventh lemma is stated as follows: “Things which are changeable are, at same the time, divisible. That is why everything that moves is divisible, and necessarily corporeal; but that which is indivisible cannot move, and cannot therefore be corporeal” (*Guide for the Perplexed*, ed. Atay, p. 249). The eighth lemma asserts that: “anything that moves accidentally will necessarily come to rest” (*ibid.*, p. 251). The ninth, that “a body that sets another corporeal thing in motion can only effect this by setting itself in motion at the time” (*ibid.*, p. 252). The exposition of the preliminary propositions goes on in like manner; the fourteenth postulates that locomotion precedes all motions, and the twenty-fifth that each compound substance consists of matter and form.

These twenty-five lemmas, some of which have just been mentioned, all belong to the Aristotelian philosophy. But they are not homogeneous: their origin separates them as much as their logical complexity. Maimonides acknowledges this heterogeneity, since he generally gives us his sources: “*Physics* and its commentaries”, and “*Metaphysics* and its commentary”. The books of *Physics* and *Metaphysics* are easy to identify: the third and the eighth book of *Physics* and the tenth and the eleventh of *Metaphysics*. But to identify exactly which commentaries on *Physics*, and which commentary on *Metaphysics*, is another matter, though not our concern here. The logical complexity of the lemmas is described by Maimonides as follows: “some lemmas are obvious by the least reflection and by demonstrative premises and by primary intelligible notions or by those close to them”, while “others require more proofs, many premises, all of which, however, have been established by indubitable proofs” (*Guide for the Perplexed*, ed. Atay, p. 272). In other words, there are lemmas which are so close to

axioms that they become self-evident by applying only the “merest reflection (التأمل الأيسر)”; others which are so remote that their proof requires many intermediary propositions, a task which has been accomplished by Aristotle, his commentators and his successors. The twenty-five lemmas of the system belong to one type or the other.

Maimonides is aware that, to be worth the name, a proof has to be both universal and compelling. But that is not the case for the question examined here regarding the irreducible opposition between the two truths, revealed and philosophical, concerning the eternity of the world. For the proof to have the form of a mathematical proof, that is, be truly apodictic, it should always be valid, whether one believes in the eternity of the world or not. Maimonides thus introduces into the system, as a mathematician so to speak, and also against his own conviction, the eternity of the world as a postulate, bringing the number of the preliminary propositions up to twenty-six. Regarding this, he says without the slightest ambiguity:

To the above lemmas one lemma must be added which enunciates that the universe is eternal, which is held by Aristotle to be true, and which has to be believed first and foremost. We therefore admit it by convention (على جهة التقرير) only for the purpose of demonstrating our theorem (*ibid.*, p. 272).

Maimonides thus introduces the eternity of the world as a necessary postulate for the completion of the system and, subsequently, for the deduction of his “theorem”. The conventional — but non-arbitrary — aspect of the proposition is in sharp contrast with his rejection of the doctrine of the eternity of the world. Here, for example, is what he has to say on this matter:

The true method, which is based on a logical and indubitable proof, consists, in my opinion, in demonstrating the existence of God, His unity, and His incorporeality by philosophical methods, but founded on the theory of the eternity of the universe; I do not propose this method as though I believed in the eternity of the universe, for I do not follow the philosophers on this point, but because by the aid of this method the proof can be valid; and certainty can be reached concerning these three principles, viz., the existence of God, His unity and His incorporeality, irrespectively of the question as to whether the universe is eternal or created (*ibid.*, p. 187).

In fact, Maimonides knew that the problem of the eternity of the universe cannot have a positive solution. Some were to say later that dialectical reason comes up against an antinomy, since the properties of things which do not yet exist have to be determined.

The architectonic of this part of the *Guide* is surely conceived of as a mathematical exposition, following the order of geometry. In fact, this order appears to be a condition for the certainty of metaphysical knowledge, namely that of God, of His existence and of His incorporeality. This seminal idea, already present in al-Kindī, will be found later in Spinoza. But, as noted by Crescas, the big problem still remains as to whether these twenty-five propositions have effectively been proved; and, whether, even then, the “theorem” can really be deduced. These two questions will keep on haunting Maimonides’ successors. Al-Tabrīzī’s commentary is designed to prove these propositions, and Crescas’ attempt is motivated by the same intention. Maimonides himself attempts this deduction, which we will expound in broad terms, while emphasising the spirit in which it is carried out.

According to the twenty-fifth lemma, each composite individual substance needs for its existence a motor which properly prepares matter and enables it to receive form. But, according to the fourth lemma, there exists necessarily another motor which can be of a different class and which precedes the first motor. Following the third lemma, this chain of motors/mobiles is necessarily finite: motion finishes in the celestial sphere and then comes to rest. The celestial sphere establishes the act of locomotion, since this motion precedes all the other kinds of motion for the four categories of change, according to the fourteenth lemma. But the celestial sphere must have a motor since each moving object has necessarily a motor according to the seventeenth lemma. And this motor either resides within or without the

moving object. This is a necessary division. If the motor is outside, then either it is an object outside the celestial sphere, or it is not in an object; in the latter case, the motor is said to be “separate” from the sphere. If the motor is within, it must be either a force distributed throughout, or an indivisible force, like soul in man. Four cases have then to be examined; three of them have been rejected by Maimonides since he shows their impossibility with the help of different lemmas. He is then left with only one possibility, of an incorporeal object outside and separate which is the cause of locomotion of the celestial sphere in space. Maimonides concludes his long proof in these words:

It is therefore proved (فقد تيرهن) that the motor of the first Orb, if its motion be eternal and continuous, is necessarily neither itself corporeal nor does it reside as a *potentia* in a corporeal object for this motor to move, either of its own accord or accidentally; that is why it must be indivisible and unchangeable, as it has been mentioned in the fifth and the seventh lemmas. This prime Motor of the sphere is God, praised be His name. It is impossible that He could be two or more [...]. That is what had to be proved (*ibid.*, p. 276).

We have just shown that according to Maimonides, mathematics can be considered as a condition for metaphysical knowledge in three senses. The most obvious one is that mathematics is an exercise for the mind. In the second place, it offers a construction model — an architectonic — which can lead to certainty. And finally, it provides theoretical-proof techniques, mainly, the apagogic method. But these are not the only connections between mathematics and metaphysics that we can find in the *Guide*. We have quite recently drawn attention to another connection which is by no means less important: mathematics can play the role of an argumentation method in metaphysics. The most famous example, and the most relevant, is precisely taken from Apollonius’ *Conics*: the problem of the relation between imagination and conception can best be dealt with by taking the example of an asymptote to an equilateral hyperbola. In his criticism of *Kalām*, Maimonides intends to refute the following thesis: “everything conceived by imagination is admitted by the intellect as possible”. His strategy is to establish the negation of the thesis: there are unimaginable things, that is, things that can in no way be imagined though their existence can be proved. This shows that, for Maimonides, there is no principle which licenses a move from imagination to the metaphysical reality. He expresses his thesis as follows:

Know that there are certain things, which would appear impossible, if tested by man’s imagination, being as inconceivable as the co-existence of two opposite properties in one object; yet the existence of those same things, which cannot be represented by imagination, can nevertheless be established by proof, and their reality brought about (*ibid.*, p. 214).

We have had the opportunity of showing (Rashed, 1987) that in these terms Maimonides takes up the problem of proving what cannot be conceived, a problem posed in the 10th century by the mathematician al-Sijzī. The example invoked by Maimonides to make his point is the same as the one discussed by his predecessor — proposition II. 14 of Apollonius’ *Conics* concerning asymptotes to an equilateral hyperbola: the curve and its asymptotes will always come closer to each other if they are prolonged indefinitely, but they never meet.

This is a fact, writes Maimonides, which cannot easily be conceived, and which does not come within the scope of imagination. Of these two lines the one is straight, the other curved, as stated in the aforementioned book. One has consequently proved the existence of what cannot be perceived or imagined, and would be found impossible if tested solely by imagination (*ibid.*, p. 215).

The imagination invoked here by Maimonides is the mathematical imagination: nothing ensures even the way to metaphysical reality. But it can be stated with certainty that what is true for the mathematical imagination is *a fortiori* also true for all other forms of this faculty. Invoking the *Conics* proposition seems, in Maimonides’ mind, to have more force than just

that of mere example: it is an argumentation technique that the metaphysician borrows from mathematics.

To conclude: as did his predecessors from the time of al-Kindī, Maimonides finds in mathematics an architectonic model, proof techniques and model argumentation methods. The role of mathematics is in no way reduced to that of a propaedeutic to philosophical teaching: if Maimonides devoted time and energy to acquiring a mathematical knowledge — however modest one — it is because he conceived of it, as did his predecessors, as a deeply philosophical task: that of resolving metaphysical problems mathematically.

2. Mathematics in the philosophical synthesis and the “formal” modification of ontology: Ibn Sīnā and Naḥīr al-Dīn al-Ḥūṣī

In his monumental *al-Shifā'*, as in his book *al-Najāt*, and in his *Danish-Nameh*, Ibn Sīnā gives mathematics a particular prominence. To take the *Shifā'* alone, Ibn Sīnā (980-1037) devotes no fewer than four books to mathematical sciences. To this must be added some independent papers in astronomy and music. In all these writings, it has not been sufficiently understood that the presence of mathematics is significant in two respects. We have seen that al-Kindī was interested in mathematics on two accounts, in his capacity as a philosopher, and as a mathematician. So when he treats of burning mirrors, optics, sundials, astronomy, and when he comments on Archimedes, he does so as a mathematician. Mathematics is also a source of inspiration and an argumentation model for the philosopher. While al-Kindī's tradition survived him in the writings of Muḥammad ibn al-Haytham, Ibn Sīnā belongs only in part to this tradition. His mathematical knowledge, as one can see, is fairly wide-ranging though traditional. He probably knew the works of Euclid, of Nicomachus of Gerasa, and of Thābit ibn Qurra on the amicable numbers. He was also familiar with elementary algebra, with the theory of numbers and with certain works in Diophantine analysis. He seems not to have been well informed about contemporary research, as is shown by his claims about the regular heptagon. We can say, then, without fear of contradiction that Ibn Sīnā had a solid mathematical knowledge which allowed him to deal with certain applications, though not to undertake true mathematical research. This means that it is just as inaccurate to reduce his mathematical knowledge to Euclid's *Elements* and to Nicomachus of Gerasa's *Introduction to Mathematics*, as it is to represent him as a major mathematician of the 10th century. For this great logician, metaphysician and physician, mathematics plays a different role from that in al-Kindī since it is not only a source of inspiration for philosophical research but an integral part in a philosophical system. This explains the presence of four books in *al-Shifā'* devoted successively to the disciplines of the *quadrivium*. The question therefore is to assess the philosophical implications of this state of affairs.

If we consider Ibn Sīnā's theoretical views on the status of mathematics, the nature of its objects and the number of disciplines of which it is composed, we can conclude that he is the direct heir to a tradition: the status of mathematics is defined accordance with the Aristotelian theory of the classification of sciences, itself founded on the famous doctrine of Being; its objects are defined thanks to abstraction theory; as for the number of its disciplines, it is the well-known number passed on by the ancient Greek tradition. This concerns the three disciplines of the intermediary science (علم الوسط), which make up theoretical philosophy the objects of which are distributed among physics, mathematics and metaphysics — an order that the composition of *al-Shifā'* follows as a function of the materiality and mobility of the objects studied. Therefore mathematics considers objects abstracted from experience, separated from mobile, material and physical objects. The four disciplines which form mathematics are called the Quadrivium: Arithmetic, Geometry, Astronomy and Music. Ibn Sīnā always comes back to this doctrine, in the *Isagoge* as well as in the *Metaphysics* of *al-*

Shifā's, and also in an opuscle devoted to the classification of sciences, among other writings.

The types of sciences set out to consider beings either as moving objects, according to their conception and constitution, and as having to do with particular species and matters; either as separated from matters, according to the conception but not the constitution; or as separated according to the constitution and the conception. The first part of these sciences is physics; the second part is pure mathematics which includes the famous theory of numbers. As for the nature of numbers as numbers, they do not belong to this science. The third part is metaphysics. Since beings are by nature according to the three parts, theoretical philosophical sciences are those ones. Practical philosophy has to do either with the teaching of opinions whose use makes it possible to order the participation in common human things, and <this part> is known as the city's organisation; it is called politics; or with what makes it possible to order the participation in private human things, and <this part> is known as the home's organisation, <economics>; or finally what makes it possible to order the state of one person in order to build his soul : that is called ethics (*al-Shifā', Isagoge*, p. 14).

There is nothing new in this conception. If we stop at this Aristotelian bias of Ibn Sīnā, the real role that mathematics plays in *al-Shifā'* cannot be captured. Perhaps we should wonder, first and foremost, whether such a position of principle corresponds to the philosopher's mathematical knowledge and whether the theoretical classification reflects a possible *de facto* classification. But to assess and to understand the distance, if it exists, between these two classifications, it is necessary to refer first to Ibn Sīnā's mathematical studies. Only arithmetic will be considered, even if geometry provides the philosopher with further opportunities for reflection (the fifth postulate for example, as in *Danish-Nameh*).

If we first consider purely biographical details, we know that while receiving his philosophical teaching, Ibn Sīnā was learning Indian arithmetic and algebra. It is only later that he was to learn logic, Euclid's *Elements* and the *Almagest*; an account given by many biobibliographers such as al-Bayhaqī, Ibn al-'Imād, Ibn Khallikān, al-Qifī and Ibn Abī U{aybi'a. Al-Bayhaqī reports for example:

When he was ten years old, he knew certain fundamental texts of literature by heart. His father was studying and reflecting upon an opuscle of the Brothers of the Purity. He also reflected over it. His father took him to a greengrocer named Ma1mūd al-Massā1 who knew Indian calculation and algebra and al-muqābala (*Tārīkh Zukamā' al-Islām*, ed. Kurd Ali, p. 53).

Ibn al-'Imād gives this biographical anecdote in the same words and, quoting Ibn Khallikān, he writes: "When he was ten years old, he improved his knowledge in the science of the Glorious Qur'ān, literature, and he knew certain religious foundations by heart, Indian calculation and algebra and al-muqābala" (*Shadharāt al-dhahab*, III, p. 234; see also Ibn Khallikān, *Wafayāt al-A'yān*, II, pp. 157-158). As for Ibn Sīnā himself, he writes: "My father took me to a greengrocer who practised Indian arithmetic so that he could teach me." (al-Qifī, *Tārīkh al-Zukamā'*, p. 413 and Ibn Abī U{aybi'a, *'Uyūn al-Anbā' fī [ābaqāt al-Attibā'*, ed. 1965, p. 437).

But these new disciplines — Indian arithmetic and algebra — unknown to the Alexandrians, cannot find their place in the traditional framework of the classification of sciences without at least changing its general outline, if not changing drastically its underlying conceptions. But in Ibn Sīnā's classification, they appear under the sole title of "secondary parts of arithmetic (الأقسام الفرعية)". Ibn Sīnā gives no explanation whatsoever of this notion; contents himself simply with their enumeration. Here is what he writes:

The secondary parts of mathematics — branches of the [science of] numbers: the science of addition and of separation of the Indian arithmetic; the science of algebra and al-muqābala. And the branches of the science of geometry: the science of measurement, the science of ingenious mobile techniques; the science of the traction of heavy bodies; the science of weights and scales; the science of instruments specific to arts; the science of perspectives and mirrors; the hydraulic science. And the branches of astronomy: the science of astronomical tables and of calendars. And

the branches of music: the use of wonderful and curious instruments as the organ and the like
(*Parts of rational sciences*, p. 112).

Thus we learn only that arithmetic has as secondary parts Indian arithmetic and algebra. But the number of arithmetic disciplines invoked by Ibn Sīnā is not limited to the last two given in his classification of sciences. We have in fact already mentioned the volume that he devotes, in *al-Shifā'*, to the science of calculation called *al-Arithmā }īqī*. To this two further disciplines have yet to be added: one, though named, has never had its status fixed by Ibn Sīnā — it is al-2isāb; the other is only present through its objects: integral Diophantine analysis.

The theory of numbers, *al-Arithmā }īqī*, Indian arithmetic, algebra, al-2isāb and integral Diophantine analysis: six disciplines which overlap and which are sometimes superimposed to cover the study of numbers. The reality is thus obviously much more complex than it looks in the classificatory schema of sciences. But to disentangle these disciplines and to elucidate their connections, we must briefly recall the works of the mathematicians at the time. The latter in fact distinguished, by denoting them under two different names, the Hellenistic tradition of arithmetic and its Arabic development: the number theory (علم الأعداد) on the one hand, and the discipline denoted by the phonetic transcription of ¹ φριθμητικ on the other. If their connotation was not altogether unrelated, each of these terms did however refer to a distinct tradition. The expression “number theory (علم الأعداد)” referred to the arithmetic books of Euclid’s *Elements*, and also to later works such as those of Thābit ibn Qurra, for example. Meanwhile, the phonetic transcription of ¹ φριθμητικ (*al-arithmā }īqī*) denoted the arithmetic tradition of the Neo-Pythagoreans, that is, the tradition as Nicomachus of Gerasa understands it in his *Introduction*; a term translated nevertheless by Ibn Qurra under the title *Introduction to the Number theory* (□□□□□ □□□ □□□ □□□□) (see References). Without being systematic, the terminological difference between the 9th and 10th centuries seems to measure the gap which separated the two disciplines at the time. To understand how this gap was perceived later, let us read what Ibn al-Haytham writes.

There are two ways in which the properties of numbers appear: the first is induction, since if we follow numbers one by one, and if we distinguish them, we find all their properties by distinguishing and by considering them, and to find the number in this way is called *al-arithmā }īqī*. This is shown by [Nicomachus’] *al-arithmā }īqī*. The other way in which the properties of numbers appear is by proofs and deductions. All the properties of numbers seized by proofs are contained in these three books [of Euclid] or in what is related to them (Rashed, 1980, p. 236).

This eminent mathematician deems both approaches to be scientific; a remark all the more important since Ibn al-Haytham demanded, everywhere and without restriction, rigorous proofs. And in fact, from the 10th century at least, these two traditions offered mathematicians the same conception of the object of arithmetic: an integer arithmetic represented by line segments. But while in number theory the norm of proof is restrictive, in *al-arithmā }īqī* a simple induction can be used. For scientists of the 10th century, the difference between the two traditions was reduced to a distinction between methods and norms of rationality.

It is precisely this conception of the connection between the two disciplines which is expressed by Ibn Sīnā. In *al-Shifā'*, arithmetic appears twice: the first time in the geometry of *al-Shifā'* in which he merely summarises Euclid’s books on arithmetic. On the second occasion, he writes his own book of *al-arithmā }īqī* — which will be read and taught for many centuries — and whose real foundations, according to the author himself, can be mainly found in the *Elements*. Perhaps it is also this vision of the relationship between the two disciplines which explains why, in his *al-arithmā }īqī*, Ibn Sīnā is not content with a simple summary of Nicomachus, as he had been for the theory of numbers, with Euclid’s *Elements*. It would thus become clear how far he departs in this regard from the Neo-Pythagorean tradition. From now on, all the ontological and cosmological considerations which burdened the notion of number

are *de facto* banned from *al-arithmā}īqī*, considered thus as a science. What is left is the philosophical intention common to all branches of philosophy, whether theoretical or practical, that is, the perfection of the soul. Ibn Sīnā thus directs his attacks against the Neo-Pythagoreans

It is customary, for those who deal with this art of arithmetic, to appeal, here and elsewhere, to developments foreign to this art, and even more foreign to the custom of those who proceed by proof, [developments which are] closer to the exposition of rhetoricians and poets. It should be abandoned (*al-Shifā'*, *al-Arithmā}īqī*, ed. Maz*har, p. 60. It should be noted that few lines earlier, Ibn Sīnā clearly mentions them by their name, i.e. the Pythagoreans).

He can even partly abandon traditional language, and adopt that of the algebraists, to express the successive powers of an integer. The terms “square (مال)”, “cube (كعب)”, “square-square (مال مال)”, which used to denote the successive powers of the unknown, were thus employed by the philosopher to name the powers of an integer (*ibid.*, p. 19).

In these conditions, nothing prevented Ibn Sīnā from including in his *al-arithmā}īqī* theorems and results obtained elsewhere, without repeating the proof (if there was one). That is what he did when he adopted (without proof) Thābit ibn Qurra’s theorem on amicable numbers, in the Thābit’s pure Euclidean style. Ibn Sīnā mentions as well several problems of congruence.

If you add even-even four numbers and a unit, if you get a prime number, provided that, if the last of them is added, and if the preceding one is taken away, and if the sum and the remainder are prime, then the product of the sum by the remainder, and the total by the last added numbers, yields a number which has a friend; its friend is the number obtained by adding the sum and the remainder, multiplied by the last of the added numbers, and by adding the product to the first number which had a friend. These two numbers are amicable (after correction of some errors in the Cairo edition, p. 28).

To these two traditions, a third also mentioned by Ibn Sīnā should be added which concerns the integral Diophantine analysis. In the logical part of *al-Shifā'* devoted to the proof, Ibn Sīnā considers the example of the first case of Fermat’s conjecture, already dealt with by at least two mathematicians of the 10th century, al-Khujandī and al-Khāzin. Ibn Sīnā writes:

When we wonder [...] whether the sum of two cubic numbers is a cube, in the same way as the sum of two square numbers was a square, we pose then an arithmetic problem (حساب or *lisāb*) (*al-Shifā'*, ed. Afifi, V, pp. 194-195).

We realise specifically that the term *lisāb* seems to designate here a discipline which includes disciplines other than the Euclidean theory of numbers and *al-arithmā}īqī*. By *lisāb*, Ibn Sīnā seems to mean a science which includes all those which deal with numbers, rationals or algebraic irrationals; the last paragraph of his *al-Arithmā}īqī* is unambiguous in this respect.

That is what we meant in the science of *al-arithmā}īqī*. Certain cases have been left aside since we consider that mentioning them here would be extrinsic to the rule of this art. There remains in the science of *al-lisāb* what suits us in the use and determination of numbers. What ultimately remains in practice is like algebra and *al-muqābala*, the Indian science of addition and separation. But for the latter, it would be best to mention them among the derivative parts (*al-Shifā'*: *al-Arithmā}īqī*, p. 69).

Everything thus indicates that, in *al-Arithmā}īqī* as in the summarised Euclidean arithmetic books, Ibn Sīnā, like his predecessors and contemporaries, restricts his study to natural numbers. As soon as he meets some problems which would urge him to examine the conditions of rationality, whether it comes to searching for a positive rational solution or, more generally, to considering a class of irrational numbers, he finds himself outside these two sciences. The term of *al-lisāb* (الحساب) thus encompasses all arithmetic researches which are carried out by such disciplines as algebra, Indian arithmetic and the like. These disciplines have consequently an instrumental and, so to speak, applied aspect which puts them in opposition to the ancient number theory. And it is precisely this instrumental and applied

character which enables Ibn Sīnā, as can be verified, to distinguish in his classification the set of “derivative parts”, which are then defined as such. The “derivative parts (الأقسام الفرعية)” of physics are therefore medicine, astrology, physiognomy, oneiromancy, the divinatory art, talisman, theurgy and alchemy.

To understand the distance put by Ibn Sīnā between himself and traditional, Hellenistic and Greek classifications as well as between himself and his own theoretical classification, it is worth introducing here one of his predecessors, al-Fārābī (872-950). Whether Ibn Sīnā’s opusculum *The parts of rational sciences* is related to al-Fārābī’s classification expounded in his *Enumeration of Sciences* is a question first posed by Steinschneider, who denied that there was any such relation. Wiedeman (1970, p. 327) confirms this opinion, and claims that Ibn Sīnā lists only separated sciences, whereas al-Fārābī designates and characterises them by their mutual dependence; or, as he puts it “Ibn Sīnā zählt im wesentlichen die einzelnen Wissenschaften auf, während al-Fārābī sie in zusammenhängender Darstellung charakterisiert.”

In fact the comparison forces itself upon us anew, since the examination of “derivative parts” of Ibn Sīnā’s arithmetic shows that they are nothing but those disciplines brought together by al-Fārābī under the title “the science of ingenious techniques”, which he defines as follows:

The science of the way to proceed when we apply all whose existence is proved, by predication and proof, in the previously mentioned mathematical sciences, to physical bodies; and when we achieve and put it effectively in the physical objects (*Ilfā’ al-’Ulūm*, ed. U. Amin, p. 108).

According to al-Fārābī, the object of mathematics is lines, surfaces, solids and numbers that he considers as intelligible by themselves, and separate (منتزعة), that is, abstracted from physical objects. Intentionally to discover and show mathematical notions in the latter with the help of the art would require the conception of ingenious devices, the invention of techniques and methods capable of overcoming the obstacles posed by the materiality of empirical objects. In arithmetic, the ingenious devices involve, among other things, “the science known by our contemporaries under the name of algebra and *al-muqābala*, and what is similar to it” (*ibid.*, p. 109). He also takes notice however that “this science is common both to arithmetic and geometry” and further on adds that

It includes the ingenious devices to determine the numbers that we try to determine and use, those which are rational and irrational the principles of which are given in Euclid’s *al-Usūjusāt* 10th book, and those which are not mentioned by Euclid. Since the relation of rational to irrational numbers — to one another — is like the relation of numbers to numbers, each number is thus homologous with a certain rational or irrational magnitude. If we determine the numbers which are homologous with magnitude *ratios*, we then determine these magnitudes in a certain manner. That is why we postulate certain rational numbers to be homologous with rational magnitudes, and certain irrational numbers to be homologous with irrational magnitudes (*ibid.*, p. 109).

In this text of capital importance, algebra is distinguished from science on two accounts: although — like every science — apodictic, it nevertheless represents the domain of application not only of one science but of two at the same time, arithmetic and geometry. As for its object, it includes geometric magnitudes as well as numbers, which can be both rational or algebraic irrational. In the presence of this new discipline which has to be taken into account, the new classification of the sciences which aimed at both universality and exhaustiveness has to justify in one way or another the abandonment of certain Aristotelian theses. Names such as “science of ingenious devices”, “derivative parts” are coined so that a non-Aristotelian zone can be arranged within a received Aristotelian style of classification.

The philosophical impact caused by such a revision is on a larger scale and — especially — more profound than mere taxonomic modification. If algebra is in fact common to arithmetic and geometry, without in any way giving up its status as science, it is because its very object, the “algebraic unknown”, that is, the “thing (الشيء, *res*)” can refer indifferently to a number or

to a geometric magnitude. More than that: since a number can also be irrational, “the thing” designates then a quantity which can be known only by approximation. Accordingly the algebraists’ subject matter must be general enough to receive a wide range of contents; but it must moreover exist independently of its own determinations, so that it can always be possible to improve the approximation. The Aristotelian theory is obviously unable to account for the ontological status of such an object. So a new ontology has to be made to intervene that allows us to speak of an object devoid of the character which would none the less enable us to discern what it is the abstraction of; an ontology which must also enable us to know an object without being able to represent it exactly.

This is precisely what has been developing in Islamic philosophy since al-Fārābī: an ontology which is “formal” enough, in a way, to meet the requirements mentioned above, among other things. In this new ontology, “the thing (الشيء)” has a more general connotation than the existent. This is a distinction made more precise by al-Fārābī when he writes: “the thing can be said of every thing that has a quiddity, whether it is external to the soul or [merely] conceived of in any way” whereas the “existent is always said of every thing that has a quiddity, external to the soul, and cannot be said of a quiddity merely conceived of.” Therefore, according to him, the “impossible (المستحيل)” can be named a “thing” but cannot be “existent” (*Kitāb al-Iurūf*, ed. Mahdi, 1970, p. 128).

As regards the history of mathematics, this trend has been again confirmed between al-Fārābī and Ibn Sīnā: al-Karajī particularly gives a more general status to algebra, and emphasises the extension of the concept of number. A contemporary to Ibn Sīnā, al-Bīrūnī goes even further and writes without hesitation

The circumference of a circle is in a given proportion to its diameter. The number of the one to the number of the other is also a proportion, even if it is irrational (*Al-Qānūn al-Mas’ūdī*, I, p. 303).

As regards philosophy, Ibn Sīnā — a consistent metaphysician — includes al-Fārābī’s conception into a doctrine that he wants to be more systematic and which is expounded in his *al-Shifā’*. According to this doctrine, “the thing” is given in an immediate evidence or, in Ibn Sīnā’s own terms, is imprinted immediately in the soul, just as “the existent” and “the necessary”; along with these two other ideas, it is the principle behind all things. While the existent signifies the same meaning as “asserted (المُثَبَّت)” and “achieved (المُحَصَّل)”, the thing is, writes Ibn Sīnā, what the predication concerns (the proposition). Hence every existent is a thing but the converse is not correct, though it is impossible that a thing should exist neither as a concrete subject nor in the mind (*al-Shifā’*, *al-Ilāhiyyāt*, ed. Anawati and Zāyed, I, p. 29 sq. and p. 195 sq.). A full description of Ibn Sīnā’s doctrine is outside the scope of the present paper, but it is sufficient to recall that, being neither Platonic nor Aristotelian, this new ontology arose to, in part at least, due to the new results in mathematical sciences. If mathematics leads Ibn Sīnā to shift his ontology in a “formal” direction, so to speak, it acts in the same way on his conception of the ontology of emanation, as we shall see later with al-Ḥusī’s commentary.

The emanation from the One of Intelligences and celestial orbs and the other worlds — that of nature and corporeal things —, is one of the central doctrines of Ibn Sīnā’s metaphysics. This doctrine raises both ontological and noetic questions: how can a multiplicity emanate from one unique and simple being, a multiplicity which is also a complex, ultimately containing the matter of things as well as the form of bodies and human souls? This ontological and noetic duality sets up the question as an obstacle, as both a logical and metaphysical tangle that must be unravelled. From that point we understand, in part at least, why Ibn Sīnā returns tirelessly to this doctrine and implicitly to this question in his different writings.

The study of the historical evolution of Ibn Sīnā’s thought on this problem through his different writings would show how he was able to amend his initial formulation as a function of this difficulty. To limit ourselves to *al-Shifā’* and *al-Ishārāt*, Ibn Sīnā expounds the

principles of the doctrine and the rules of the emanation of multiples from one simple unity. His explanation looks like an articulated and ordered exposition but does not constitute a rigorous proof: Ibn Sīnā does not in fact give the syntactic rules capable of matching the semantics of emanation. This is precisely where the difficulty of the derivation of the multiplicity from the One lies. This derivation has long been seen as a problem and examined as such. The mathematician, philosopher and commentator of Ibn Sīnā, Naḥṣir al-Dīn al-Ḥūsī, not only grasped the difficulty, but wanted to offer the syntactic rules that were lacking.

To understand this contribution, we have at the outset to go back to Ibn Sīnā to recall the elements of his doctrine and also to grasp, however weakly, the formal principle in his synthetic and systematic exposition whose presence has made possible the introduction of the rules of combinatory analysis. In fact, this principle allows Ibn Sīnā to develop his exposition in a deductive style. He has to ascertain on the one hand the unity of Being, which is said of everything in the same sense and, on the other, the irreducible difference between the First Principle and His creation. He then develops a somewhat “formal” general conception of the Being: considered as a being, he is not the subject of any determination, not even that of modalities; it is just a being. It is not a genus, but a “state” of whatever there is, and can only be grasped in its opposition to non-being, without nevertheless being preceded in time by the latter — this opposition is only according to the order of reason. On the other hand, only the First Principle receives His existence from Himself (Ibn Sīnā distinguishes between existence and essence for all other beings; on this point, see Goichon, 1992; D. Saliba, 1926; Verbecke, 1977). So this existence is what is necessary, and it is in this case that existence coincides with essence. All other beings receive their existence from The First Principle by emanation. This ontology and the cosmogony that goes with it provide the three points of view under which a being is envisaged: as a being, as an emanation (see Gardet 1951, Heer 1992, Hasnawi 1990, Druart 1992, Morewedge 1992, Owens 1992) of the First Principle, and as being its quiddity (viewed from the first two angles, the necessity of the being imposes itself while its contingency reveals the third). These are, briefly mentioned, the three notions on which Ibn Sīnā is going to establish his postulates, which are:

1° There is a First Principle, a necessary Being by essence, one, in no way divisible, which is neither a body nor in a body.

2° The totality of Being emanates from The First Principle.

3° The emanation is not carried out either “according to an intention (على سبيل القصد)” or to achieve any purpose, but by a necessity of the being of the First Principle, that is, His self-intellection.

4° From the One only one proceeds.

5° There is a hierarchy in the emanation, from those whose being is most perfect (الأكمل وجوداً) to those whose being is least perfect (الأخس وجوداً).

We might think that certain postulates seem to contradict each other, as, for example, 2 and 4, or suspect that some lead to contradictory consequences. To avoid this first impression, Ibn Sīnā introduces further determinations in the course of his deduction. So from 1, 2, 4 and 5 follows that the totality of Being, in addition to the First Principle, is a set ordered by both the logical and axiological predecessor-successor relation, regarding both the priority of the being as well as its excellence. Barring the First Principle, each being can have only one predecessor (as the predecessor of its predecessor, and so on). On the other hand, each being, including the First Principle, can have only one successor (respectively the successor of its successor, and so on). But the philosopher and his commentator know that, taken literally, this order forbids the existence of multiple beings, that is, their independent coexistence, without some having logical priority over others or being more perfect than them; which makes this order clearly false, as al-Ḥūsī says (*al-Ishārāt*, p. 216). Thus it is necessary to introduce further details and intermediary beings.

But 1 and 2 in their turn exclude multiplicity to be a product of the First Principle's "momenta" (نزوعات) and "perspectives" (وجهات), since assuming momenta and perspectives in Him amounts to denying His unity and simplicity. Finally, 3, 4, and 5 imply that the emanation as an act of the First Principle is not like a human act, because its Author has neither intention nor purpose. Everything indicates then that intermediary beings (*mutawassi'a*), hierarchically ordered, no doubt, have to be used to account for the multiplicity-complexity.

Let us begin, as one should, with the First Principle, and designate it, as Ibn Sīnā does in his opusculum *al-Nayrūziyya*, by the first letter of the alphabet — *a*. The First Principle intellects itself by essence. In Its self-intellection, It "intellects" the totality of the being of which It is the very principle (*al-Shifā'*, *al-Ilāhiyyāt*, ed. Mūsā and Zāyed, II, p. 402, l. 16), without there being in Itself any obstacle to the emanation of this totality, nor to its rejection. It is in this sense only that the First Principle is said to be the "agent" (فاعل) of the totality of being.

But having admitted this, one has yet to explain how the necessary emanation of the totality of being can be achieved without having to add anything which could be inconsistent with the Unity of the First Principle. Following 1, 4, 5, from the First Principle only one being emanates, a being which necessarily belongs to the second rank in existence and perfection. But, as it is the emanation from a pure and simple unique being, at the same time pure truth, pure power, pure goodness..., with none any of these attributes existing in it independently so as to ensure the unity of the First Principle, this derived being can only be a pure Intellect. This conclusion respects 4, since, if this intellect were not pure, we should conclude that more than one emanates from the One. We have here the first separate Intellect, the first effect (معلول) of the First Principle. Following Ibn Sīnā, let us refer to it as *b*.

Everything is now in place to explain the multiplicity-complexity. By essence, this pure Intellect is an effect: it is therefore contingent. But, as an emanation from the First Principle, it is necessary since it was "intellected" by the latter. This ontological duality is superimposed upon a noetic multiplicity: this pure Intellect knows itself and knows its own being as contingent being, that is, its essence is different from that of the First Principle since the latter is necessary; on the other hand, it knows the First Principle as the necessary Being; and finally it knows the necessity of its own being as an emanation of the First Principle. I have just paraphrased here what Ibn Sīnā writes himself in *al-Shifā'* (*ibid.*, pp. 405-406). He replies in advance to a possible detractor, noting that this multiplicity-complexity is not, if we may say so, a hereditary property: the pure Intellect does not receive it from the First Principle, for two reasons. First the contingency of its being belongs to its own essence, and not to the First Principle, which gave it the necessity of being. On the other hand, the knowledge that it has of itself, as well as the knowledge that it has of the First Principle, is a multiplicity, which is the result of the necessity of its being which derives from the First Principle. In these conditions, Ibn Sīnā can reject the accusation of attributing this multiplicity to the First Principle.

Ibn Sīnā then describes how the other separated Intellects, celestial Orbs, and Souls which enable the Intellects to act, emanate from the Pure Intellect. So, from the pure Intellect *b* emanates, by its intellection of *a*, a second intellect; let it be named *c*; and by its intellection of its own essence, the Soul of the ninth celestial Orb; and by its intellection of its own being as contingent being the body of this ninth Orb. Let us denote the Soul of this Orb and its body as *d*.

Ibn Sīnā thus continues to describe the emanation of Intellects, celestial Orbs with Souls and their bodies. From now on, the matter of sublunary things emanates from every Intellect, the forms of the bodies and human souls. But even if Ibn Sīnā's explanation has the advantage of not separating the question of the multiplicity from that of complexity, that is, the ontological content of the multiplicity, it does not however lead to a rigorous knowledge of the latter,

since no general rule is given. Ibn Sīnā does nothing but lead the elements back to the Agent Intellect.

It is precisely here that al-ʿūsī intervenes. He will actually show that there emanates from the First Principle — following Ibn Sīnā’s rule and with the help of a reduced number of intermediaries — a multiplicity such that each effect will have only one cause which exists independently. We shall see that the price of such undeniable progress in knowledge of the multiplicity is impoverishment of the ontological content: from multiplicity-complexity there will in fact remain only the multiplicity.

In his commentary of *al-Ishārāt*, al-ʿūsī introduces the language and techniques of combinatorial analysis to follow the emanation to the third rank of beings. Here he stops the application of these techniques, to conclude: “if we then go beyond [the first three] ranks, there may exist an uncountable multiplicity (لا يحصى عددها) in only one rank, and go on ad infinitum” (ed. Dunyā, III, pp. 217-218). The intention of al-ʿūsī is thus clear, and the device applied to the first three ranks leaves no doubt: one must provide the proof and means lacking in Ibn Sīnā. But al-ʿūsī is at this stage still distant from his goal. Indeed it is one thing to proceed by combinations for a number of objects, and another to introduce a language with its syntax. The language in this case would be that of combinations. It is to this task that al-ʿūsī applies himself in an independent dissertation (Rashed, 1999a), whose title leaves no room for ambiguity: *On the proof of the Mode of Emanation of Things in an Infinite <number> from the Unique First Principle*. In this instance, as we shall see, al-ʿūsī proceeds in a general way with the help of combinatorial analysis. The text of al-ʿūsī and its results do not pass away with the death of its author; they are to be found in a later treatise entirely devoted to combinatorial analysis. Thus al-ʿūsī’s solution not only distinguishes a style of research in philosophy, but represents an interesting contribution to the history of mathematics itself.

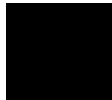
Al-ʿūsī’s idea is to subject this problem to combinatorial analysis. But, for combinatorics to be used, he has to make sure that the time variable is neutralised, which in the case of the doctrine of emanation involves either discarding Becoming, or, at least, offering a purely logical interpretation of it. This condition has already been suggested by Ibn Sīnā himself, as we have shown. It should rightly be noted that emanation does not take place in time, and anteriority and posteriority have to be understood essentially, not temporally (*al-Shifāʾ*, VI, 2, p. 266. See Hasnawi 1990, Gardet 1951, Davidson 1987, Druart 1987, Morewedge 1972). This interpretation, in our view crucial in the Avicenic system, refers to his own conception of the necessary, the possible and the impossible. Let us recall, briefly, that in *al-Shifāʾ* (see especially book 3, chapter 4 of *Syllogism*, IV, ed. Zāyed, 1964), Ibn Sīnā takes up this old problem to reject right from the start all ancient doctrines which are, according to him, circular: they use in the definition of each of the three terms one or the other of the two remaining ones. To break this circularity, Ibn Sīnā intends to restrict the definition of each term by bringing it back to the notion of existence. He distinguishes then what is considered in itself as necessary existence from what, equally considered in itself, can exist and may also not exist. Necessity and contingency are for him inherent in the beings themselves. As for possible being, its existence and non-existence depend on a cause external to it. Contingency does not appear thus as a denied necessity, but as another mode of existence. The possible being might even be, while remaining in itself, of a necessary existence as an effect of another being. Without wanting to follow here the subtleties of Ibn Sīnā’s development, it is sufficient to note that, from this particular definition of the necessary and the possible, Ibn Sīnā bases the terms of emanation in the nature of beings, neutralising from the outset — as it has been underlined above — the time variable. From these definitions, he infers some propositions, the majority of which are established by *reductio ad absurdum*. He shows that the necessary cannot but exist, that by essence it cannot have a cause, that its necessity includes all its aspects, that it is one and can in no way admit a multiplicity, that it is simple, without any

composition.... On all these points, it is opposed to the possible. Thus it is in the very definition of the necessary and the possible, and in the dialectic in which they enter, that are forever fixed the anteriority of the First Principle and its relation with the Intelligences.

If therefore emanation can be described without appealing to time, it is because its own terms are given in the logic of the necessary and the possible. This doctrine may raise difficulties, but it is not the point here: we know that the conditions for introducing a combinatorics have already been ensured by Ibn Sīnā himself.

We have said that from a emanates b ; the latter is then in the first rank of effects. From a and b together emanates c , that is, the second intellect; from b alone emanates d , that is, the celestial Orb. We have thus in the second rank two elements c and d such that each one is not the cause of the other. Up to now we have in all four elements: the first cause a and three effects b , c and d . Al-Ḥusī calls these four elements the *principles*. At this point, let us combine the four elements two by two, then three by three, and finally four by four. We successively get six combinations — ab , ac , ad , bc , bd , cd —, four combinations — abc , abd , acd , bcd —, and one combination of four elements — $abcd$. If we take into account the combinations of these four elements 1 by 1, we get a total of 15 elements; of which 12 are in the third rank of effects, without any of them being used as intermediary to obtain the others. That is what al-Ḥusī sets out in his commentary on the *al-Ishārāt*, as well as in the treatise mentioned above. But as soon as we go beyond the third rank, things quickly get complicated, and al-Ḥusī has to introduce in his treatise the following lemma:

The number of combinations of n elements is equal to



To calculate this number, al-Ḥusī uses the equality



So, for $n = 12$, he gets 4095 elements. It should be noted that to deduce these numbers, he gives the expressions of the sum by combining the alphabetical letters.

Al-Ḥusī returns later to calculate the number of elements of the fourth rank. He then considers the four principles with the twelve beings of the third rank; he gets 16 elements, from which he gets 65520 effects. To reach this number, al-Ḥusī proceeds with the help of an expression equivalent to

(*)



The value of which is the binomial coefficient



None of these elements — with the exception of a , b , and ab — is an intermediary for the others. Hence al-Ḥusī's response is general, and (*) gives a rule which permits ascertaining the multiplicity in each rank.

Having established these rules and given the example of the fourth rank, with its 65520 elements, al-Ḥusī is able to give a definitive answer to the question “of the possibility of the

emanation of the accountable multiplicity from the First Principle under the condition that from the One emanates only one and without the effects being successive (in chain). That is what had to be proved.”

Al-Īsī's achievement —to make Ibn Sīnā's ontology speak in terms of combinatorial analysis — has driven two important evolutions: both in Ibn Sīnā's doctrine and in combinatorics. It is clear that this time the question of multiplicity is kept at a certain distance from that of the complexity of being. Al-Īsī cares little about the ontological status of each of the thousands of beings which make up, for example, the fourth rank. Even more: metaphysical discourse at this point allows us to speak of a being without allowing us to represent it exactly. This somewhat “formal” evolution of ontology, which is here blatant, does nothing but amplify a trend already present in Ibn Sīnā in his considerations on “the thing (الشَّيْءَ)”, as we have emphasised above. This “formal” movement is accentuated by the possibility of designating beings by the letters of the alphabet. Even the First Principle is no exception to the rule, since It was denoted by a . In this al-Īsī once again amplifies an Avicenian practice while modifying its sense. In the epistle *al-Nayrūziyya*, Ibn Sīnā resorted to this symbolism, but with two differences. On the one hand, he attributed to the succession of the letters of the Arabic alphabet following the order *abjad hawad* the value of a priority order, of logical anteriority; on the other hand, he has used the numerical values of the letters ($a = 1$, $b = 2$, etc.). Although al-Īsī implicitly keeps the order of priority by denoting — as does Ibn Sīnā — the First Principle by a , the pure Intellect by b , he has dropped the hierarchy in favour of the conventional value of the symbol. And the numerical value has disappeared. This is necessary for the letters to be the objects of a combinatorics. A mathematician and a philosopher, al-Īsī has thought through Ibn Sīnā's doctrine of emanation in a formal sense, thus favouring a trend already present in Ibn Sīnā's ontology.

3. From *ars inveniendi* to *ars analytica*

Due to reasons internal to the evolution of the discipline, mathematicians of the 9th century confronted the problem of the duality of order: is the order of exposition identical to the order of discovery? Naturally, this question was raised concerning the very model of the mathematical composition at that time and for many centuries to come, namely Euclid's *Elements*. Thābit ibn Qurra devotes a treatise to this problem in which he claims that Euclid's order of exposition is just the logical order of proofs, and differs from the order of discovery. To characterise the latter, Thābit develops a psychological doctrine of mathematical invention. We are already in a sense within the philosophy of mathematics.

This question of order was soon to be included in a problematic of a more general nature, that of analysis and synthesis, profoundly transformed. Mentioned by Galen, Pappus, and occasionally Proclus, this topic had never assumed the dimension that it took on in the 10th century. The development of mathematics and the conception of new chapters from the 9th century were enormously significant for the breadth and understanding of this subject, giving rise to the development of a real philosophy of mathematics. Indeed we witness in succession the elaboration of a philosophical logic of mathematics, then the project of an *ars inveniendi* and, finally, of an *ars analytica*.

Everything began, apparently, with Ibrāhīm ibn Sinān (909-946). He wrote a book devoted entirely and uniquely to analysis and synthesis, entitled *On the Method of Analysis and Synthesis in the Problems of Geometry* (Rashed and Bellosta, 2000, chap. I). The importance of this is clear. From now on analysis and synthesis constitute a domain which the mathematician can occupy both as a geometer and as a logician-philosopher. Here is how Ibn Sinān describes his enterprise and his intention:

I have then, exhaustively, established in this book a method designed for students, which contains all that is necessary to resolve the problems of geometry. I have exposed in general terms the various classes of geometric problems; I have then subdivided these classes and illustrated each of them by an example; I have afterwards shown the student the way thanks to which he will be able to know in which of these classes to put the problems which will be posed to him, by which he will know how to analyse the problems — as well as the subdivisions and conditions necessary to that purpose —, and to carry out their synthesis — as well as the necessary conditions for that —, then how he will know whether the problem is among those which are solvable in one or several trials, and more generally, all that he must know in these matters. I have pointed out the kind of errors committed by the geometers when analysing because of a habit they have acquired: excessive abbreviation. I have also indicated for what reason there may seemingly be for the geometers, in the propositions and the problems, a difference between analysis and synthesis, and I have shown that their analysis is different from synthesis only due to abbreviations, and that, if they had completed their analysis as it should be, it would have been identical to the synthesis; the doubt would then have left the hearts of those who suspect them of producing in the synthesis things which had not been mentioned previously in the analysis — the things, lines, surfaces, and such, which are seen in their synthesis, without having been mentioned in the analysis; I have shown that and I have illustrated it by examples. I have presented a method thanks to which analysis is such that it coincides with synthesis; I have warned against the things which are tolerated by the geometers in analysis, and I have shown what kind of errors attach to them if they are tolerated (Rashed and Bellostà, 2000, pp. 96-98).

The intention of Ibn Sinān is clear, and his project is well articulated: to classify the geometric problems according to different criteria in order to show how to carry on, in each class, by analysis and synthesis, and to point to the occurrence of errors so that they can be avoided. Here is a broad outline of his classification.

1. The problems the assumptions of which are completely given
 - 1.1. The true problems
 - 1.2. The impossible problems
2. The problems for which it is necessary to modify some assumptions
 - 2.1. The problems with discussion (diorism)
 - 2.2. The indeterminate problems
 - 2.2.1. The indeterminate problems strictly speaking
 - 2.2.2. The indeterminate problems with discussion
 - 2.3. The overabundant problems
 - 2.3.1. The indeterminate problems to which an addition is made
 - 2.3.2. The problems with discussion to which an addition is made
 - 2.3.3. The true problems to which an addition is made

To this may further added the modal classification of propositions.

This classification is made from several criteria: the number of solutions, the number of assumptions, their compatibility and their possible independence.

A little over two centuries later, al-Samaw'āl takes up this classification, still starting from the number of solutions and the number of assumptions (Ahmad and Rashed, 1972). He further refines the classification. He distinguishes identities from the problems which have an infinite number of solutions without being identities. He furthermore introduces the notion of undecidable problems, for which no proof, either of existence or of impossibility, can be found (Rashed, 1984b, p. 52). Unfortunately the author gives no example. The least to be said however is that the mathematician was able to shift the Aristotelian notions of the necessary, possible and impossible to those of computability and semantic undecidability.

In his book, Ibn Sinān discusses other logical problems such as the place of auxiliary constructions, the reversibility of analysis, and apagogic reasoning. Analysis and synthesis thus appear in his book both as a discipline and as a method. The former is in fact a philosophical and pragmatic logic, since it makes possible the combination of an *ars*

inveniendi and an *ars demonstrandi*, the latter is a technique founded on a proof theory that Ibn Sinān endeavoured to elaborate.

One generation after Ibn Sinān, the mathematician al-Sijzī (last third of the 10th century) designed a different project, that of an *ars inveniendi* which meets both logical and didactic requirements. Al-Sijzī begins by enumerating certain methods aimed at facilitating mathematical invention — at least seven. Among them, “analysis and synthesis” figures as the principal method, which are provided with effective means of discovery by several specific methods such as the method of punctual transformations and the method of ingenious devices. All these specific methods share the idea of transforming and varying the figures as well as the propositions and solution techniques. Summarising his project, al-Sijzī writes

As the examination of the nature of propositions (الأشكال) and of their properties in themselves is surely carried out following one of these two ways: either we imagine the necessity of their properties by having their species vary, an imagining which draws on sensation or what is common to the senses; or by setting these properties and also the lemmas they necessitate, successively, by a geometric necessity [...] (Rashed, 2001, Appendice I).

For al-Sijzī, the *ars inveniendi* consists mainly of two ways. All specific methods are put together in the first way, while the second is nothing but “analysis and synthesis”. It is this distinction, the nature of the first way and this close relationship between the two, which single out al-Sijzī’s conception and reflect the originality of his contribution.

It remains to be noted that the first of the two ways is divided into two, according to the two senses of the term *shakl* (شكل). This term, chosen to render *διάγραμμα* by the translators of Greek mathematical writings, designates as the latter indifferently both the figure and the proposition. This double meaning is not too fraught with ambiguity as long as the figure graphically translates — in a static manner, if I may say so — the proposition; in other words, as long as geometry remains mainly the study of figures. But complications arise when the figures are subjected both to transformations and variations, as is already the case in certain branches of geometry at the time of al-Sijzī. The double reference then requires clarification. Let us begin with the first sense, that of “figure”.

In this treatise, al-Sijzī recommends, on three occasions, proceeding by variation of the figure: when a punctual transformation is carried out; when one element of the figure is changed, all others remaining fixed; finally, when an auxiliary construction is chosen. But several elements are common to these different techniques. Firstly the goal: we always try to reach, thanks to transformation and variation, invariable properties of the figure associated with the proposition, those which characterise it specifically. It is precisely these invariable properties which are stated in the figure as a proposition. The second element is also related to the goal: variation and transformation are means of discovery since they lead to invariable properties. The imagination takes over at this stage, a power of the soul capable of drawing upon the multiplicity suggested by the senses, through the variable properties of the figures, the invariable properties, and the essence of things. The third element concerns the particular role of the figure, as a representation this time: the role, mentioned by al-Sijzī, of fixing the imagination, of helping it in its task when it draws upon the sensation. And the last element, but not the least, deals with the duality figure-proposition: there is no one-to-one relation. To the same and sole proposition can be related a variety of figures; just as to one sole figure can be related a whole family of propositions. Al-Sijzī chose to deal at length with the last case. These new connections between figure and proposition that al-Sijzī was the first to point out, so far as I know, require that a new chapter of *ars inveniendi* be thought through: the analysis of figures and their connections to propositions. This is precisely what seems to have been inaugurated by al-Sijzī.

One generation later, Ibn al-Haytham (d. after 1040) conceives another project: founding a scientific art, with its rules and vocabulary. Ibn al-Haytham begins by recalling that

mathematics is founded on proofs. By proof, he means “the syllogism which necessarily indicates the truth of its own conclusion” (Rashed, 1991, p. 36). This syllogism is made up in its turn “of premises whose truth and validity are recognised by the understanding, without its being troubled by any doubt about them; and of an order and arrangement of these premises such that they compel the listener to be convinced of their necessary consequences and to believe in the validity of what follows on their arrangement” (*ibid.*). The Art of analysis (صناعة التحليل) offers the method to obtain these syllogisms, that is, “to pursue the research of their premises, to contrive to find them, and to try to find their arrangement” (*ibid.*). In this sense, the Art of analysis is an *ars demonstrandi*. It is also an *ars inveniendi*, since it is because of this art that we are led to “discover the unknowns of mathematical science and how to carry on seeking the premises (literally ‘to hunt (تصيد) for the proofs’), which are the material of proofs indicating the validity of what is discovered from the unknowns of these sciences, and the method to reach the arrangement of these premises and the figure of the combination” (*ibid.*, p. 38).

For Ibn al-Haytham, it is indeed an *Ars* (τέχνη, صناعة) *Analytica*, which has to be conceived and constructed. But to my knowledge nobody before him considered analysis and synthesis as an art or, more precisely, as a double art, of proof and discovery. In the former, the analyst (المحلل) has to know the principles (أصول) of mathematics. This knowledge has to be backed both by an “ingenuity” and an “intuition formed by the art” (حدس صناعي). Indispensable for discovery, this intuition is equally proved to be necessary when the synthesis is not the strict reversal of the analysis, but requires further data and properties which have to be discovered. That the knowledge of principles, ingenuity and intuition are numerous means that the analyst must have at his disposal the ability to discover mathematical unknowns. The “laws” and “principles” of this analytical art remain yet to be ascertained. This necessary knowledge is the subject of a discipline which bears on the foundations of mathematics, and which deals with the “knowns”. It must itself be constructed. The latter is a feature peculiar to Ibn al-Haytham, since nobody before him, not even Ibn Sinān, had considered elaborating an analytical art founded on a specific mathematical discipline. To this Ibn al-Haytham devotes a second treatise, *The Knowns* (Rashed, 1993d), one that he had promised in his treatise on *Analysis and Synthesis* (Rashed, 1991, p. 68). He himself presents this new discipline as that which offers the analyst the “laws” of this art and the “foundations” in which discovery of properties and apprehension of premises are brought to completion; in other words, it reaches the basis of mathematics, the prior knowledge of which is in fact, as we have said, necessary to the completion of the art of analysis: these are the notions called the “knowns” (*ibid.*, p. 58). It should be observed that whenever he deals with a foundational problem, as in his treatise *On Squaring the Circle* (Rashed, 1993c, pp. 91-95), Ibn al-Haytham comes back to the “knowns”.

According to Ibn al-Haytham, a notion is said to be “known” when it remains invariable and admits no change, whether or not it is thought by a knowing subject. The “knowns” refer to the invariable properties, independent of the knowledge that we have of them, and remain unchanged even though the other elements of the mathematical object vary. The aim of the analyst, according to Ibn al-Haytham, is precisely to lead to these invariable properties. Once these fixed elements have been reached, his task ends, and the synthesis can then start. The *Ars inveniendi* is neither mechanical nor blind, it should lead to the “knowns” through sustained ingenuity.

The analytical art thus requires for its construction a mathematical discipline, itself to be constructed. The latter contains the “laws” and the “principles” of the former. According to this conception, the analytical art cannot be reduced to any logic, but its own logical component is immersed in the mathematical discipline. We immediately discover the limit of the range of this art.

The contributions briefly sketched here indicate several situations where mathematicians deal with the philosophy of mathematics. We have previously examined other situations where philosopher-mathematicians and mathematician-philosophers contribute to the philosophy of mathematics. These contributions are obviously part of the history of philosophy and the history of the sciences, the history of the mathematical thought of classical Islam. To neglect these contributions is both to impoverish of the history of philosophy and to cut short the history of mathematics.

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The Birth of Scientific Controversies The Dynamics of the Arabic Tradition and its Impact on the Development of Science: Ibn al-Haytham's Challenge of Ptolemy's *Almagest*

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و الواجب على الناظر في كتب العلوم، إذا كان غرضه معرفة الحقائق،
أن يجعل نفسه خصما لكل ما ينظر فيه، و يجيل فكره في متنه و في جميع حواشيه،
ويخصمه من جميع جهاته و نواحيه، و يتهم أيضا نفسه عند خصامه فلا يتحامل عليه و لا يتسمح فيه.
فإنه إذا سلك هذه الطريقة انكشفت له الحقائق، و ظهر ما عساه وقع في كلام من تقدمه من التصير و الشبه.

Al 2asan ibn al-Haytham
(*Al-Shukūk 'ala Batlamyūs*, p. 4)¹

Abstract. The so-called Copernican revolution is Kuhn's most cherished example in his conception of the non-cumulative development of science. Indeed, in his view not only has the Copernican model introduced a major discontinuity in the history of science but the new paradigm and the old paradigm are incommensurable, i.e. the gap between the two models is so huge that the changes introduced in the new model cannot be understood in terms of the concepts of the old one. The aim of this paper is to show on the contrary that the study of the Arabic tradition can bridge the gap assumed by Kuhn as a historical fact precisely in the case of Copernicus. The changes involved in the work of Copernicus arise, in our view, as a result of interweaving epistemological and mathematical controversies in the Arabic tradition which challenged the Ptolemaic model. Our main case study is the work of Ibn al-Haytham who devotes a whole book to the task of refuting the implications of the *Almagest* machinery. Ibn al-Haytham's *al-Shukūk* had such an impact that since its disclosure the *Almagest* stopped being seen as the suitable model of the heavenly bodies. Numerous attempts have been made to find new alternative models based on the correct principles of physics following the strong appeal launched by both Ibn al-Haytham and, after him, Ibn Rushd. The work of Ibn al-Shātir, based exclusively on the concept of uniform circular motion, represents the climax of the intense theoretical research undertaken during the thirteenth and the fourteenth centuries by the Marāgha School (which owes its name to the observatory of Marāgha in north-western Iran). The connection point, in our view, between the works of Ibn al-Haytham and Ibn al-Shātir is that while the *al-Shukūk* gives the elements to build a countermodel to the *Almagest*, the work of Ibn al-Shātir offers a model which takes care of the objections triggered by the work of Ibn al-Haytham. Furthermore, not only has the basic identity of the models of Ibn al-Shātir and Copernicus been established by recent researches, but it was also found out that Copernicus used the very same mathematical apparatus which was developed by the Marāgha School over at least two centuries. Striking is the fact that Copernicus uses without proof mathematical results already geometrically proven by the Marāgha School three centuries before. Our paper will show that Copernicus was in fact working under the influence of the two streams of the Arabic tradition: the well known more philosophical western stream, known as physical realism, and the newly discovered eastern mathematical stream. The first relates to the idea that astronomy must be based on physics and that physics is about the real nature of things. The second relates to the use of mathematics in the construction of models and countermodels in astronomy as developed by the Marāgha School. The case presented challenges the role of the Arabic tradition assigned by the standard interpretation of the history of science and more generally presents a first step towards a reconsideration of the thesis of discontinuity in the history of science. Our view is that major changes in the development of science might sometimes be non-cumulative, though this is not a case against continuity understood as the result of a constant interweaving of a net of controversies inside and beyond science itself.

1. The problematic assessment of the Arabic-Islamic tradition

No scientific and philosophical tradition has raised such passion and so many heated debates among historians of science than the Arabic-Islamic tradition. The disagreement begins first with the problem of how to label the tradition adequately. Some prefer to call it the Arabic tradition because of the overwhelming dominance of the Arabic language used in scientific and philosophical writings. Others are more inclined to qualify the tradition as Islamic since the majority of those who contribute to the development of science and philosophy are non-Arabs. But this is not the dividing issue. It should be noted though that for the Arabs themselves this is not an issue, and they happily agree to use the more inclusive expression *Arabic-Islamic tradition*. The latter term includes Arabs (whether Moslems, Christians, Jews, Sabeens, etc.), Moslems (Persians, Asians, etc.), and any other writer who works under the direct influence of Arabic-Islamic thought. For the sake of convenience, Arabic and Islamic are considered in the present paper as interchangeable qualifications. A far more serious question is the assessment of this vast heritage. The never-ending point of controversy is: what is the real contribution of the Arabic tradition in the development of science? This question sharply divides historians of science into two disjoint classes of historians; the latter one is a recent stream which proposes a new perspective in relation to the role of the Arabic tradition. For the sake convenience, let us give them the two following labels: the *Non-Innovationists* and the *Innovationists* (with regard to the contribution of the Arabic tradition).

1.1. The Non-Innovationists

The underlying methodology of the Non-Innovationists is to try to determine to what extent the resulting novelties, if any, contribute to the emergence of modern science. The conclusion is almost always the same.

Let us begin with the assessment of Arthur Koestler; in his popular and widespread *The Sleepwalkers*, he writes

But the Arabs had merely been the go-betweens, preservers and transmitters of the heritage. They had little scientific originality and creativeness of their own. During the centuries when they were the sole keepers of the treasure, they did little to put it to use. They improved on calendrical astronomy and made excellent planetary tables; they elaborated both the Aristotelian and the Ptolemaic models of the universe; they imported into Europe the Indian system of numerals based on the symbol zero, the sine function, and the use of algebraic methods, but they did not advance theoretical science (Koestler 1968, p. 105).

Furthermore:

From the twelfth century onwards, the works, or fragments of works, of Archimedes and Hero of Alexandria, of Euclid, Aristotle, and Ptolemy, came into Christendom like pieces of phosphorescent flotsam. How devious this process of Europe's recovery of its own past heritage was, may be gathered from the fact that some of Aristotle's scientific treatises, including his *Physics*, had been translated from the original Greek into Syriac, from Syriac into Arabic, from Arabic into Hebrew, and finally from Hebrew into medieval Latin. Ptolemy's *Almagest* was known in various Arab translation throughout the Empire of Harun Al Rashid, from the Indus to the Ebro, before Gerardus of Cremona, in 1175, retranslated it from the Arabic into Latin. Euclid's *Elements* were rediscovered for Europe by an English monk, Adelard of Bath, who around 1120, came across an Arabic translation in Cordova (*ibid.*, p. 104-105).

For Koestler, then, the Arabs have added nothing new, their role is simply limited to bringing the Greek scientific and philosophical tradition from the East to the West. In the same spirit we find the following remark:

Insofar as science is concerned, the first six hundred years of established Christendom were a glacial period with only the pale moon of Neoplatonism reflected on the icy steppes. The thaw came not by the sudden rise of the sun, but by ways of devious Gulf-stream which wended its way from the Arab peninsula through Mesopotamia, Egypt and Spain: the Moslems. In the seventh and

eight centuries, this stream had picked up the wreckage of Greek science and philosophy in Asia Minor and in Alexandria, and carried in a circumambient and haphazard fashion into Europe (ibid., p. 104).

This and the following are the only passages where the Arabs are mentioned in the work of Koestler. John Dreyer, who at least took care to consult some original sources, devoted more space (one whole chapter) in his *History of the Planetary Systems from Thales to Kepler*² to the contribution of the Arabic tradition to the development of astronomy. However, his final verdict is not very different from Koestler's:

In this rapid view of Arabian astronomers we have only mentioned those whose work we shall have to allude to in the following pages [...]. It cannot be denied that they left astronomy pretty much as they found it. They determined several important constants anew, but they did not make a single improvement in the planetary theories (Dreyer 1953, p. 248-249).

Enough has been said to show that when Europeans again began to occupy themselves with science they found astronomy practically in the same state in which Ptolemy had left it in the second century (ibid., p. 279-280).

François Nau reviews the work of Bar Hebraeus and his Arab colleagues of the thirteenth century in the same non-innovationist spirit as the authors mentioned above:

A l'époque où écrivait Bar Hebraeus, les Arabes s'occupaient d'astronomie depuis près de quatre siècles et notre auteur cite un certain nombre de leurs résultats ; mais ces résultats semblent peu importants ; les auteurs arabes que nous connaissons furent surtout des commentateurs et des astrologues amateurs, on ne les a admirés que faute de connaître les œuvres grecques, leurs modèles. On peut donc considérer le présent *Cours d'astronomie* comme un résumé des œuvres de Ptolémée (avec quelques *adjuncta* dus aux Arabes) (Nau 1899, p. xiv).

Among the Non-Innovationists we even find Thomas Kuhn. Indeed, in his *The Copernican Revolution*, Thomas Kuhn clearly sums up the received view of the contribution of the Islamic tradition:

The Moslems were seldom radical innovators in scientific theory. Their astronomy, in particular, developed almost exclusively within the technical and the cosmological tradition established in classical antiquity. Therefore, from our present restricted viewpoint, Islamic civilisation is important primarily because it preserved and proliferated the records of ancient Greek science for later European scholars (Kuhn 1957, p. 101).

1.2. The Innovationists

The Innovationists have a more positive view with regard to the contribution of the Arabic tradition. Roshdi Rashed, one of the most eminent members of the group of Innovationists, vehemently opposes the received view and especially the Non-Innovationists' account of the evolution of astronomy.

Following in the wake of the western doctrine of classical science, he [the historian of science] can view Arabic science as a repository of Hellenic science, a belated Hellenic science as it were ... According to this doctrine Arabic science constitutes an excavation site, in which the historian is the archaeologist on the track of Hellenism (Rashed 1996, p. x).

Régis Morelon points out that there is a discontinuity in the transmission of the Greek tradition since the composition of the *Almagest* in the second century B.C. Moreover he remarks that the translation of the Greek astronomical writings is not sufficient by itself to establish a genuine, efficient and lasting tradition in the practice of astronomy.

Cette discipline [i.e. astronomy] n'était plus vivante dans le bassin méditerranéen depuis plusieurs siècles: il n'y a que quelques observations isolées qui aient été enregistrées entre le IIIe et le VIIIe siècle, et les successeurs de Ptolémée ne furent globalement que des commentateurs. Il y avait donc discontinuité dans une tradition. Lorsqu'il s'est agi de la revivifier à Bagdad sous al-Ma'mūn (813-833), les sources écrites de travail étaient évidemment hellénistiques, mais il a fallu retrouver

quelle bases et quelles méthodes convenaient pour cette discipline, donc de les recréer (Morelon 1998, p. 104).

One of the main features of the Arabic astronomical tradition, according to Morelon, is the institution of a research programme which can be summed up in the three following points: (i) the importance given to the role of observation which manifests itself in the construction of many observatories. The aim of observatories is to permit the continuous collection of empirical data; (ii) the increasing application of the mathematical apparatus to astronomy due to the development of spherical astronomy; (iii) the sustained conflict between “astronomical physics” (i.e. the research for a physical representation of the universe) and “astronomical mathematics” (dealing with the calculation of the position of the planets).

The studies of Rashed and Morelon strongly suggest that the role of the Arabic tradition is far from being a mere imitation of the Greek tradition. More generally, the Innovationists complain that, in the received view, the Arabic tradition is never examined for its own sake but always in relation to the Greek tradition and as an appendix to the history of Greek science and philosophy. The result is then the expected one: to reduce the role of the Arabic tradition to that of a mere intermediary or transmitter of the Greek heritage. Moreover, it seems that the Non-Innovationists have a peculiar view of the discontinuity of the history of science, which in their view begins with the work of Copernicus. Rashed denounces what he calls the “oblique view of an historical ideology which views classical science as the achievement of European humanity alone” (ibid., p. xii).

But the Non-Innovationists might fight back and take the historical sources as irrelevant for their main argument. The point is not, of course, to compare the results of the Arabic tradition and those of the seventeenth century. The question is rather: why has Arabic astronomy failed to live up to its promises? In other words, the comparison must be between the kind of activities undertaken by the Arabic and medieval European traditions; the latter is rich and profound since it leads to the Copernican revolution, while the former leads nowhere besides the achievements of the Greek science. More generally, the underlying idea is that major achievements of the Renaissance are intrinsically linked to the medieval European tradition and ultimately to the Greek heritage. According to the Non-Innovationists, things suddenly and radically changed when the Europeans of the Middle Ages became acquainted with the Greek works. Koestler claims for example that

With Euclid, Aristotle, Archimedes, Ptolemy and Galen recovered, science could start again where it had left off a millennium earlier (Koestler 1968, p. 105).

Koestler adds: “As soon as it was reincorporated into Latin civilization, it bore immediate and abundant fruit” (ibid.). And without the slightest hesitation, he concludes: “The heritage of Greece was obviously of no benefit to anybody without some specific receptiveness for it” (ibid.). Carra de Vaux, another Non-Innovationist, finds sharper words to qualify the contribution of the Arabic tradition. Carra de Vaux undertook the translation, at the end of the nineteenth century, of what appears to be the most original chapter of al-Tūsī’s *Tadhkira* (or *Memoir of Astronomy*). The purpose of the translator is very clear: to explicitly refute the following statement made by an Arab biobibliographer (quoted by de Vaux): “N. E. Attūsī met le sceau à l’interprétation de l’*Almageste*, mais il est si bref et il ajoute des gloses si profondes que les esprits les plus perspicaces en restent étonnés” (Tannery 1893, p. 341). The translated chapter, included as an appendix in Paul Tannery’s extremely influential historical book *Recherches sur l’histoire de l’astronomie ancienne*, is preceded by an introduction in which de Vaux gives this overall assessment:

Le chapitre dont nous allons donner la traduction suffira peut-être à faire sentir ce que la science musulmane avait de faiblesse, de mesquinerie, quand elle voulait être originale. N. E. Attūsī est un des hommes qui l’ont le plus illustrée (ibid., p. 338).

And why this lack of originality? “Elle [Arabic science] a manqué d’un élément non moins nécessaire que la liberté : la force du génie” (ibid.). It looks as if there is something intrinsic to Arabic-Islamic thought which makes it incapable of any creativity. “Le génie” is not something that can be shared by all human beings or that can be acquired, through hard labour, by human nature, but it is endowed to only one restricted class of human beings; that is why Western science will never leave its natural soil since it is the making of the Europeans and will remain so. And de Vaux presents his translation as objective evidence of his claim since he concludes

La portée de ce chapitre n’est donc pas très grande; il mérite néanmoins d’être lu à titre de curiosité. [...] Arrivé à ce point, nous n’avons plus à intervenir ni comme historien ni comme critique ; nous nous faisons simple interprète, et nous souhaitons d’être fidèle (ibid., p. 347).

Kuhn further explains how the indirect discovery of the Greek heritage through third languages gave the medieval Europeans a strong desire to go back to the authentic texts:

Peuerbach, for example, began his career in astronomy by working from second-hand translations of the *Almagest* transmitted via Islam. From them he was able to reconstruct a more adequate and complete account of Ptolemy’s system than any known before. But his work only convinced him that a truly adequate astronomy could not be derived from Arabic sources. Astronomers, he felt, would have to work from Greek originals, and he was about to depart for Italy to examine manuscripts available there when he died in 1461 (Kuhn 1957, p. 125).

The cultural and social conditions are then favourable for a man with a mission to make his appearance:

Copernicus is among that small group of Europeans who first revived the full Hellenistic tradition of technical mathematical astronomy which in antiquity had culminated in the work of Ptolemy. [...] With Copernicus we return for the first time to the sort of technical astronomical problem (ibid., p. 135).

Faced with what the Non-Innovationists declare as being hard evidence, the Innovationists seem to be more on the defensive, seeming to have no choice but to accept the conclusions of their opponents. After expressing, like the previous historians, his astonishment as to the lack of creativity in the Arabic tradition, Koestler asserts that the problem is no longer of philosophical or epistemological consideration but of the history of civilisations and closes herewith any further discussion on the issue.

It is a curious fact that the Arab-Judaic tenure of this vast body of knowledge, which lasted two or three centuries, remained barren. [...] How this readiness to rediscover its own past, and be fertilized by it as it were, arose in Europe is a question that belongs to the field of general history (Koestler 1968, p. 105).

The discovery from 1957 onwards of the writings of a series of astronomers of the Marāgha School of the thirteenth century (which owes its name to the observatory of Marāgha in north-western Iran) dramatically changes the situation of the dispute between Innovationists and Non-Innovationists. George Saliba, one of the scholars who studies these works closely, describes the sensation caused by the discovery:

Research conducted in the History of Arabic astronomy, within the last three decades, has brought to light a group of texts, that were hitherto unknown, and which radically altered our conception of the originality and scope of Arabic astronomy. The works of astronomers such as Mu’ayyad al-Dīn al-‘Ur*ī (d. 1266), Na{īr al-Dīn al-ūṣī (d. 1274), Qu}b al-Dīn al-Shīrāzī (d. 1311), and Ibn al-Shā}ir (d. 1375), to name just a few were barely known in the nineteenth century or in the early part of the present [twentieth] century. Only ūṣī was mentioned in nineteenth-century literature (Saliba 1996, p. 245).

The work of the Marāgha astronomers was motivated by the same aim: to find alternative models to the Ptolemaic system. Their results have far-reaching consequences on our understanding of the history of astronomy.

First, the Marāgha results were, from the very beginning, deemed extremely important on account of their relationship to the works of Copernicus. But one should say that this relationship did not touch upon the Copernican notion of heliocentricity. That feature of Copernican astronomy entails the transformation of geocentric mathematical models into heliocentric ones by the reversal of the vector connecting the sun to the earth, while leaving the rest of the mathematical models intact. It is rather the similarity of the Copernican geocentric versions of those models to those of the Marāgha astronomers that invited curiosity (*ibid.*, pp. 265-267).

The sensation in these new findings is not so much the identity of the Copernicus models with those developed by the Marāgha astronomers three centuries earlier, since it is perfectly conceivable that Copernicus could have developed his own models independently. The existing links between Copernicus and the Marāgha School became more and more evident when it was found that Copernicus used the same technical apparatus as that developed by the Arabic astronomers during the thirteenth and fourteenth centuries (for more detail see Saliba 1996, p. 269). Moreover, Saliba explains what is at stake on the historical level as far as the development of astronomy is concerned.

What is clear is that the equivalent model of Ibn al-Shātir seems to have a well established history within the results reached by earlier Muslim astronomers, and could therefore be historically explained as a natural and gradual development that had started some three centuries earlier. The same could not be said of the Copernican model (Saliba 1994, p. 304; also Saliba 1996, p. 113).

And he concludes that Copernican astronomy cannot be very well understood, on the mathematical and technical level, without careful study of the achievements of the earlier Marāgha astronomers. This explains why de Vaux failed to understand the originality of the translated chapter in which al-Tūsī proves an important theorem, establishing the generation of rectilinear motion from two circular motions, which is then systematically used by his successors including Copernicus. The time is now ready for the historians of science and its evolution through successive civilisations to step in to respond to Koestler's invitation. The distinguished historian Otto Neugebauer, who has closely studied the development of astronomy from its beginning with the Babylonian civilisation up to the renaissance, has this to say:

The recovery of the planetary theory of the astronomers of the Marāgha School [...] is not only of great interest in itself, but has also demonstrated that much of what had been taken for Copernicus's own planetary theory is actually of medieval Arabic origin, and was transmitted to western Europe by an unknown route, perhaps by way of late Byzantine sources, to Italy at some time in the fifteenth century (Neugebauer 1984, p. 290).

Neugebauer adds "the question therefore is not whether but when, where, and in what form, he [Copernicus] learned of Marāgha theory" (*ibid.*, p. 47). It looks as if the history of science is about to be rewritten. The status of Copernicus seems to be hanging in the balance more than ever, thus blurring the sharp distinction which he is traditionally associated with. The Non-Innovationists find in the *De Revolutionibus* a clear-cut between the medieval European era and the Renaissance, since it contains some significant results which make his author the starting point of a revolution. According to Kuhn, for example, Copernicus' aim in *De Revolutionibus* is to solve the problem of the planets which he felt Ptolemy and his successors had left unresolved.

None of the "Ptolemaic" systems which Copernicus knew gave results that quite coincided with good naked-eye observations. They were no worse than Ptolemy's results, but they were also no better. After thirteen centuries of fruitless research a perceptive astronomer might well wonder, as Ptolemy could not have, whether further attempts within the same tradition could conceivably be successful (Kuhn 1957, p. 139).

Furthermore, in Kuhn's view, the nature of Copernicus's revolution is more technical than conceptual in the sense that it is the use of mathematics which distinguishes him from his predecessors:

In recognizing the need for and in developing these new techniques, Copernicus made his single original contribution to the Revolution that bears his name. [...] Copernicus' mathematics distinguishes him from his predecessors, and it was in part because of the mathematics that his work inaugurated a revolution as theirs had not (ibid., p. 143).

Since, as already mentioned, the technical novelties essential to the development of the Copernican system have been shown to have their origin in the Marāgha School, we might say, using Kuhn's own paradigm, that the Copernican revolution is in fact the Marāgha School revolution. Far more significant is the fact that Copernicus appears to be working under the influence of both the traditionally well known Arabic western realist tradition and the newly discovered Arabic eastern tradition. This paper will show in what way the *De Revolutionibus* can be seen as a synthesis of both these traditions. The first relates to the idea that astronomy must be based on physics and that physics is about the real nature of things; the second relates to the use of mathematics in the construction of models and countermodels in astronomy as developed by the Marāgha School.

The availability of the works of the Marāgha astronomers gives more sophisticated ammunition to counterbalance the Non-Innovationists' strong weaponry: hard facts. Armed with these new findings, the Innovationists go on the offensive in the dispute. In the face of this new evidence, it seems that the received view on the periodisation of science, already challenged by Duhem, needs to be revised as suggested by the following statement of Neugebauer: "in a very real sense, Copernicus can be looked upon, as if not the last, surely the most noted follower of the Marāgha School" (Neugebauer 1984, p. 47). Saliba for his part calls for the abandonment of the following periodisation paradigm underlying the Non-Innovationist reading of the Arabic tradition.

The prevailing periodization could be summarized along the following stages: (1) the translation stage, when Greek astronomy passed into Arabic, and that seems to have been understood as *just* a translation stage; (2) a stage of additional minor commentaries of a type that Nau called *adjuncta* to Greek astronomy; and finally (3) a stage of general decline in Arabic scientific creativity, which must have started sometime during the twelfth century just as Europe was in the process of acquiring the Greek heritage, especially the astronomical and the mathematical one, through the translation from Arabic into Latin. From then on, there was no longer any need to pay attention to the Arabic tradition, for Europe was developing science on its own (Saliba 1994, p. 247).

Like Saliba, Rashed identifies the problem in the dogmatic periodisation of the history of science. Their way of looking at the history of science, not as a process but as a jump from ancient to modern science, creates a paradox: heavily underestimated, the Arabic tradition ends up denying its existence in its own right, while at the same time the working historian of science cannot afford to ignore the hard facts he is confronted with in his practical studies. Rashed invokes Duhem's major historical work as "merely the expression of a profound [historical] necessity" (Rashed 1996, p. x) of the role of the Arabic tradition, without which the medieval scientific and philosophical writings could not be understood. And he concludes that the Non-Innovationists' account creates a gap so huge between ancient and modern science that it makes it impossible to bridge.

Presented as a postulate, and in the absence of authentic knowledge of the works of the School of Marāgha and of its predecessors in astronomy – of al-Khayyām and of Sharaf al-Dīn al-Tūsī in algebra and algebraic geometry, of the Arabic infinitesimalists from Ibn Qurra to Ibn al-Haytham – this absolute pre-eminence has naturally created a vacuum prior to the works of the seventeenth century, and has resulted in a model of Arabic science that flattens its most remarkable peaks of achievement (ibid., p. x).

As a result of this stark contrast introduced in the periodisation of the history of science, there is not only one science but two wholly different kinds of science. According to the Non-Innovationists' view, a new kind of science emerges whenever a major upheaval occurs in the fundamental concepts of science. Rashed's underlying objection has far-reaching

consequences: (i) what is science seems to be a more problematic question than ever, and as a result there is no way to distinguish science (since there is no such thing as a science) from other forms of theories and beliefs; (ii) the rigid periodisation of the evolution of science, which leads to the fragmentation not only of the various scientific disciplines but also of the multiple scientific theories in each scientific branch, leaves no room for talking about the unity of science or at least understanding how the so-called modern science and its rapid ramifications are brought about. The fact of the matter is that our awareness of the deep interdependence between the various scientific disciplines is getting stronger as more scientific branches are linked together through the rapid exchanges of scientific writings and our understanding of the evolution of science increases as more records are made available. The full significance of Duhem's enterprise can now be seen in a new light, since his systematic analysis of all the available scientific and philosophical documents shows how this task can be achieved. Driven by another motivation, Duhem wants to make the following point: the way in which the major achievements of the Renaissance can be seen as the necessary evolution of the scientific research framework developed during the Scholastic period. The nineteenth century historian's attempt (the first eminent historian to challenge the received view) is at odds with the prevailing paradigmatic periodisation of modern historians of science according to which the seventeenth century is the beginning of the era of modern science. To defend his claim and his faith, the strongly religious physician has to innovate. This explains why he devotes eight out of ten volumes of his *Le Système du Monde* to the analysis of the so-called sterilised medieval European writings. The result: a lively and dynamic account in which he describes science in the making by systematically exposing the various controversies leading to the formation of modern scientific concepts. By presenting newly discovered material (Ibn al-Haytham's *Al-Shukūk 'ala Batlamyus*), our contribution is designed to update *Le Système du Monde* and to fill the gap in our understanding of Duhem's exposition of the evolution of science in general and of the emergence of modern science in particular. Our aim is to show how our fruitful and dynamic interpretation of the progress of science can be a way out of this long dispute between the Non-Innovationists and the Innovationists. One last remark: the general lines of this interpretation are based on Shahid Rahman's research project "La science et ses contextes" (MSH-Nord-pas de Calais), already suggested in Rahman/Symons 2004, (pp. 3-16) and developed in my thesis in relation to the history of mathematics, where the gap between the history and philosophy of mathematics is closed by the systematic study of scientific controversies.

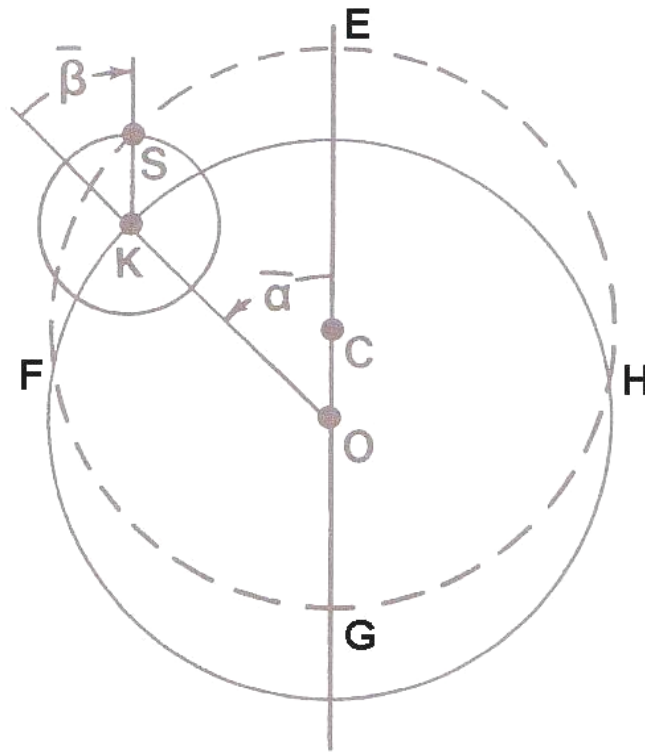
2. Plato's astronomical doctrine

According to Duhem, if we want to find the first clear definition of the subject matter of astronomy, we have to go back to the teachings of Plato as reported by Simplicius in his *Commentary*

Plato assumes in principle the motion of celestial bodies is uniform circular and perfectly regular [i.e. constantly in the same direction]; he then poses to the mathematicians the following problem: What uniform circular motions are convenient to be taken as hypotheses in order to save the appearances presented in the wandering planets? (Duhem 1913, volume I p. 103).

The task of the astronomer is clearly defined: (i) he may only use uniform circular motion; (ii) he has to account for any other kind of motion by the combination of uniform circular motions such that the resulting motion resembles the motion of the star; (iii) he has to choose further hypotheses in such a way that the resulting motion is in conformity with the observed motion of the heavenly bodies. These are the principles which guided the works of Eudoxus and Kallipsus. The latter's homocentric system does not succeed in saving the appearances due to the complexity of the motion of the wandering planets. Successor astronomers such as

Apollonius and Hipparchus succeeded in giving a satisfactory response to Plato's problem. Strictly following Plato's principle, they retained uniform circular motion as the principle of motion, but they introduced some hypotheses which gave rise to the famous theory of circular motions eccentric to the earth and the deferent-epicycle theory. These two models, which turn out to be equivalent, have been used successfully to account for the motion of the sun (Figure 1).



$\bar{\alpha}$ and $\bar{\beta}$ are called the mean anomaly since they measure the angular distance of the mean sun from the apogee. The two models are mathematically equivalent if $SK = OC$ and $\bar{\alpha} = \bar{\beta}$.

Figure 1

In the eccentric model ($EFGH$ circle), the sun moves with uniform speed along $EFGH$, but the centre of the circle is no longer assumed to coincide with that of the earth. The sun S is at its greatest distance from the earth at apogee E and at its closest to the earth at its perigee G . This model allows the sun to travel at constant speed describing equal arcs at equal times but it appears to an observer supposed to be at O to travel more quickly when in the lower half of the eccentric FGH and more slowly when in the upper half HEF (its slowest point being of course at apogee E), because of its varying distance from the earth. It happens that an entirely different model will produce the same result if the sun is assumed to be moving on an epicycle with centre K in the direction contrary to the order of the signs (i.e. clockwise as indicated by the arrow). Point K is assumed to be moved on a circle centred on the earth O called the deferent (broken circle) in an equal and contrary motion to that of the epicycle. The deferent circle is said to be concentric to the earth. The equivalence of the eccentric and the deferent-epicycle models, and therefore the resulting motions, was first proved by Apollonius and is reproduced by Ptolemy in the *Almagest* Book III Chapter 3.

3. Shift in the theory of astronomy: Ptolemy's *Almagest*

To account for the more complex motion of the wandering planets, Ptolemy uses further hypotheses which introduce a major shift in the evolution of astronomy. His description of the motion of Venus illustrates the importance of these changes (Figure 2).

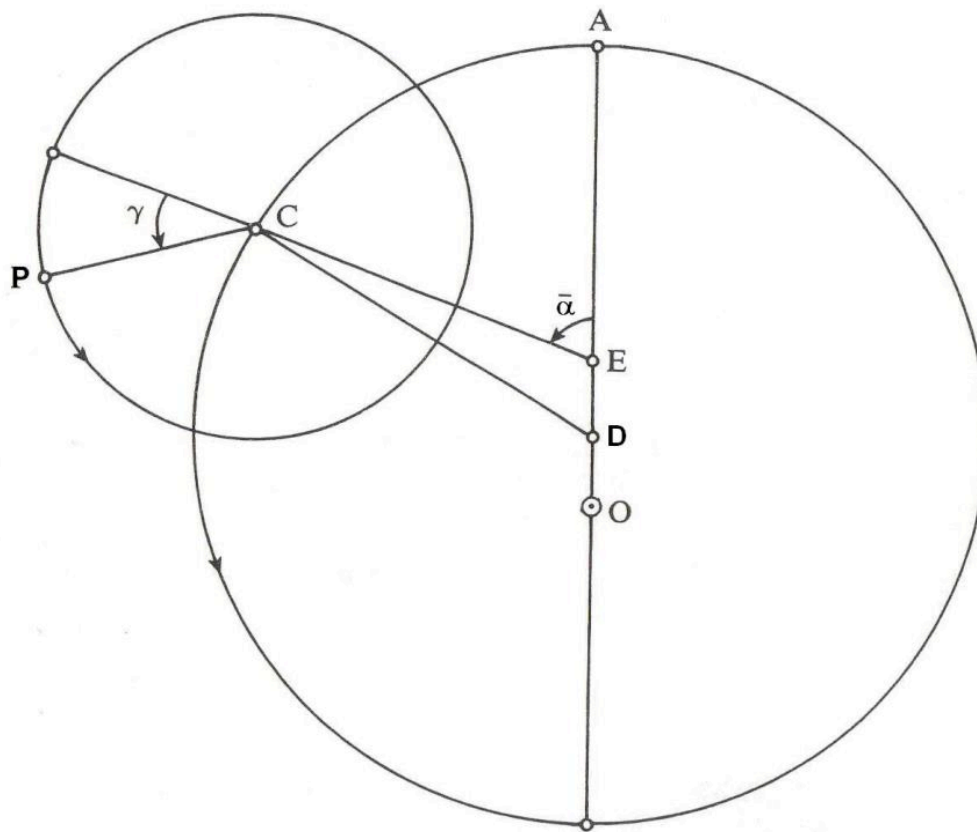
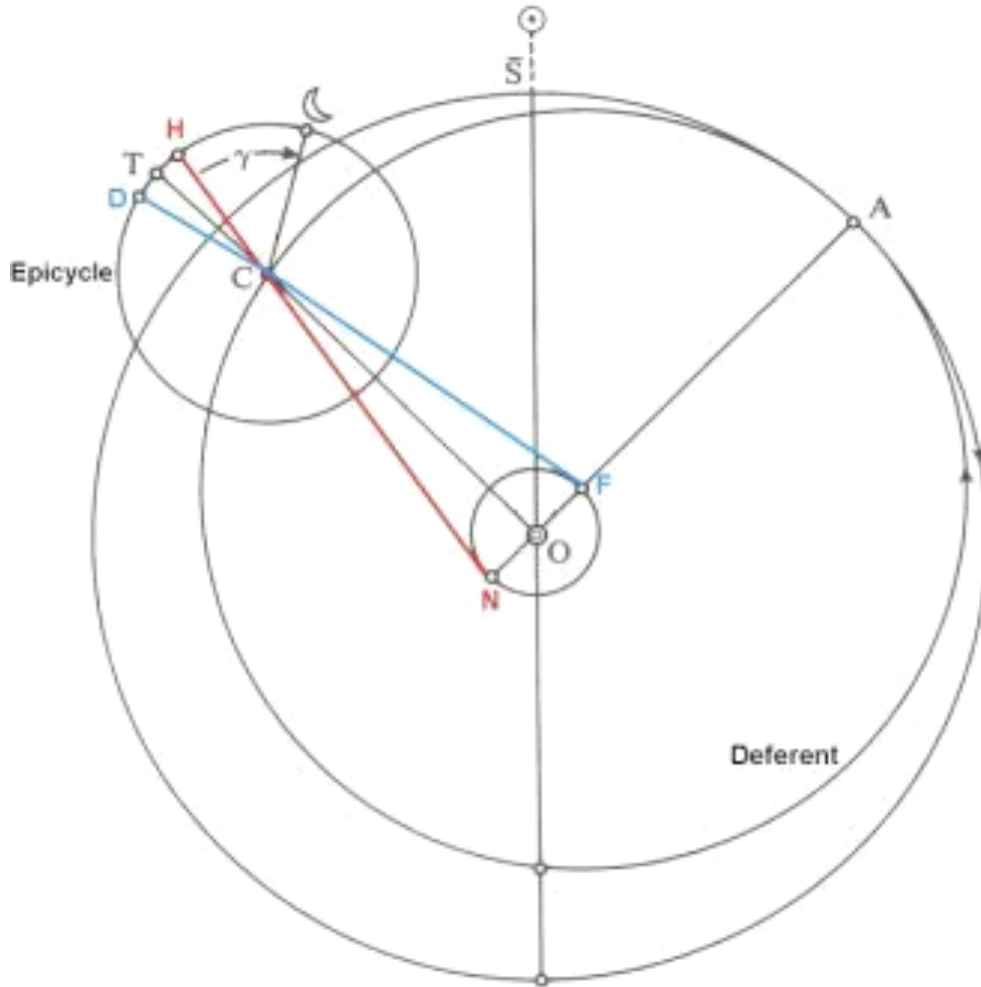


Figure 2

To account for the double anomaly for each planet ((i) an anomaly which varies according to the planet's position in the ecliptic, and (ii) which varies according to its position relative to the sun), Ptolemy assumes that the planet P is carried by the epicycle in its backward motion at uniform speed measured by anomaly γ , while the centre of the epicycle is moved by the deferent around its centre D .

Now, according to the principle of uniform circular motion, point C must move uniformly around the deferent and the planet P must also move uniformly around the epicycle; i.e. the line CP must describe equal arcs in equal times and the epicycle, which carries the planet, is invariably linked to the vector CE describing equal arcs in equal times around D . In the case of Venus, Ptolemy declares: "but since it is not clear whether the uniform motion takes place around D ..." (Ptolemy, p. 473). Ptolemy assumes instead that point C , the centre of the epicycle, describes equal arcs in equal times not around the centre of the deferent D as it should but around a point E such that $ED = OD$; E is called the equant point. In other words the deferent is assumed to move uniformly around a point different from its centre.

His account of the motion of the moon confirms this trend (Figure 3).



\bar{S} is the mean apogee (i.e. a fictitious body that moves uniformly around the earth O); F is the centre of the deferent; C is the centre of the epicycle on which the moon moves uniformly; T , the apogee seen from O , is called the true apogee; H , the point from which the mean anomaly is measured, is called the mean apogee; N is the prosneusis point.

Figure 3

This description of the motion of the moon consists of the following features:

(i) the deferent moves around its centre F such that $\bar{S}OA$ and $\bar{S}OC$ are equal and opposite, i.e. like the motion of Venus, we have here a deferent moving uniformly around a point O other than its centre; (ii) instead of counting the anomaly γ which determines the distance of the moon M on the epicycle from the apogee D , it had to be measured from point H (called the mean apogee for this reason) such that the radius HC has a direction towards a point N which is always located diametrically opposite to point F .

It should be noted that H is a variable apogee since N , called the prosneusis point, has to be constantly in motion to remain opposite to the moving point F . The difference between the equant and the prosneusis is thus that the description of the motion of the moon starts from a non-stable point. The motion of Mercury is accounted for by more complicated motions since Ptolemy uses a combination of the equant and prosneusis features.

The admission of the equant and prosneusis hypotheses signals a significant departure from both Plato's astronomical tradition and Aristotle's physical principles, since we have in both cases a uniform motion which takes place around an axis that does not pass through the centre of the sphere generating it. This is what Ptolemy calls $\pi\alpha\rho\iota\ \tau\tilde{\omicron}\nu\ \lambda\tilde{\omicron}\gamma\omicron\nu$ translated by Toomer as "not in strict accordance with [ancient] theory" (Ptolemy 1984, p. 422), in effect rejecting the conception of his predecessors. The *Almagest* is to show here that astronomy needs new

principles and a new way of reasoning: (i) Plato's astronomical doctrine should be modified by abandoning uniform circular motion; (ii) the Aristotelian physical principles should be restricted to the sublunar phenomena.

One of the major consequences of Ptolemy's work is that it puts an end to the unclear relationship between mathematics and physics. His decision is to subordinate the latter to the former; this philosophical approach involves a particular conception of science which has, as one of its severe side effects, widened the gap between mathematics and physics. It remains to be seen whether it is the right approach. These difficult questions are not raised by the author of the *Almagest*. Given Ptolemy's exposition of his theory, the reader should not expect the mathematician-astronomer to discuss its philosophical and epistemological implications. A controversy is needed to challenge its underlying assumptions and to bring to the forefront these foundational issues.

4. The beginning of the controversy over the foundations of astronomy: Ibn al-Haytham's *Doubts against Ptolemy's Almagest*

The composition of the *Almagest* represents the climax of Greek astronomy. It was Ptolemy who brought the Greek planetary theory to its final and definitive form. The original name of *Almagest*, which according to Toomer is originally derived from a Greek form *μεγίστη* meaning the "great [treatise]", is *μαθηματική σύνταξις* *Mathematical Systematic Treatise* (Ptolemy 1984, p. 1); by this Ptolemy means to give a comprehensive mathematical account for the motion of heavenly bodies. Beside some important results of his own, Ptolemy includes practically all astronomical achievements of his predecessors which could be reached with the mathematical methods of antiquity. Ptolemy's work reigns supreme over the cosmological scene for many centuries. His *Almagest* is to astronomy what Euclid's *Elements* is to geometry. But surprisingly, *Almagest's* life is much shorter than that of the *Elements*, since it was the first important Greek scientific work to be successfully disputed. By the eleventh century, we begin to notice a serious shift in the astronomical field, a shift which has far-reaching consequences for the development of philosophy and science as a whole. The starting point of this shift is the relentless and systematic attack levelled against Ptolemy's approach to science. The domination of *Almagest* was strongly challenged for the first time by an eminent Arabic scientist, al-2asan ibn al-Haytham, in his famous book entitled *al-Shukūk 'ala Batlamyus* in which the author raises serious doubts about Ptolemy's claims concerning the nature of astronomical theory. Ibn al-Haytham does not seem to be impressed at all by the *Great Mathematical Treatise*, since what interests him is not so much the formal account for the motion of heavenly bodies, successful though it may appear, but rather the justification of the geometric constructions underlying the *Almagest* machinery.

Our analysis of the controversy between Ibn al-Haytham and Ptolemy is largely based on *al-Shukūk*, whose argumentative presentation will be followed closely, and its impact on the history of astronomy will be clearly exposed by taking as our guide Duhem's interesting dynamic approach to the history of science which underlies his monumental work *Le Système du Monde*. The structure of Ibn al-Haytham exposition of the controversy follows the style of what later has been formalized as disputations or obligations by presenting Ptolemy as a proponent while he plays the role of an opponent or a challenger. This original dialogical method of exposition adopted by Ibn al-Haytham in his *al-Shukūk* which can be characterised as a dispute based-approach not only inaugurates a new way of arguing in the history of science and philosophy, to be followed later by his successors, aimed at putting to the test Ptolemy's fundamental claims but captures the nature of scientific and philosophical practice. It should be noted that the object of the controversy concerns the motion of the planets, which

is by far the main controversial issue raised by *al-Shukūk*; that does not mean that the other issues discussed are devoid of any interest.

4.1. The controversy over the structure of a planetary theory

4.1.1. The refutation of the prosneusis hypothesis

The first point raised in this respect by Ibn al-Haytham is that concerning the movement of the moon in which Ptolemy uses the prosneusis hypothesis. He begins his discussion by presenting a concise formulation of Ptolemy's argument.³

[Ptolemy] says in Book V, chapter 5, which is on the inclination of the diameter of the moon's epicycle, that the diameter of the moon's epicycle, whose extremity is the epicycle apogee, always inclines towards a point below the centre of the world, a point whose distance from the centre of the world is as the distance of the centre of the world from the eccentric centre. But since the eccentric sphere moves the epicycle, the epicycle's diameter, whose extremity is the apogee when the epicycle is at the eccentric apogee, will always point to the eccentric's centre. [For] when the eccentric deferent moves, thereby moving the epicycle, there will move, together with the epicycle, the eccentric's diameter that passes through the epicycle apogee. This diameter cannot therefore be directed at any time to a point other than the eccentric's centre unless it moved and changed its position so as to be oriented towards another point (p. 15).

Ibn al-Haytham stresses here that Ptolemy's account presupposes that the diameter of the epicycle which carries the moon moves to a point other than its natural one. And from this assumption, he concludes: "now the epicycle's diameter is an imagined line." This assertion seems to do no harm to Ptolemy's theory since he claims that some concepts such as the prosneusis point could be considered so. Indeed, Duhem, as we shall see, articulates a sophisticated theory which could be used as a justification of the conceptual apparatus underlying the *Almagest*. But Ibn al-Haytham replies with a surprise counterargument, since he continues

And an imagined line does not move by itself with a sensible movement that produces something existing in the world. Similarly, the plane of the epicycle is an imaginary plane; and an imaginary plane does not have a sensible motion. Nor does anything move with a sensible movement that produces something in the world unless it be a body that exists in the world. From this it follows that it is the body of the epicycle that moves, thereby giving rise to the change of position of the epicycle's diameter in such a way as to be directed towards a point other than that towards which it would [otherwise] be directed (p. 15).

This is a devastating attack against Ptolemy's theory. Furthermore, Ibn al-Haytham rejects Duhem's interpretation of Ptolemy's approach according to which the whole theory should be considered as pure fiction because a planetary theory could by no means be homogeneous.

Ptolemy had gathered all the motions that he could verify from his own observations and from the observations of those who had preceded him. Then he sought a configuration of real existing bodies that exhibit such motions, but could not realise it. He then resorted to *an imaginary configuration based on imaginary circles and lines*, although *some* of these motions could *possibly exist in real bodies*. But if one imagines a line to be moving in a certain fashion according to his own imagination, it does not follow that there would be a line in the heavens similar to the one he had imagined moving in a similar motion. Nor is it true that if one imagined a circle in the heaven, and then imagined the planet to move along that circle, that the [real] planet would indeed move along that circle (p. 41, my emphasis).

Ibn al-Haytham considers *Almagest* as a mixed theory because motion, which is a physical notion, divides its concepts into two classes: (i) physical entities which possess a uniform circular motion. These are abstract entities since they can be associated with real existing bodies; (ii) imaginary entities which are, by contrast, not capable of acquiring the property of motion. By distinguishing entities according to the motion criterion, Ibn al-Haytham rejects Ptolemy's attempt to blur the two kinds of entities. As for imaginary entities in particular, Ibn

al-Haytham is not opposed at all to the introduction in a physical theory of entities to which no physical reality corresponds — we have to bear in mind that he is also a mathematician. But what the Arabic physician denies to the Greek mathematician is the attribution to imaginary entities by the latter of properties which are of a purely physical character. It is like attributing to i the property of a real number although everybody knows that there is no real number whose square is equal to -1 . For Ibn al-Haytham to assume thus the existence of imaginary objects in motion is an absurdity: a conclusion he draws from his discussion of the motion of the inferior planets in latitude.

This is an absurd impossibility, in direct contradiction with his earlier statement about the heavenly motions – being continuous, uniform and perpetual – because this motion has to belong to a body that moves in this manner, since there is no perceptible motion except that which belongs to an existing body (p. 36).

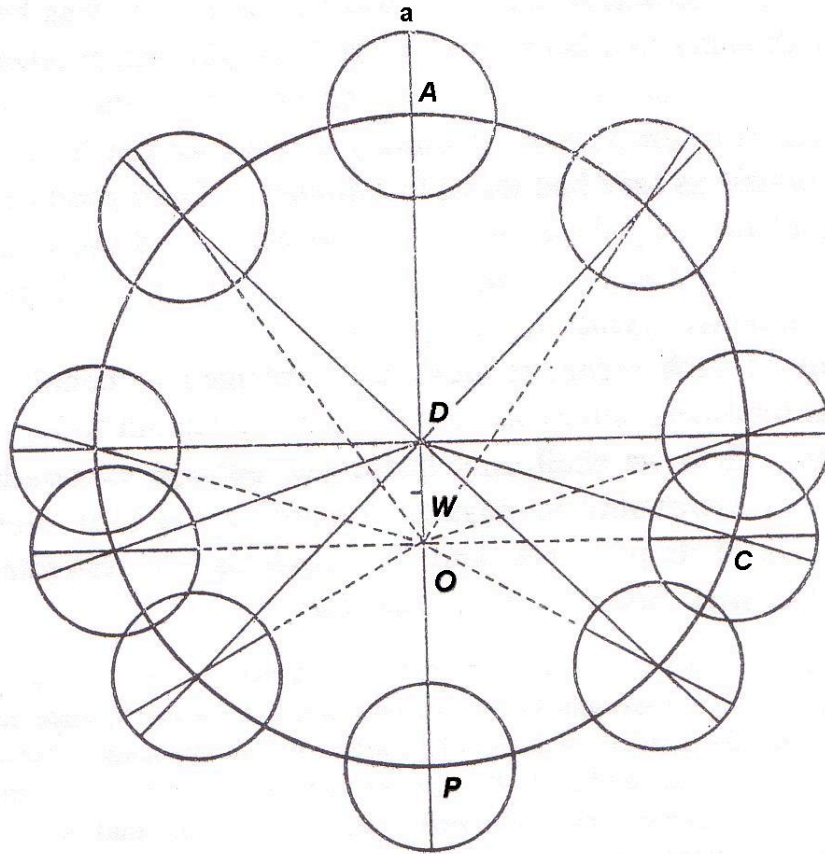
Now how could the motion of the lunar diameter be justified? It should be noted that the diameter is moved by the body of the epicycle. But for the position of the diameter to be changed, the epicycle must move in such a way that the diameter's position should always be directed towards the prosneusis point. Ibn al-Haytham examines an assumption made by Ptolemy in his *Planetary Hypotheses* in which he introduces a body (a sphere or a disc) that moves the epicycle. But he points out that this body moves uniformly, and consequently the diameter moves with uniform circular motion around the centre of the epicycle, and he concludes:

Therefore if this diameter always points, as he [Ptolemy] assumed, to one and the same point, while the body of the epicycle moves with a circular, uniform and continuous movement, then this diameter needs another mover which always orients it towards the assumed point (p. 16).

In other words the prosneusis hypothesis requires a body other than that introduced by Ptolemy, and he continues:

Ptolemy, however, does not assume in the *Planetary Hypotheses* a body that brings about this movement. Moreover, if, for the sake of this movement, a body is assumed to move the epicycle, then this body must need to possess two opposite movements. (p. 17)

He then shows by *reductio ad absurdum* that the assumed existence of such a body leads to a contradiction. Below is a brief summary of how he arrives at this conclusion. I think using a slightly modified version of Sabra's figure (Sabra 1994, XIV p. 125) makes it easier for the reader to follow Ibn al-Haytham's abstract argument.



a: apogee of epicycle, A: apogee of deferent, D: deferent centre, W: world centre, O: proneusis point, P: perigee of deferent.

Figure 4

For the lunar diameter to be always directed towards O , the assumed body B must move in two opposite movements: (i) from A , where the centre of the epicycle coincides with the eccentric apogee, to C , where the lunar diameter is perpendicular to OA (the world diameter that passes through all the centres), B must move contrary to the epicycle movement in order to maintain the diameter directed towards O . At this position, the angle OCD reaches its maximum. (ii) As the epicycle continues its movement, the angle becomes smaller and smaller and at the perigee of the eccentric P , the prosneusis line OC coincides with the world's diameter OA . To move the diameter OC towards the perigee, B must move in the opposite direction to its previous movement, i.e. in the same direction as that of the epicycle. Within a period of half a lunar month, the assumed body B has thus to perform two opposite movements. The same can be shown for the other half of the lunar month.

Ibn al-Haytham thus shows that the prosneusis hypothesis needs another assumption itself: an entity which can perform not only two kinds of motion, but two motions which are required to occur in opposite directions. What is the status now of what can be called an assumption of second-order? Can this assumption be accepted by assuming that the denoted object is by no means a physical reality but rather an imaginary one? We have already mentioned that Ibn al-Haytham rejects the idea of assuming motion in an imaginary entity. But the second-order assumption is worse, since it attributes to an entity the capacity of performing two contrary motions. Can we accept it nevertheless on the grounds that the object which performs two contrary motions is a more imaginary entity than the object which moves uniformly around a point other than its centre? In short, the second-order assumption is a property of second-order entities. It appears then that the imaginary status not only further widens the gap between mathematics and physics but seems to give an absolutely free hand to the mathematician-astronomer to assert whatever he imagines suitable for his calculations. Is there no restraint

whatsoever on the kind of assumptions he may make or on the conditions they should satisfy? If this is the case, this means that to account for the empirical phenomena, the mathematician-astronomer does not seek to make the right assumptions since there is no such idea of right assumptions, that is, there is no formal criterion on which he has to base his assumptions. But since he declares that his assumptions are, accordingly, not necessarily about the actual world, how can we thus determine whether his language machinery makes sense or not? It is in this context that we have to understand why the most virulent attack ever launched by Ibn al-Haytham against Ptolemy's arguments is the one involving the second-order assumption.

Now this is an absurd impossibility: I mean that one and the same body should possess two opposite, natural and permanent motions. And if it is said that the two motions are voluntary, it will follow that one part of the heaven makes two opposite choices and therefore its substance must consist of two opposite substances or of a multitude of opposite substances. And this is regarded as impossible by *all philosophers* □□□□□□□□ □□□□ عند و هذا محال (p. 19, my emphasis).

And further ahead he says about this same assumption according to which a body can have two contrary substances: "this is an impossibility which we must refrain from considering; and it is still more the case for heavenly bodies because that [assumption] is worse than a [mere] contradiction *و هو من المحال الممتنع. و هو في الأجسام السماوية أشدُّ امتناعاً، لأن تلك أبعد من التضاد* (p. 36). This is not the only place where Ibn al-Haytham explicitly mentions the philosophers. He also appeals to them when he discusses a similar point.

And the *inquirers among philosophers believe* □□□□□□□□ □□□□□□□□ □□ □□□□□□□□ □ that no two opposite motions can exist in the heavens. It is one of the most absurd impossibilities *المحال الممتنع* that they can be possessed by one and the same body of the heavenly bodies (p. 37, my emphasis).

By bringing the philosophers into the controversy, Ibn al-Haytham wants to make two important points: (i) the controversy has reached a point they can no longer ignore. The point is no longer a technical matter, as was first thought, but is now of philosophical interest since the question concerns the foundation of astronomy; and as a result (ii) he urges them to take a firm stance on the issue. This is rather a clever and powerful move which could have serious consequences. By involving the philosophers into the discussion, Ibn al-Haytham no doubt hopes they will take up the matter more deeply — an interesting attempt aimed at further radicalising the controversy: if it succeeds, it can only speed up the collapse of Ptolemy's system by bringing to the surface all its hidden assumptions and weaknesses. The aim of Ibn al-Haytham's argument is to establish that "each body having to have only one kind of motion" is a second fundamental principle of motion; the first being uniform circular motion. It is now easy for him to show by *reductio ad absurdum* that the assumed body required by the *prosneusis* hypothesis cannot exist.

(i) Assume that there is such a body *B*;

(ii) *B* must have two opposite motions;

(iii) But according to the second principle of motion, a body can have only one kind of motion;

(iv) Therefore *B* cannot exist.

And from the non-existence of *B* it follows that the *prosneusis* hypothesis cannot be justified. Ibn al-Haytham uses a similar line of reasoning to reject the equant hypothesis introduced by Ptolemy in his account of the motion of the superior planets.

4.1.2. The refutation of the equant hypothesis

Ibn al-Haytham begins by presenting Ptolemy's argument.

He says in Book IX, chapter 2, which is on the principles that need to be laid down for the wandering planets: 'Since it is our aim to show in the case of the five wandering planets, as we

showed in the case of the sun and the moon, what all their apparent irregularities are, and that they are brought about by means of regular and circular motions, inasmuch as such motions are conformable to the nature of divine bodies and do not admit of disorder and irregularity.'

And he says in Book IX, Chapter 5, which is on what needs to be put forward with respect to the principles employed for the five wandering planets, that he assumes for each of the five planets an eccentric sphere and an epicyclical sphere, and that he made the eccentric move the epicycle. Then he says at the end of this Chapter:

'And we also found that the centres of the epicyclical spheres move on circles equal to the eccentric spheres, I mean those that produce the irregularity. But these circles are not about the same centres. Rather, in the case of all the five planets except Mercury, they are about centres that bisect the straight lines between the centres of eccentrics and the ecliptic's centre. And, in the case of Mercury, [the circle is] about a centre distant from the centre that turns it around by as much as this [latter] centre is removed towards the apogee from the centre about which the motion of the anomaly takes place, of as much as this [last] centre is removed from the centre where the eye is placed. (p. 23-24)

After giving a faithful description of Ptolemy's general account for the motion of the five planets (Chapter 6), Ibn al-Haytham adds: "That which we have mentioned is the truth of what Ptolemy asserted regarding the motions of the five planets. But this is a notion that leads to the contradiction." In other words, Ibn al-Haytham considers that what Ptolemy states in Chapter 5 (implemented in Chapter 6), in which he introduces the equant notion as simply the point around which the centre of the epicycle moves uniformly, is in flagrant contradiction with the principle of uniform motion of the same book, reaffirmed just 3 chapters before.

The proof of the contradiction is constructed as follows:

- (i) according to the principle of uniform circular motion: a spherical body moves uniformly and around its centre;
- (ii) a spherical body cannot move uniformly and at the same time around two points; Ibn al-Haytham stresses here that it is Ptolemy himself who establishes the validity of this statement in Book III Chapter 3, devoted to the motion of the sun;
- (iii) Ptolemy assumes that the centre of the epicycle moves uniformly around the equant;
- (iv) from (ii) and (iii): the centre of the epicycle does not move uniformly around the centre of its own deferent;
- (iv) contradicts (i)

As in the case of the prosneusis hypothesis, Ibn al-Haytham envisages cases in which the motion of the centre of the epicycle towards the equant point and not the centre of the deferent could be brought about by an assumed body. And he ends up with the following conclusion: the assumed body should not only have two opposite motions but would also be the cause of an irregular motion.

If he assumes for the epicycle a body which moves it in such a way as to direct its diameter towards the farther point [i.e. the equant], as we assumed in the case of the moon epicycle with respect to the opposite point [i.e. the prosneusis], it will follow that this body has two opposite motions, just as this followed in the other case. It will also follow that [the assumed body] will move the diameter about the farther centre with an irregular motion, given that the epicycle's motion about the centre of the deferent is regular, as was shown earlier. But to assume the existence of a body of this description is impossible. (p. 28-29)

The clash between the mathematician-physician and the physician-mathematician over the structure of a planetary theory reflects in fact two underlying opposite philosophical approaches to the aim of a physical theory.

4.2. The controversy over the aim of a physical theory: saving the appearances or explaining their underlying regularities

Ptolemy is well aware of the objections that could be raised against his approach. By adopting the prosneusis and equant hypotheses, he knows very well that he is departing from the

traditional way of studying astronomy. To defend his new approach against possible attacks, Ptolemy uses the following argument:

Let no one, considering the complicated nature of our devices, judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what [could one compare] more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself? (Ptolemy 1984, p. 600)

It is interesting to note that to justify his new hypotheses Ptolemy uses, ironically, the well known dogma of ancient Greece. By insisting on the radical distinction between the supra and sublunar phenomena, he tries to abort any possible rapprochement between his hypotheses required by the motion of heavenly bodies and those required by terrestrial bodies. It is the attempt to establish this kind of rapprochement through the interpretation of his imaginary entities by physical objects that leads to the kind of absurdities pointed out by Ibn al-Haytham.

Why should anyone think it strange that such complications can characterise the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another? Thus, quite simply, all the elements can easily pass through and be seen through all other elements, and this ease of transit applies not only to the individual circles, but to the spheres themselves and the axes of revolution. We see that in the models constructed on earth the fitting together of these [elements] to represent the different motions is laborious, and difficult to achieve in such a way that the motions do not hinder each other, while in the heavens no obstruction whatever is caused by such combinations. (ibid., pp. 600-601)

In his commentary of this passage, Duhem expresses more clearly the thinking behind Ptolemy's approach.

C'est donc folie de vouloir imposer aux mouvements des corps célestes l'obligation de se laisser figurer par des mécanismes de bois ou de métal. [...] Les mouvements multiples qu'il compose, dans la *Syntaxe*, pour déterminer la trajectoire d'un astre, n'ont aucune réalité ; le mouvement résultant est le seul qui se produise dans le ciel. (Duhem 1913, volume II p. 85)

Ptolemy and his commentator warn us against the temptation of interpreting *Almagest's* complex machinery since it is the combination of purely fictional entities that have nothing to do whatsoever with the real physical world. Ibn al-Haytham, for his part, seems to accept Ptolemy's argument that the *Almagest* is an imagined theory, but he infers from this that his system cannot be regarded as an account of the actual motion of the heavenly bodies. And since Ptolemy admits that his theory is the product of his mind, Ibn al-Haytham concludes that the resulting motion of the various planets occurs solely in his own imagination, "that configuration produces in his own imagination the motions that belong to the planets ما هي عليه (p. 38). But for Ptolemy, that is not the point. His hypotheses should not be judged other than by the result they produce i.e. the conformity of their consequences with empirical phenomena, and the simplicity criterion.

One should try, as far as possible, to fit the simpler hypotheses to the heavenly motions, but if this does not succeed, [one should apply hypotheses] which do fit. For provided that each of the phenomena is duly saved by the hypotheses, [...] (Ptolemy 1984, p. 600)

This passage contains some keywords which have given rise to the title of Duhem's popular booklet *Sauver les Apparences*, an extremely condensed version of his monumental work *Le Système du Monde*. Duhem further develops the saving appearances doctrine to defend Ptolemy's approach.

Les diverses rotations sur des cercles concentriques ou excentriques, sur des épicycles, rotations qu'il faut composer entre elles pour obtenir la trajectoire d'un astre errant, sont seulement des

artifices ; ces artifices sont combinés en vue de sauver les phénomènes à l'aide des hypothèses les plus simples qui se puissent trouver. Mais il faut bien se garder de croire que ces constructions mécaniques aient, dans le ciel, la moindre réalité. (Duhem 1913, volume II p. 85)

Consequently, the evolution of astronomy has, according to Duhem, led Ptolemy to propose — for the first time since Plato — a major shift in the task of the astronomer.

L'astronome de Péluse [i.e. Ptolemy] dut reconnaître que des règles aussi rigides laisseraient malaisément construire une théorie capable de sauver les apparences ; ces règles, il les assouplit peu à peu jusqu'à les fausser ; il en vint enfin à professer cette doctrine : l'astronome qui cherche des hypothèses propres à sauver les mouvements apparents des astres ne doit connaître d'autre guide que la règle de la plus grande simplicité. (ibid., p. 84)

The saving appearances doctrine looks so powerful that it is hard to imagine stronger counterarguments capable of challenging it, let alone defeating it. This explains why it dominates the astronomical scene for so many centuries. It remains to be seen whether Ibn al-Haytham can change the situation in his favour by successfully refuting or at least undermining the saving appearances position, and if so by what means this change could be brought about. Ibn al-Haytham adopts a clever strategy to attack Ptolemy's approach by considering the price to be paid for saving the appearances. He begins as usual by quoting a passage from the crucial Book IX, Chapter 2 of the *Almagest* in which Ptolemy admits that his conception is a departure from the intuitive way of reasoning.

And this, I suspect, appeared difficult even to him – he means to Hipparchus. The point of the above remarks was not to boast of our own achievement. Rather, for we are at some point compelled by the nature of our subject to use a procedure not in strict accordance with the intuitive way of reasoning. For instance, when we carry out proofs using without further qualification the imaginary circles described in the planetary spheres by the movement of the body. (p. 33)

And he then shows that this departure from the intuitive way of reasoning is a consequence of the contradiction of his hypotheses with the extant principles.

Ptolemy has thus admitted that his assumption of motions along imaginary circles is the intuitive way of reasoning, then it would be more so for imaginary lines to move around assumed points. And if the motion of the epicyclic diameter around the distant centre [i.e. the equant] is also a departure from the [intuitive] way of reasoning, and if the assumption of a body that moves this diameter around this centre is also a departure from the [intuitive] way of reasoning for it contradicts the principles, then the arrangement, which Ptolemy had organised for the motions of the five planets, is a departure from the [intuitive] way of reasoning. (p. 33)

What is the point of quoting this passage, however important it may be, for Ibn al-Haytham? Even if Ptolemy has admitted that he has contradicted the established principles, this admission seems to have no effect on his overall position. On the contrary, Ptolemy seems to justify his departure from the intuitive way of reasoning by appealing to pragmatic reasons: the need to offer a workable account for the apparent irregularities of the motions of the planets. So far, Ibn al-Haytham's argument can be seen as a weak argument. But the situation changes dramatically when he adds

Ptolemy has admitted he had used in [the construction of] the configurations of the motions of the planets, some considerations which depart from the intuitive way of reasoning, and it is these considerations that necessarily lead him to the contradiction. For the contradiction which is necessarily involved in his configurations of the motions of the planets is due to his assumptions of attributing motion to imaginary circles and lines and not to existing bodies. But once the existence of real bodies is assumed, the contradiction then became clear (pp. 38-39).

Ibn al-Haytham seems to be saying in this crucial passage that the contradiction is already there before the interpretation of the imaginary entities in terms of concrete objects. And that this contradiction follows immediately from his adopting a mode of reasoning which is contrary to the intuitive way of reasoning; in other words the contradiction is internal since it is of a conceptual nature. But where does he see the contradiction? What is the sign that

indicates that Ptolemy's system contains contradictory elements? We can find some clue to the answer in Ibn al-Haytham's use of the plural "configurations" rather than the frequently singular "one configuration." And further, because here the plural noun is used twice and in conjunction with assumptions (or considerations) — since different assumptions give rise to different configurations. The consequence of departing from the intuitive way of reasoning is the fragmentation of theoretical astronomy. Ptolemy believes that he can escape the difficulty by stressing that his imaginary entities are not subject to interpretation. For Ibn al-Haytham, this approach can only make science a more confused and complicated enterprise. By radicalising the distinction between the supra and the sublunar world, Ptolemy makes it impossible to find common principles, let alone unifying ones, by which the motion of heavenly bodies could be explained. The task of the astronomer is rather hopeless: not only are there no common principles which could explain the behaviour of the planets, but different planets have different assumptions since each planet has its own peculiar course of motion.

We know, finally, that some variety in the type of hypotheses associated with the circles [of the planets] cannot plausibly be considered strange or contrary to reason especially since the phenomena exhibited by the actual planets are not alike [for all] (Ptolemy 1984, p. 423).

Furthermore, the motion of heavenly bodies cannot be understood in the same way as the motion of terrestrial bodies, since the principles of supralunar phenomena have nothing to do with those of sublunar phenomena. Ptolemy expresses this idea unambiguously when he discusses the notion of simplicity.

We should not judge 'simplicity' in heavenly things from what appears to be simple on earth, especially when the same thing is not equally simple for all even here. For if we were to judge by those criteria, nothing that occurs in the heavens would appear simple, not even the unchanging nature of the first motion, since this very quality of eternal unchangingness is for us not [merely] difficult, but completely impossible (ibid., p. 601).

The resulting account of the *Almagest* is not an account of the structure of the planetary system but rather a mere collection of geometric constructions of the imaginary motion of individual planets that bear no relation whatsoever to each other. Let us give a brief summary of the account for the motion of the heavenly bodies and their corresponding assumptions:

The motion of the sun is accounted for by the principle of uniform circular motion.

The motion of the moon is accounted for by the prosneusis hypothesis.

The motion of the four planets is accounted for by the equant hypothesis.

The motion of Mercury is accounted for by mixed prosneusis-equant hypotheses.

There is no contradiction if the *Almagest* is considered as a piecemeal theory. But as a unified theory, the prosneusis and equant hypotheses contradict the principle of uniform circular motion as we have already explained.

Ibn al-Haytham opposes Ptolemy's cosmological conception with the following:

The truth that leaves no room for doubt is that there are correct configurations for the movements of the planets, which exist, are *systematic*, and entail none of these impossibilities and contradictions حركات الكواكب هيئات صحيحة موجودة □□□□□□ لا فيها شيء من المحالات و لا من المناقضات but they are different from the ones established by Ptolemy (p. 64, my emphasis).

And he concludes more forcefully:

It becomes clear, from all that we have shown so far, that the configuration, which Ptolemy had established for the motion of the five planets, is a false configuration, and that the motions of these planets must have a correct configuration, which includes bodies moving in a uniform, perpetual, and continuous motion, without having to suffer contradiction, or be blemished by any doubt. That configuration must be other than the one established by Ptolemy. (p. 34)

What is required is not a piecemeal configuration that accounts individually for the motion of the heavenly bodies but a single systematic configuration whose validity is determined by its ability to be interpreted by means of physical objects. Here Ibn al-Haytham shifts the course of argumentation. It is not merely the question of subordinating the principles of physics to those of mathematics (or vice versa) but as Sabra⁴ points out the problem is much deeper than that: it is the question of the possibility of the existence of astronomy as a theoretical physical science. Astronomy cannot exist as a truly physical science without a coherent theoretical structure based on some fundamental principles that could explain a wide range of apparently unrelated phenomena. Ptolemy's conception yields an astronomy of irregularity/account: his starting point is that the heavenly bodies present certain irregularities, and the problem of the astronomer is how to account for what is taken as merely apparent anomalies so that he can determine as accurately as possible the future positions of the various planets. But for this we do not need the whole machinery of the *Almagest* since the Babylonians have succeeded in predicting some cosmological phenomena such as lunar eclipses with remarkable accuracy by simply accumulating a large amount of data over many centuries. As one scholar of the history of ancient astronomy remarks: "the first successes in predicting the behaviour of the planets came from the recognition that, over long enough time intervals, the patterns repeat." (Evans 1998, p. 312). This could be seen as a failure on the part of Ptolemy since the motivation of the whole *Almagest* enterprise represents a step backwards. Ibn al-Haytham's approach, on the other hand, yields an astronomy of explanation and understanding, as Neugebauer has already noticed:

[Ibn al-Haytham's] objections are right on the mark if Ptolemy's models are to be taken seriously as physical bodies in the heavens. And there is no doubt that Ibn al-Haytham and other astronomers wished to do so, that is, they were not content with a mathematical representation of the apparent motions of the planets using models that were to be taken only as geometry, but wished to *understand the structure of the physical mechanisms*, composed of rotating spherical bodies, that carried the visible bodies of the planets through their apparent motions. (Neugebauer 1984, p. 44; my emphasis)

Ibn al-Haytham's remarkable insight is not only that our actual world could be understood but that it should be understood, and mathematics is just a tool helping us reach that goal by putting to the test the various mathematical theories. That is what his optical works taught him.⁵ According to this view, to account for irregularities is to explain them: why does the motion of the heavenly bodies appear irregular? Why is the behaviour of the inferior planets more irregular than that of the superior planets? It is the answers to questions like these which can lead to a theoretical system in order to explain the order underlying the apparent anomalies. When such an explanation is given, the so-called anomalies will no longer be regarded as such due to the increase in our understanding of the relationship between the heavenly bodies. In sharp contrast with the saving appearances doctrine, Ibn al-Haytham states time and again that there is a correct single configuration for the motion of the heavenly bodies. But how can he be sure of the existence of such a configuration? And by what means can it be found? As for the first question, Ibn al-Haytham claims that "it is not true there should be uniform, perceptible and perpetual motion which does not have a correct configuration in existing bodies" (*al-Shukūk*, p. 42). Ibn al-Haytham's strong conviction of the existence of a correct configuration is based on a more general principle: the regularity of motion of the planetary system. Furthermore his firm conviction of the existence of a "correct configuration in existing bodies" reflects the close connection between Ibn al-Haytham's structural conception of the universe and his model-theoretical approach in the investigation of natural phenomena. Such a systematic configuration can only be discovered by adopting certain basic assumptions as principles of its construction. And the same passage also indicates the means by which the correct configuration could be brought about: it is the

principle of uniform circular motion. But why should uniform circular motion be regarded as the best candidate?

- (i) It is impossible for the motion of the planets, which is perpetual, uniform, and unchanging to be contrary to the intuitive way of reasoning. (ii) Nor should it be permissible to attribute a uniform, perpetual, and unchanging motion to anything other than correct principles, (iii) الذي لا شبهة فيه واجبة بالقياس المطرد (p. 34, with the numbers added).

The crucial sentence of this passage is translated by Saliba as “(other than correct principles) which are necessarily due to accepted assumptions that allow no doubt” (Saliba 2000, p. 78) and by Sabra as: “(except in accordance with true hypotheses) entailed by consistent reasoning that is subject to uncertainty” (Sabra 1998, p. 302). The difference between the two translations is an indication of the difficulty of what seems to be an extremely condensed Arabic expression. To put it in broad terms, it seems that Saliba attempts to capture the intended meaning while Sabra’s translation tries to be literally closer to the original sentence. Let us look more closely at the whole passage in which Ibn al-Haytham makes the three following points.

(i) If there are correct principles other than uniform circular motion then the latter cannot be contrary to the former.

(ii) Hence uniform circular motion can be added to the correct principles and by doing so it becomes itself a correct principle.

Now a theory of motion whose assumptions are contrary to uniform circular motion is a theory which disputes in fact the correct principles. The dispute then moves to the higher level i.e. to the meta-theoretical level since we have two sets of principles which are contrary to each other. Which of the two theories has thus the burden of proof: the established or the challenging theory? (iii) It is in this context that we have to understand Ibn al-Haytham’s third point and the whole enterprise of *al-Shukūk*. He tries to show that the principles of the extant theory are not accepted or taken as correct dogmatically but that these established principles are correct because they can be justified. It remains to be seen how. A linguistic analysis of the Arabic expression is the first step in this direction.

(1) *Wājiba* (واجبة): Sabra translates this word by “entailed”, but this term alone does not render the idea of necessity contained in the original term. *Wājiba* is from *wājib* which commonly means duty. The word is used here in the passive present participle, and the combination of both the meaning and the grammatical form suggests the following translation: (something) required or necessarily due (by virtue of something, usually a condition or a rule). This is the translation proposed by Saliba (“necessarily due”).

(2) *Al-qiyās al-mu}arid* (القياس المطرد): Saliba’s translation is “accepted assumptions”, and “... *wājiba bil-qiyās al- mu}arid*” is rendered by “correct principles which are necessarily due to accepted assumptions”. In some sense he is right by virtue of the use of the word *wājiba*. Saliba prefers however to use “assumptions” instead of stronger terms such as rules or conditions as implied by the Arabic word *wājib*, as explained above. Furthermore, Saliba’s translation establishes a logical consequence relation between (accepted) assumptions and (correct) principles; this leaves the question of the justification of accepted assumptions unanswered. That is why here we follow Sabra, who tries to be closer to the proper meaning of *al-qiyās al-mu}arid* by translating it as “consistent reasoning”. Sabra does not specify, however, in what way “consistent” should be understood, since it is much more than the mere non-contradiction meaning which is intended here. *Mu}arid* usually means systematic, regular, but it can also mean general, such in, for example, قاعدة مطردة a general rule. The latter seems to be more convenient since the subject of reasoning is the principles of a given theory. From this point of view, *mu}arid* can be understood as simply referring to the level of reasoning, i.e. meta-theoretical reasoning. We shall come back to *al-qiyās al-mu}arid* later.

(3) *Lā shubhata fih* (لا شبهة فيه): by translating it as “uncertain”, Sabra is clearly far away from the intended meaning since this expression conveys in no way the idea of certainty. Sabra probably chooses this word because of the necessarily logical consequence that he establishes in his translation between “true hypotheses” and “consistent reasoning”. According to this reading, hypotheses are true because they are (necessarily) entailed by uncertain reasoning. We suggest proceeding the other way round: it is rather *lā shubhata fih* which specifies the quality of reasoning involved. Here we follow Saliba’s translation of this expression: “(allow) no doubt”. But the proper meaning of *lā shubhata fih* is “unambiguous”, and this meaning is complementary to that proposed by Saliba since it specifies the way by which doubt is raised. So *lā shubhata fih* indicates the nature of meta-theoretical reasoning involved in establishing the principles of a scientific theory, i.e. the meta-theoretical reasoning should be so unambiguous that no doubt can be raised. To put it positively, the meta-theoretical reasoning should carry conviction or more simply “a convincing meta-theoretical reasoning”. This translation agrees with the translation of *khārija ‘ani al-qiyās* by “intuitive way of reasoning.” Furthermore our interpretation is confirmed by some passages of Ibn al-Haytham where he uses explicitly “conviction” and similar words in discussing Ptolemy’s arguments. In page 45, he says for example: “then he [Ptolemy] mentions the proportions of the distances of the planets from the earth and their values in a convincing way بطريق إقناعي”; while in another passage he says that the value of eccentricity used by Ptolemy is unreliable غير موثوق به (p. 32). According to our interpretation, the translation of the full Arabic expression is: correct principles which are necessarily due to a convincing meta-theoretical reasoning. But there is a difficulty here: how is something (principles) necessarily due to something (convincing reasoning) which is merely probable? This explains why Saliba substitutes accepted assumptions for *al-qiyās al-mu}arid* to avoid the necessarily logical consequence relation between principles and convincing reasoning. We have already mentioned the problem posed by the substitution of accepted assumptions for *al-qiyās al-mu}arid*. On the other hand Saliba is right in using some more general propositions (to which the correct principles are due) by virtue of the use of the grammatical form *wājib*. The underlying and interesting idea of Saliba’s is that *wājib* implicitly introduces an intermediary notion between the principles of a theory and the meta-theoretical reasoning. As explained above, however, Saliba prefers to use “assumptions” instead of stronger terms such as conditions or rules as implied by the Arabic word. By sticking to the grammatical use of *wājib*, the resulting translation will shed new lights on Ibn al-Haytham’s thought: “correct principles that should satisfy conditions which are established by a convincing meta-theoretical reasoning”. Is this not precisely what Ibn al-Haytham is doing in his *al-Shukūk*? Is he not trying to establish, by means of a convincing meta-theoretical reasoning, the conditions which should be justified by the principles of a scientific theory? From our analysis of the controversy, Ibn al-Haytham seems to identify at least two conditions which should be justified by assumptions designed to play the role of principles of an astronomical theory: (1) They should have some universality or generality feature so that they can work as unifying principles. (2) They should be consistent with the principles of physics. Ibn al-Haytham mentions two kinds of such principles: (i) a body cannot have two contrary properties at the same time; (ii) imaginary entities can be used provided they are not given physical properties. These are formal conditions which are imposed on the construction of an astronomical theory, and it is up to the physician-mathematician to specify their content. To Ibn al-Haytham, it seems that uniform circular motion could be such a principle since it satisfies both conditions and there is no evidence up to now that heavenly bodies move otherwise. This explains why any configuration of the planetary system should be set up on the principle of uniform circular motion. It also explains why the challenging theory has the burden of proof:

(1) it must first make its principles explicit;

(2) it must show that its principles are correct, i.e. that they satisfy the conditions established in the meta-theory;

(3) it must show that the principles of the established theory are not correct either by challenging one or several conditions underlying its principles or by showing that the established theory does not fully satisfy the underlying conditions.

This is the underlying method followed by Ibn al-Haytham in his optical studies. In sharp contrast, Ptolemy fails to satisfy any of the three criteria mentioned above. As a result, Ibn al-Haytham excludes Ptolemy's *Almagest* from being a possible model of our actual world. Nor is he interested in a scientific theory which is about an imaginary world and not about our world, as Saliba rightly put it:

To Arabic astronomers the world was constituted in only one of two ways: either it was made of real physical bodies that retained their physical properties throughout the process of accounting for their observable behavior, or of imaginary mathematical concepts that do not apply to this particular world that we see. One could not have it both ways, as Ptolemy seemed to be doing (Saliba 2000, p. 331).

As for the saving appearances doctrine invoked by Ptolemy as a justification for his hypotheses, Ibn al-Haytham responds:

Ptolemy assumed an arrangement that cannot exist, and the fact that this arrangement produces in his imagination the motions that belong to the planets does not free him from the error he committed in his assumed arrangement, for the existing motions of the planets cannot be the result of an arrangement that is impossible to exist [sic.] (p. 38).

And a little further after quoting Ptolemy's following statement from Book IX, Chapter 2: "if something assumed without proof is later found to agree with the phenomena, then it could not have been discovered without following one of the ways of scientific knowledge, even though it would be difficult to describe how it was apprehended", he comments

The way Ptolemy followed was indeed a legitimate beginning, but since it led him to what he himself admitted to be a departure from the intuitive way of reasoning, he should have declared his assumed arrangement to be false (p. 39).

(1) Ibn al-Haytham interprets the saving appearances argument as an admission of failure by Ptolemy since he does not show that the motion of the heavenly bodies could not be accounted for by the principle of uniform circular motion, i.e. Ptolemy fails to show that the planets can perform a motion other than uniform circular motion. (2) The second reason, akin to the first, is that Ptolemy fails to recognise the possibility of finding a more consistent configuration with the principles of physics, thus giving the impression that there is no other theory than his own. It is because of the second reason that Ibn al-Haytham uses the following harsh words against Ptolemy.

Ptolemy either knew of the impossibilities that would result from the conditions that he had assumed and established, or did not know. If he had accepted them without knowing of the resulting impossibilities, then he would be incompetent in his craft, misled in his attempt to imagine it and to devise configurations for it. And he would never be accused of that. But if he had established what he established while he knew the necessary results – which may be the case befitting him – with the reason being that he was obliged to do so for he could not devise a better solution, and [on top of that] he went ahead and knowingly fell into these contradictions, then he would have erred twice: once by establishing these notions that produce these impossibilities, and the second time by committing an error when he knew that it was an error. When all is considered, and to be fair, Ptolemy would have established a configuration for the planets that would have been free from all these impossibilities, and he would not have resorted to what he had established – with all the resulting grave impossibilities – nor would he have accepted that if he could produce something better. The truth that leaves no room for doubt is that there are correct configurations for the movements of the planets, which exist, are systematic, and entail none of these impossibilities and contradictions, but they are different from the ones established by Ptolemy.

And Ptolemy could not comprehend them, nor could his imagination come to grips with them (p. 63-64).

The controversy stretches over to the epistemological and philosophical field, considerations which motivate first and foremost the composition of *al-Shukūk*. By reflecting on his job and on his discipline, the scientist turns surprisingly into a profound philosopher of science.

5. The epistemological dimension of *al-Shukūk*: science as an open system and the progress of science as a process

According to Ibn al-Haytham, Ptolemy's crime is not so much that he knowingly proposes a false configuration — after all no human being is immune from error, as he points out in his introduction: "God has not preserved the scientist from error and has not safeguarded science from shortcomings and faults." Ptolemy's crime is that he presents his work as an achieved science by overlooking the difficulties he encounters and denying thus, at least implicitly, the possibility of the existence of more acceptable theories. For ignoring the existence of a correct model has the effect of preventing Ptolemy from making any suggestion which could lead to the discovery of much more improved theories than his own. In short, Ptolemy's approach is not the right path to be followed if science is to make any progress at all. The synthetic approach, which characterises the Greek way of exposing science, has no doubt the advantage of systematically presenting a scientific theory. But its drawback is that it hampers more than it contributes to the progress of science by closing the door to further theoretical research. It is this synthetic feature of the *Almagest* which prompts Ibn al-Haytham to warn his fellow colleagues not to be lured by the mathematical systematic exposition of any scientific theory by taking the truth of its results for granted or by naïvely following the methods of their predecessors.

Truth is sought for its own sake. And those who are engaged upon the quest for anything that is sought for its own sake are not interested in other things. Finding the truth is difficult, and the road to it is rough. For the truths are plunged into obscurity. It is natural to everyone to regard scientists favourably. Consequently a person who studies their books, giving a free rein to his natural disposition and making his object to understand what they say and to possess himself of what they put forward comes to consider as truth the notions they had in mind and the ends which they indicate (p. 3).

To prevent such a situation from happening, Ibn al-Haytham makes it the duty of every scientist to strongly and thoroughly challenge the claims of his predecessors. In fact the progress of science requires of a scientist not only that he adopt the systematic exposition of science but also that he complement this synthetic method by the systematic critical analysis of his theory and of the theories of his opponents through the exchange of arguments and counterarguments.

It is not the person who studies the book of his predecessors and gives a free rein to his natural disposition to regard them favourably who is the [real] seeker after truth. But rather the person who in thinking about them is filled with doubts, who holds back with his judgement with respect to what he has understood of what they say, who follows proofs by argumentation (الحجة) and demonstration (البرهان) rather than the assertions of a man whose natural disposition is characterised by all kinds of defects and shortcomings. A person who studies scientific books with a view of knowing the real facts (الحقائق), ought to turn himself into an opponent of everything that he studies, he should thoroughly assess its main as well as its margin parts, and oppose it from every point of view and in all its aspects. And while thus engaged in his opposition, he should also be suspicious of himself and not allow himself to become abusive or be indulgent [in his assessment]. If he takes this course, the real facts (الحقائق) will be revealed to him, and the possible shortcomings and flaws of his predecessors' discourse will stand out clearly (pp. 3-4).

Ibn al-Haytham is not a naïve or narrow-minded realist as he is often portrayed by some historians such as Duhem. He not only envisages the possibility that the same phenomena could be accounted for by more than one theory, but he seems to be well aware of the fact that producing a new conflictual theory is not sufficient to destroy its predecessor. The destruction of the established theory can only be brought about by defeating its central arguments. This view is confirmed to us by what al-Bayhaqi reports him to have said

We have imagined certain modes appropriate to the celestial movements, if we now imagine other suitable to the same movements, there would be no objection to them, as long as it has not been proved that the first modes imagined are the only ones tenable (al-Bayhaqi 1946, p. 87).

Many historians of science who review *al-Shukūk* seem to be disappointed in stressing that Ibn al-Haytham adopts a merely negative stance in astronomy in contrast with his outstanding positive contributions in optics. This shows considerable misunderstanding of Ibn al-Haytham's real intention. And I think we cannot fully understand the epistemological motivations of Ibn al-Haytham if we isolate *al-Shukūk* from his other writings, and mainly from his optical works, as many historians have done. Since Ibn al-Haytham has already produced a major treatise in which he proposes an optical theory alternative to that of Ptolemy, what is the point of devoting the last chapter of *al-Shukūk* to a critical examination of Ptolemy's optical theory? This seems just another detail since it is ignored by nearly all historians. By including a critical review of Ptolemy's optics similar to that of astronomy, Ibn al-Haytham wants to inject some form of dynamism into astronomy through the power of argumentative analysis. In particular, he wants to remind his readers that his new and successful optical theory does not come out of the blue, but is the result of challenging Ptolemy's theory by putting to the test his fundamental claims. His critical analysis of many of Ptolemy's arguments proves beyond any doubt that the latter's theory is false and that a new theory is needed for optics. And Ibn al-Haytham is strongly convinced that the same can be done for astronomy. In *al-Shukūk*, he has done the hard part of the job by showing that the *Almagest* cannot be a correct configuration of the planetary system. It is up to later generations of astronomers and philosophers to finish the job.

When we examine the writings of a man who, is famous for his excellence, shows great ingenuity in his mathematical ideas, and who is [always] being cited in the true sciences, I mean Claudius Ptolemy, we find in them many scientific doctrines and precious, most instructive and useful ideas. But when we oppose and we assess them, and we enquire into doing justice to him and taking a fair decision between him and the truth, we find in them obscure passages, improper terms, and contradictory notions; there are however fewer of them, if they sit beside the correct notions which he hit upon. In our view, to disregard all this is not to serve the truth, illegal, and to act unjustly towards *those who will examine his writings after us* by not disclosing all that. We find that first and foremost we have to mention their places, and to make them explicit for those who afterwards want to make an effort in filling the gaps and correcting these notions by any means susceptible to lead to the truth (p. 4, my emphasis).

As Morelon rightly points out (Morelon 2000, p. 110), this sounds like a research programme proposed by Ibn al-Haytham to his successors: to find a correct and systematic configuration, based on the correct principles of physics, which can explain the regularities underlying the apparent motions of the planets through its interpretation into real spherical bodies. *Al-Shukūk* is not simply a book about astronomy as is generally believed; it is much more profound than that. It is an unprecedented philosophical and epistemological doctrine on how progress in science can be achieved. The book is clearly divided into two parts; the first of which is an introduction, extremely condensed and poetical (to ensure that his discourse will have as much effect as possible), in which Ibn al-Haytham exposes his dynamic approach to the development of science. The rest of the book is an implementation of this approach to astronomy, the most advanced scientific discipline.

6. The impact of *Al-Shukūk*: the deepening of controversies over the foundations of astronomy

Al-Shukūk was widely known and quoted both in eastern and western Arabic countries. His strong counterarguments to the *Almagest*'s hypotheses have had the desired effect, since his forceful appeal has been answered by both astronomers and philosophers. Sabra describes in these terms the extent of the influence of Ibn al-Haytham's book in the East:

We now know that Abū 'Ubayd al-Jūzjānī, the pupil of Avicenna, not only discussed the equant problem with a view to solving it, as was first made known by Saliba in 1980, but did so almost certainly under the influence of Ibn al-Haytham's writings. [...] That Tusi was acquainted with at least some of Ibn al-Haytham's writings about Ptolemy's configurations is known from his direct references to the latter's *Treatise on the Winding Movement* both in the *Memoir on Astronomy (al-Tadhkira)* and in the early Persian treatise. In his "Book on al-Hay'a", 'Urdu mentions Ibn al-Haytham by name as one of two astronomers who had raised doubts against the Ptolemaic configurations for planetary motions but stopped short of any solution (the other astronomer being "Ibn al-'Aflah al-Maghribī", who flourished in the middle of the twelfth century). His discussion before and after this explicit mention contains statements and expressions that leave no doubt that he not only read Ibn al-Haytham's *Aporias Against Ptolemy [al-Shukūk]*, but more importantly, that he shared the book's premises and its general diagnosis (Sabra 1998, pp. 304-306).

It is remarkable that after intense theoretical research carried out by eastern astronomers of the Marāgha School, a positive answer has been given to the technical part of Ibn al-Haytham's demand. The climax of this intense activity was reached in the fourteenth century by Ibn al-Shātir who constructs mathematical models, alternative to those of Ptolemy, that contain no eccentrics whatsoever (see Saliba 1994, pp. 233-241 and pp. 299-302; also Saliba 1996, pp. 100-103, pp. 108-113 and pp. 120-121).

Interestingly, the fever of Ibn al-Haytham's *Doubts* spread to the West in Arabic Spain. *Al-Shukūk* is explicitly mentioned by the great Andalusian philosopher Ibn Bājja in a letter to Abū Ja'far Yusūf ibn Hasday (Pines [1962] 1964). In his *Summary of the Almagest*, the other great Andalusian philosopher Ibn Rushd explicitly invokes doubts expressed by Ibn al-Haytham concerning the movement of the moon (quoted in Duhem, volume II of *Le Système du Monde*, p. 127). The western Arabic philosophers deepen the controversy on the foundations of astronomy by explicitly rejecting out of hand the *Almagest* machinery. To illustrate such an outright rejection by the western tradition, we present the final verdict of Ibn Rushd taken from his *Commentary on Aristotle's Metaphysics* in which he condemns once and for all the Ptolemaic system.

The astronomer should build an astronomical system from which the heavenly motions follow and such that there is no impossibility from the physical standpoint ... Ptolemy has not succeeded in putting astronomy on its true foundations. The epicycle and the eccentric are impossible. It is then necessary to conduct new research for this true astronomy whose foundations are the principles of physics. In my opinion, this astronomy is based on the motion of a single orb simultaneously around two or several different poles; the number of these poles is suitable for the explanation of phenomena; such motions can account for the acceleration and the retardation of the stars, for their accession and recession motion, in one word for all appearances that Ptolemy failed to explain by means of a correct astronomy. In my youth I had hoped to accomplish this investigation, but now in my old age I have despaired of that, having been impeded by obstacles.

And he ends up with the same conclusion as that of Ibn al-Haytham

But let this discourse spur someone else to inquire [further] into these matters. For nothing of the true science of astronomy exists in our time, the astronomy of our time being only in agreement with calculations and not with what exists (in Duhem 1913, volume II, p. 138).

The new astronomy should thus be founded on the strict principles of physics: this task is left to a younger generation of astronomers. It is in this context that al-Bitrūjī composes his *on the*

Principles of Astronomy in which he proposes a new conception of the universe based on the principles of dynamics

The supreme body [the ninth orb] is distinct from the power which it bestows on those spheres below it, just as one throws a stone or shoots an arrow is distinct from the object thrown, and he is not bound to the power which he imparts to them. The stone propelled by a staff and the arrow shot by an archer continue to move as long as their power remains. But that power becomes weak as they move away from their mover until finally it is exhausted and they fall (to the ground). Similarly, the power which the supreme sphere imparts to those spheres below it continues to diminish until it reaches the earth, which is at rest by its nature. (in Duhem 1913 volume 8, p. 173; also Al-Bitrūjī 1971, p. 61)

Here Al-Bitrūjī seems to apply the two interesting dynamic ideas suggested by Ibn Bājja in his famous challenge to the Aristotelian argument on the non-existence of the void: (i) a body (e.g. planets and fixed stars) moves, even in the void, with a finite velocity; (ii) when motion takes place in a medium, it suffers retardation which is proportional to the density of the medium. An idea which could be used to explain the immobility of the earth due to the slowness or deceleration observed in projectile motion: the underlying idea is that of a power imparted to the celestial bodies which makes the latter continue in motion until that power is exhausted. We will give Ibn Bājja's explanation of the immobility of the earth to indicate that al-Bitrūjī was writing under the influence of the dynamics of his time which is Ibn Bājja's dynamics.

Ibn Bājja states, as a matter of digression, first the immobility of the earth in his *Commentary to the Aristotelian Meteorology*: "There is a reason for the fact that the earth is at rest and does not rotate (around its axis through its centre) in a circle; the place suitable for its discussion is *De Caelo*" (Lettinck 1997, p. 457). And he stresses that this reason is quite different from that given by Aristotle and the ancient philosophers which is based more on speculations (what is said) than on observations.

Let us establish the matter according to what is observed and to what is given in the account and leave the investigation of the cause of this condition of rest to another place. For (one should note) that this (kind of) rest is different from the one studied in *De Caelo*. In *De Caelo* it is investigated whether the earth as a whole has a rectilinear motion. Thus, (the earth) as a whole it has no motion at all, neither rectilinear, nor circular... The ancient natural philosophers have especially studied the earth (and investigated) what kept it at rest and why it was at rest here, for they thought that every part of it was moving in the air. Also, he who thought that it was not circular and thought that it is extended without limit, conceived a bearer for it, such as the Greeks talked about Atlas (Lettinck 1997, p. 457).

After attributing to only the fifth body, i.e. the celestial sphere, an internal motive power, Ibn Bājja goes on to explain how the other three elements are generated from fire by the weakening of the received fire's motion due to the distance traversed

The fifth body has a nature or a soul by which it moves, and mover and moved are in the (same) moving body such the doctor who heals himself. The mover [of fire], however, is external, such as the hand which moves the pen. This is the cause of the perishing of fire, for it only perishes by getting wet or cold, and this only occurs when it is at rest. Therefore it moves with these motions until it gets further away from the daily motion; the motion in it becomes weaker, it arrives in another place and becomes air. If it occurs that it gets even farther away, so that it comes to rest completely, it becomes water or earth. This account is more fitting for *De Generatione et Corruptione*, for it gives the cause of the continuous generation of elements of each other (ibid., p. 461).

And in the following passage he gives his own reason why, unlike fire, the earth cannot be moved

What is light can easily be divided and shaped and is in general easily receptive of motion, whereas earth by itself does not move at all, unless something strong forces it (to move) قاهر قوي
وألأرض غير متحركة جملة بذاتها إلا أن يقهرها. If a part of a fire is always larger than a part of the earth, it

has this kind of quantity not because its depth, length and width have a known proportion to each other – that is also the case for the earth, but there they exist permanently and have certain proportions which do not change unless by something which causes them to change; then it changes. Fire can easily be divided, and if it *meets the least of resistance* these proportions do not remain the same, but the proportions of the sizes change *لَمْ تَبْقَ عَلَى تِلْكَ النِّسْبَةِ بَلْ يَتَغَيَّرُ نَسَبُ أَقْطَارِهَا* ... The earth is not easily divisible; it can be divided only by something which *overpowers it in an appreciable time* *لَمْ يَتَقَسَّمْ إِلَّا بِشَيْءٍ يَهْلِكُ فِي زَمَانٍ مُّجْتَمِعٍ* ... *وَالْأَرْضُ لَيْسَ لَهَا هَذَا التَّقْسِيمُ سَرِيعًا إِلَّا* This is the essence of thickness and thinness (pp. 463-465, my emphasis).

According to Ibn Bājja, two reasons explain the immobility of the earth: (i) the weakening of motion coming from the fifth body; (ii) the inertial nature of matter: the strong resistance of the earth due to the high degree of density of the matter of which it is composed. The connection between Ibn Bājja's dynamic principle mentioned above and the explanation of the supralunar phenomena is very clear; all the main ideas are present: the weakening or retardation of motion in terms of the distance, the finite magnitude of velocity, the resistance function of the milieu and of the body to be moved (this is explicitly stated and discussed by Ibn Bājja). A full exposition of Ibn Bājja's interesting dynamics, which is different from that traditionally attributed to Philoponus, is beyond the scope of this paper. Duhem is perfectly right when he points out that "la *Théorie des planètes* [al-Bitrūjī's book] ne suggérait cette pensée [the passage quoted above] que d'une manière fugitive, dans le seul but de développer une comparaison" (*Le Système du Monde*, volume VIII, p. 175).

This brings the physical status of the earth more closely into the scientific discourse since the immobility of the earth is no longer assumed dogmatically but is integrated into the global explanation of the universe; and it appears to be a consequence of Ibn Bājja's dynamic principles. It thus appears that the importance given by al-Bitrūjī to the qualitative nature of his system is an attempt to fulfil the other part of Ibn al-Haytham's appeal: the explanation of the regularities of the movements of the heavenly bodies by adopting a more systematic physical approach to the study of empirical phenomena. With al-Bitrūjī, astronomy and dynamics become more closely linked than ever and the frontier between celestial and terrestrial phenomena is forever abolished. This is implicitly recognised by Duhem in his comment on al-Bitrūjī's conception of the universe:

Au mouvement de la huitième sphère, al-Bitrūjī rattache ainsi les grandes variations de la surface terrestre, les déplacements des continents et des mers, dont les anciens philosophes grecs avaient affirmé la réalité et qu'Aristote, au second livre des *Météores*, réduisait aux proportions plus modestes d'inondations causées par l'abondance des pluies (Duhem 1913, volume II p. 156).

The underlying fruitful idea of al-Bitrūjī is that both sublunar and supralunar worlds, which were sharply distinguished by Ptolemy, have to be explained by a universal dynamics. This explains why al-Bitrūjī's system enjoyed an enthusiastic reception among medieval European philosophers. Albertus Magnus, for example, expresses his "fascination by a very simplified model of the theory of al-Bitrūjī i.e. the attempt to explain all celestial appearances by means of a single driving force that would carry all the celestial bodies in a more or a less rapid motion towards the west, which would account for their apparent motions towards the east" (in Rashed 1996, p. 294). And Duhem confirms, at the end of his account of la *Théorie des planètes* or *On the Principles of Astronomy*, that al-Bitrūjī's conception has succeeded in being favoured by some European astronomers and mainly the Italian Averroists up to Copernicus.

Tel est dans ses grandes lignes, cet ouvrage d'al-Bitrūjī qui devait, jusqu'au temps de Copernic, inspirer tous les adversaires, frayant ainsi la voie à l'astronome de Thorn (ibid., p. 156).

It is because of this deep foundational crisis in which astronomy finds itself that Copernicus proposes his heliocentric system.

Après que j'eus longtemps roulé dans ma pensée cette incertitude où se trouvent les traditions mathématiques, touchant la théorie des mouvements célestes, il me prit un vif regret que les philosophes dont l'esprit a si minutieusement scruté les moindres objets de ce Monde, n'eussent trouvé aucune raison plus certaine des mouvements de la machine du Monde (quoted by Duhem 1994, pp. 73-74).

And if Copernicus succeeds in finding the heliocentric system, it is not because he takes the idea of placing the sun at the centre and making the earth move around it as a purely fictional hypothesis, but because he is strongly convinced that the new system is the true one; in other words it is because Copernicus, and the entire School of Padua which was the challenging capital to the domination of the Ptolemaic system in the rest of Europe, philosophically adheres to what Duhem calls the Arabic realist tradition.

Copernic conçoit le problème astronomique comme le conçoivent les physiciens italiens dont il a été l'auditeur ou le condisciple; ce problème consiste à *sauver les apparences au moyen d'hypothèses conformes aux principes de la Physique*. (ibid., p. 73)

7. The controversial nature of the progress of science: *Al-Shukūk* vindicated by *Le Système du Monde*

We can now understand the sharp contrast between the Greek and the Arabic approach to the foundations of astronomy so much emphasised by Duhem in his introduction to the second chapter of *Le Système du Monde*, devoted to the Arabic physicians and astronomers.

Le génie géométrique des Grecs s'était efforcé, avec autant de persévérance que de succès, à décomposer le mouvement compliqué et irrégulier de chaque astre errant en un petit nombre de mouvements circulaires simples. Leur génie logique et métaphysique s'était appliqué, de son côté, à l'examen des combinaisons de mouvements imaginées par les astronomes; après quelques hésitations, il s'était refusé à regarder les excentriques et les épicycles comme des corps doués, au sein des cieux, d'une existence réelle; il n'y avait voulu voir que des fictions de géomètre, propres à soumettre au calcul les phénomènes célestes; pourvu que ces calculs s'accordassent avec les observations, pourvu que les hypothèses permissent de sauver les apparences, le but visé par l'astronome était atteint; les hypothèses étaient utiles; seul, le physicien eut été en droit de dire si elles étaient ou non-conformes à la réalité; mais, dans la plupart des cas, les principes qu'il pouvait affirmer étaient trop généraux, trop peu détaillés pour l'autoriser à prononcer un tel jugement. Les Arabes n'ont pas reçu en partage la prodigieuse ingéniosité géométrique des Grecs; ils n'ont pas connu davantage la précision et la sûreté de leur sens logique. Ils n'ont apporté que de bien minces perfectionnements aux hypothèses par lesquelles l'Astronomie hellène était parvenue à résoudre en mouvements simples la marche compliquée des planètes. Et d'autre part, lorsqu'ils ont examiné ces hypothèses, lorsqu'ils ont tenté d'en découvrir la véritable nature, leur vue n'a pu égarer en pénétration celle d'un Posidonius, d'un Ptolémée, d'un Proclus ou d'un Simplicius; esclaves de l'imagination, ils ont cherché à voir et à toucher ce que les penseurs grecs avaient déclaré purement fictif et abstrait; ils ont voulu réaliser, en des sphères solides roulant au sein des cieux, les excentriques et les épicycles que Ptolémée et ses successeurs donnaient comme artifices de calcul; mais, dans cette œuvre même, ils n'ont fait que copier Ptolémée (Duhem 1913, volume II pp. 117-118; also Duhem 1994, pp. 27-28).

Unfortunately for Duhem, history has proved him wrong especially regarding the last claim. When he was writing these lines, he obviously did not know of Ibn al-Haytham's *al-Shukūk* nor of the works of the Marāgha School. What he calls Arabic realism does not appear by chance, nor by lack of imagination (interestingly Ibn al-Haytham has responded to the imagination argument), but is triggered by the desire for understanding on the part of Arabic physicians and astronomers. *Al-Shukūk* is a landmark in the history of science since it has the unprecedented effect of a priori destroying what seems to be an undisputed scientific theory (on empirical grounds) by successfully challenging its dogmatic and philosophical assumptions. For Arabic philosophers and astronomers both in the East and in the West, Ptolemy's approach, aiming at merely saving appearances, is dead. The influence of *al-*

Shukūk has far exceeded the author's original expectations. Not only his successors, philosophers and astronomers alike, unanimously rejected the Ptolemaic model, but the criticism of the *Almagest* becomes widespread and extends to other areas not discussed by Ibn al-Haytham such as the problem of the order of the planets or the discrepancy between the moon's observation and Ptolemy's calculations. *Al-Shukūk* has changed the epistemological status of the *Almagest* forever: before Ibn al-Haytham's book, the Ptolemaic model was considered a well established and confirmed scientific theory; after *al-Shukūk*, it was no longer the case since it became instead the subject of intense theoretical and observational investigations aimed not only at finding new alternative models but more importantly at making astronomy a genuine scientific discipline by firmly basing it on sound and universal dynamic principles. It is then wrong to believe that it is Ibn al-Shātir or even Copernicus who satisfactorily answers Ibn al-Haytham-Ibn Rushd's unprecedented historical appeal since they both fail to provide the correct physical principles on which their model is founded. Nonetheless their achievements are undoubtedly a great step in the right direction, and more research which means more controversies is needed before this task can be accomplished by Kepler and Newton.⁶ Furthermore the appearance of *al-Shukūk* on the scientific scene creates a new environment which is described to us by Saliba in the following terms:

In a very interesting additional comment, Ibn al-Akfānī goes on to say: 'The ancients continued to restrict themselves to pure circles in regard to the representations of the configurations of the celestial spheres until Abu Ali Ibn al-Haytham explicitly stated the corporeality of the latter and mentioned the conditions and the implications resulting therefrom. The later [astronomers] followed him in that.' In that regard, the content and composition of the *Tadhkira* [al-Tūsi's *Memoir*] made it an excellent introduction to that type of theoretical astronomy [...] and thus may have become suitable for School instruction. The number of commentaries written on it from within those Schools, and the direct evidence from later astronomers who studied commentaries on the *Tadhkira* as part of their School curriculum, attest very well to its popularity. This type of astronomical literature allowed people to discuss highly sophisticated astronomical matters, but this time in terms of real physical bodies, as was required by Ibn al-Haytham. It is this intersection of the mathematical and physical disciplines that formed the core of this type of texts. From that perspective, the inclusion of the *Tadhkira*, along with other *hay'a* [astronomical] texts, in the School curriculum must have meant that the subject matter of *hay'a* was no longer restricted to the few astronomers who were interested in reforming Ptolemaic astronomy. It must have then become the subject of various discussions by jurists and theologians who would have been among the regular students and teachers of such Schools. In that regard, the faults of Ptolemaic astronomy and the need to reform it must also have been well understood by the larger community (Saliba 1994, pp. 34-35).

This controversial attitude towards the Greek scientific writings, which characterises the Arabic literary tradition long before the advent of Islam, is not limited to astronomy but is common to nearly all the various scientific disciplines: in medicine and biology (Rāzī's *al-Shukūk 'ala Jālinus (Galen)*), in philosophy and metaphysics (al-Ghazālī's *al-Tahāfut*), in optics (Ibn al-Haytham's *Optics*), in physics (Ibn Bājja's famous refutation of the Aristotelian argument on the non-existence of the void), in logic (Ibn Taymiya's *Against the Greek Logicians*), to name just the most famous writings. It is this thorough, systematic and sustained challenge of the Greek scientific and philosophical works which, through its transmission to medieval European scholars, opens the way for modern science. And for this to happen, it is of cultural necessity that some pieces of the Greek scientific and philosophical shipwreck should be recovered from the sea's dark depths and brought to a more favourable milieu, and of historical necessity that science and philosophy should flourish once again in the middle East before they move westwards due this time to the rapid spread of knowledge. It is wrong, though, to infer from this that modern science is the result of the fruitful cultural exchanges between solely the two great civilisations of the Greeks and the Arabs; many ancient civilisations such as the Sumerians, the Babylonians, the Chaldeans, the Egyptians, the Phoenicians, the Persians, the Chinese, the Indians, have also contributed in one way or

another to our present understanding of the world we live in. Interaction between various scientific disciplines has been universally recognised as indispensable to the good progress of science, but the role of intercultural scientific exchanges between different civilisations throughout history in the development of science has yet to be fully appreciated by historians of science and philosophy. If, however, we pay a little attention to the history of science, we shall find out that each civilisation, since the invention of writing, has tried, according to its cultural, historical and geographical background, to benefit from and build upon the achievements and inventions of its predecessors. And the construction of this magnificent scientific edifice does not progress monotonically, i.e. by accumulation or by steady and constant increase; this static interpretation assumes naïvely that history moves along a straight line which is really not the case. It is obvious that the evolution of science is much more complicated than that since there are huge gaps in the making of science which cannot be explained by the monotonic approach. This explains why the logic of the history of science and philosophy follows, on the contrary, a rather nonmonotonic pattern: a scientific theory, astronomy for example, reaches a point where controversies are necessary for its further development. More precisely, a thesis (or theory), no matter how firmly it is established, will, sooner or later, be attacked by the construction of one or more arguments. The strength of the thesis is then put to the test depending on the arguments it produces to defend itself. The thesis preserves its position as an established or dominant doctrine if it succeeds in producing one or more counterarguments capable of defeating the arguments which are hostile to it. If, on the contrary, its counterarguments are defeated, this has an impact on the status of the thesis which finds itself refuted. Is the refutation of the former established thesis definitive? Not at all — the refuted theory can be reinstated, either totally or partially, by the future emergence of one or several counterarguments strong enough to defeat the previous argument which defeats it. The status of the thesis is then assessed with respect to the set of all arguments available at a certain stage of the process of the exchanges of arguments and counterarguments; a status which is subject to change as soon as more arguments are constructed with the passage of time. In short nonmonotonic logic allows us to draw a provisory conclusion which can be modified or completely withdrawn when new information becomes available. It is this main feature of nonmonotonic logic that makes it a suitable instrument for capturing the structure of scientific controversies which are the driving force behind the dynamic development of science.

Unlike many modern historians of science who retain a narrow interpretation of the evolution of science, this point has not been missed out by Duhem since he recognises the necessary role of scientific and philosophical controversies in the advancement of science. The instrumentalist philosopher reflects on the consequences of the bitter controversy produced by al-Bitrūjī's doctrine on the development of astronomy throughout the Middle Ages up to Copernicus, and though throughout his exposition he makes full use of his good writing talent at the service of the saving appearances camp with the aim of persuading the reader of the superiority of the non-realist approach (giving thus more credit to the Non-Innovationists), he cannot resist drawing the right lesson:

Il est une proposition qu'on peut formuler sans réserve et que la suite de cet écrit justifiera: cette œuvre qui n'est qu'une tentative et qui ne s'achève pas, aura la plus grande influence sur l'évolution de l'Astronomie occidentale. Cette influence, nous la reconnaitrons partout et pour toujours, côtoyant celle qu'exerce la doctrine de Ptolémée, la contrariant et l'empêchant de ravir l'acquiescement unanime des astronomes. Le perpétuel conflit de ces deux influences entretiendra le doute et l'hésitation à l'égard de chacune d'elles ; il ne permettra pas aux intelligences d'être asservies par l'empire incontesté de l'une ou de l'autre d'entre elles ; il assurera aux esprits curieux la liberté de recherche sans laquelle la découverte d'un nouveau système astronomique fût demeurée impossible (Duhem volume II, p. 171).

And this is what *al-Shukūk* and other similar challenging arguments are all about. It is amazing to see that *Le Système du Monde* is just a vindication of Ibn al-Haytham's philosophical approach to the formation of science: the correct structure of the universe that he has called for is not reached overnight by the discourse of an individual scientist or even by the work of a single School of thought but it is the product of a long process in which philosophers and scientists, belonging to various currents of ideas, have taken part in a series of controversies through the exchange of arguments and counterarguments.

In his attempt to explain why there is a lack of originality in the Islamic tradition, de Vaux does not deny that Islamic thinkers enjoy free thinking which manifests itself in the critical attitude towards religious and scientific writings: "la science arabe", he admits, "avait vis-à-vis de la parole révélée aussi bien que vis-à-vis de l'enseignement antique, toute la liberté de pensée nécessaire à son développement et à sa transformation" (in Tannery 1893, p. 337). But it seems to de Vaux that free thinking and the critical spirit are not sufficient to produce a scientist of genius such as Copernicus because the Arabic tradition lacks "la force du génie". Commenting on a letter sent by Copernicus to the Pope, Duhem tells us, on the other hand, how not men of genius but ideas of genius such as that of Copernicus are born, and why that gift of la "force du génie" of Copernicus appears in Padua and not in Torun or elsewhere.

Ce passage évoque à nos esprits les grands débats qui agitaient les Universités italiennes au temps où Copernic est venu s'asseoir sur leurs bancs : D'une part, les discussions touchant la réforme du calendrier et la théorie de la précession des équinoxes ; d'autre part, l'ardente querelle entre les Averroistes et les partisans du Ptolémée ; du choc entre ces deux écoles a jailli l'étincelle qui a allumé le génie de Copernic (Duhem 1994, p. 73).

In view of the newly discovered eastern influence, however, we now know why the Copernicus model could not make its appearance before the fourteenth century: the time was needed for the mathematical apparatus to be fully developed by the Marāgha School.

The questions raised in the introduction have now been fully answered. As a conclusion, let us briefly recapitulate our views. It seems to me that one of the main achievements of the Arabic-Islamic civilisation is the institution of a lasting and an unprecedented dynamic research tradition, stretching from Samarkand in the East to Toledo in the West, in which diversity is the driving force behind its open and controversy based-approach to scientific and philosophical learning. Its contribution lies simply in its huge impact on the rest of the world. And it seems that it is the neighbouring south-western European countries which have benefited the most from the unprecedented globalisation of knowledge and learning in which Arabic was the vehicle language par excellence. The scientific and philosophical orient express can now continue uninterrupted its journey that started when it was first indefinitely propelled and unmistakably set on the right track.

A forthcoming paper will use concepts of the logic of defeasible argumentation, developed in our unpublished thesis and successfully applied to the study of the controversies on the foundations of mathematics,⁷ as an instrument for capturing the different levels of argumentation by sharpening the analysis of the exchange of arguments and counterarguments between Ptolemy and Ibn al-Haytham.

Acknowledgements

Notes

¹ “A person, who studies scientific books aiming at the knowledge of the real facts (الحقائق), ought to turn himself into an opponent of everything that he studies, he should thoroughly assess its main as well as its marginal parts, and oppose it from every point of view and in all its aspects. And while thus engaged in his opposition, he should also be suspicious of himself and not allow himself to become abusive or be indulgent [in his assessment]. If he takes this course, the real facts (الحقائق) will be revealed to him, and the possible shortcomings and flaws of his predecessors’ discourse will stand out clearly.” (p. 4)

² The number of pages refers to the 1953 Dover edition reprinted as *A History of Astronomy from Thales to Kepler*.

³ All Ibn al-Haytham quotations refer to *al-Shukūk* unless stated otherwise. We have greatly benefited from the translation by G. Saliba and A. Sabra of some passages of Ibn al-Haytham’s book.

⁴ Sabra 1998, p. 312. But Sabra attributes these views to ‘Urdu. I think that we do not have to wait until the thirteenth century to find this kind of argument concerning the foundations of astronomy. They are already present in the *al-Shukūk*.

⁵ For more details on Ibn al-Haytham’s epistemologically original approach in optics, see Simon 2003, mainly pp. 88-113.

⁶ See Duhem’s volume VIII of *Le Système du Monde* on the impact of another famous controversy concerning the principles of dynamics between Ibn Rushd and Ibn Bājjā which is sparked off by the latter’s outstanding refutation of the Aristotelian argument on the non-existence of the void that signalled the beginning of the end of the Aristotelian physical system.

⁷ A summary of the thesis entitled “Essai pour un rapprochement entre la philosophie et l’histoire des sciences” is available on the following website http://stl.recherche.univ-lille3.fr/dernières_nouvelles.html

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PART II
LOGIC PHILOSOPHY AND GRAMMAR

The *Jiha/Tropos-Mādda/Hūlē* Distinction in Arabic Logic and its Significance for Avicenna's Modals*

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mīnū nā māte jēna līdhe
ā shakya banyū che

Abstract. The word, *tropos*, translated in Arabic as *jiha*, is understood in the field of logic as mode. Though investigations of modals in the medieval Arabo-Islamic logical tradition trace their lineage back to Aristotle, the Greek word designating this concept was never used in this manner by the Stagirite. The closest word that the Arabic *jiha* translates from Greek is *tropos*, which was a technical term that gradually developed with Aristotle's commentators. The word came to be understood as part of a dichotomy, *tropos-hūlē*, which was inherited by the Arabs as *jiha-mādda*. This dichotomy seems to have become a determining factor for conversion rules of modal propositions and thus for modal syllogistic. After an investigation outlining the evolution of the term *tropos* and the development of the dichotomy *tropos-hūlē* in the Commentary tradition of modal logic, the article presents philological evidence for their influence on Avicenna. It then briefly discuss the ramifications of this influence for his modal conversion rules and syllogistic. In sum, the article argues that the *jiha-mādda (tropos-hūlē)* division was part of a larger dichotomy that allowed Avicenna to construe propositions in various ways. How he understood a given proposition determined the validity of its conversion and so of its place in his modal syllogistic.

1. Introduction

Interest in Arabic modal logic first bloomed in the second half of the twentieth century with the works of Nicholas Rescher, who offered a preliminary syntax and semantics for the statistical (and some alethic) models of a few medieval logicians writing in Arabic.¹ As Rescher's studies were geared mainly towards providing us with a systematic interpretation of various modal systems, they remained largely ahistorical in their approach. After Rescher – and until the turn of the century – Arabic modal logic was studied at a steady but slower pace, either in short articles devoted exclusively to the subject² or as a tangential part of some larger study.³ A good number of the scholars of this period turned to historicization and contextualization. It seems that over the past five years, studies in Arabic modal logic have come to maturity with large strides, with works that combine the systematization of Rescher with the historical bent that followed after him.⁴ Two things, however, are missing from the latest approach: a focus on the chronological development of the modal systems of individual medieval logicians, and attention to some important underlying assumptions that might explain those systems.⁵

This short article — something of a sketch extracted from the first part of a larger study I have been preparing on the chronological development of Avicenna's modal logic — is a contribution to filling these lacunae. I have divided it into three parts. In the first, (1) I present a central topic of discussion in the logical systems of Aristotle's commentators that Avicenna had to consider before setting down his own pronouncements on modalities and modal syllogistics. This was the perennially appearing distinction between *jiha (tropos)* and *mādda*

(*hūlē*) that we find clearly articulated in *al-Shifā'* and *al-Najāt*.⁶ I present, in this first part, some of my notes on the development of these concepts and on the associated technical terminology. I begin (1a) by focusing on the term *tropos* in Aristotle and then (1b) move on to discuss it and its counterpart, *hūlē*, in pre-Avicennan philosophy — in the commentary tradition and in al-Fārābī. In the second part, (2) I offer Avicenna's appropriation of these concepts, paying close attention to his language to get a precise sense of how he inherited them. Finally, in the third part, (3) I discuss how these concepts were important for Avicenna's understanding of modalities. I do so by (3a) zeroing in on Avicenna's comments on the quantification of possible propositions. For distinctions in his quantification scheme seem to run parallel to the *jīha-mādda* one. (3b) How Avicenna understood the quantification of modalities affected his stance on the conversion of propositions. With respect to the e-conversion of the ever-ambiguous absolute propositions, (3c) his stance may have changed over time. (3d) I think that this change may again be explained with reference to the *jīha-mādda* distinction.

2. Jīha (tropos)

2.1. Technical Terminology in Aristotle

In the field of Arabic logic, *jīha* is a technical term understood as mode. Goichon tells us that it was the translation of the Greek *tropos*, as it occurs in Aristotle.⁷ She explains the term further as “mode”. Now I am uncertain whether, by this term (usually given as mood in English), Goichon means the modes Barbara, Celarent, etc., or modality. In the case of the only logical work she cites, namely, *Topics* 106a3, *tropos* is used in neither way: *to de posachōs pragmateuteon mē monon hosa legetai kath' heteron tropon...* (regarding how many ways it is employed, not only those many which are said in a different way...). The same general sense of manner/way goes for her citations of *Metaphysics*, 1052a17 and *De Generatione*, 318b8. Goichon also cites the *Prior Analytics*, 43a10, for an occurrence of *tropos* as “mode (du syllogisme)”.⁸ The Greek reads: *kai gar en pleiosi schēmasi kai dia pleionōn tropōn* (<they can be proved> with more figures and moods). This is certainly not a reference to modalities. Goichon gives the Arabic equivalent of d#arb for this use of *tropos*. D!arb is the standard word used in Arabic to convey moods; and *tropos* as mood appears fairly frequently in Aristotle and should be considered a technical term.⁹ Be that as it may, the word *tropos* was often translated in Arabic as *jīha*, even if it did not mean mode.¹⁰

The fact of the matter is that there is no technical term in Aristotle that means mode. *Tropos* (way/manner), like *jīha*, is a vague enough term to have a wide semantic range. In my own survey of the works of Aristotle, I have been able to find the following types of uses:¹¹ (1) general type/way/manner/means: (1a) *Prior Analytics*, 32b5: *duo legetai tropous* (it is said in two ways); (1b) Id., 45a4: *eis tous tropous* (<it will reduce> to the types); (1c) Id., 45a7: *ek...tropou* (from the type); (1d) Id., 25b15: *kath' hon tropon diorizomen to endechomenon* (according to which manner we define the possible); (1e) *Posterior Analytics*, 74a29: *ton sofistikon tropon* (in a sophistic sense); (1f) *oute gar ho rētorikos ek pantos tropou peisei* (for the rhetorician will not persuade with every means); (2) Aristotle often uses the following or similar phrases to avoid repetition: (2a) Id., 24a30: *ton eirēmenon tropon* (in the aforementioned way); Id., 25a27: *ton auton tropon* (the same way); (3) in the technical sense of mood, mentioned above and at *Prior Analytics*, 43a10, 52a38; *Posterior Analytics*, 85a11, etc.;¹² (4) in order not to be redundant, Aristotle often uses the expression *ho autos tropos ho tēs deixeōs* (the same method of proof), which reduces elliptically at *Prior Analytics*, 65a18, *to ouch ho tropos* (not this method <of proof>); this may be related to an expression like *ho sofistikos tropos* (the sophistic method <of proof>) at *Topics*, 111b32; again, at *Topics*, 128a37: *ton auton de tropon kai epi tōn allōn tōn toioutōn* (<you must use> the same method

<of proof> in other cases of this kind);¹³ (5) Genus at *Metaphysics*, 996b5 (equivalent of *eidē*); (6) figures (of syllogisms) at *Prior Analytics*, 45a4-7: *tropoi syllogismōn* (types, i.e. figures, of syllogisms).¹⁴

Now as for (7) the use of *tropos* in the sense of modality, we have some indications of the seeds of this technical use already in Aristotle: at *Prior Analytics*, 41b35, we read, “*kai hoti sullogismou ontos anagkaion echein tous horous kata tina tōn eirēmenon tropōn* (and that when we have a syllogism, it is necessary for the terms to be according to one of the aforementioned ways/relations).” The reference of course has to do with the universal or particular relation of the terms of the premises, not with their modal relation. What I wish to point out here is that the loose semantic range of *tropos* has allowed Aristotle to use it to indicate *some kind of relation between terms*. A less stretched indication of a quasi-modal use of *tropos* in Aristotle is found at *Topics*, 135a7. Aristotle begins the discussion by pointing out that errors regarding properties occur because there is often no indication given as to how and to what things these properties belong. Thus one often fails to mention that *x* belongs to *y* naturally, actually, specifically, etc. At the end of this discussion, we read, “*allou men own houtōs apodidontos to idion epicheirēteon autōi d’ou doteon esti tautēn tēn enstasin all’ euthus tithemenōi to idion dioristeon hon tropon tithēsi to idion* (if someone else gives the property thus one must stand against it, but for oneself this attack should not be given; rather, immediately upon setting it down, one must define in what way one is setting down the property).” Again, *tropos* is certainly not used as a technical term, but it loosely refers to the manner in which a property holds of a subject. In other words, Aristotle does not say that one should indicate the *tropos* of a proposition (that would be a technical use), but the *tropos* in which a predicate holds of a subject. I imagine that it is only a small step that would get us to the technical sense from this usage in Aristotle.¹⁵

2.2. The Peripatetic Tradition: *Eidos-hūlē* and *Tropos-hūlē*

quanti apud Syros Arabesque Alexandri scripta aestimata fuerint

(Wenrich, quoted in Steinschneider, *Die arabischen Übersetzungen*, 1960, p. 94)

I have found no clear signs of the development of *tropos* as a technical modal term within the surviving logical writings of Alexander of Aphrodisias.¹⁶ Rather, the term appears there in its fully matured technical sense; this suggests that it was used to indicate modalities well before his time.¹⁷

In the post-Aristotelian logical tradition, *tropos* as mode comes to oppose *hūlē*. In my view, this distinction became central to Avicenna’s understanding of modified propositions. But before I turn to a discussion of this dichotomy, I think it would be useful to say something about the other related *eidōs-hūlē* one. In his commentary on the *Topics*, Alexander writes at 2,1 that one kind of syllogism does not differ from another *qua* syllogism, but “*kata ta eidē tōn protaseōn, tēn de kata tous tropous kai ta schēmata tēn de kata tēn hūlēn peri hēn eisin* (according to the form of the premises — according to the moods and the figures — and according to the matter about which they are).” Thus we have a difference, on the one hand, between the form of a syllogism, consisting of its mood and figure, and its matter, on the other. Alexander says something similar in his commentary to the *Prior Analytics* at 6, 16: the figures <of the syllogisms> are like a sort of common matrix. You may fit matter (*hūlēn*) into them and mold the same form (*eidōs*) for different matters. Just as, in the case of matrixes, the matters fitted into them differ not in respect of form or figure, but in respect of matter, so too is it with the syllogistic figures.”¹⁸ Further clarification of what is meant by matter is found in Ammonius’ commentary on the *Prior Analytics*, 4,9-11, where he says that the matter is

analogous to objects (*pragma*), whereas the form is analogous to the figures of syllogisms.¹⁹ It is then clear that at least in one sense matter is to be taken as the objects for which the figures serve as a common logical matrix.²⁰

Let me now turn to another dichotomy. Commenting on *Prior Analytics*, 25a2-3 (on which see note 16), Alexander writes that by the expression *kath' hekastēn prosrēsīn* Aristotle intends “*kath' hekastēn katēgorias diaphoran kai kath' hekastēn tropou prosthēkēn* (with respect to each difference of predication, i.e. with respect to each attachment of mode).” The word *prosrēsis* of course carries the sense of adding a designation to something. Thus, for Alexander, the mode of a proposition is something that is added to it. For he writes that the modality of a proposition does not depend on what is set down,²¹ but on that which is joined to it. Here then we have the emergence of a dichotomy between Aristotle’s system for classifying modalized premises on the one hand, which operates on the explicit presence of a modal copula-modifier (*prosrēsis/tropos*), and the modal facts on the other hand, e.g. necessary truths, that inhere in the objects that form the subject-matter of the premise (*apo tōn hupokeimenōn*).²²

The distinction between the *tropos* (attached mode) and *hūlē* (subject matter) of a proposition is now at hand; but Alexander has so far drawn up this dichotomy only conceptually, without recourse to a clear distinction in his technical apparatus. But this is not far off. Following his comments on *Prior Analytics* 25a3-5, Alexander says the following:

axion de edoxen episkepseōs einai moi ti dē pote peri sullogismōn kai schēmatōn ton logon en toutois tois bibliois poioumenos paralambanei kai tas tōn protaseōn kata tēn hūlēn diaphoras hūlikai gar diaphorai to houtōs ē houtōs huparchein (it appeared to me to be worthy of investigation why, when speaking in these books about syllogisms and figures, he also sets out the distinctions of premises according to the matter; for to hold thus or thus is a material difference).²³

In the proposition, “x is y”, how y is predicated of x is determined by the natures of x and y themselves. This is something *hūlikē* or material. The fact of pointing out truly or falsely in speech that it holds in such and such a manner means adding a mode to this proposition.²⁴ The truth-value of the articulated mode will be judged against the material relation.²⁵

By the time we get to Ammonius, *hūlē* becomes a technical shorthand for material relation.²⁶ For he writes in his commentary on the *De Interpretatione*, 88,17, “*tautas de tas scheseis kalousin...tōn protaseōn hūlās, kai einai autōn phasi tēn men anagkaian tēn de adunaton tēn de endechomenēn* (these relations <between subject and predicate> they call the matter of propositions and they say that they are necessary, impossible, or possible).” He then explains that they are called matters because they depend on the objects posited in the propositions and – this is important to note – they are not so due to our opinions or predication, but due to the very nature of the objects (*ouk apo tēs hēmeteras oiēseōs ē katēgorias all' ap' autēs tēs tōn pragmatōn lambanontai phuseōs*). That which is due to our opinion or predication is the modal expression; that which is due to the nature of the things is the matter.²⁷

This second dichotomy is of course related to the first. In the first, the form of a syllogism comprised its figure and mood. Its matter was that from which it was constructed, namely, the proposition.²⁸ The form of the proposition was a contrast to its matter, which was its subject and predicate. Everything else, including the modal expression, was its form. Thus *tropos* was a part of *eidos* and conceptually stood apart from the *hūlē*. *Hūlē*, in the sense of material relation, was derived from *hūlē* in the sense of matter. It stands to reason, then, that the *eidos/hūlē* dichotomy should incorporate the *tropos/hūlē* one.

Both pairs of distinctions were known to al-Fārābī and both have been preserved in the Arabic translation of Alexander’s treatise on conversions. For al-Fārābī, both uses of matter (i.e. as material modality and as things about which statements are made) are found in one passage, which has an echo of Ammonius’ explanation of the emergence of *hūlē* as a technical term

(i.e. meaning material modality). Zimmermann translates al-Fārābī's *Commentary on the De Interpretatione*, 164,11, as follows:

Modes are not the same as matter²⁹. Modes signify how the predicate holds of the subject, while the matters³⁰ are the things connected when brought together in an informative way by a statement: their connexion produces the qualities (signified by modes). This is why modes belong to the part of logic which examines the composition of statements – for they are modes and qualities of composition –, and not the part which examines the subject-matters. Accordingly, these modes can occur in statements whose material (modalities)³¹ are contrary to those signified by their modes, which signify the mode and quality of the connexion alone.³²

Modes are connected with the composition of propositions; they are formal. Matters constitute what the propositions are about; the former are both the content and the quality of the latter.

Before I move on to discuss the significance of the *jihā/tropos-mādda/hūlĒ* distinction for Avicenna's modal logic, I would like to point out briefly that it must have been available to him also in the Arabic translations of the commentators.³³ In his *Fī in'ikās al-muqaddamāt*,³⁴ Alexander points out that particular negative propositions do not convert because the conversion is sometimes true and sometimes false, depending on the matter:

fa-ammā al-muqaddamātu allatī lā yūjadu fihā's)-s)idqu 'inda inqilābi al-h\$udūdi fa-laysa tan'akisu ra'san lākinna rubbamā s)adaqat wa-rubbamā kadhabat min qibali khās)iyati al-māddati wa-kayfiyyatihā wa-hādhihi hiyā al-muqaddamātu's-sālibatu al-juz'iyatu al-mawjūdatu minhā wa-d#arūriyyatu wa-dhālika fī al-muqaddamāti allatī tan'akisu hiyā' llatī tas)diq min qibali annahā bi-hādhihi al-h\$āli mina al-kayfiyyati wa-sh-shakli lā min qibali annahā fī hādhihi al-māddati wa-fī hādhihi li-annā al-muqaddamāti allatī tantaqilu bi-intiqāli al-māddati laysat tan'akisu wa-in s)adaqat mirāran kathīratan fī ba'd#i al-mawāddi wa-dhālika anna al-in'ikāsa li al-muqaddamāti innamā huwa min qibali'sh-shakli wa-s\$-s)ūrati ka-l-h\$āli fī tantjī al-qiyāsati lā min qibali al-māddati wa-li-dhālika wajaba an takūna h\$āluhu fī jamī'i al-mawāddi h\$ālan wāh\$idatan (as for those propositions in which there is no truth when the terms are transferred, they do not convert absolutely (ra'san?); rather, they are sometimes true and sometimes false on account of the special property of the matter and its quality. These are the hyparctic and necessary negative particular propositions. Regarding propositions that convert, they are true on account of the fact that they have this quality and form, not on account of the fact that they are with respect to this and that matter. For propositions which transfer <their truth-value> with the transference of the matter do not convert, even if they are true many times with respect to some matters. This is because conversion of propositions is only with respect to the form (ash-shakl wa-s\$-s)ūra), like the condition with respect to extracting conclusions in syllogisms, not with respect to the matter. For this reason, it is necessary that its condition with respect to all the matters be one.)"

Conversions of propositions, then, should be judged in accordance with their forms, not in accordance with their matters or the qualities of the matters. Better put, they should be judged with respect to abstracted forms that can support all matters, not just some. And we already know that *tropos* is a formal aspect of a proposition.³⁵

3. Avicenna: *Jihā/Mādda*

The ground for studying Avicenna's position on these matters has now been prepared, so that it should be fairly simple to see what he is up to. To the best of my knowledge, Avicenna does not speak much about the general *eidōs-hūlĒ* type of distinction in his logical works. However, he writes the following in the *Kitāb al-'Ibāra* of *al-Shifā'*:³⁶

<Mode>

The least of the conditions (*ah\$wāl*) of propositions is that they are two-fold. Then the <copula is> made explicit (*yus)arrah\$u bi'r-rābit,a*) so that they become three-fold. Then a mode (*al-jihā*) may attach to them so that they <i.e. the propositions> become four-fold. The mode is an (A) utterance (*lafz)a*) which indicates the relation (*al-nisba*) which the predicate has with respect to the subject. It specifies (*tu'ayyinu*) that it is a relation of necessity or non-necessity; thus it indicates a

firmness (*ta'akkud*) or a <mere> allowance (*jawāz*) <of their relation>. The mode may <also> be called a kind (*naw'*) <of relationship?>.37 There are three modes: (1) one which indicates the suitability (*istih\$qāq*) of the perpetuity of existence, i.e. the necessary; (2) another which indicates the suitability of the perpetuity of non-existence, i.e. the impossible; and (3) another which indicates that there is no suitability of the perpetuity of existence and non-existence, i.e. the possible mode.

<Difference between Mode and Matter>

The difference between mode and matter (*al-mādda*) is that the mode is an (A) utterance — added (*lafz)a zā'ida*) to the subject, predicate, and the copula — which is made explicit and which indicates the strength or weakness of the copular connection. <It indicates this> (B) sometimes falsely by means of an utterance. As for the matter — and it may be called an element (*'uns)uran*) — it is the (C) condition of the predicate (*h\$āl al-mah\$mul*) in itself (*fī nafsihī*) in an affirmative relation to the subject (*bi al-qiyās al-ijābī ilā al-mawd#ū'*) regarding the (D) nature of its existence (*fī kayfiyyati wujūdihi*) <for that subject>. If an utterance were to indicate <this nature>, it would do so by means of a mode. It may be that a proposition with a <certain> mode differs from its matter. For, if you say, “It is necessary that every man be a writer,” the mode would be necessary and the matter would be possible.

Almost all the important elements of the commentary tradition discussed above are found in this passage: (A) Avicenna states that modes are added expressions that indicate the nature of the relationship between the subject and the predicate (*lafz)a zā'ida = prosrēsis*). (B) What is indicated by the modal expression may sometimes be false. This implies that these expressions are subject to our *tas)dīq*, and may at times fail to get it. And this means that they are in the category of judgments that we pass regarding things (*apo tēs hēmeteras oiēseōs ē katēgorias*). In other words, they function on the level of signs, not on the level of things of which they are signs. Things themselves cannot be false, but signs as part of complex statements, which indicate thoughts about things, may be. (C) As opposed to this, the matter of a proposition is *h\$āl al-mah\$mul fī nafsihī* in its relation to the subject. This rings of *ap' autēs tēs tōn pragmatōn lambanontai phuseōs* above. (D) The expression *fī kayfiyyati wujūdihi* also reminds us of the Arabic translation of Alexander's *On Conversion*, which I quoted above: *min qibali khās)iyiyati al-māddati wa-kayfiyyatihā*. There are thus two kinds of modalities: those which are due to us and our attachment of an expression to a proposition (which may be false); and those which are the nature of the things themselves (which are always true).

4. Avicenna's Modals: An Excursus

So what significance does this distinction have for Avicenna's modal logic? I think that it is at the base of the *dhātī-was)fī* dichotomy of assertoric propositions. A *dhātī* assertoric of the *khās)sī* type conveys with “All A is B” that “All As pick out that which Bs pick out at some time and fail to pick it out at some other time.” The *'āmmī* assertoric conveys only the first half of this conjunction. A *was)fī* proposition, for example, states that “All As, for as long as they are As, are Bs.”38 A *dhātī* proposition then speaks about things, a *was)fī* about things insofar as they are defined in this or that manner; the latter, therefore, is conditioned. This dichotomy is easily extended to the modified propositions in general: with regard to this distinction, we can speak, for example, about Avicenna's various types of necessities,39 or we can speak about possibilities (a) insofar as they express the nature of the relationship between two things or (b) insofar as they have something to say about what obtains in this world (i.e. for those things that come under a certain description).40 The former has to do with things themselves, the latter with how they are in relation to us. That which is in relation to us is susceptible to our judgment and may have any modality attached to it.41 At the base of these dichotomous manners in which one can look at a proposition lies the *jiha-mādda* distinction. It explains how the various propositions should be read. How propositions are read, in turn,

determines how they convert. And since conversion is the *sine qua non* of Avicenna's syllogistics, we can say that *jīha-mādda* lies at the very core of his logic. My cryptic and summary comments here will become clear below by way of a case study in what Avicenna has to say about the quantification of problematic premises. I will then move on to consider some conversion rules in the light of what he says about these quantifications and what we have so far learned about *jīha-mādda*.

In the *Kitāb al-'Ibāra* of *al-Shifā'*,⁴² Avicenna points out that, as the complexity of a proposition increases, each new added element comes to modify that which preceded it. Thus, when a copula is added to a binary proposition, it comes to modify the predicate. When a mode is attached to a ternary proposition, it modifies the copula. It seems then that each new building block of a proposition applies to the most recent one that was added before it. However, quantifiers are much like particles of negation. Although they modify a proposition, they do not contribute to its ordered complexity. Thus, just as particles of negation attach to the copula in a ternary proposition, so a quantifier attaches to the subject; however, neither makes the proposition five- or six-fold. But what happens when a modified proposition is quantified? Does the mode apply to the quantifier or to the copula?

Avicenna says that both the quantifier and the copula can be modified. He does not disqualify one position in favor of the other,⁴³ but he does realize that two different kinds of propositions will result with the two different modifications. When the mode is applied to the quantifier, we get the statement, "*Yumkinu an yakūna kullu wāhšidin mina an-nāsi kātiban* (it is possible for each one of men to be a writer)." When it is applied to the copula, we get, "*Kullu insanin yumkinu an yakūna kātiban* (every man, it is possible <for him> to be a writer)." Likewise, with the particulars, we get, "*Yumkinu an yakūna ba'd#u an-nāsi kātiban* (it is possible for some of men be writers)" as a modified quantifier. And for the modified copula, we get, "*Ba'd#u an-nāsi yumkinu an yakūna kātiban* (some men, it is possible <for them> to be writers)". Avicenna states that there is nothing in the Arabic language that can express universal negative propositions with possibility modified copulae.⁴⁴ The idea of a thing possibly not being a thing can be expressed, but the statements produced "resemble" affirmatives. The particular negatives pose no problems.⁴⁵

The difference between the two kinds of propositions is that universal modified quantifiers pick out every single member of a given class and state that a predicate holds or fails to hold of each one of them. The universal modified copula, on the other hand, is about the relationship that holds between a predicate and all members of a class. It is obvious that modified quantifiers carry existential import, whereas modified copulae indicate the nature of class relations.⁴⁶

It is in drawing these distinctions that Avicenna's language becomes very interesting: for he says that modifications of the copula suggest the *kayfiyya* of the copular connection; that the copular reading is *t,abī'ī*; and that the copula is the *mawd#i' t,abī'ī* of the mode. Likewise, modification of the copula would tell us about the *t,abī'a* of the subject. On the other hand, modification of the quantifier would suggest the fact of something obtaining or not obtaining. Statements about such facts, unlike true statements about the nature of things, can be challenged. For example, the idea that it is possible for each and every man to be a writer is something about which we may raise doubts;⁴⁷ but the idea that writing applies in a possible fashion to all men cannot be questioned. In other words, true statements of the first type may be false at some (indeed most) times; but true statements of the second type, once true, will always be true.

In this same section, Avicenna speaks about the distinction between the modes *mumkin* and *muhšamal*. He says that the former is that which is with reference to the thing itself (*mā huwa fī nafs al-amr kadhālika*) and the latter is that which is with reference to us (*mā huwa 'indanā kadhālika*). According to an alternative interpretation, the *mumkin* is also that which

has no perpetuity of existence or non-existence, without any view to whether it does or does not now obtain. Thus there is neither any necessity nor impossibility that is attached to this reading of the possible. This is the classic statistical reading of the possible, called *al-mumkin al-khās)sī* elsewhere by Avicenna.⁴⁸ The *muh\$amal* is that which does not exist now, but will exist in the future. In other words, it has existential import with reference to the present time of the speaker. It is worth noting that Avicenna informs us that the appellations of these two types of possibilities are reversed for some people. But he also adds that they are not consistent in their technical conventions.

At this point, one cannot help but notice the emergence of a larger dichotomy within the fold of the *jiha-mādda* one. *Jiha*, *al-muh\$amal*, and the possibility modifications of quantifiers in quantified propositions are all things that happen with reference to us. As such they also carry an existential import. On the other hand, *mādda*, *al-mumkin al-khās)sī*, and possibility modifications of copulae in quantified propositions happen with reference to the thing itself (i.e. with reference to its nature).

Let us now see if any of this has explanatory value for Avicenna's system of conversions. A proposition like "It is possible for all A to be B" can be read in two senses. First, it can be saying that B holds of all A in a possible manner. This would be a statement about the nature of the things involved; the possibility would apply to the nature of the copula that brings them together. The *jiha* "possible" here stands as a sign for the *mādda* and the possibility expressed is of the *khās)sī* type. For it implies that B may or may not hold of A non-perpetually (in a statistical reading). Now the fact that B holds of all A in a possible manner does not mean that A holds in the same manner of B. For writer holds in a possible manner of all man, but man holds in a necessary manner of writer. It is perhaps for this reason that a universal affirmative *mumkin khās)sī* proposition converts to a particular affirmative *mumkin 'āmmī* one, and not to a *mumkin khās)sī* one.⁴⁹ In terms of its *mādda*, in terms of the *mumkin khās)sī*, and the modification of copulae — all parts of one side of the dichotomy — this proposition does not convert with its original mode.

Second, the proposition converts insofar as the quantifier is modified — i.e. insofar as we are speaking about the possibility of all men actually being writers — with reference to a *jiha* that expresses not the nature of things, but *mā huwa 'indanā*⁵⁰. The *jiha*, as a formal part of a proposition, may be redefined by us in terms of select material instances, so that a new formal system is abstracted from this delimitation. Thus, whereas the conversion will not be true for all material instances, it will naturally go through for those that participate in the formal structure of the new system. So we may say with the modified quantification reading that "Possibly, all men are writers" and "Possibly, all men are animals", for it is conceivable that all men exist as writers and animals. The *jiha* "possible" is the *mumkin 'āmmī* type. It takes into its fold both the possible *mādda* of "writer" to "man" and the necessary *mādda* of "animal" to "man". Thus redefined, a universal affirmative possibility premise does convert to a particular affirmative, while retaining the mode. Again, the conversion is possible because the modified quantification speaks about the possibility of factual existence, which is compatible both with necessity and possibility. This, in turn, allows for the redefinition of the *jiha* "possible" as a mode that applies to a larger class of material instances.

For possibilities of the sort that have obtained in the present, i.e. for assertoric propositions, we can no longer speak directly about the nature of the relation that the predicate has with the subject. Instead, we must speak about the fact of the predicate holding of the subject.⁵¹ This means that there would be a shift from the application of the possibility mode to the copula to its application to the quantifier. This, in turn, means that we have moved from things as they are naturally to things as they are for us. This is a perfectly legitimate move. For if W holds possibly of M, it may be true that M is W. And so we may assume W to hold of M, without any logical contradiction⁵² Thus, we have an assertoric affirmative universal, which converts

both under *mādda* and *jiha* readings: for if we inquire about the manner in which man and writer hold of each other, the proposition will convert as an assertoric, because, insofar as they are about things that exist, assertorics can amplify both with implied necessity (“some writers are men”) and implied possibility (“every man is a writer”).⁵³

If, on the other hand, we speak about modality as a *jiha* that does not correspond to the *mādda*, i.e. which is not due to the nature of things, but with reference to the way things are for us and in our judgment (*indanā*), we will turn to look at it with regard to its form only (*jiha* as a part of *eidos*). Thus we would say that if W can hold of M and actually does come to hold of it, M and W being so defined only with reference to each other, M also holds of W. For, on a purely formal level, I may choose to qualify my propositions without reference to *mādda*, and with a formal view to how a material instance has come to be. This technique allows for formal conversions. Thus, without speaking about the nature of the relation that a substrate has with its predicate, I can speak about it insofar as it is picked out by a certain subject term. I will then only be making a statement about a thing with regard to the description applied to it. Thus, “Everything moving is changing (i.e. insofar as it is moving)” converts to “Something changing is moving (i.e. insofar as it is changing).” These propositions are considered *was)fi* assertorics and fall, in some categorizations, under the necessary. This necessity does not refer to the nature of things; it is a formal necessity and is indicated by a *jiha* in accordance with the way things obtain and the way they are for us.⁵⁴

Finally, let us take the Avicennan example I offered above, but this time as a universal assertoric negative: “No man is a writer.” This proposition may be true, since writer applies in a possible manner to man – i.e. since the *mādda* of this proposition is possible, it may fail actually to obtain. To put it differently: $(Ax) (Mx \rightarrow P(Wx))^{55} \rightarrow (P) ((Ax) (Mx \rightarrow Wx)) \& (P) ((Ax) (Mx \rightarrow \text{not}(Wx)))^{56}$ is a valid supposition. The implied conjunction is compatible with (although it does not necessarily imply): $(Ax) (Mx \rightarrow Wx) \& (Ax) (Mx \rightarrow \text{not}(Wx))^{57}$. As Avicenna has already said, universal negative possibility propositions in the pure negative form can only support the modification of the quantifier. In other words, “No man is a writer” must be read as, $(P) ((Ax) (\text{Man } x) \rightarrow \text{not}(\text{Writer } x))$, which is implied by the possible manner in which writer holds of man, and insofar as this possibility actually obtains.

Now if we take the existential statement to be an implication of the possible manner in which writer holds of man, i.e. if we pay attention to the *mādda*, “No man is a writer” fails to convert. For, just as with the universal affirmative possible proposition above, writer applies possibly to man, but man applies necessarily to writer. Thus there is an exception to the rule of conversions, namely, that the converted should maintain the quality of the proposition: the necessary predication of man to writer means statistically that writer can never fail to be man.⁵⁸ So, with regard to the *mādda* in a special absolute *dhātī* reading, this proposition does not convert.

However, if we consider the matter from the perspective of how things are judged by us, the proposition can certainly convert. For formally, I may choose to say, “All M fails to be W, for as long as it is M” (this would be a *was)fi* reading of the assertoric). Thus, without reference to the material relation of M and W, I may also say that formally all W is excluded from M (with the same qualification). This would be a reading of the proposition in accordance with our judgment, i.e. a *jiha* reading, and would be perfectly legitimate if the assertoric proposition is seen as something obtaining only from the possibility modification of a quantifier in a quantified universal negative proposition. I cannot redefine the *māddī* proposition, but I can maneuver it once I speak about it as something that implies a modified quantification (not a modified copula) which, in turn, is compatible with the assertoric. Once I have the second half of the assertoric conjunction, I may redefine the *jiha* (i.e. as a necessary *was)fi*) in the manner above and get a formal conversion. This is the business of ampliation.⁵⁹

Before closing, I would like to point out that until his late phase, Avicenna seems to have accepted both ways of looking at propositions as legitimate. This means that the generally accepted view that assertoric e-propositions do not convert for Avicenna applies only to this late phase.⁶⁰ He is familiar with the different ways of reading a proposition in *al-Shifā'*, *al-Najāt* and *al-Ishārāt*. In *al-Shifā'*, he points out that the conversion works if the proposition is understood as that which is used in the sciences⁶¹ and is taken with reference to common speech (*ta'āruf*). He then goes on to give *was)ft* readings of e-propositions; and they convert.⁶² He says very similar things in *al-Najāt*.⁶³ I say with some hesitation that, when we get to *al-Ishārāt*, Avicenna's attitude seems to have changed. For, although he does speak about e-conversion with respect to *was)ft* readings, he straightaway and categorically calls the latter and the principle of *mubāyana* tricks.⁶⁴ Avicenna had offered severe criticism of the method of *mubāyana* in *al-Shifā'*⁶⁵ and had similarly said some harsh things of other alleged proofs for e-conversions.⁶⁶ But, to the best of my knowledge — and provided my understanding of *hṣiyal* is correct — he dismissed e-conversion of *was)ft* propositions only in the *Ishārāt*.

I am not sure why he took this position in his later work. I would venture a less than confident guess that it had to do with how he understood the function of modes in propositions. In the section on the *mawādd* of propositions in *al-Ishārāt*, Avicenna does not acknowledge the latitude that we have with respect to the *jihāt* of propositions. He simply says, “By *mādda* we mean these three conditions <i.e. necessary, possible, impossible> about which these three words <i.e. necessary, possible, impossible>, if expressed, are true with respect to affirmation and negation.” Certainly, *jihāt* should ideally express the true *mawādd* of propositions. But, as we saw above, they can do more than that. As part of our judgment, they can indicate modalities in any number of ways. In *al-Ishārāt*, Avicenna does not say anything about how *jihāt* may be with reference to us or to our judgment. In fact, it is al-Tūūsī who tells us this, extracting the information, it seems, from *al-Shifā'*.⁶⁷ I doubt that Avicenna meant to rule out the possibility of the creation of formal systems according to things as they obtain for us, but if he did mean to pose this limitation, it might explain his possible refusal to acknowledge the conversion of assertoric e-propositions. For, as we saw, on a *māddī* reading, they do not convert.

5. Conclusion

This article began with a word (*tropos*); it explored how this word became a technical term for the Commentators; how, as part of *eidos*, it came to be dichotomous with *hūlē*; how the *eidos-hūlē* and *tropos-hūlē* dichotomy was known to al-Fārābī; how Avicenna inherited this dichotomy; and finally, what role this dichotomy, along with several associated concepts, had to play in Avicenna's modal logic.

Although I must confess that there is no completely neat dichotomy that gathers *jihā*, modified quantifications, *muhṣtamal*, and *was)ft* assertoric propositions under one head, and *mādda*, modified copulae in quantified propositions, *mumkin khās)s)ī* propositions, and *dhātī* assertorics, under another, I hope that I have presented enough evidence in this article to cause us to recognize that they *generally* constitute two distinct communities of notions. Each side consists of related ideas, a number of which are simultaneously deployed by Avicenna to accept or reject a reading for a given proposition. How he reads the propositions determines, in turn, what he tells us about their conversions.

Select Translations from *al-Shifā'*, III: 112-118

<Quantification and Modal Propositions>

Just as it is suitable for the quantifier that the subject be delimited/encompassed by it (*an yujāwara bihi*) and for the copula that the predicate be delimited/encompassed by it, likewise it is suitable

for the mode that the copula be delimited/encompassed by it if it <i.e. the copula> is not quantified. If it is quantified, it <i.e. the mode> would have two places (*mawdhi'ān*) <of application>, whether the sense remains one or differs, one of them being the copula, the other the quantifier. And it is up to you to connect it <i.e. the mode> with the one and the other. For you say, “It is possible for each one of men to be a writer.”⁶⁸ And you say, “Every man, it is possible <for him> to be a writer.”⁶⁹ Likewise you say, “It is possible for some men be writers.”⁷⁰ And you say, “Some men, it is possible <for them> to be writers.”⁷¹

As for the negative universal, there is only one utterance that is found in the language of the Arabs, which is, “It is possible that there not be one among men a writer.”⁷² There is no other <utterance> in which <the mode> is attached to the copula, to the exclusion of the quantifier, unless you say, “There is not one among men except that it be possible that he not be a writer.”⁷³ Or you say, “Every man, it is possible <for him> not to be a writer.”⁷⁴ However, this utterance resembles more the affirmation.

As for the negative particular, with respect to it, we say both statements. For we say, “It is possible that it not be that each man is a writer”⁷⁵ and “Some men, it is possible <for them> not to be writers.”⁷⁶ Before we verify the statement regarding these < matters > and investigate whether the meaning of that in which the utterance of the mode is connected with the copula and of that in which the utterance of the mode is connected with the quantifier are one or not (and if not one, whether they follow from each other or not), it is necessary that you know something else.

<General Comments Regarding Negation>

We say that just as when you do not insert the copula in the individual (*shakhs)iyya*) proposition, when you intend a negation it is a natural necessity that you attach the particle of negation to the predicate; and when you insert the copula of the predicate⁷⁷ and intend the negation, it is necessary that you attach the particle of negation to the copula, so that the negation of our statement, “Zayd is just”⁷⁸ is not “Zayd is not-just,” but “Zayd is not just.” For how could this <not be so> if both your statements may be false if Zayd is non-existent.⁷⁹

<Negation of Modal Propositions>

Likewise, when you attach the mode to the copula and intend a negation, it is necessary that you attach the particle of negation to that which stands in front,⁸⁰ thereby removing the totality of that which follows, not some of it. Thus when you say, “It is possible for Zayd to be a writer,” its negation is not the possibility of the negation, but the negation of the possibility. I mean, it is not your statement, “It is possible not to be...” but “It is not possible to be...” For how could it <not be so> while your statement,⁸¹ “It is possible not to be...” is mutually sound with your statement, “It is possible to be...” if true.⁸² Likewise, if you say, “It is necessary for Zayd to be a writer.” Its negation is not, “It is necessary not to be a writer.” For both of them would be mutually sound if false. Rather, <the negation is,> “It is not necessary to be...” Likewise, if you say, “It is impossible for Zayd to be a writer.” Its negation is not, “It is impossible for Zayd not to be a writer.” For your statement, “It is impossible for Zayd not to be a writer,” is mutually sound with it <i.e. the former statement> if false. Rather, the negation of “It is impossible for Zayd to be a writer” is “It is not impossible for Zayd to be a writer.” As for “It is possible <for x> to be...” with “It is not possible <for x> to be...” and “It is necessary <for x> to be...” with “It is not necessary <for x> to be...” and “It is impossible <for x> to be...” with “It is not impossible <for x> to be...” — these <pairs> do not occur <together> at all after all the conditions obtain either <if both members of each pair are> true or <if both> are false. Likewise, “It is possible⁸³ <for x> to be...” with “It is not possible² <for x> to be...”

<Differences between Possible and Possible2>

It seems that by possible2 is meant that which is for us thus and so <i.e. possible> (*mā huwa 'indānā kadhālika*) and that possible is that which is thus and so <i.e. possible> by the very nature of the thing.⁸⁴ It seems that another meaning is meant by it, i.e. possible2, is that in which is expressed the condition of the future, while it <i.e. the condition> is non-existent at the <present> time. Possible is that which has no perpetuity in existence or non-existence, whether it exists or not. A group (*qawmun*) says that by possible is meant the common <possible> and by possible2 is meant the special <possible>. But their statement is not consistent with respect to the utterances <used for> it <i.e. possibility>.⁸⁵ It seems that there is another difference between possible and possible2, a difference which is not accessible to me; <but> there is not much of a need for elaboration and for seeking it.

<To What Does the Mode Attach>

We say that it is suitable for the mode to be attached to the copula. This is because it indicates in an absolute fashion (*mut, laqan*) the nature (*kayfiyya*) of the copular connection, which the predicate has in relation to a thing. Or <it is suitable for it to be attached> to a quantifier which generalizes or specifies (*mu'ammim aw mukhas*)*is*). For the quantifier explains the quantity of the predication and conditions the copular connection (*mukayyif al-rabt.*). So if we say, “Every man, it is possible <for him> to be a writer,” this is natural (*al-t, abī'iyu*) and its meaning is, “Each one among men, it is possible for him to be a writer.”⁸⁶ For if it <i.e. the mode> is attached⁸⁷ to the quantifier and by this is not intended <its> removal from its natural place by way of expansion,⁸⁸ but the indication that its natural place is the delimitation/encompassing of the quantifier is intended, then the mode would not be for the copular connection. Rather, it would be a mode for generalization and specification. And so its meaning would change: the possible would come to <indicate> that the existence of each one of men — all of them — as writers is possible. The proof that the meaning has changed is that there is no doubt in the first <statement> in the minds of people generally.⁸⁹ For it is known that the perpetuity of writing or not writing is not necessary for each single man with respect to his nature. As for your statement, “It is possible for every man to be a writer” — taking the possibility to be a mode of the universality and the quantifier — there may be doubt regarding this. For there are those who say, “It is absurd that all men should be writers,” i.e. “It is absurd that it should be⁹⁰ that every man is a writer.” But then it comes to be that it so happens⁹¹ that there is not one among men except that he is a writer. Thus there is a difference between the two meanings.⁹²

As for the particulars, with respect to them, the two ways <of modifying> function in one way, both on the surface and underneath. But it may be known, despite this, that there is a difference in the two meanings if recourse is taken to the reality of that which is understood and if, with regard to it, consideration of the universal is relied upon.

As for the universal negation, there is nothing in the language of the Arabs which indicates truly the negation of the common possible.⁹³ Rather, common usage (*muta'arif*) in it <i.e. the language> only indicates the possibility of the negation of the common.⁹⁴ For this reason it is ambiguous to say, “It is possible that not one among men be a writer.”⁹⁵ Someone may say that it is not possible for this to be true; rather, it is necessary in an absolute fashion⁹⁶ that the disciplines exist in some <men>. Our discussion here is not about whether this statement is true or false; for the knowledge of this is not a part of the discipline of logic. Rather, our intention is that something regarding which there may occur a doubt is not that regarding which there occurs no doubt. That regarding which there occurs a doubt is the possibility of the negation of writing from each single man. However, there is nothing in the language of the Arabs that indicates this except by way of an affirmation, such as their statement, “Each one among men, it is possible <for him> not to be a writer.” As for their statement, “Not every man is a writer,” it is not possible to insert in it the mode of possibility except <that it again governs> the quantifier. Thus its meaning comes to be, “It is possible for every man not to be a writer.” So it indicates the possibility of the quantifier.

As for our statement, “Some men, it is possible <for them> not to be writers,” in some way, it may be equal to our statement,⁹⁷ “It is possible for some men not to be writers.” And it may differ from it — although they mutually follow from each other — so that the intention of one of them comes to be that some men are described by the possibility of the negation of writing from them; the <intention of the> second is that it is possible — upon the verification of the statement, “Some men are not writers.”⁹⁸

<The Meanings of Possible>

Now that you know these states, when you investigate the state of the implication⁹⁹ of these propositions, it is necessary that you investigate the state of the implication of these four-fold propositions, which have modes, keeping in mind that they are modes of the copular connection, not modes of the quantifier.

Also, the true nature of the affair will not be revealed for us with regard to them <i.e. the propositions> until after the state of homonymity existing in the utterance “possible” is known. We say that the utterance “possible” was used by the common people¹⁰⁰ in a <certain> sense and is now used by the philosophers in another sense. The common people used to mean by the possible that thing which is not impossible¹⁰¹ insofar as it is not impossible, and they did not turn <to ask> whether it was necessary or not-necessary.¹⁰² Then it occurred that there were things regarding which it was true to say that they were possible to be and possible not to be, i.e. <that they were>

not impossible to be and not impossible not to be. So when the specialists found things in which the possibility of being and the possibility of not being were combined, i.e. as possibility commonly occurs¹⁰³, they specified its state by the name of possibility. So they made that thing in which the two possibilities existed together, i.e., of negation and affirmation, to be specifically designated by the name of possibility. It is that thing in which there is no Necessity.¹⁰⁴ Thus these specialists agreed regarding that <conception> which was among them and coined the technical term¹⁰⁵ “possible” for that thing whose existence and non-existence is not impossible.

So for them things came to be of three types: impossible of existence; impossible of non-existence; that which is neither impossible of existence nor of non-existence. If you wish, you can say: Necessary of existence; Necessary of non-existence; that which is neither Necessary of existence nor of non-existence. The meaning of Necessary is the perpetual, for as long as that which is described has an essence that exists, as we will explain in another place with proof.

If by the possible is meant the common meaning, everything would be either possible or impossible; and everything that is not possible would be impossible and that which is not impossible would be possible. There would be no third type. And if the special meaning is intended by it, everything would be either possible or impossible or necessary and that which is not possible would not be impossible but Necessary, either with respect to existence or non-existence.

Thereafter, another coinage was concocted among the specialists with regard to that which was among them <i.e. another concept>: they made the possible indicate a meaning more specific than this one. It is that the judgment about which is non-existent at the time the speaker speaks about it.¹⁰⁶ Rather, it is not-Necessary of existence or not-existence in the future, i.e. at an imagined time. Elaboration of the statement with respect to this meaning will be given in what is to follow.

Thus the possible is said of three meanings, some of which are ordered above some others, the more common above the more special. Statements about it, both with regard to the more common and more special <meanings>, <will occur> homonymously. <Possible> in the more special manner is said in two ways: one of them is with regard to that which is specific to it; the other is by way of the predication upon it of that which is more common. This is something you already knew from what preceded. The common meaning is that the judgment regarding a thing is not-impossible; I mean by judgment that which is judged about it of affirmation or negation. The meaning of the special is that its judgment is not-Necessary. The third meaning is that judgment about it is non-existent <for the present> and is not Necessary for the future.¹⁰⁷

The existent affair, the existence of which is not necessary, is not included in the most special possible, and only in the special and common <possible>. The necessary is neither included in the most special nor in the special; it is included in the common. A group of people raised doubts against themselves,¹⁰⁸ saying that the necessary must either be possible or must not be. If it is possible — and the possible to be is also the possible not to be — then the necessary becomes possible not to be. This is absurd. They answered with the following account: they said that the possible is homonymous; for it is said of that which is *in potentia* and of that which is Necessary. The former possible cannot be included in that which is said of the Necessary. <In the case of the latter,> the possible to be does not occur together with the possible not to be; rather, <only> the possible to be <obtains>. As for the possible which is said of the *in potentia*, it is that regarding which possible to be and possible not to be are true together. Thus it is not the case that “possible not to be” is true of everything of which “possible to be” is said. For the possible is said of the Necessary. Likewise, it is not the case that everything of which possibility is denied must be impossible. For the possible <in the sense of> *in potentia* is negated of the Necessary; but from this it does not follow that it is impossible.

Notes

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¹ See his *Studies in Modality*, Oxford, 1974 (with R. Manor, et al.) and his *Temporal Modalities in Arabic Logic*, Dordrecht: 1967.

² For references, see my “Avicenna’s Reception of Aristotelian Modal Syllogistics” in *Before and After Avicenna*, ed. David C. Reisman (Leiden: E. J. Brill, 2003), pp. 3-24.

³ See, for example, al-Fārābī, *al-Fārābī’s Commentary and Short Treatise on Aristotle’s De Interpretatione*, trans. with intro. and notes F. W. Zimmermann (London: Oxford University Press, 1981). Relevant pages cited in this article below under Zimmermann; Joep Lameer, *al-Fārābī and Aristotelian Syllogistics*, (Leiden: E. J. Brill, 1994), pp. 106-7.

⁴ See, for example, Tony Street, “Avicenna and Tūṣī on the Contradiction and Conversion of the Absolute”, *History and Philosophy of Logic*, **21**, 45-56 (2000); Paul Thom, *Medieval Modal Systems* (England: Ashgate, 2003), pp. 65-80.

⁵ An exception is Tony Street, “Fakhraddīn ar-Rāzī’s Critique of Avicennan Logic” in *Logik und Theologie: Das Organon im arabischen und im lateinischen Mittelalter*. Eds Dominik Perler and Ulrich Rudolph (Brill, 2005), pp. 99-116 (in the series *Studien und Texte zur Geistesgeschichte des Mittelalters*).

⁶ For this article, I have used Avicenna, *al-Najāt min al-Gharaq*, ed. M. Dānispāzūh (Tehran, Dānīshgāh-i-Tīhrān: 1364 A.H.).

⁷ A. M. Goichon, *Vocabulaires comparés d’Aristote et d’Ibn Sīnā*, (Paris: Desclée, 1939) #384.

⁸ Goichon, #757.

⁹ See next paragraph.

¹⁰ In addition to the citations in Goichon, see, for example, a rare instance of *tropos-jiha* translation in the *Prior Analytics*, 32b15: *antistrephei men oun kai kata tas antikeimenas protaseis hekateron tōn endechomenōn, ou mēn ton auton ge tropon = fa-kullu wāhšidin min s)anfay al-mumkini qad yan ‘akisu ‘alā al-muqaddamāti al-mutanāqid#ati ghayra anna dhālika laysa ‘alā jihatīn wāhšidatin bi-‘aynihā*. It is interesting to note that most of the instances of *tropos* in the Arabic translation of the *Prior Analytics* do not occur as *jiha* or some variant of it. In many cases, when speaking about the manner of something, the word *nahšw* is used. On the other hand, most instances of the word *tropos* that I have checked in the *Categories* occur as *jiha* or as some variant of the root. Its rendering as *nahšw* occurs rarely. I do not know whether this had to do with the particular tastes of the translators (Ishšāq b. H!unayn for the *Categories* and Tadhārī for the *Prior Analytics*) or with the standardization of translation techniques. I opt for the latter, given the following: Tadhārī has been identified by Lameer, *al-Fārābī*, p. 4, as the brother of Is)t,ifan b. Basīl, a translator known to have collaborated with H!unayn b. Ishšāq. Lameer reports that this translation was submitted by Tadhārī to H!unayn for corrections. The *Prior Analytics* translation was therefore done some time in the second or third quarter of the ninth century and it was a product of H!unayn’s generation. It is true that H!unayn’s correction of this translation depended on the Syriac translation prepared by his son. This does not necessarily mean that it should be counted as a product of the next generation of translators. For some thirty-five years elapsed between Ishšāq’s and his father’s deaths. It is thus imaginable that Ishšāq standardized the translation of the term in question after he had prepared the Syriac translation. The difference in translation then very likely has to do with the stage of the translation movement. I would guess that it is only by Ishšāq’s time that *tropos* came to be translated in a standard fashion by *jiha*. See, for example, *Categories*, 4a29; 4b2; 9b10; 12b3; 12b11; 13a16; *awjuh = tropous*, at 14b22, etc. With respect to the *jiha-tropos* translation, the *Topics* is very similar to the *Categories*. The work was translated by Abū ‘Uthmān al-Dimashqī, who was from the generation of Ishšāq b. H!unayn. See 101b29 (*kull wajh = pantos tropou*); 101b36; 102a12; 106a4; 108a34, etc. Abū ‘Uthmān also translated Alexander’s treatise on the conversion of propositions (see below). To the best of my knowledge, there is no extant Greek for this work. The term *jiha* makes several technical appearances in this treatise and is very likely a rendering for *tropos*. This lends further support to the claim that the *jiha-tropos* translation had become fairly standard by Abū ‘Uthmān’s generation. All references to the Arabic translations come from Aristotle, *Mant,iq Arist,ū*, ed. A. Badawī (Kuwayt: Wakālat al-Mat,bū‘āt, 1980). See also al-Fārābī, *Short Commentary on Aristotle’s Prior Analytics*, tr. N. Rescher (Pittsburgh: University of Pittsburgh Press, 1963) p. 32.

¹¹ This list is not exhaustive and I am certain that there are many other contexts in which this word is used. The purpose of this list is to give the reader a sense of the wide contextual and semantic range of the word.

¹² Same for the Stoics: see Henry George Liddell and Robert Scott, *A Greek-English Lexicon* (Oxford: Clarendon, 1996) “*tropos*”.

¹³ This use definitely becomes technical in this sense of “method of proof” by the end of the Hellenistic period: *ho kata tēn homoiotēta tropos* (the method of proof according to similarity), which is opposite to *ho kat’ anaskeuēn tropos tēs sēmeioseōs* (the method of proof according to denial of a visible sign). See Liddell-Scott, *tropos*.

¹⁴ The usual term for figures is of course *schēma*.

¹⁵ We can imagine how the frequent loose usage of a word in a given context might lead to its development into a *mot d'art* specific to that context. Thus, with regard to *tropos* or *modus* in the Greek-inspired medieval theory of modes in musicology, we have a rather late development of this word as a technical term. See Calvin Bower, "The Modes of Boethius", *The Journal of Musicology*, (III, 3: 253); Henri Potiron, "Les notations d'Aristide Quintilien et les harmonies dites Platonicienne", *Revue de musicologie*, (47e, 124e: 160). At *Prior Analytics*, 25a2, Aristotle says the following: "*hai men kataphatikai hai de apophatikai kath' hekastēn prosrēsīn* (<Some are> affirmative, others negative, according to the each adjunct)." This phrase occurs within the context of a discussion of modes; "adjunct" thus refers back to them and is the closest we get to the use of a single expression that denotes them. *Prosrēsīs* is translated as mode by A. J. Jenkinson (*The Complete Works of Aristotle*, ed. Barnes (Princeton: Princeton University Press, 1984) vol. 1: 40. Unfortunately, this is the only occurrence of this word in all of Aristotle's logical works. It is interesting to note that the Arabic translation of the *Prior Analytics* ignores the expression *kath' hekastēn prosrēsīn*. It reads, "*wa-kullu wāhšidatin min hādhihi* (i.e. the three kinds of necessary, problematic, and assertoric propositions) *immā an takūna mūjibatān wa-immā sālibatān*. (each one of these is either affirmative or negative)." In the Greek usage of the period, *prosrēsīs* signified the manner in which one addressed someone. In other words, it was a word or expression used to speak about something. Thus "adjunct" may not be a suitable translation here and it is possible that only with Alexander was it glossed as such (see pp. 7-8 below). I thank M. Crubellier for this comment. See *Mant,iq Arist,ū*, vol. 1: p. 109. On Alexander's comment on *prosrēsīs*, see Section I, ii below.

¹⁶ There is a discussion at *In Aristotelis Topicorum*, 38, where he uses *tropos* to refer to the different modes of predication (essential, accidental, etc.), i.e. he uses the word to refer to the kind of relation that holds between subject and predicate, but not with reference to the logical constants "necessary" and "possible" of his formal system. Thus he comes somewhat close to Aristotle's *Topics*, 135a7, mentioned above.

¹⁷ On Alexander's use of *tropos* as mode, see his *In Aristotelis analyticorum priorum librum i commentarium*, 197,2 (*ton gar tropon tēs huparxeōs ou tēn huparxin anairein epaggelletai*); 202,6 (*ean de metatethōsin hoi kata tas protaseis tropoi*), etc. This of course does not mean that the word did not continue to be used in several other ways: *tropos* as mood occurs at his *In Aristotelis Topicorum*, 2,4; as manner of expression (*tropos kata tēn lexin*) at *Id.*, 37,17; 40,18, etc.

¹⁸ This is the translation given by J. Barnes, "Logical Form and Logical Matter" in *Logica, Mente e Persona*, ed. A. Alberti (Firenze: L. S. Olschki, 1990), p. 41.

¹⁹ See Barnes, 'Logical Form' p. 41.

²⁰ For other senses of *hūlē* in the logical tradition, see Barnes, "Logical Form" p. 41. For Avicenna, it is not matter in the sense of *pragmata*, but in the sense of modal relation that is important. This is one of the meanings indicated by Barnes. He discusses this further at pp. 44-45, for which see below. According to Barnes, it is very likely that this distinction existed before Alexander, but there is no solid evidence to suggest it. See Barnes, 'Logical Form' p. 43.

²¹ *Ta hupokeimena*, i.e. the objects referred to by the terms.

²² I thank Tad Brennan for discussing this passage in an e-mail communication.²² I doubt, with Zimmermann, *al-Fārābī's Commentary*, p. 243, n. 1, that Aristotle himself envisioned this distinction. It is very likely a Peripatetic invention.

²³ This passage seems somewhat out of place and seems to be an afterthought. See Alexander, *On Aristotle's Prior Analytics*, trans. Barnes et al. (Ithaca: Cornell University Press, 1991) p. 81, n. 165.

²⁴ As in the previous paragraph.

²⁵ This discussion reduces to the old and well-known point of Porphyry's school that entities qua entities were the subject matter of metaphysics and that logic was about statements regarding these things. See Zimmermann, xxxix. It was perhaps this formal space created for logic that lay behind its survival in the Neoplatonic curriculum. See Richard Sorabji, "The Ancient Commentators on Aristotle" in *Aristotle Transformed*, ed. R. Sorabji (Ithaca: Cornell University Press, 1990). For an identical argument in Themistius, see Rosenberg and Manekin, 'Themistius' p. 97.

²⁶ At least this is how I understand the evolution of the term. Barnes points out that, for Ammonius, *hūlē* was the equivalent of *pragma*; the latter was defined as something signified by words in a *logos*. The *significans* must either be an *onoma* or a *rēma*. Thus the *hūlē* must be that which is referred to by the subject and predicate terms. This brings us back to *hūlē* as matter, as opposed to the form of a proposition/syllogism. Barnes then says that the other items of a sentence do not signify *pragmata*, even though they do signify something, including relations. Perhaps he means that *hūlē* as relation is something entirely distinct in Ammonius. See Barnes, "Logical Form" pp. 45-6. For the latter sense of the term, see this paragraph. See also C. Ehrig-Eggert, "Zur Analyse von Modalaussagen bei Avicenna und Averroes" in XXII, *Deutscher Orientalistentag*, 1983, p. 196. Here Themistius identifies *hūlē* with *schesis* (relation; Verhältniss).

²⁷ Thus the mode of "Necessarily, every man is an animal" and "Necessarily, every man is a writer" is the same, i.e. necessary; but the matter in the former is "necessary" and it is "possible" in the latter. The mode is "due to

us” and the matter is “due to the nature of things”. In the case of the first proposition, what is due to us corresponds with what is due to the nature of things. Not so in the second. See also Stephanus, *In de Interpretatione*, 25, 20.

²⁸ The same idea is expressed by Themistius. See Rosenberg and Manekin, “Themistius” p. 92.

²⁹ Zimmermann translates *mādda* here as material <modalities>. I am not sure that this is what al-Fārābī means. For he goes on to explain that matters are things which produce qualities when connected. Certainly material modalities do not produce qualities, for the latter are themselves those qualities. I suspect though that this is a slip in translation, for Zimmermann knows that al-Fārābī is not only familiar with both uses of the term, but also with how they relate to each other. For he writes, “These <i.e. necessity, possibility, impossibility> had been called <<the three <kinds of> matters>>. Al-Fārābī not only shows himself familiar with this usage, he also contrasts a proposition’s <<matter>> (rendered as <<material modality>> in my translation) with its <<mode>> (i.e. modality expressly specified by means of words like ‘necessarily’) as features of <<matter>>, i.e. content, and <<composition>>, i.e. structure.” This is not very different from the suggestion I made above regarding the appearance of the *tropos-hūlē* dichotomy as a concomitant of the *eidōs-hūlē* one. For the distinction between form and matter in Fārābī, see Zimmermann, p. B. For material and formal contrariety, see *Id.*, xl; for a treatment of the same subject in Alexander, see K. Flannery, *Ways into the Logic of Alexander of Aphrodisias*, (Leiden: Brill, 1995), Chapter Three. In speaking about the subject matter of logic, al-Fārābī states that the *De Interpretatione* is about the compositions (*ta’līf*), not the matter (*mādda*) of propositions. The former is the form (*sūra*) of sentences. It is clear on the basis of philological analysis that his inspiration comes from the commentary tradition: for *ta’līf* = *sumplokē*; *mādda* = *hūlē*; *sūra* = *eidōs*. See Barnes, “Logical Form” 42, where these distinctions (along with the aforementioned Greek terms) are found throughout the Greek and the Greek-inspired Latin traditions. Zimmermann (p. xxxix) seems not to be familiar with these connections (though he does have a definite hunch about them): “Striking an individual note in the very first sentence of his *Commentary* al-Fārābī says that the *De Interpretatione* is about <<composition>>...I do not find this opposition of terms, which recurs as a kind of *leitmotiv* throughout the work, in the Greek commentaries.” Indeed he seems to think that the notion of matter as content was a Fārābīan invention, extracted from his understanding of material modalities (or at least this is what I understand him to be saying (xxxix-xl)): “He <i.e. al-Fārābī> thus appears to have arrived at his own term ‘matter’ in the sense of content simply by extending an earlier usage from a particular aspect of subject-matter <i.e. material modality?> to subject-matter in general.”

³⁰ Again, Zimmermann has ‘material <modalities>’

³¹ Here I agree with Zimmermann’s translation.

³² See Zimmermann, pp. 243-4.

³³ I do this lest all this talk about the commentary tradition should be considered obsolete in the absence of relevant Arabic translations.

³⁴ In *Commentaires sur Aristote perdus en grec*, ed. A. Badawī (Bayrūt, Dār al-Mashriq, 1971) p. 62.

³⁵ The distinction between *tropos* as form and *mādda* is also found in the Hebrew translation of Themistius: “A sentence such as ‘Every man is an animal,’ though considered necessary ‘according to the nature of things’ or ‘according to the materials (*ha-hšomerim*)’ is *de inesse* simply because the modal qualification ‘necessarily’ is absent.” See Rosenberg and Manekin, “Themistius” p. 87. On the matter of premises, see also *Id.*, pp. 92, 96.

³⁶ *Al-Shifā*, 3: 112 (Avicenna, *al-Shifā*, ed. I. Madkour (Cairo, 1991)). This is Avicenna’s most comprehensive treatment of the subject.

³⁷ *Naw’* usually corresponds to *eidōs*. Now the latter, when translated by the former, means species. But *eidōs* is also translated into Arabic as *sūra*. The two senses of *eidōs* are of course related to the extent that they refer to things on an abstract and formal level. It is a long shot, but I wonder if Avicenna is not thinking about *naw’* not in its very specific sense as species/kind, but in the related sense as form. If this is the case, my claim that the *tropos-hūlē* dichotomy is subsumed under the larger category of the *eidōs-hūlē* is further substantiated. See Zimmermann, l; Goichon, #372, #723. Another possibility is that this text was dictated to a scribe who mistook *naw’* for *nahšw*. The latter was widely attested as a translation of *tropos* in H!unayn b. Ishšāq’s generation. But I am unfamiliar with the use of *naw’* as a technical term used to translate *tropos* in the sense of mode. See footnote 11.

³⁸ Something like this distinction was already found in Aristotle’s commentators, e.g., Ammonius. See Thom, *Medieval Modal Systems*, p. 67; pp. 74-5. See also Street, “Avicenna and T□ūsī”, pp. 45-7.

³⁹ See my ‘Avicenna’s Reception’, pp. 17-18. See also Rosenberg and Manekin, “Themistius” pp. 94-5.

⁴⁰ The division bears some *loose and surface* similarity to Abelard’s divided/compound readings. But I do not think that there is really anything underneath the surface. For example, a *de rebus* compound reading (given as a comparison to Avicenna by Thom) does not correspond much to a *was)ft* reading: “It is possible for those standing to sit while remaining standing,” for “while remaining standing,” although descriptive, is not a condition that allows for the predication of sitting for the subject. What might have misled Thom in this instance is that both the *dhātī* and *was)ft* readings may be conditioned in Avicenna. And it is this condition that

determines the truth of the predication. However, they are conditioned in different ways: the *dhātī*, by the existence of the subject's essence, the *was/jfī*, by descriptions of that subject. See Thom, *Medieval Modal Systems*, pp. 47, 68. See Tony Street, "An Outline of Avicenna's Syllogistic" in *Archiv für Geschichte der Philosophie*, 84: p. 133.

⁴¹ I explain below the two types of possibilities hinted at here.

⁴² *Al-Shifā'*, III: 112-118. See translation of selected passages from this section at the end of this article.

⁴³ Although he does say that one reading is more natural than the other and that, as far as inferences are involved, one ought to be concerned with modes of copular connection. See translation, p. 30 below.

⁴⁴ I think that the inability to modify copulae with the possibility mode in universal negative propositions is a meta-linguistic problem. For one can say that the relationship of A and B is a possible, impossible, or necessary one. But how does one assert that no A is possibly B in terms of the relationship that holds between A and B? "No A is possibly B" is a concomitant of the possible relation between A and B, and it must be expressed with existential force. The assertion of the possibility modified copulae in such statements must be affirmative, as such statements can only express the nature of the relationship between something A and something B. I will say more on this below.

⁴⁵ I don't see why not.

⁴⁶ The Abelardian *de sensu/de rebus* distinction seems to have similar implications. See Thom, *Medieval Modal Systems*, p. 47.

⁴⁷ For a similar doubt expressed by Alexander (according to Themistius), see Rosenberg and Manekin, "Themistius" p. 96.

⁴⁸ See my "Avicenna's Reception", pp. 15-16. See also Street, "Outline" p. 135.

⁴⁹ See Street, "Outline" p. 145. Avicenna does not offer an argument in my fashion in the *Ishārāt*, p. 385. In fact, he uses *ekthēsis* for his proof. However, he does hint that he has my kind of reasoning in mind when he points out that possibility e-conversions do not go through because the subject may be necessary for the predicate, but the predicate may only be accidental for it. For this article, I have used *al-Ishārāt wa-t-tanbīhāt*, ed. S. Dunyā (Egypt: Dār al-Ma'ārif, 1957-68).

⁵⁰ I.e. as we judge things to be and perhaps also as they are with reference to us with their existential import.

⁵¹ There is of course always the possibility of speaking about how a predicate holds of a subject in assertoric propositions, since they are understood temporally by Avicenna and can amplify with the statistical readings of necessary and possible propositions. I explain myself further below. See also note 56 below. From the statistical readings, we can revert to the alethic ones. For a statement on the relation between necessity, possibility, and assertoric propositions, see *Ishārāt* p. 322.

⁵² See my "Avicenna's Reception". This move, a false but possible supposition, is used by Avicenna in some syllogistic proofs. See Tony Street, "An Outline", p. 141.

⁵³ This is of course if we revert to a consideration of the *mādda*. This is a problematic conversion because it understands assertoric propositions in two different ways – with a necessary and possible *mādda* – and places both readings under one head. Conversion of possibility universal affirmatives, as we saw in the previous paragraph, may be supported by a similar argument.

⁵⁴ See *al-Shifā'*, 4: p. 91, where Avicenna gives the example of "Every writer is awake" which converts to "Some awake are writers". Avicenna seems not to agree with this conversion. But I am not sure whether this is a general rejection of the conversion of a *was/jfī* proposition. It is more likely that he is interested here in giving a more precise manner of understanding the conversion. In summary form, he reasons as follows: if all writers, insofar as they are writers and for as long as they exist, are precisely those that are awake, then some that are awake are writers, for as long as their essence exists. Now, the fact of some As being Bs does not rule out the possibility of some As not being Bs. Likewise, some As being necessarily Bs does not rule out the possibility of some As being non-necessarily Bs. Thus, if by the argument above, some awake are writers by necessity, there may also be some that are so without necessity. Given this, we need not accept that writers, <only> insofar as writers, are awake. For some awake may be writers (i.e. those things that are writers) without this condition (i.e. of being a writer). In other words, "All writers are always awake while writers" converts to "Some who are awake are only sometimes writing while awake" and not to "Some who are awake are always writing while awake".

⁵⁵ This is *mā huwa fī nafs al-amr*. It is certainly expressed by a *jīha*, but only insofar as this *jīha* is a sign for the *mādda*. "A" is the universal quantifier.

⁵⁶ With its existential import, this is *mā huwa 'indanā*. It is not a statement about the nature of things, but about how they obtain for us and how we judge them to obtain. This judgment, expressed in the *jīha*, may or may not be compatible with the *mādda*. Since it is not a statement about the nature of relationships, but about the possibility of subjects coming to be with certain predicates, this proposition is open to be defined in a manner suitable to the speaker. As before, I am tempted to add an existential quantifier to this proposition.

⁵⁷ I am tempted to add an existential quantifier to this proposition.

⁵⁸ *Al-Ishārāt*, p. 369. Here he also offers a proof by *ekthēsis* aimed at proving that this conversion follows. The proof goes as follows: “No A is B”; so “No B is A”; if not, then “Some B is A”; let that B which is A be J. So, “All J is B” and “Some J is A”; so some of A is B, namely, that which is J. But since “No A is B,” this is absurd. Avicenna explains that this proof is perfectly fine in itself, except that “No A is B” is compatible with “Some A is B” – presumably under the *dhātī* reading, which is compatible with modified possibility quantifiers, which reflect a contingent *māddī* relation. For, “All things picked out by As (whenever that may be) are at least once picked out by Bs and at least once not picked out by Bs” is the case only if it is possible for all As to be Bs and not to be Bs, which, in turn, is possible if B holds contingently of A. See also Street, “Outline”, p. 135.

⁵⁹ In Alexander’s *On Conversion*, e-conversions go through, pp. 63-65. This happens also in his *In Aristotelis analyticorum priorum librum i commentarium*, 30,1ff. For a long discussion of challenges to conversion in propositions like “No drink is in a jug” and “No jug is in a drink” and the manipulation of “in” required for this e-conversion to work, see his *On Conversion*, pp. 69-74. This is also discussed by Avicenna at *al-Shifā’*, 4: 87.

⁶⁰ See Street, ‘Outline’, p. 143, where says that e-conversion fails, without pointing out that Avicenna makes room for this conversion in his middle period for *was)fi* readings. Cf. Street, T., “Avicenna and T□ūsī”, p. 47. Avicenna does allow the traditional square under *was)fi* readings at *Ishārāt*, p. 358, but, in this same work, he seems not so amenable to these readings when it comes to conversions (see below).

⁶¹ I wonder if the reference is to propositions that are possible and true in most cases. They approximate the necessary and are isomorphic with the assertorics. See “Themistius on Modal Logic”, p. 102.

⁶² Here he also includes the *dhātī* necessity (“All A is B for as long as the essence of A exists”) among the kinds of assertoric e-propositions that convert. See *al-Shifā’*, 4: 75-6. Along the way, he also offers several criticisms of those who argue for the conversion of e-propositions on the basis of *ekthēsis* proofs that involve the conversion of particular affirmatives, of those who offer proofs via the principle of *mubāyana*, of those who argue for the conversion of assertorics insofar as the latter can be taken to be limited by the period in which something fails to obtain, etc. See *al-Shifā’*, 4: 76-85.

⁶³ *Al-Najāṭ*, pp. 45-6. It is true, as Street says (“Outline” p. 155), that Avicenna gave a “rather cavalier treatment of, and claims for, the syllogistic with propositions in the descriptive reading.” But I do not think that in his middle period he was averse to them (see e-conversions mentioned in this paragraph). Street tells us that these readings became very important in the post-Avicennan logical tradition.

⁶⁴ *Wa-l-h\$aqqu laysa lahā ‘aksun illā bi-shay’ in mina al-h\$iyal*. See *al-Ishārāt*, I:369. *Mubāyana*, according to Avicenna, was an argument invented by “recent philosophers” to prove e-conversions. The underlying principle it worked with was: that which separates from something which is separated is separated from it (*mubāyin al-mubāyan mubāyan*). He discusses it in *al-Shifā’*, 4: 77-9. A very similar argument is also found in Themistius as a proof for the conversion of absolute e-propositions. See Rosenberg and Manekin, “Themistius” p. 98. For al-Fārābī’s use of a *mubāyana* proof (apparently appearing also in Theophrastus and Eudemos), see Lameer, *al-Fārābī*, pp. 101-103. See also Alexander’s *On Conversion*, pp. 64-65. See also Alexander’s *In Aristotelis analyticorum priorum librum i commentarium*, 31,1.

⁶⁵ See *al-Shifā’*, 4: 77-79.

⁶⁶ See, for example, *Ibid.*, 4: 77.

⁶⁷ See T□ūsī’s commentary on this passage in *Ishārāt*, I: 307: *mā yufhamu wa-yutas)awwaru minhā <i.e. al-mādda> bi-h\$asbi mā tu’t,ihī al-’ibāratu min al-qad#iyyati allatī hiya al-jihā*.

⁶⁸ *Yumkinu an yakūna kullu wāh\$idin mina an-nāsi kātiban*. The mode is being applied to the quantifier.

⁶⁹ *Kullu insanin yumkinu an yakūna kātiban*. In this case, the mode is applied to the copula.

⁷⁰ *Yumkinu an yakūna ba’d#u an-nāsi kātiban*.

⁷¹ *Ba’d#u an-nāsi yumkinu an yakūna kātiban*.

⁷² *Yumkinu an lā yakūna ah\$adun mina an-nāsi kātiban*. As an analogy to the affirmatives, this would be a mode applied to the quantifier.

⁷³ *Wa-lā wāh\$ida mina an-nāsi illā wa-yumkinu an lā yakūna kātiban*.

⁷⁴ *Kullu insānin yumkinu an lā yakūna kātiban*.

⁷⁵ *Yumkinu an lā yakūna kullu insānin kātiban*.

⁷⁶ *Ba’d#u an-nāsi yumkinu an lā yakūna kātiban*.

⁷⁷ I would have much preferred to read “*adkhalta ar-rābit,ata ‘alā al-mah\$mul’*” i.e. “when you insert the copula to the predicate,” as below, “*alh\$aqta al-jihata ‘alā’r-rābita*.” No such reading is offered in the apparatus.

⁷⁸ This is the first instance in this discussion when the copula is indicated by *w-j-d* and not *k-w-n*.

⁷⁹ He must mean the statements, “Zayd is just” and “Zayd is not-just” would be false if Zayd did not exist. Thus the latter cannot be a negation of the former. I would much prefer to read *wa-tānika* as *wa-qawlānika* in analogy to *fa-kayfa wa-qawluka* below.

⁸⁰ *Bi-mā taqaddama*. The idea is that each new element attached to the growing proposition comes to govern the character of the whole. So, in order to change the proposition, one needs to operate on that new element.

⁸¹ I would have much preferred to read *fa-kayfa wa-qawluka* in place of *wa-kayfa wa-qawluka* but no such alternative reading appears in the apparatus.

⁸² I.e. with regard to their truth-value “True”.

⁸³ *Muh\$amal*.

⁸⁴ Possible = *mumkin*. Possible2 = *muh\$amal*. The Arabic of the last two phrases is, “*al-muh\$amalu innamā yu'nā bihi mā huwa 'indanā kadhālika wa-l-mumkinu mā huwa fī nafsi -l-amri kadhālika.*”

⁸⁵ *Lākinna qawlahum ghayru mustamirrin fī alfāz)ihī*.

⁸⁶ According to the earlier paradigms, the mode attaches to the copula in the first statement. I would expect the mode to apply to the quantifier in the second statement. But part of it is worded in the manner where the quantifier takes the mode and part of it where the copula does: *Kullu wāh\$idin mina an-nāsi yumkinu an yakūna kātiban*. I would expect, *yumkinu an yakūna kullu wāh\$idin mina an-nāsi kātiban*. But perhaps Avicenna means to say, “There is a possibility for each member of the class ‘man’ to be a writer” which is different from “It is possible for each one among men to be a writer.” The former modifies each member of a larger class separately; and this can be generalized as the possibility attached to the class as a whole. The latter, on the other hand, modifies each and every member of the class.

⁸⁷ Reading *qurinat* for *qurina*.

⁸⁸ I.e. of *kullu insān* to *kullu wāh\$idin mina an-nās*.

⁸⁹ *'inda jumhūri an-nās*.

⁹⁰ *Yūjada*.

⁹¹ *2attā yakūnu' ttafaqa*.

⁹² The texts says *farqān*, but I did not record two differences. The difference between the two applications of the mode may be summed up symbolically (Ax = universal quantifier; Ex = existential quantifier; P = possibility): (1a) modified quantifier: P ((Ax) (Mx --> Wx)). Since the argument carries existential import, which is exactly what carries the doubt, a better rendering might be: (1b) (P) ((Ax) (Mx --> Wx)) & (Ex) Mx. Modified copula: (2) (Ax) (Mx --> P(Wx))

⁹³ Perhaps because the common possible, as not-I, is itself ambiguous, as it can be isomorphic with the necessary. Therefore its negation poses the following problem: P --> not-I; not-I --> N or non-N; but non-N = not-N (if one fails to make a distinction between contingency and possibility); not-N --> P or I. Thus P --> I. See, for example, my ‘Avicenna’, pp. 15-16. A nice tree for this as Boethius’ understanding of Stoic positions is also offered in Benson Mates, *Stoic Logic* (Berkeley: University of California Press, 1961) p. 37, n. 51.

⁹⁴ *Imkān salb al-'āmm*. Perhaps he means the possibility of the negation of the predicate of all subjects, as in the example that follows, not the negation of the possibility relation between subject and predicate. The problem with the latter negation is discussed in the previous footnote.

⁹⁵ The form of this statement allows for the modification of the quantifier. As Avicenna said, this means that the possibility of the negation of the predicate from all subjects is being conveyed, not the negation of the mode of possibility in the relation of the subject and the predicate: *yumkinu an lā yakūna wāh\$idun mina an-nāsi kātiban*.

⁹⁶ *Lā mah\$ālata*

⁹⁷ *Qad yusāwī min jihatin qawlanā*.

⁹⁸ *Annahu mumkinun ih\$qāqa qawli al-qā'ili laysa ba'd#u an-nāsi kātiban*. I read *laysa* with MS readings S, H, instead of omitting it and *kātiban* with S, H, instead of *kātibun*.

⁹⁹ *Talāzum*.

¹⁰⁰ *Al-jumhūr*.

¹⁰¹ *Mumtani'*.

¹⁰² *Wājib aw ghayr wājib*.

¹⁰³ *Al-imkān al-'āmmī*. I am tempted to translate this as “the common possible”, but this is not what Avicenna could have meant, since the common possible is that of the common people and the description of the possible given here is that of the special possible.

¹⁰⁴ *arūra*. I translate *d#arūra* with Necessity, using a capital letter to indicate that this encompassed both the *wājib* and *mumtani'*.

¹⁰⁵ *Is)t,alah\$ū 'alā an yusammū*.

¹⁰⁶ I.e. its present existential status is not in question.

¹⁰⁷ This of course implies that the *state of affars*, say, a man’s being white, does not obtain at the present. Otherwise, we would be able to affirm or negate the predicate of man.

¹⁰⁸ *Tashakkakū 'alā anfusihim*.

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Islamic Logic

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Abstract. A current ideology has it that different cultural traditions have privileged sources of insight and ways of knowing. Prizing one tradition over another would reek of cultural imperialism. In this vein we have those pushing for a unique status for Islamic philosophy: it should have its rightful place alongside Western philosophy — and no doubt alongside Chinese philosophy, Indian philosophy, African philosophy.... I begin by examining what could be meant by ‘Islamic philosophy’. I argue that embracing a multiculturalism that makes the philosophic enterprise relative to particular cultural traditions ignores a quite important part of the Islamic philosophical tradition itself: the quest for a transcultural, universal objectivity. The major Islamic philosophers embraced this ideal: al-Fārābī and Ibn Sīnā (Avicenna), for instance. They held that some cultures are better than others at attaining philosophical wisdom, and some languages better than others at expressing it. They advocated selecting critically features from the different cultures for constructing a general theory. I illustrate their method by considering their treatment of paronymy and the copula. I end by advocating a return to this Islamic tradition.

Professor Rahman has formulated the program for this book thus:

The thinking underlying our proposal is the following. It is a common place today to say that philosophical thinking not merely articulates questions within a framework but also sometimes seeks alternative frameworks in order to dissolve or reframe the familiar questions. That is, one of the interesting procedures of research is to ask: How could our familiar questions look differently if we attempt to articulate them within a framework that is not familiar to us? It is in context that Non-European traditions acquire their interest: they can be reasonably expected to contain idioms and frameworks not familiar to us so that making available those frameworks for consideration today would invigorate our intellectual debate by making new intellectual instruments available. Obviously the Arabic text tradition is one such resource where alternatives to the current idioms of thinking can be sought. More particularly, we are thinking of the fact that both the Arabic and European text traditions took off from a reception of the text corpus of ancient philosophy, yet the historical circumstances within which the reception took place are different. This makes our focus all the more interesting: Since the problems are stimulated by the same corpus of texts inherited from the ancient Greeks, they may appear at first sight to be identical. But it is reasonable to assume that differing historical circumstances of reception resulted in different articulations. So we want to focus on bringing out not the similarity but the difference, that is, to show how the idioms and frameworks in the Arabic text-tradition, though they appear similar at first sight, differ in fact from those idioms and frameworks that are familiar to us today from our acquaintance with the European philosophical tradition.

Oddly and ironically, I can indeed find that Islamic philosophy of the classical period does indeed provide “one such resource where alternatives to the current idioms of thinking can be sought.” For the *falāsifa* — and indeed even the great mystics like Ibn ‘Arabī later — stressed a common human experience, an objective human nature and truths, across cultures. To be sure, they were aware of linguistic and cultural diversity. They took some pains to analyze the differences. Yet their goal, as for the Greek philosophers living in polyglot, sophisticated and sophisticated imperial Athens, was to find the objectivity in the diversity. Perhaps a quixotic task, yet they claimed to succeed. So then, to reclaim the perspectives of the diversity of cultures, past and present, we may need to reacquaint ourselves with the very objectivity dismissed today in certain circles (at least in talk).

I do not mean to criticize our editor by quoting him. Rather, I am examining his views as a significant cultural artifact, representative of a certain ideology in our culture. For I find our current intellectual stance curious. We are to approach cultures other than our own with the intention of finding what is distinctive, valuable and non-Western about them. In doing so we seek to “embrace diversity” (to coin a phrase). From our viewpoint we know, seemingly a priori, that we have something to find and how to look for it.

Now what I find paradoxical, although not necessarily inconsistent,¹ concerns this very approach. For is it not just one more instance of imposing a Western ideology upon a non-western culture? Instead of looking immediately for what we Westerners find, or should find, significant and distinctive about, say, Islamic culture, would it not be more responsible and respectful of that very culture to see what the people in that culture have to say about this issue? Indeed we should resist the temptation to legislate and say ‘what they *should* say’ about that issue.

I cannot of course carry out this whole project here. I confine my inquiries to looking at a few instances of how some major Islamic philosophers, Al-Fārābī, Avicenna (Ibn Sīnā), and Averroes (Ibn Rushd), deal with cross-cultural comparisons and objective truth. Now these philosophers may not represent the majority views of their own culture(s). (Someone — a Muslim from Malaysia — once remarked to me that philosophers have more in common with each other than with people in their own cultures.) Moreover, the standards that they propose in logic, for instance, concern how people should reason, not how people in fact do reason. Likewise, in epistemology they propose standards that few people actually ever follow. As the First Teacher (as Aristotle was commonly known in some classic Islamic circles²) said:

Perhaps, as difficulties are of two kinds, the cause of the present difficulty is not in the facts but in us. For as the eyes of bats are to the blaze of day, so is the reason in our soul to the things which are by nature most evident of all.³

But then philosophy has generally been, whether by fiat or in practice, an elitist activity, in which few, even the philosophers, measure up to their task. We should not then expect the views of the philosophers always to reflect the majority views of their culture.

In any event, my case of the Golden Age of Islam concerns a cosmopolitan society with such diversity that I would be hard pressed to find much cultural consensus on particular details. Peoples from many traditions mixed freely in a fairly tolerant milieu.⁴ At the least the philosophers were a group often strange by the standards of their culture.

Traditionally logic has been thought to deal with the structure of human thought — if not actual human thought, at any rate, the ideal human thought. For surely, as philosophers from Parmenides and Socrates delighted in pointing out, many if not most people reason fallaciously. The goddess herself held Parmenides back from the way of mortals on which they know nothing.⁵ Plato has Socrates go so far as to compare most people with children and the philosopher with a doctor trying to give them a nutritious diet, in competition with a pastry-cook.⁶ The children will prefer the pastry, and will offer many strident reasons for their preference. Of course, such “reasoning” frequently contradicts itself as well as conflicting with what facts we know about nutrition and health.

Now a champion of the children, of the people, may well object: you are using an adult logic assuming an ideology from the health sciences. We prefer our reasoning. In the interests of diversity, you should admit our children’s logic as an equal of your own logic. Moreover, as more of us use it and like its conclusions than like and use yours, surely ours should become the ruling standard for human thinking — as in fact human history and the actual lives of actual people attest. (Indeed we can see glimmers of such an ideology in current American educational practices, like fuzzy math.)

The befuddled philosophers, fresh down from the clouds of reason and science, might reply: you have just committed another fallacy, the *ad populum*. Yet, as the very standards of logic

have come into question, does not their very reply by their own standards commit the fallacy of begging the question?

At any rate, such are the questions that we have to face if we ask: can “Islamic philosophy” provide a unique, valuable perspective? Islamic philosophy itself challenges this question. For the perspective some Islamic philosophers provide is that the perspective of reason is privileged, objective, and relatively independent of particular cultural circumstances. We have then a perspective rejecting the equal validity of other perspectives. As Rahman says, and as we shall see Islamic logicians agreeing, different cultures will articulate the principles of this perspective differently. Nevertheless, whereas current post-modern ideology insists on diversity for all perspectives whether those perspectives like it or not, in classical Islamic philosophy we have objectivity being asserted for all perspectives within the context of a single perspective.

I shall deal here with logic, which was and still is held to provide the basic standards for reasoning, be it human or otherwise. I shall be taking ‘logic’ in a broad sense, which was traditional in Islamic and other cultures.

In a logical spirit, first I shall analyze the expression “Islamic philosophy”. I shall then proceed to examine how some classical Islamic philosophers themselves dealt with questions of multicultural perspectives. In doing so, I shall use as test cases their doctrines of derivative words and the copula. I shall be going into some technical detail, in order to give some indication of the depth and sophistication of their views. It is perhaps a sad reflection of the current state of Islamic studies today that we tend to shy away from such details, as I shall have occasion to remark again below. For surely any distinctive worth of Islamic philosophy, like Islamic mathematics and science, lies in the details. Likewise, an atomic theory proclaiming that atoms compose everything has little merit without explaining how so, in great detail.

1. ‘Islamic’ is said in many ways

Rahman speaks of the “Arabic text-tradition” and asks us to seek what is distinctive in it. Taken literally, the “Arabic text-tradition” concerns anything written in Arabic, including translations of texts from Western and other non-Arabic traditions. Still, in its usual connotation, the “Arabic text-tradition” signifies texts distinctive of Arabic, sc., Islamic culture(s).

We have a bevy of related issues here, including: To what extent does translation into a new language change the content of a text or doctrine, here principally a philosophical one? To what extent does the content of the text or doctrine change, relative to the culture associated with and embodied in that language? Do then Arabic translations of Greek philosophy count as “Islamic philosophy”? Do the works of Maimonides written in Arabic, like *The Guide for the Perplexed*, count as Islamic philosophy? Does a paraphrase of Aristotle’s works?

Today such questions routinely appear in a post-modern context. Claims of objectivity and a privileged viewpoint are dismissed as cultural and even political imperialism. We can see the same issues arising in current histories of philosophy from different cultural traditions. A recent history of Islamic philosophy has some polemical discussions about just what is “Islamic philosophy” [1162-9] as opposed to “philosophy” [i; 2-4; 21-2; 40-1; 497-8; 598-9; 796-7] or to “Muslim philosophy” [37 n.1; 1084] or to “Arab philosophy” [11; 17] or to “theosophy” [35; 638].⁷ Nasr in particular seems to have a defensive bias against types of “modern philosophy” “which has reduced philosophy to logic and linguistics”.⁸ Indeed, this bias may obscure the technical sophistication of Islamic philosophers. For this history mostly neglects the technical details of Islamic philosophers, who excelled at logic and linguistics.⁹ Indeed, such neglect may be due to most current scholarship on Islamic philosophy being

done by Orientalists and not by philosophers proper.¹⁰ As Gutas says, the view of Islamic philosophy as focusing on the spiritual and the religious and ignoring the logical and scientific has to do largely with what texts Westerners have chosen to translate and focus upon.¹¹ As a result, the focus has been on the cultural and religious contexts more than on the technical work — including that in logic and linguistics — of Islamic thinkers. To be sure, many chapters of this *History* make claims about “new” logical theory being advanced.¹² Yet, in most cases (unlike, say, the *Cambridge History of Later Medieval Philosophy*), too little detail is given to assess the originality or the logical acumen of these new theories.

This understanding of Islamic philosophy makes “Islamic philosophy” have a different use than ‘Greek philosophy’ or even ‘medieval philosophy’. Moreover, it makes it resemble instead the thought of Peter Damian and Bernard of Clairvaux, or of later scholastics in the West, or of the theologically inclined like Calvin and Luther, or perhaps of neo-scholastics like Poincaré and Maritain. It would also tend to exclude those like al-Fārābī, and al-Rāzī, who see little use for their own religious traditions in their philosophical work.¹³ Likewise it makes far more use of Avicenna’s treatises on prophecy than of his voluminous output on logic and natural science.¹⁴ The authors of this history tend to focus much more on those like Ibn ‘Arabī, Suhrawardī, and Mulla ḡadrā, who indeed are much more congenial to current Islamic “philosophical” practices. In this way, Hossein Ziai complains of the standard Western view of Islamic philosophy, that it stagnated or devolved into theosophy after Ibn Rushd.¹⁵ Yet he too admits that it is a future task to determine whether Suhrawardī’s thought is “philosophically sound” as opposed to being polemical and devoted to justifying the existence of “extraordinary phenomena” in the “imaginal” world, like “reviving the dead” and “personal revelations”.¹⁶

This relative neglect of the technical work written in Arabic may thus reflect our biases more than the state of Islamic philosophy. It also tends to make Islamic philosophy rather uninteresting to philosophers today, apart from being one more multicultural phenomenon. Moreover it has a basis as a reaction to the older Orientalist view, that those in Semitic cultures had no philosophical ability.¹⁷ Thus Renan says that the Arabs, like all the Semitic peoples, had no idea of logic as they were enthralled by poetry and prophecy.¹⁸ So we get the view that the Muslims contributed nothing new to Greek philosophy but were merely its caretakers.¹⁹ The current view does not contest this assessment of the logic and philosophy, so much as to insist on the superiority of Islamic philosophy in the poetical, mystical, and religious areas of Islam.

Such problems do not pertain to “Islamic philosophy” alone. We can see the same issues arise for other areas. I use here the historically ironic example of “Jewish philosophy”. In *The Cambridge Companion to Medieval Jewish Philosophy* Oliver Leaman starts off by worrying just what can be meant by “Jewish philosophy”: is it any philosophy done by Jews?²⁰ Philosophy using materials from Jewish culture? Philosophical comments on Jewish culture? I share the worry.

After all, would we want to speak of “Jewish physics”? To be sure, Hitler did so, but most of us do not find such talk palatable or useful. Rather, some people do physics, and physics consists of the theories they come up with. Some of these people happen to have a Jewish heritage. As many Jews have been or are prominent modern physicists, we hardly need to emphasize or even to remark upon the fact that Jews do physics by speaking of “Jewish physics”. Again basing a physics on Jewish culture seems off-target. To develop a physics based on the *Talmud* and to present it as “Jewish physics” seems ludicrous.

Why, then, is it not any less ludicrous to speak of “Jewish philosophy”? As Leaman remarks, this talk does not work well, and generally is not applied to, certain areas of philosophy, like logic or (I hope) epistemology. The areas to which it is applied, and with which this *Companion* predominantly deals, are those like religion, ethics, and political theory.

But why then is this material not philosophy but theology? The definition given by David Shatz of ‘Jewish philosophy’, as “an interpretation in philosophical terms of beliefs, concepts and texts bequeathed to medieval Jews by the *Bible* and by rabbinical literature”, certainly makes it seem so.²¹ Likewise Menachem Lorberbaum says, “Jewish philosophy must begin by attending to Jewish existence, to the meaning of Judaism confronting history.”²²

This view makes Jewish or Islamic philosophy have religion as its main focus.²³ Sabra calls this the marginality thesis: the technical, marginal work being done in Islamic philosophy was done by “...a small group of scientists who had little to do with the spiritual life of the majority of Muslims.”²⁴ Yet, as Sabra goes on to note, most of the philosophical works were preserved in the religious schools [*madrassa*], and every mosque had a resident astronomer-mathematician.²⁵ As George Sarton has remarked, Islamic science lasted longer than Greek, medieval, or modern science has (600 years).

I find Lenn Goodman’s discussion in the *Routledge Encyclopedia of Philosophy* better than these. Goodman distinguishes “Jewish philosophy” from “the philosophy of Judaism”, the latter amounting to a Jewish theology and theodicy. He says: “Jewish philosophy is philosophical inquiry informed by the texts, traditions and experiences of the Jewish people...What distinguishes it as Jewish is the confidence of its practitioners that the literary catena of Jewish tradition contains insights and articulates values of lasting philosophical import.”²⁶ In these terms, a lot of the discussions in this *Companion* consist in the philosophy of Judaism, and not Jewish philosophy.

So too we may then define “Islamic philosophy” as “a philosophical inquiry informed by the texts, traditions and experiences of Muslims”. In this way Islamic philosophers need not be devout Muslims. Likewise they need not write in Arabic: some like al-ġūsī and Avicenna wrote in Persian. Still they will be reacting to and thinking in the motifs of the prevailing culture. Thus the Arabic language will have importance in Islamic philosophy due to its social and religious significance in the society.

But all this would mean that those from Islamic or Jewish backgrounds find some materials useful there for developing and defending their own philosophical positions. Such materials may inspire them. Yet their sources do not *ipso facto* justify their claims.

Gutas suggests that we use “Arabic philosophy” instead of “Islamic philosophy” because Arabic was the language of Islamic civilization and some philosophers writing were not Muslims. Indeed, up to the tenth century (A.C.E.) logicians writing in Arabic were mostly Christian. Moreover Arabic was deliberately made into a philosophical language.²⁷ Still, as noted above, those like Maimonides writing in Arabic are not considered part of the Arabic text-tradition, and others like al-ġūsī are, even when they do not write in Arabic. My conception of Islamic philosophy does give Arabic a prominent place while not making its use a necessary condition.

Nevertheless my classical conception of “Islamic philosophy” does not have a place for discussions grounded on the revealed truth of the *Qur’ān*. Perhaps we can find a place for such Islamic or Jewish discussions, mostly as data to be explained, in anthropology, political science, history of religion, or philosophy of culture. But where is the philosophy as traditionally conceived: a pure pursuit of truth, going wherever the *logos* takes us? Where in this do we need an appeal to the culturally contingent practices of a particular culture? I doubt that “Jewish existence” needs a special existential quantifier or calls for a “Jewish logic”.

I do not mean to be too facetious here. Yet the issue has become serious. Even the head of the British commission on racism has asked recently: is the current version of “multiculturalism” a new, politically correct racism? Such talk of “Jewish philosophy” becomes a case in point. (We might think too of Spinoza’s remark in his *Letters* that Jews encourage anti-Semitism via their dietary laws and by celebrating themselves as the chosen people.)

Likewise for “Islamic philosophy”: al-Rāzī and even al-Fārābī viewed Islam as superstitious claptrap, at best fit for popular use and propaganda. In what sense are they “Islamic philosophers”? What impels us to say so? I suspect that more our present perspective than the material being studied might motivate the classification of works even of those of Arab ancestry and Islamic culture into “Islamic” and non-Islamic” philosophy etc. Cultural pride can motivate people to insist that their philosophers are as good as other philosopher. (At times I wonder whether the current development of “Islamic philosophy” has developed mostly from the tendency of Muslim donors and certain foundations to fund positions and programs of “Islamic” philosophy etc.)

To make my point with less controversy, consider the history of mathematics (or medicine or astronomy!). Here we can say, confidently, that Islamic mathematicians did much original work in trigonometry and algebra: a real advance on Greek and Roman science. We do not need to speak of “Islamic mathematics”. Rather there is mathematics, and it turns out that many Muslim authors, mostly writing in Arabic, have contributed significantly to the field — much more so than many other cultures, it turns out. (Ancient Greek arithmetic is terrible!) I think that the same could be maintained for Islamic philosophy.

Thus, although it is fashionable today to speak of “Islamic philosophy”, “African-American philosophy”, “Lesbian philosophy” etc., let me ask: is such talk racist? Is it a way to demote philosophy to mere ideology — to admit implicitly that certain traditions are second-rate?

For on Paul Grice’s theory of conversational implicature, the overemphasis on “Islamic” suggests this.²⁸ Apart from contexts where we wish to specify that we wish to study the history or the culture of Islam, what is the point of insisting that certain philosophy is “Islamic”, “Arabic” etc.? We do not need to speak of Islamic algebra: after all, as algebra is an Islamic invention, there is not need to insist upon the importance of Muslims in algebra. Likewise, we do not need to insist upon “African-American jazz musicians”, although we might for “Afghani jazz musicians”. To continue to speak of Islamic philosophy is to acknowledge that there are strong reasons to think that it has not, or does not, measure up to the standards of the field, and that we must defend its legitimacy. Yet if we turn to the technical details of the philosophers themselves, we find that we have nothing to defend. As we shall see briefly, the content speaks for itself.

2. Linguistic Determinism

The dependence of Western ontology on the peculiarities of the Indo-European verb ‘to be’ is evident to anyone who observes from the vantage point of languages outside the Indo-European family.²⁹

The Sapir-Whorf hypothesis lurks behind many later views on the relation of language to its culture, including the philosophy done there.³⁰ On it a language embodies a culture. Different cultures have no objective common ground, nor can a neutral observer find such ground in order to make objective comparisons and translations. We can add to this some descendants of the Sapir-Whorf hypothesis: Wittgenstein’s conception of different ways of life being different *language* games, each with its internal standards, and Quine’s doctrine of the indeterminacy of translation: no exact translation between languages, or indeed between idiolects in the same language, is possible, given that under-determination of stimulus meanings — even assuming that different human beings could have the same stimulus meanings at the same time, given their different vantage points and their different physiologies and past experience — and the wide variety of different, mutually incompatible sets of analytic hypotheses to supplement those meanings.³¹ Small wonder then that those doing comparative philosophy will say:

If Whorf is right...[if] the philosopher is trapped in his native language, then every cognitive insight he provides can do nothing else but redescribe the fundamental structures of his linguistic outfit.³²

On this view philosophy amounts to an articulation of the values of the culture, whether these be grammatical or political. Knowledge amounts to what we can experience from our particular viewpoints without ever being able to go beyond their limits. At best the philosopher can articulate, analyse, and make consistent the general principles presupposed by her perspective. Aristotle, “the master of all who know”, did no better. For instance, in coming up with his list of the categories, Aristotle unconsciously took as his criterion the existence of the corresponding expressions in Greek, the distinctions in the language, without noticing what he was doing.³³

Like many others, Jean-Paul Reding flirts with such a linguistic determinism, although he ultimately shies away from it. Thus Reding accepts to a great extent the Whorf thesis of linguistic relativism. Still he continues to hold that “philosophy is not entrapped in language” and we may find common cognitive insights in different traditions.³⁴ Still he is a “soft” linguistic determinist: e.g., he suggests that atomic theory tends to arise only in languages that are alphabetic.³⁵ Reding goes on to say that the comparison of Chinese and Greek philosophy is our only chance to see to what extent philosophy is independent of language, and test the Whorf hypothesis. For the other sorts of philosophy that we have come from Indo-European languages.³⁶

Reding sees fundamental profound differences between ancient Greek and Chinese, ones that have to influence the logical theory. Unlike Greek, Chinese has no ‘is’ to serve as a separate copula and no inflections, and indicates time and frequency differently.³⁷ Moreover, what Graham takes to be the main difference, Chinese distinguishes sharply between nominal and verbal sentences.³⁸ Reding sees as the main linguistic difference between classical Chinese and Greek that in Chinese temporal markers are expressed at the start of a sentence and temporal frequency markers come at the end, and, secondarily, that Greek has a distinctive word for the copula.³⁹ The difficulty in comparing this claim to Greek is that Greek has no fixed word order. Graham’s point about the big difference in structure in Chinese between the verbal and the nominal sentence seems more apropos than the common point about Greek having a distinctive word for the copula.

All this may be so. The irony of Reding’s position is that much the same grammatical points can be made about Arabic, which Islamic philosophers adapted, quite self-consciously, to express the truths of the Greek tradition that they inherited and expanded. The striking point is that Arabic differs from Greek in much the same ways as Reding says that Chinese does: a difference between the nominal and the verbal sentence, and not having a copula.

Moreover, Islamic philosophers like al-Fārābī and Avicenna explicitly note the differences between Greek and Arabic, and discuss which language gives a better description of what is real. They then make up some structures in Arabic to side with the Greek, while discarding some of the Greek structures in favor of what they judge to be the more perspicacious ways of signifying things in Arabic. We shall see the former happening with the copula, and the latter with paronymous expressions.

All this does not look like the activities of simple-minded insects trapped in their linguistic web. On the contrary, it looks just as sophisticated as what we can do today in comparing different traditions, and judging whether these or those philosophers are trapped in the illusions of their language games.

Now Reding follows A. C. Graham in taking Arabic philosophy to “descend from” the Greek.⁴⁰ The claim is that Islamic philosophers received the Greek materials, translated more or less accurately, and then tried to defend and articulate their doctrines without much original thought.⁴¹ This view dovetails with the view that Islamic philosophy has intrinsic flaws, from

having no direct knowledge of Greek and from having received neo-Platonist works as those of Aristotle. We have the picture of Ortega y Gasset: al-Fārābī or Avicenna or Averroes becomes a Quixote, trapped in a dream of commenting upon the *Poetics* of tragedy without knowing any plays.⁴²

All this many have found convincing. But, I submit, it convinces you the less you know of the technical details of Islamic philosophy. Moreover it ignores the independence of thought of those like Avicenna. For instance, after explaining Aristotle's claim of the priority of the first figure in demonstration, he ends by saying that he does not agree with it and that it should not be accepted.⁴³ In short, I reject Reding's taking Islamic philosophy as a mere slavish fiefdom of the Greek. But to show the originality and sophistication of Islamic philosophy I must get down to some details.

3. Paronymous Terms

Aristotle discusses paronymy in his *Categories*.

Whatever differ by inflection are called paronyms: They have their appellation in virtue of the name, as the grammatical [man] from grammar and the brave [man] from bravery.⁴⁴

As paronyms have appellation, they are called by names, and are real objects, not expressions.⁴⁵ The basic object is signified by an abstract name, like 'grammar' and 'whiteness'; the derivative object by a concrete one, like 'white' and 'grammatical'. Aristotle uses the masculine singular definite article here (e.g., τὸ γραμματικὸν) to indicate that the derivative term signifies a man. Thus paronyms are two objects referred to by two grammatically related terms.⁴⁶ In terms of Aristotle's theory of categories, the abstract, base term usually refers to an item in a non-substantial category, while the concrete, derivative term refers to a substance having that item. E.g., 'white' names the substance having whiteness, while 'whiteness' names the quality. 'The dog is white' is true, while 'the dog is whiteness' is false. In contrast, the (essential) predication of a species of a genus in any category requires that non-derivative terms be used. Thus Aristotle says that 'whiteness is a color' is true, while 'whiteness is colored' is false.⁴⁷

This doctrine conflicts with Greek as with Arabic grammar. Abstract terms are not basic grammatically and are usually derived from more concrete terms. Rather, Aristotle is making a logical point, about which expressions signify directly and primarily existing objects and which do not. Other expressions are "inflections" of these primary ones. Ordinary language confuses: it takes as primary "what is primary and evident to us" and not what is so in itself.⁴⁸ Aristotle is well aware of departing from common usage, e.g., in distinguishing between the abstract term designating the quality, like 'whiteness', and the term derived paronymously from it, 'white':

Those stated above are the qualities, while the *qualia* are those said paronymously in virtue of these or in some other such way from these. In most cases, even nearly in all, they are said paronymously, like 'white [man]' from 'whiteness', and 'grammatical [man]' from 'grammar', and 'just [man]' from 'justice', and likewise for the other cases. In some cases on account of there not being available names for the qualities it is not possible for them to be said from them paronymously. For example the runner or boxer... Other times, even when the name is available, the *quale* said in virtue of it is not said paronymously. For example, the good man is so called from virtue...⁴⁹

Qualities belonging to the category are usually signified by abstract terms; their associated *qualia*, derived paronymously from them, are predicated of a subject, in the category of substance. Instances of the two exceptions in the category of quality are 'boxer', in the sense that someone is said to have a talent for boxing, by nature and not by training, and 'good' respectively. Aristotle is noting that there is no name in the ordinary Greek language presently

for boxing-ability, and that ‘good’ is the *quale* for the quality ‘virtue’. So here ordinary language is inadequate or its grammar misleads.⁵⁰ In developing his own position Aristotle develops a technical vocabulary that departs from common usage.⁵¹ In this sense, at least, Aristotle’s thought is developmental: starting from ordinary language, he is creating his technical language.

Note that in discussing paronymy Aristotle often has to invert this grammatical order: e.g., although logically the paronym whiteness is basic and the paronym the white derivative, grammatically the paronymous term, ‘whiteness’, is not basic but derives from ‘white’. Once again for the philosopher ordinary language misleads: what is primary and evident in it is least primary and evident in itself.

Islamic philosophers continued Aristotle’s project. Even just in translating Greek texts into Arabic, often via Syriac, Arabic had to be adapted to the reception of Greek locutions and technical terms.⁵² For the languages differ greatly. The translators had to invent new terms and even new syntactic structures. By the time we come to al-Fārābī the terminology had stabilized.⁵³

Also by this time there was already an indigenous tradition of Arabic grammar.⁵⁴ The grammarians sometimes clashed with the philosophers about who had the best methods for analyzing and interpreting texts, particularly religious texts. For instance there was a famous debate in 932 between Mattā and Sīrafī. Sīrafī the grammarian won “due to the incongruities of creating a language within a language,” as Sabra puts it.⁵⁵ Yet perhaps philosophy won out in the long run. After all, science also progresses by creating artificial linguistic structures and notations.

This translation and assimilation of the Greek corpus did not amount to slavish, second-rate imitation. One way in which Islamic logicians differ from the Greeks commenting on Aristotle’s logical works concerns their approach to the Aristotelian material and above all the style in which they do so. We need only compare the commentary of al-Fārābī on *On Interpretation* with the one by Ammonius. With al-Fārābī we have a much clearer style, and a strong hint that the author has systematic views, sometimes differing from Aristotle’s, that he will be developing quite clearly — without mixing them up with Aristotle’s or other commentators. In contrast, with Ammonius and other Greek commentators (perhaps not Porphyry) — and likewise with the Latin Boethius — we get the sense that they are dutifully collecting and recording what texts they have and what thoughts they might have without much regard to overall consistency or theory. In contrast, Islamic philosophers sought progress. As al-Rāzī says about the philosopher:

Readily mastering what his predecessors knew and grasping the lessons they afford, he readily surpasses them. For inquiry, thought and originality make progress an improvement inevitable.⁵⁶

Moreover, Islamic philosophers espoused the theory of Greek philosophers like Aristotle, who held that all human beings have a common mental language of thought, while having differing spoken languages signifying those thoughts.⁵⁷ Those like al-Fārābī accordingly saw quite different roles for logic and grammar:

Grammar shares with it to some extent and differs from it also, because grammar gives rules only for the expressions which are peculiar to a particular nation and to the people who use the language) whereas logic gives rules for the expressions which are common to all languages.⁵⁸

In this it is hard to see the philosophers’ uncritically reflecting the structure of their language games. Indeed al-Fārābī makes claims that may well be embraced by a cognitive scientist today:

That is to say, the thoughts all men understand when expressed in their different languages *are the same for* them. The sense-objects which those thoughts are thoughts of are *also common to all*. For whatever individual thing an Indian may have a sensation of — if the same thing is observed by an Arab, he will have the same perception of it as the Indian.⁵⁹

Unlike their Greek predecessors, Islamic philosophers regularly discussed the different ways in which different languages would express the same claims. Since they held to objective standards of thoughts mirroring the realities of the world, they could look at the conventions of different natural languages and judge them as being more or less adequate and perspicacious:

...since the inventors of different languages had endeavored to capture the same logical structures in different ways some could be expected to have been more successful than others from case to case; and that where the grammatical conventions of a given language failed to arrange for the display of the logical structure of thought with optimum perspicuity it was the logician's task to amend them.⁶⁰

If their indigenous language(s) did the job, they used them. But, if they did not measure up, they felt free to use the conventions of another language or to make up new structures to express the truths. Al-Fārābī does just this when he discusses the names of the categories: they have conventional names in various languages and the technical ones reserved for the elite philosophers. He also admits an intermediate level of names, where the paronymous term, derived from the true name of the item in the categories, is used instead. As Aristotle had noted in his account of paronymy in the category of quality, al-Fārābī says that we might use 'noble' instead of 'nobility', even though 'nobility' names the quality whereas 'noble' names only the nobility presented in an unnamed subject.⁶¹

Looking at how Aristotle's paronyms are signified in Greek, Greek grammarians had already discussed these derivative terms, which they called "paronymous". In explaining how to generate the derivative forms, they had to make many classes and exceptions. (Here suffixes are added onto the roots or verb stems.⁶²) Priscian divides the grammatically derivative terms into the inchoative, meditative, figurative, desiderative, diminutive etc. Dionysius Thrax speaks of prototypes and derivatives of nouns. The Islamic philosophers and grammarians inherited these distinctions.⁶³

The Greek commentators on Aristotle also classified expressions signifying Aristotle's paronyms.⁶⁴ Like some grammarians, they took the infinitives as indeclinable names and as the basic forms from which other expressions were derived or "inflected".⁶⁵ Here the philosophy has influenced the grammar: the former determines which terms are basic from which of the two paronymous things is basic while the latter then shows how to make names up for the paronymous things in some language.

Grammatically, Arabic forms derivative terms much more systematically and regularly than Greek does: from trilateral or quadrilateral consonantal roots.⁶⁶ Classical Arabic grammarians derived names not from these roots themselves but from the ma{dar, the verbal noun.⁶⁷ The ma{dar is not as basic morphologically as the trilateral and quadrilateral roots of Arabic but comes quite close. Indeed, perhaps these grammarians took the ma{dar as basic because their grammatical theory was following the later Greek theory, which was in turn following logical or philosophical theory more than ordinary language.⁶⁸ That is, perhaps they used the ma{dar as the equivalent of the verbal infinitive in the later Greek grammatical theory, itself influenced by logic and philosophy.⁶⁹

Be that as it may, still the fact remains that Arabic forms its concrete nouns and adjectives from a verbal root, the ma{dar or the trilateral stem. Thus those like al-Fārābī saw Arabic to have a much better fit than Greek in the case of expressing the doctrine of paronymy: the ma{dar is basic not only grammatically but also logically. Moreover, because of the regularity of derivations in Arabic, the grammar has a much better match with the logic than in Greek. In contrast, often in Greek terms derivative in meaning have no morphological connection, as in Aristotle's example of 'good' and 'virtue'. From the logical point of view, Greek takes what is ontologically basic, e.g., names of qualities, to be grammatically derivative and

making the ontologically derivative grammatically basic, as in the regular formation of the abstract nouns. In contrast, Arabic has its grammar matching the logic.

However, al-Fārābī modifies this grammatical account of paronymy, perhaps so as to bring it in line with Greek philosophical terminology. As R. M. Frank puts it,

Against the pure formalism of the grammarians... al-Fārābī recognises a more basic, conceptual derivation according to which he conceives the ma{dar or root term as the abstract underlying the concrete and composite specific.⁷⁰

For instance he takes *insānīya* [*humanitas*; ‘humanity’] as the root for *insān* [*homo*, ‘man’], and even derives the personal pronoun *huwa* (he) from *huwīya*.⁷¹ This aligns his terminology with the late Greek custom of forming abstract nouns by adding a suffix, like „sòthj (equality) from i`son (equal).⁷² Yet unlike the grammarian he takes the abstract noun as basic as it signifies the basic thing. In either way, Arabic can express the relationship between the paronyms more clearly than the Greek.

Following al-Fārābī, Avicenna says that a derived name has an indefinite or undetermined subject.⁷³ Comparing Farsi and Arabic, he says that different languages take different structures as primary but this does not concern the logician although it can make translation difficult.⁷⁴ So he says that the ma{dar is derivative logically regardless of how it is thought to function grammatically.⁷⁵ For it never signifies a substance but only an accident in a substance. Logically, the simple name is the concrete noun signifying the thing having that accident. Here, if the ma{dar is taken as basic, “the Arabic language is a hindrance...”⁷⁶

Thus those like al-Fārābī were aware of the differences between Arabic and other languages like Greek and Farsi in a sophisticated way. In this doctrine of paronymy we have an instance of Islamic philosophers distinguishing the objective truth of philosophy and the ideal technical language of logic from the conventions of a particular culture and the grammar of its language.

Zimmermann claims that al-Fārābī confuses here two conceptions of “paronymy”: the Aristotelian logical and the Arabic grammatical.⁷⁷ He complains that all this is ungrammatical and confuses different traditions. Zimmermann goes on to question al-Fārābī’s expertise. Perhaps not even being a native Arabic speaker, al-Fārābī probably did not know the other languages that he mentions: Greek, Persian, Syriac, Soghdian.⁷⁸ He may have been relying on informants who did not know much either.

Yet this is not the point here. Rather look at al-Fārābī’s method. Perhaps he does make many mistakes in what he claims for the various languages and in the doctrines with which he ends up. Still the method itself looks sophisticated. Given how al-Fārābī *et al.* understood their task, I see no simple-minded confusion here. If it is one, then so too those like Frege and Russell equally have erred in trying to construct an ideal language.

So al-Fārābī may have made many mistakes in his grammatical and philological claims. He may have been using second-hand reports from informants who were not expert grammarians or linguists by our standards. He may have endorsed a technical way of speaking that deviated from ordinary Arabic for no good purpose. Yet all this misses my point here. Rather, al-Fārābī has a sophisticated method. To be sure, its actual results may need improvement. But this makes no fundamental criticism of what al-Fārābī is doing.

To make this point clear, consider the history of a relatively recent period in science. Most of the theories and even some of the experimental claims made in twentieth-century physics, geology etc. have been discredited. Still, that work continues to be treated as “scientific”, as being in the same world-view and even in the same research tradition as the current work.⁷⁹

Thus, in physics we have cases like the “discovery” of N-rays and perhaps of cold fusion accepted and championed by reputable scientists using reputable methods, and later rejected. Likewise, the theory of continental drift was standard geological theory in the early twentieth century, and then discredited — but then reestablished later on. All these changes came about

using roughly the same experimental methods and theoretical assumptions. The point is that this discredited work still amounts to science, albeit to discredited or false science.

Likewise, I submit, in evaluating al-Fārābī's theory of knowledge and method, we should focus more on his method than on the actual results that he presents. After all, we have the advantage of having a later perspective, presumably a more adequate one. At the same time, on inductive and historical grounds, we should suspect that some of our claims, even ones about Arabic, Greek and Persian grammar, themselves will come to be discounted, modified, or rejected in the future. We ourselves do not now seem to be in a tradition of a different type. In sum, Islamic philosophers inherited Greek doctrines about paronymous terms. They distinguished the grammatical from the logical level. They sought an objectivity across the cultures. Aware of differences in the languages, they used whatever grammatical structures best represented the logical structure of terms signifying objective realities. In this case, they judged Arabic superior to Greek, although they rejected the *maḍar* of the Arabic grammarians in favor of the simple noun signifying substance. With the copula, they judged Greek superior and sought to modify Arabic accordingly.

4. The Copula

It is somewhat improper to speak of a Chinese copula...in Greek, the juxtaposition of two nominal elements to form a sentence leaves the impression that this sentence is somehow incomplete and in need of a verb...the verb 'to be'. In Chinese, however, there are two basic types of sentences: the verbal sentence, negated by *bu* and the nominal sentence negated by *fei*. Nominal sentences, however, are not felt as incomplete sentences in Chinese. Although classical Chinese does not have a 'positive' copula, nominal sentences are nonetheless marked by the final particle *ye*.⁸⁰

Once again, *contra* Reding, Chinese has no distinctive structures here. Like Chinese, Arabic does not have an explicit word for the copula, the 'is' of predication, and has both nominal and verbal sentences. Arabic may have, instead of a final particle, an initial particle like *inna*, and also will tend today to insert a pronoun like *huwa* in a nominal sentence when the subject and predicate have definite forms.⁸¹ However the insertion of *huwa* seems to have been introduced into Arabic late, largely on account of the philosophers developing structures to express Greek thought.⁸²

In Aristotle's logic and indeed in his metaphysics of being, 'is' as a separate element plays a large part.⁸³ In seeking to render Aristotelian philosophy into Arabic, the translators had to fix on some word corresponding to 'is', and for the nominal sentence settled on *mawjūd* with the predicate complement being expressed in an accusative of respect, so as to get the form, 'S (is) existent (as) a P'.⁸⁴ All this was not elegant or even colloquial Arabic. Yet, given the philosophical goal of expressing truths in whichever linguistic conventions displayed them accurately, this was hardly an issue.

Accordingly, al-Fārābī discusses how the Arabic language has a structure different from other [mostly Indo-European] languages.⁸⁵ It has no distinctive word serving as an "expression of existence" or copula. For in the (nominal) Arabic proposition, a definite noun serving as subject is followed by an indefinite name (the predicate complement), as in "the man just". Al-Fārābī says that this holds both for the Arabic people and for the Arabic grammarians.⁸⁶ He goes on to say that in Arabic (nominal) denials would then be expressed as "the man not just" and "Zayd not walking". He points out that in the other languages such statements would be the metathetic affirmation, 'man is not-just' and 'Zayd is not-walking', as Aristotle says in *On Interpretation*.⁸⁷ Al-Fārābī notes how different languages — Arabic, Persian, Syriac, Greek, and Soghdian — have *copulae* in different grammatical types of statements, mostly the nominal and verbal ones.⁸⁸ He goes on also to discuss the verbal proposition having a verb with a pronominal subject affixed to it.⁸⁹

Al-Fārābī again is distinguishing the technical language from ordinary language.⁹⁰ His technical word for the copula, *mawjūd*, he says, has been transferred from common usage of the people where it means ‘found’.⁹¹ Unlike Greek, Arabic does not have a special word for the copula and so does not reveal clearly the logical structure of statements:

And there was not in Arabic ever since its imposition was explicated an expression substituting for the *hast* in Farsi and for the *estin* in Greek not for what are comparable in the rest of the languages. And these are needed necessarily in the theoretical sciences and in the logical art. So, since philosophy has been transferred to the Arabs, and the philosophers who discourse in Arabic and make their interpretations from the senses [concepts] that are in philosophy and in logic with the language of the Arabs and do not find, in the language of the Arabs ever since what was propounded [in it] was explicated, an expression by which they translated the places in which the *estin* used in Greek and the *hasta* in Farsi, they make a substitute for those expressions in the places where the rest of the peoples use them.⁹²

The point here is that al-Fārābī is first distinguishing what is true from what is stated easily in Arabic. The idea is that in this case the grammar of Arabic is less transparent than the ideal, mental language, and that Persian or Greek comes closer to that ideal. Likewise, he says, the common people speak (in Arabic) of the ‘non-existent’ inaccurately and figurative, saying it is ‘wind’ and ‘dust’.⁹³ Moreover, he says, ordinary Arabic confuses the existent in potency with the existent in act.⁹⁴

Al-Fārābī goes on to discuss the use of ‘*huwa*’ in constructing sentences in Arabic. He extends the grammatical use of the ma{dar to signify what is logically although not necessarily grammatically the base form from which paronymous inflections are made. So too, in discussing paronymy, he takes ‘humanity’ and ‘manhood’ as ma{dar for ‘man’. Here al-Fārābī departs from the ma{dar of the Arabic grammarians and, like the Greek grammarians before him, attaches an abstractive suffix (‘*iyya*’) to the concrete noun. When he makes up names for items in the categories, their essences and paronyms, he is clear that he is extending the notion of the ma{dar analogously. So too then here. For he goes so far in rejecting the natural forms of Arabic for the copula as to make ‘*huwīya*’ the ma{dar for *huwa*.⁹⁵

Even more than al-Fārābī, Avicenna insists upon *mawjūd* making an assertion of existence.⁹⁶ He agrees with al-Fārābī that Greek is better than Arabic in displaying the logical structure of the tripartite proposition (of form ‘S is P’).⁹⁷ He goes on to discuss Farsi and three different ways of expressing the copula in Arabic.⁹⁸

Again we can find problems with the details of such accounts of the copula: lack of expertise in the languages cited, confusing logical and grammatical doctrine etc.⁹⁹ Yet I am focusing on the method. Here once again we find the Islamic philosophers looking for objective truth across cultures — and finding it more in Greek and Farsi than in the Arabic favored by Allah for the *Qur’ān*.

The absence of a separate syntactic structure for the copula has prompted some Orientalists to consider Arabic a primitive language. For instance, L. Massignon, takes Arabic to be a primitive language with a native grammar admitting exceptions as opposed to the artificial conventions of Greek logic.¹⁰⁰ Arabic got its abstract nouns from the influence of the Greek grammarians. Madkour says that philosophical reflection demands a copula, which only the most civilized languages have, after a great effort of abstraction.¹⁰¹ Most Orientalists today reject such claims of linguistic inferiority almost *a priori* on the grounds of multiculturalism, to avoid charges of cultural imperialism. It is odd to see the Islamic philosophers themselves being less slavish to an ideology and more open to possibilities.

5. Islamic Ways of Knowing

In these two cases, of paronymy and the copula, we see some of the great Islamic philosophers discussing differences between languages and cultures. In this diversity they sought to find objective truth, and then to express it in the clearest language possible: sometimes Arabic, sometimes not.

It is hard to locate in all this a distinctively “Islamic” way of knowing. Indeed to insist upon their being one smacks of foisting upon the Islamic philosophers one more foreign ideology. For their way of knowing does not give *a priori* primacy to their own culture(s). Just look at al-Fārābī’s own attitude towards Islam and its popular culture:

Some people have come and eliminated possibility from things, not by arguing from primordial knowledge, but simply by legislation and indoctrination...When we know something because it is engrained in us, no attention can be paid to the opinion of people who disagree because they think that the Law decrees otherwise. The process of investigation in logic, and in philosophy altogether, builds on, and proceeds from, knowledge engrained in us, or what follows from such knowledge. Premises decreed for following from something decreed, or views which have become commonly accepted in a community as following from the opinion of a man whose word carries authority among its member, are not employed in this process.¹⁰²

Likewise in his account of the ideal state, al-Fārābī reserves the philosophical truth for the rulers, and leaves religion as popularized philosophy and propaganda for the masses. Still al-Fārābī does not reject his culture entirely. goes on to talk about a view of possibility more congenial to a fatalistic religion than would be allowed strictly in philosophy.¹⁰³ He still insists on having objective truth prevail: the philosopher can have a view detrimental to people and rejected by all religions.¹⁰⁴ Yet he seeks to reconcile the objective truth of philosophy with the conventions of his culture: “We must therefore find a solution to these dilemmas that does not entail anything objectionable on account of reality, common sense [*endoxa*], or religion.”¹⁰⁵

Now current Islamic affairs resembles Islamic history a lot. Even in Baghdad at the height of the Flowering of Islam, there were successive waves of liberal and repressive regimes. One ruler would encourage the development of philosophical learning, invite scholars, build observatories and so forth, while his successor would halt these movements and purge some people.¹⁰⁶ These changes might occur under a single ruler, often due to his need to please various constituencies. The same happened in Muslim Spain: Averroes himself was encouraged in his philosophical pursuits, then censured and exiled, and later recalled according to the sect of Islamic prevailing in the politics of the Almohadic court of Abū Yūsuf.¹⁰⁷

Again in the Mid-East, then as now, it was hard to avoid a multicultural perspective. A city like Baghdad would contain people from many cultures and of many religions—especially on account of the famous Muslim tolerance — at least they did not usually seek to exterminate or even to convert by force those differing from themselves — unlike the Christians of the time. Thus the last of the Greek commentators moved from a Byzantine, Christian court to a Muslim one, and continued their studies in the tradition of Greek philosophy for over a century.¹⁰⁸

Islamic philosophers during this Golden Age could not avoid being aware of there being many traditions, cultures, and competing claims of insight into the truth and the good. We have seen some examples of their confronting and adjudicating this multiculturalism. They found success in seeking to extract what each tradition offered, where not all traditions had an equal amount to offer on each subject. They would ignore the Greeks in history and arithmetic, but instead developed algebra, while studying them in geometry, astronomy and philosophy. They extracted universal truths and objective structures from their multicultural studies. They sought to mold their language so as to match up with reality, and not blindly

follow the structures of Arabic grammar. Such are the lessons we can learn today from Islamic philosophers.

In the last resort the point of his [al-Fārābī's] comparative remarks is to underline the need, in the face of the diversity of human language, for a transgrammatical approach to meaning.¹⁰⁹

Notes

¹ Sc, the original Greek sense of 'paradox', 'something contrary to common opinion, strange, marvelous, unexpected'.

² Dimitri Gutas, *Avicenna and the Aristotelian Tradition* (Leiden, 1988), p. 206 n. 19

³ *Metaphysics* 993b9-11.

⁴ Cf. *Grundrisse der Arabischen Philologie*, Vol. III, ed. W. Fischer (Wiesbaden, 1993).

⁵ Fr. 6,2-5.

⁶ *Gorgias* 521e-522a.

⁷ Page numbers in the *History of Islamic Philosophy*, ed. S. Nasr & O. Leaman (London, 1996). So too Edward W. Said, *Orientalism* (New York, 1978), pp. 2-3; 203, on the older concept of 'Orientalism'.

⁸ *History of Islamic Philosophy*, p. 13; cf. 23 — & 192.

⁹ Cf. the treatments of logic in general in this *History of Islamic Philosophy*, ed. S. Nasr & O. Leaman, pp. 802-19, with the technical work of the great logician Ibn Sīnā, pp. 234-6. Again, later logicians like Al-Qazwīnī and Al-Shirwānī are paid little attention, pp. 1038-9; still cf. p. 441.

¹⁰ Cf. pp. 1143-8.

¹¹ Dimitri Gutas, "The Study of Arabic Philosophy in the Twentieth Century", *British Journal of Middle Eastern Studies*, Vol. 29.1 (2002), p. 13.

¹² Cf. *History of Islamic Philosophy*, pp. 441; 488-9; 553; 589.

¹³ *History of Islamic Philosophy*, pp. 202-3.

¹⁴ Dimitri Gutas, "The Study of Arabic Philosophy in the Twentieth Century", *British Journal of Middle Eastern Studies*, Vol. 29.1 (2002), p. 16, makes the same point about two other recent histories: Majid Fakhry, *A History of Islamic Philosophy*, Second Edition (New York, 1984); *History of Islamic Philosophy*, ed. S. Nasr & O. Leaman, 2 vols. (London: Routledge, 1996).

¹⁵ *History of Islamic Philosophy*, pp. 442; 466-7. Likewise Edward W. Said, *Orientalism* (New York, 1978), p. 305, complains about focusing only upon the centuries of greatness of the Golden Age. Cf. also his *Culture and Imperialism* (New York, 1993).

¹⁶ *History of Islamic Philosophy*, pp. 439-40; 448. Given the present state of research, I would have to agree with I. Sabra, "The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement", *History of Science*, Vol. 25 (1987), p. 238, that there are four main periods of Islamic science and philosophy: reception; flowering; naturalization; decline.

¹⁷ M. Mahdi, "Orientalism and the Study of Islamic Philosophy," *Journal of Islamic Studies*, Vol. 1 (1990), pp. 79-93; Dimitri Gutas, "The Study of Arabic Philosophy in the Twentieth Century", p. 11.

¹⁸ Ernest Renan, *Averroès et l'Averroïsme, œuvres complètes*, Vol. 3 (Paris, 1949), p. 85. So too A. Merz, *Historia artis grammaticae apud Syros* (Leipzig, 1889), pp. 137-53; Léon Gauthier, *Introduction à l'étude de la philosophie musulmane* (Paris, 1923).

¹⁹ A. Elamrani-Jamal, *Logique aristotélicienne et grammaire arabe* (Paris: J. Vrin, 1983), pp. 10-11, notes (and protests against) this view that there is no Islamic logic but only a slavish repetition of the Greek corpus by a few Muslims. So says, e.g., R. J. De Boer, *The History of Philosophy in Islam*, trans. E. R. Jones (London, Luzac, 1903); repr. New York, Dover, 1967), p. 23. Ironically, around the same time Carl Prantl, *Geschichte der Logik in Abendlande* (Leipzig, 1855), Vol. 2, pp. 263ff., claimed, also erroneously, that Latin medieval logic got all its apparent originality from the Greek just by copying Arabic materials.

²⁰ *The Cambridge Companion to Medieval Jewish Philosophy* (Cambridge, 2003), ed. D. Frank & O. Leaman, "Introduction."

²¹ *The Cambridge Companion to Medieval Jewish Philosophy*, p. 16.

²² *The Cambridge Companion to Medieval Jewish Philosophy*, p. 179.

²³ Dimitri Gutas, "The Study of Arabic Philosophy in the Twentieth Century", *British Journal of Middle Eastern Studies*, Vol. 29.1 (2002), p. 12, notes that Islamic philosophy is often presented just as a fight with religion over truth. So Renan's *Averroès et l'Averroïsme*; so too Alain de Libera and Maurice-Ruben Hayoun, *Averroès et l'Averroïsme* (Paris, 1991), who also, pp. 3-8 give a list of such sources; so too Oliver Leaman, *An Introduction to Medieval Islamic Philosophy* (Cambridge, 1985).

- ²⁴ I. Sabra, "The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement", *History of Science*, Vol. 25 (1987), p. 228.
- ²⁵ I. Sabra, "The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement", *History of Science*, Vol. 25 (1987), p. 232.
- ²⁶ *Routledge Encyclopedia of Philosophy* (London, 1998), s.v. 'Jewish philosophy'.
- ²⁷ Dimitri Gutas, "The Study of Arabic Philosophy in the Twentieth Century", *British Journal of Middle Eastern Studies*, Vol. 29.1 (2002), pp. 17-8; n. 11.
- ²⁸ H. P. Grice, "Logic and Conversation" in *Syntax and Semantics*, Vol. 3, ed. P. Cole & J. Morgan (New York, 1975).
- ²⁹ A. C. Graham, *Relating Categories to Question Forms in Pre-Han Chinese Thought: Studies in Chinese Philosophy and Philosophical Literature* (Singapore, 1986; reprint of [1960, *The Verb Be and its Synonyms, Philosophical and Grammatical Studies*, Part VI (Dordrecht, 1967)], p. 406; cf. F. Jullien, *Du 'temps': éléments d'une philosophie de vivre* (Paris, 2001), pp. 29-31.
- ³⁰ E.g., see Edward Sapir, "The Status of Linguistics as a Science" in *Selected Writings of Edward Sapir* (Berkeley, 1956), p. 162; Benjamin Whorf, *Language, Thought, and Reality* (Cambridge, Mass., 1956).
- ³¹ Ludwig Wittgenstein, *Philosophical Investigations* (New York, 1953); W. V. O. Quine, *Word and Object* (Cambridge, Mass., 1960).
- ³² Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking* (Aldershot, 2004), p. 12.
- ³³ E. Benveniste, "Catégories de pensée et catégories de langue," in *Problèmes de linguistique générale* (Paris, 1966), Vol. I, p. 70.
- ³⁴ Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking*, pp. 10-3.
- ³⁵ Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking*, p. 94.
- ³⁶ Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking*, p.9; n. 17.
- ³⁷ Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking*, pp. 80-1.
- ³⁸ A. C. Graham, "Relating Categories to Question Forms in Pre-Han Chinese Thought," p. 373.
- ³⁹ Jean-Paul Reding, *Comparative Essays on Early Greek and Chinese Rational Thinking*, pp. 80-1; 190.
- ⁴⁰ *Disputers of the Tao* (La Salle, 1989), p. 389.
- ⁴¹ Even the relatively sympathetic F. W. Zimmermann, trans. & comm., *Al-Fārābī's Commentary [= in de Int.] and Short Treatise [= Short Treatise] on Aristotle's De Interpretatione* (London, 1981), p. xxiii, says that al-Fārābī et al. "dutifully wrote a commentary on the *Rhetoric*" and even the *Poetics*, because they were following Alexander's "highly eccentric inclusion" of them into the logical *Organon*. Still he says about his commentaries also that "[t]hey reorganize, and select from, Aristotle's subject-matters with considerable freedom; and they draw on the whole range of known philosophical tradition."
- ⁴² Cf. J. Ortega y Gasset, *Meditaciones del Quijote* (Madrid, 1914), trans. E. Rugg and D. Marín, *Meditations on Quixote* (New York, 1961). Avicenna and Averroes both comment upon Aristotle's *Poetics* without having read or seen a Greek play.
- ⁴³ *Al-Burhān* 180,9-11.
- ⁴⁴ *Categories* 1a12-5.
- ⁴⁵ In this way, paronymy, like homonymy and synonymy, is a relation between objects. Cf. J. L. Ackrill, *Aristotle's Categories and De Interpretatione*, pp. 72-3.
- ⁴⁶ 'Inflection' here is meant to be taken in a general sense. Cf. Anonymous, in *De Int.* 2,10-3,5.
- ⁴⁷ *Topics* 109a39-b12.
- ⁴⁸ Cf. *Metaphysics* 1029b8-12.
- ⁴⁹ *Categories* 10a27-b7; cf. 6b11-4.
- ⁵⁰ Al-Fārābī, *Kitāb al-Ḥurūf* 81,22-82, makes this point too: "And there may be connected in the Greek a curious thing, namely that there may be some name significative of a category and a species abstracted from its subject where the subject is not named by it insofar as there is taken for it that species by a name derived from the name of that species, but rather by a name derived from the name of another species, like 'excellence' in the Greek. So what is qualified by it does not have said 'excellent' said of it like what is said in Arabic. Rather there is said 'diligent' [spoudaios?] or 'desirous'."
- ⁵¹ See Allan Bäck, *Aristotle's Theory of Predication* (Leiden, 2000), p. 144.
- ⁵² Richard Walzer; *Greek into Arabic* (New York, 1962); Shukri Abed, "Language" in the *History of Islamic Philosophy*, ed. Nasr & Leaman.
- ⁵³ Zimmermann, *Al-Fārābī's Commentary and Short Treatise*, p. xlix.
- ⁵⁴ How original Arabic grammar was is also a matter of dispute. Earlier Orientalists like A. Merz, *Historia artis grammaticae apud Syros* (Leipzig, 1889), pp. 137-53, held that Arabic grammar came from the Persians, and the latter from Greek logic, as opposed to Greek grammar. A. Elamrani-Jamal, *Logique aristotélicienne et grammaire arabe* (Paris: J. Vrin, 1983), pp. 21-4, opposes this, but does admit sources from Greek grammarians

like Dionysius of Thrace. For a major example of the debate about such influence, see the discussion below about the ma{dar.

⁵⁵ A. I. Sabra, "Avicenna on the Subject Matter of Logic", *Journal of Philosophy* (1980), p. 747. See his n. 4 for further references.

⁵⁶ *Al-Munāz,arāt*, in *Abi Mohammadi filii Zachariae Roghensis opera philosophica gragmentaque quae supersunt* (Cairo, 1939) 301; trans. Lenn Goodman in the *History of Islamic Philosophy*, ed. Nasr & Leaman, p. 202.

⁵⁷ *On Interpretation* 16a2-11.

⁵⁸ *Al-Fārābī's Introductory Risalah on Logic* ed. D. M. Dunlop, *Islamic Quarterly*, Vol. 4 (1957), (4), 228,8-10 [trans. p. 233]. Cf. *Al-Fārābī's Paraphrase of the Categories of Aristotle* ed. & trans. D. M. Dunlop, *Islamic Quarterly*, Vol. IV.4 (1958), 172,28-173,8 [trans. p. 187 §9]: "What we have mentioned exists in all languages, and it is possible to find the like of it in the existing Arabic language. For the experts in Arabic call the short syllables 'movent' letters, and the long syllables and what resembles them they call *asbāb* or 'cords'. What can be combined in their language of both kinds of syllables they call *autād* (pegs). Then they combine some of these with others and make of them measures greater than these, by which they measure their metrical expressions and discourses, e.g., *fa'ūlun*, *mafā'ūlun*, *mustaf'ilun*. If this is so, then every expression can be measured by a long or short syllable or a combination of both. Syllables are the smallest of the parts by which expressions can be measured, and the combination of them is greater than they are. These things in the expressions are like the cubits among the lengths." See W. Wright, *A Grammar of the Arabic Language*, II §358.

⁵⁹ Al-Fārābī, *Al-'Ibāra*, ed. Kutsch & Morrow (Beirut, 1960); trans. & comm. *Al-Fārābī's Commentary [= in De Int.] and Short Treatise [= Short Treatise] on Aristotle's De Interpretatione* (London, 1981), 27,25-28,2; trans. 12-3; cf. 27,5-26; 27,8-10.

⁶⁰ Zimmermann, *Al-Fārābī's Commentary and Short Treatise*, p. xlv.

⁶¹ *Kitāb al-Hurūf* §§19-20, 71,2-15.

⁶² Cf. Herbert Smyth, rev. G. Messing, *Greek Grammar* (Cambridge, Mass., 1920), §§822ff.

⁶³ *Institutiones* VIII.14; Dionysius Thrax, *Ars Grammatica, Grammatici Graeci*, Part I., Vol. 1, ed. G. Uhlig (Leipzig, 1883), 25,3-5. Zimmermann, *Al-Fārābī's Commentary and Short Treatise on Aristotle's De Interpretatione*, p. xxxi n. 2, claims that the *Ars Grammatica* was known to Al-Fārābī via Syriac translation.

⁶⁴ Simplicius, in *Cat.* 38,1-6.

⁶⁵ Ammonius, in *De Int.* 50,15ff.; Stephanus, in *De Int.* 13,15-8. M. Chase, trans. & comm., *Simplicius on Aristotle's Categories 1-4* (Ithaca, 2003) n. 283, describes infinitives as indeclinable nouns/names

⁶⁶ Wolf Dietrich Fischer, *A Grammar of Classical Arabic*, Third Ed., trans. J. Rodgers (New Haven, 2002) p. 35.

⁶⁷ Elsaïd Badawi, M. C. Carter, & Adrian Gully, *Modern Written Arabic* (London, 2004) 1.11.1

⁶⁸ Perhaps not Aristotelian: taking the infinitive as the basic form might appeal to a neo-Platonist.

⁶⁹ Cf. Wright, *A Grammar of the Arabic Language*, Vol. I §195: "...most Arab grammarians derive the compound idea of the finite verb from the simple idea of this substantive. We may compare with it the Greek infinitive used as a substantive." Likewise C. H. M. Versteegh, *Greek Elements in Arabic Linguistic Thinking* (Leiden, 1977), p. viii, and I. Madkour, *L'Organon d'Aristote dans le monde arabe*, second édition (Paris, 1969), pp. 16-9, argue that Arabic grammar had much Greek influence, Aristotelian and Stoic. However, A. Elamrani-Jamal, *Logique aristotélicienne et grammaire arabe* (Paris: J. Vrin, 1983), pp. 13; 72, claims that there is no direct Greek influence on Arabic grammar: cf. the nominal phrase and the ma{dar.

⁷⁰ R. M. Frank, Review of *Alfarabi's Book of Letters (Kitāb al-Zurūf)*, ed., Intro, & comm. M Mahdi (Beirut, 1969) in *Journal of the American Oriental Society*, Vol. 92.2 (1972), p. 394. See *Kitāb al-Zurūf* 112ff; §§33; 28; 35.

⁷¹ *Kitāb al-Zurūf* §83.

⁷² Earlier Greek tended to use substantives, like τὸ ἄσων, for the abstract nouns. Cf. Plato, *Phaedo* 74c8; Aristotle, *Metaphysics* 1029b32.

⁷³ Avicenna, *Al-'Ibāra*, ed. M. al-Khudayri (Cairo, 1970; Part One, Volume Three of *Al-Shifā'*), 18.7.

⁷⁴ *Al-'Ibāra* 19,16-21,6. He gives some details at *al-Maqūlāt*, ed. G. Anawati, A. El-Ehwani, M. El-Khodeiri, & S. Zayed (Cairo, 1959) (Part One, Volume Two of *al-Shifā'*) 16,12-17,14.

⁷⁵ *Al-'Ibāra* 26,3-27,4.

⁷⁶ *Al-'Ibāra* 27,5.

⁷⁷ Zimmermann, *Al-Fārābī's Commentary and Short Treatise*, pp. xxix; xxxvii.

⁷⁸ Zimmermann, *Al-Fārābī's Commentary and Short Treatise*, p. xlvii. R. Walzer, "L'éveil de la philosophie islamique," *Revue des Etudes Islamiques*, Vol. 38.1-2 (1970), p. 41; *Greek into Arabic*, p. 130 n.4, says that al-Fārābī did not know Greek, even though he was traditionally credited with knowing the languages that he mentions. Still Avicenna followed al-Fārābī's method and makes many of the same points. Perhaps he too erred on his Greek, but less likely on his native Persian.

⁷⁹ I am taking the distinction of “world-views” and “research traditions” from Larry Laudan, *Progress and its Problems* (Berkeley, 1977).

⁸⁰ Jean-Paul Reding, “‘To Be’ in Greece and China,” in *Comparative Essays on Early Greek and Chinese Rational Thinking* (Aldershot, 2004), p. 190.

⁸¹ Wright, *A Grammar of the Arabic Language*, Vol. II §§124-5. Cf. *Al-Fārābī’s Introductory Sections on Logic*, ed. & trans. D M Dunlop, *Islamic Quarterly*, Vol. 2 (1955), 272,17-273, trans. p. 280, on the need to insert ‘*huwa*’ in Arabic nominal sentences.

⁸² A. Elamrani-Jamal, *Logique aristotélicienne et grammaire arabe* (Paris: J. Vrin, 1983), pp. 132-4.

⁸³ See Bäck, *Aristotle’s Theory of Predication*.

⁸⁴ Less frequently with the verbal sentence using *kāna*. Cf. *Al-Fārābī’s Introductory Sections on Logic* ed. & trans. D M Dunlop, *Islamic Quarterly*, Vol. 2 (1955), 272,2-6, trans. p. 280: “These and what stands in their place are called existential vocables since they are used to signify the existence of a thing in relation to another and to connect the predicate with the subject of predication, as when we say Zaid exists (*yūjadu*) going away, when he is (*kāna*) going away. These existential vocables are employed as connectives when the predicate and the subject of predication are both names we wish to signify the three tenses as when we say Zaid was (*kāna*) eloquent, Zaid will be eloquent, Zaid is eloquent.”

⁸⁵ Al-Fārābī, in *De Int.* 102,16-23. *Kitāb al-Ḥurūf* §§80-1 110,9-21; *Iisā’ al-’Ulūm*, ed. Amin (1968) 61,9-13.

⁸⁶ See Zimmermann, *Al-Fārābī’s Commentary and Short Treatise*, p. cxix, for a discussion of what Al-Fārābī knew of Arabic grammarians.

⁸⁷ In *De Int.* 102,24-103,2; 44,21-3. At 46,9-20 he takes ‘Zaydun mawjūdun ‘ādilan’ as a proper sentence — which it is not in normal Arabic; cf. Zimmermann, p. xlv-v; likewise for the negative forms proposed by al-Fārābī — p. 98 n. 2.

Cf. Aristotle, *On Interpretation* 10; *Prior Analytics* 51b33-4; 52a24-6.

⁸⁸ *Kitāb al-Ḥurūf* §82 111,5-21; in *De Int.* 46,13-20.

⁸⁹ In *De Int.* 46,13-20; 103,2-20 *Kitāb al-Ḥurūf* §82 111,5-21.

⁹⁰ Thus *Kitāb al-Ḥurūf* §81 115,13-4 distinguishes the use in the theoretical sciences from the common use.

⁹¹ *Kitāb al-Ḥurūf* §84 113,15-9.

⁹² *Kitāb al-Ḥurūf* §83 112,1-8.

⁹³ *Kitāb al-Ḥurūf* §96 122,11-21.

⁹⁴ *Kitāb al-Ḥurūf* §94 120,8-121,6.

⁹⁵ *Kitāb al-Ḥurūf* §83 112,15-113,5.

⁹⁶ *Al-’Ibāra*, ed. M. Al-Khudayri (Cairo, 1970), 34,7-9.

⁹⁷ *Al-’Ibāra* 37,12ff.

⁹⁸ *Al-’Ibāra* 39,14-40,4; 77,3-9.

⁹⁹ Zimmermann, *Al-Fārābī’s Commentary and Short Treatise*, pp. xlv-v; cxxxi-cxxxiii, remarks on the grammatical artificiality of ‘*mawjūd*’ as copula, and observes that al-Fārābī has not followed customary Arabic usage here in forming the negative statement in Arabic.

¹⁰⁰ L. Massignon, “Reflexions sur la structure primitive de l’analyse grammaticale en arabe”, *Arabica*, Vol. 1.1 (1954), pp. 3-16.

¹⁰¹ I. Madkour, *L’Organon d’Aristote dans le monde arabe*, seconde édition (Paris, 1969), p. 162.

¹⁰² In *De Int.* 83,16-24; trans. p. 77.

¹⁰³ I.e., strictly we can know only that an event is necessary given a contingent act of free will. Cf. in *De Int.* 100,2-13; 100,24-5, trans. pp. 95-6.

¹⁰⁴ In *De Int.* 98,18-9, trans. p. 93.

¹⁰⁵ In *De Int.* 98,20-1.

¹⁰⁶ Gerhard Endress, “Die Wissenschaftliche Literatur” in *Grundrisse der Arabischen Philologie*, ed. H. Gätje (Wiesbaden, 1987), Vol. 2, pp. 423-31.

¹⁰⁷ Oliver Leaman, *Averroes and his Philosophy* (Oxford, 1988), pp. 2-6.

¹⁰⁸ Gerhard Endress, “Die Wissenschaftliche Literatur”, p. 430; cf. Dimitri Gutas, *Greek Thought and Arabic Culture* (London, 1998), pp. 14-27.

¹⁰⁹ Zimmermann, *Al-Fārābī’s Commentary and Short Treatise*, p. xlvi.

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Logical fragments in Ibn Khaldūn's *Muqaddimah*

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Abstract. In this short contribution we briefly present life and times of Ibn Khaldūn, his magistral accomplishment in the *Muqaddimah*, and present *Muqaddimah* fragments related to logic and epistemology from the perspective of modern modal logic.

1. Life of Ibn Khaldūn

Ibn Khaldūn was a 14th century historiographer and author of the well-known *Muqaddimah*, equally well-known by its Latin title *Prolegomena*. He lived from 1332 to 1406. Though born in Tunis, his family originated in Seville, where they lived prior to its conquest by the king of Castille, the king of Spain so to speak. This conquest was part of the grander scheme that became later known as the Reconquista. His life is rather well-documented, as he wrote an autobiography (the autobiography is included in the French edition by de Slane (de Slane 1934-38). This autobiography already makes for absolutely fascinating reading. Ibn Khaldūn lived an itinerant life serving as a magistrate for — in modern geographic terms — Spanish, Moroccan, Tunisian and Egyptian Islamic courts. In that function in Granada, Spain, he negotiated treaties with the Christian Spanish crown (with Pedro the Cruel, which does not sound too encouraging). The autobiography follows a stupefying cyclic pattern: Ibn Khaldūn goes to state *X* to serve ruler *A*; then, unfortunately, ruler *A* dies / is murdered / is deposed, due to intervention of his son / his prime minister / other family or court official *B*. Ibn Khaldūn then: flees from state *X* to state *Y* in case he remained loyal to the former ruler *A*, or, alternatively, remains in state *X* in case he had switched allegiance to the new ruler *B* in time. This suggests, rather improperly as it is the undersigned suggesting it, a somewhat flighty character, but the picture in fact emerging from these repetitive sequences of events is that of a steady mind living in troubled times, who chooses according to principles of justice and fairness, with the greater good of the population and the desirability of a stable society very much in mind. He writes utterly matter-of-factly about the continuous change of power and focuses on his achievements to the administration: his mutterings about immorality in Cairo, where he deposed corrupt judges (irregularities at trials and inheritances, such as appropriation of religious bequests, were a great illegitimate source of income), could be equally found in today's Watergates and the like.

A well-known exploit during the later period of his life in the politically more stable environment of Cairo, where he also taught at the renowned Al-Azhar University, is his meeting with the Turkish conquerer Tamerlane (a.k.a. Timur) during the siege of Damascus. The story goes that Ibn Khaldūn dared outside the city walls to propose parley with the attacking army — by no means a safe pursuit that might already cost one one's life. But — according to his own and contemporary documentation — he succeeded to contact the army's leader Tamerlane and had a discussion on history, philosophy and very practical matters such as rules and customs of peoples still to conquer further West. Whether this contributed to the delivery of Damascus on more favourable terms is not really known, but of course suggested.

Part of the historical evidence is that he interacted with Tamerlane by way of an interpreter ‘Abd Al-Jabbār Al-Khwārizmī.

2. The *Muqaddimah*

Ibn Khaldūn’s major heritage to civilization is his encyclopedic overview of science and philosophy, and of as well — and mainly so — the history of North-African and Andalucian Islamic culture and politics at the time. The encyclopedic approach was in the Arabic tradition of the general philosophical project of the 9th century known as the translation project, which was implemented by the House of Wisdom in Baghdad and directed by Al-Kindī (Tahiri, Rahman and Street 2007, fn 6). According to the author himself he wrote his voluminous compendium (mainly) in a period of five months in ‘the year 779’ (AH, i.e. 1377 AD). He continued to expand this for the remainder of his life. An obvious bibliographic source for the history of the *Muqaddimah* is (Rosenthal 2005). Ibn Khaldūn’s entry in Wikipedia (http://en.wikipedia.org/wiki/Ibn_Khaldun) also gives fairly precise references concerning the genesis of the *Muqaddimah*. The first Western language edition was the French translation by M. de Slane from 1863, that was reprinted in the 1930s (de Slane 1934-38). The German and English translations are from the 20th century, much later. One has to be careful about one’s wording of *first* here: Western, European, Modern? There is a Turkish translation from 1745, published in Istanbul, in Europe... And of course Granada, whose rulers Khaldūn served, lies very Westerly in Europe anyway — which makes Arabic a Western European language at that time. My apologies for the digression.... A wonderfully concise — for the present-day itinerant scholar — English edition is the 2005 Princeton University Press reprinted abridgment (Rosenthal 2005) of the 1958 Rosenthal translation. This source was used for the quotations involving logic below — although we performed that search in the unabridged French translation by de Slane.

This brings us closer to our research question: what evidence does the *Muqaddimah* provide for epistemic logical concepts, and theoretical or otherwise precise treatment of knowledge and related concepts? We further focussed this question as follows: is there any evidence in the *Muqaddimah* of the three postulates of epistemic modal knowledge — truthfulness, awareness of knowledge and awareness of ignorance, a.k.a., respectively, *the postulate of truth*, *the postulate of positive introspection*, and *the postulate of negative introspection*? Now this concerns *modern* epistemic logic, in which these postulates makes sense given a Tarskian distinction between syntax and semantics and a Kripke semantics for modal logic. And apart from that this concerns epistemic logic in its rather contested appearance that became popular in areas as computer science and artificial intelligence, which makes rather encompassing simplifications about the nature of knowledge and truth. There is no reason a priori to assume that this perspective makes sense in a medieval setting that is much more concerned with truly epistemological investigations, that question the nature of knowledge rather than its formal or structural behaviour given some simplifying assumptions. On the other hand, the interest for not necessarily epistemic modal logic but for the more purely modal logic of necessary and possible throughout early modern times, with roots back in Aristotle and well-known from later medieval authors as Thomas Aquinas, suggests that some such pursuit might not be totally in vain.

And apart from looking ahead, we might as well look further back in time, closer to the roots of the Translation Project. The 8th century Arabic (Iraq/Oman area) philologist Al-Khalīl Ibn Aḥmad composed the first Arabic dictionary and is credited with the following famous epigraph that as well adorns the introduction (Tahiri, Rahman and Street 2007) to this volume:

الرَّجُلُ أَرْبَعَةٌ: رَجُلٌ يَدْرِي وَ يَدْرِي أَنَّهُ يَدْرِي فَسَلُوهُ،
 وَ رَجُلٌ يَدْرِي وَ لَا يَدْرِي أَنَّهُ يَدْرِي فَذَاكَ نَاسٌ فَذَكِّرُوهُ،
 وَ رَجُلٌ لَا يَدْرِي وَ يَدْرِي أَنَّهُ لَا يَدْرِي فَذَلِكَ مُسْتَنْزَعٌ فَعَلْمُوهُ،
 وَ رَجُلٌ لَا يَدْرِي وَ لَا يَدْرِي أَنَّهُ لَا يَدْرِي فَذَلِكَ جَاهِلٌ فَأَرْفُضُوهُ.

There are four kinds of men: men who know and know that they know; ask them.
 Men who know and do not know that they know, they are forgetful; remind them.
 Men who do not know and know that they do not know, they search for guidance; teach them.
 And men who do not know and do not know that they do not know, they are ignorant; shun them.

(Al-Khalīl ibn Aḥmad al-Farāhīdī, in Ibn Qutayba *ʿUyūn al-akhbār* 1986, II, p.142)

It is therefore clear that the epistemological enterprise is at the very heart of Islamic philosophy, and as this is so obviously related to the postulates of introspection we can expect to find some relation to them in the *Muqaddimah* or in contemporary early medieval writings. Section 3 provides some essential formal background to understand the three postulates of knowledge. Section 4 reports on the fragments found. Section 5 discusses these results in relation to known other work from the era relating to the knowledge postulates and to reasoning about knowledge in general.



1. Autograph of Ibn Khaldūn (upper left corner)
 From MS. C (Atif Efendi 1936)

Muqaddimah manuscript fragment
 (<http://www.muslimphilosophy.com/ik/klf.htm>)

3. Modern epistemic logic

Modern epistemic logic starts with Hintikka's *Knowledge and Belief — An introduction to the logic of the two notions* (Hintikka 1962). The postulates of knowledge, and the names under which they are commonly known, are that:

- what you know is true (truthfulness),
- you are aware of your knowledge (positive introspection),
- you are aware of your ignorance (negative introspection).

Such linguistic utterances are, firstly, formalised and, secondly, interpreted in their formal logical appearance on a relational structure representing ‘the information’. This structure is also known as a *Kripke model*. It consists of *possible worlds*. A feature of these worlds is that, unlike the real world, they can be completely described by enumerating factual truths. Assume a very simple world in which only two facts are relevant: whether it rains in Bonn, and whether it rains in Cairns. Given two such facts we can only base four different world descriptions on them: it rains in Bonn and in Cairns, it rains in Bonn but not in Cairns, it does not rain in Bonn but rains in Cairns, and it neither rains in Bonn nor in Cairns. I am currently not in Bonn, so I have no idea whether it rains there. I am currently in Cairns, so I know that it rains here: I am getting wet. We are therefore concerned with only two of these four different worlds: one where it rains in Bonn and in Cairns, abbreviated as (rainBonn, rainCairns) and another one where it rains in Bonn but not in Cairns, abbreviated as (norainBonn, rainCairns). Only one of these can be the case, assume (surely...) that this is (rainBonn, rainCairns). Now what?

The idea is that we continue to reason from the perspective of the rational agent about what is possible and what is not possible. In fact, we can think of the rational agent as ourselves. With ‘possible’ in this epistemic context is meant: what facts that we know to be relevant, are conceivably false or true given our observations about the world, our background knowledge, and our deductive abilities. As an observation counts that we are wet in Cairns — where it rains. In this case we assume that there is no background knowledge at all, except our ‘realization’ that the only other relevant fact that we care to be uncertain about is: whether it rains in Bonn. In this setting surely the actual world (rainBonn, rainCairns) is considered possible. But also the world (norainBonn, rainCairns) is possible: even though it rains in Bonn, we cannot observe it. Now as a rational observer we do not actually know from which world we reason. Therefore, also if (norainBonn, rainCairns) had been the actual world, we would have considered that possible and also, in that case: (rainBonn, rainCairns). On an abstract structure with a domain of two objects we have so described a binary relation between worlds consisting of four pairs. This is called the accessibility relation. For example, given that in (norainBonn, rainCairns) we consider it possible that (rainBonn, rainCairns), this means in other words that the (ordered) pair [(norainBonn, rainCairns), (rainBonn, rainCairns)] is in this accessibility relation.

Now we proceed to modal logic. In the actual world you are said to *know* a proposition if and only if it holds in *all* worlds that are *possible* given that actual world. For example, you know that it rains in Cairns, because it rains in Cairns in world (rainBonn, rainCairns) and in world (norainBonn, rainCairns) and those are the *only* worlds you consider possible in actual world (rainBonn, rainCairns). You don’t know something if and only if it is not the case that you know it. That one’s easy. And you consider something possible (the diamond form of the modal operator) if and only if it you don’t know that it is not the case. In relational terms this means that you consider something possible if and only if there is (at least) an accessible / possible world where it holds. For example, in the actual world (rainBonn, rainCairns), where it rains in Bonn, you consider it possible that it does not rain in Bonn, because the world (norainBonn, rainCairns) is accessible from the actual world.

An interesting aspect of this interpretation schema is that it can be applied iteratively — and that will be where the three postulates of knowledge also come in. We have already computed that in the actual world (rainBonn, rainCairns) you know that it rains in Cairns. Now if

(norainBonn,rainCairns) had been the actual world we could have similarly computed that you know that it rains in Cairns. Consider (rainBonn,rainCairns) again... This world is considered possible — and you know there that it rains in Cairns. The other world is also considered possible — and you know there as well that it rains in Cairns. Therefore, in the actual world you know that (you know that it rains in Cairns), because the proposition 'you know that it rains in Cairns' is possible in both accessible worlds! In other words, in the actual world you are aware of your knowledge that it rains in Cairns. (We will use 'knowing that you know' and 'being aware of your knowledge' as interchangeable.) If this holds regardless of which is the actual world, and regardless of the proposition known, the *postulate of positive introspection* is satisfied.

The first postulate of knowledge prescribes that what you know should be true. And this is also the case for our rainy example. We have just computed that the modal proposition 'you know that it rains in Cairns' is true in actual world (rainBonn,rainCairns). But this is also really the case: the proposition 'it rains in Cairns' is evidently true in this actual world. If this holds regardless of the actual world and regardless of the proposition, the *postulate of truth* is satisfied. Note that, as for the positive introspection example, the true propositions can also be modal. For example, you know that (you don't know whether it is raining in Bonn), and this is also true: it is indeed the case that you don't know whether it is raining in Bonn.

This also brings us to the third postulate, awareness of ignorance: consider the example argument in the previous sentence in reverse: you don't know whether it is raining in Bonn, and indeed it is also true that you *know* that. In other words, you are aware of your ignorance. If this is always the case, and for every proposition, the *postulate of negative introspection* is satisfied.

When the three knowledge postulates are satisfied the accessibility relations between worlds are always equivalence relations. This means that we can think of the domain of possible worlds as partitioned into non-overlapping subsets called equivalence classes. Your equivalence class consists of all the worlds that are indistinguishable from your point of view — where 'your point of view' is the real world: one of those in that class.

A difference between knowledge and belief is that beliefs may be false. Clearly, the truth postulate can in that case not be satisfied. There is a wealth of alternatives to this simplifying setting for the analysis of knowledge and belief and it has been contested from various sides. The original (Hintikka 1962) is still an excellent reference for that. In particular, negative introspection is unrealistic, as it requires us to be aware of all our ignorance: there are many things that we don't know of which we are unaware. In the words of a soon forgotten American government official: there are *unknown unknowns*. The technical reason for this discrepancy is that our assumption that *we are aware of all the relevant facts but just not their truth value*, is incredibly unrealistic for human reasoning.

Let us not proceed into this direction, but finish by mentioning a puzzling phenomenon of this logic of knowledge, often called a paradox (one of many epistemic paradoxes). Consider the actual world (rainBonn,rainCairns) again. If I tell you that (it rains in Bonn and you don't know that), then (a) this is *true*, and (b) after having told you that it is *false*: you now know that it rains in Bonn, so it's not longer true that you don't know it! A somewhat different way to address this matter, is to say that the following proposition is inconsistent, or incoherent: *you know that (it rains in Bonn and you don't know that)*. Already in the Middle Ages this was known as the *Knower Paradox*, e.g. in the works of Thomas Bradwardine (Read 2007, Read 2007c). Or at least it relates to the complexities, normally explained as truth-functional, involved in this paradox. This should at least make us hopeful to find similar phenomena of epistemic interest in Khaldūn's work.

4. *Muqaddimah* text fragments

I consulted Ibn Khaldūn's *Prolegomena* from A to Z searching for references to logic or knowledge. My source was the authoritative French translation by de Slane from the 1860s (de Slane 1934-38), the first complete edition of the *Prolegomena* in a western language. In particular I was interested to find out whether Ibn Khaldūn considered the three postulates of knowledge as formalised in the logic *S5*: truthfulness, positive introspection, and negative introspection. My recent publication not accidentally entitled *Prolegomena to dynamic logic for belief revision* refers in a long footnote (the main text is on purely modern epistemic matters) to such text fragments and suggests that the answer to that tripartite question is: yes, yes, no. I am now even less certain of the two 'yes's. I will here present and discuss these fragments — they were only afterwards matched with their English counterparts in the Rosenthal translation (Rosenthal 2005).

I found four relevant fragments. They are all in Chapter 6 of the *Muqaddimah*, entitled: *The various kinds of sciences. The methods of instruction*. As said, the main part of the *Muqaddimah*, the content of most other chapters, is a history of North-African and Andalucian peoples of the era, including various sociological ramifications that are praised by others loudly (and justifiedly) enough already. But his overview of the academic accomplishments of his era — or rather metaphysics and natural philosophy — is certainly also very much worth reading. In Chapter 6, the relevant fragments are in Section 1 — *Man's ability to think*, Section 2 — *The world of things that come into being as the result of action ...*, Section 3 — *The knowledge of human beings and the knowledge of angels*, and Section 22 — *The science of logic*. It turns out that for our purposes the last is not the most interesting of the four! Our observations in the quotations are between [and].

Chapter 6, Section 1: Man's ability to think

God distinguished man from all the other animals by an ability to think (...). This comes about as follows. Perception — that is, consciousness on the part of the person who perceives — is something peculiar to living beings to the exclusion of all other possible and existent things. (...) Man has this advantage over other beings: he can perceive things outside his essence through his ability to think, which is something beyond his sense. (...) The ability to think is the occupation with pictures that are beyond sense perception, and the application of the mind to them for analysis and synthesis. The ability to think has several degrees. The first degree (...) mostly consists of perceptions. (...) The second degree (...) mostly conveys apperceptions. (...) This is called the experimental intellect. The third degree (...) is the speculative intellect. (Rosenthal 2005, p.333-334)

This serves as an introduction to our further observations on positive introspection ad fragment three, below. We should point out that the corresponding French terms in (de Slane 1934-38) for 'perception', 'ability to think', and 'apperception' are: *perception*, *réflexion*, and *affirmation*. The French terminology appears to lend itself more to an interpretation suggesting a link to epistemic logic and positive introspection. (See <http://www.cnrtl.fr/lexicographie/reflexion>: *Réflexion*: Faculté qu'a la pensée de faire retour sur elle-même pour examiner une idée, une question, un problème. Freely translated: Faculty of thought that introspectively considers an idea, question, or problem.) It seems peculiar (to an non-francophone) not to find the terms *pensée* and *apercevoir* instead. My apologies for not being familiar with the distinctions intended in the originally used Arabic terminology. The transliteration of the Arabic term corresponding to 'ability to think' or 'réflexion' is *fu'ād*. This is the singular of *afīda*, for heart. See (de Slane 1863, v.2, p.427).

Chapter 6, Section 2: The world of things that come into being as the result of action.

The ability to think is the quality of man by which human beings are distinguished from other living beings. The degree to which a human being is able to establish an orderly causal chain determines his degree of humanity. Some people are able to establish a causal nexus for two or

three levels. Some are not able to go beyond that. Others may reach five or six. Their humanity, consequently, is higher. For instance, some chess players are able to perceive (in advance) three or five moves (...). (Rosenthal 2005, p.335)

This is the citation I like best, although it is not really related to epistemic logic. Again, it is tempting to compare causal chains of reasoning to iterations of knowledge operators, where you know that you know that you know that... But this relation only exists to the extent that in either case a chain of reasoning is necessary to make an argument. A present-day philosopher or cognitive scientist immediately thinks of Turing tests and intelligent computers when reading this! Given that computers now exceed humans in computational power, computational power ceased to be seen as a sign of intelligence per se. And present-day philosophers prefer to see the *creativity* of humans as what makes them human, and not their *rationality*... I do find the observation above uncannily accurate though: this is not just some wild guess but an experimental observation, that one could as well easily link to the often suggested limit of six or seven items that can be concurrently processed in short-term working memory. Ibn Khaldūn makes good reading!

Chapter 6, Section 4: The knowledge of human beings and the knowledge of angels.

We observe in ourselves through sound intuition the existence of three worlds. The first of them is the world of sensual perception. We become aware of it by means of the perception of the senses, which the animals share with us. Then, we *become aware of the ability to think* [our emphasis] which is a special quality of human beings. We learn from it that the human soul exists. This knowledge is necessitated by the fact that we have in us scientific perceptions which are above the perceptions of the senses. They must thus be considered as another world, above the world of the senses. [The third world is the world of spirits and angels.] (Rosenthal 2005, pp.337-338)

This fragment is the most pertinent to our quest. From ‘become aware of the ability to think’ it may seem a big step to ‘awareness of knowledge’ in the epistemic sense, but a small case can be made. This step seemed on the whole a lot smaller in the French translation where ‘réflexion’ is used to denote ability to think, although for this particular passage the French version is less striking than the English version: La réflexion, faculté spéciale à l’homme, nous enseigne de la manière la plus positive l’existence de l’âme humaine; (elle nous le fait savoir) au moyen des connaissances acquises et enfermées dans notre intérieur; connaissances bien au-dessus de celles qui proviennent des sens (de Slane 1934-38, v.2, p.433). It is surely comforting to a modal logician that awareness of knowledge provides proof of the existence of the soul.

Chapter 6, Section 22: The science of logic.

(Logic concerns) the norms enabling a person to distinguish between right and wrong, both in definitions that give information about the essence of things, and in arguments that assure apperception. (...) Eventually, Aristotle appeared among the Greeks. He improved the methods of logic and systematized its problems and details. (...) (Rosenthal 2005, pp.382-383)

This concerns a roughly 2000 word overview of Aristotle’s *Categories*, and how they found their way into the Arab world by way of translations and commentaries, such as by — using their latinized names — Averroës and Avicenna. (Ibn Khaldūn was of course familiar with the works of these philosophers — he received a classical Arabic education in Tunis at an early age and his teacher was a Al-Abili (http://en.wikipedia.org/wiki/Ibn_Khaldun.) For our epistemic logical concerns this part is of no interest.

There seems to be a missing logic treatise by Ibn Khaldūn (de Slane 1934-38) on which contents it would be unwise to conjecture. So, the above is all. What can we conclude? Concerning the *postulate of truth*: that real knowledge is true seems easily read into various phrases, as some kind of reliability or certainty corresponds to the connotation of the word knowledge anyway. E.g., we give a final corroborating quote: “Their knowledge [of prophets]

is one of direct observation and vision. No mistake or slip attaches itself to it, and it is not affected by errors or unfounded assumptions.” (Rosenthal 2005, p.339). Concerning the *postulate of positive introspection*: It is tempting to see reflection on acquired knowledge as a form of introspection in the modern epistemic logical sense, but it is not found in some general form that involves arbitrary iteration or reflection on knowledge. Concerning the *postulate of negative introspection*: I did not find a reference to negative introspection. In fact, the main context of the knowledge postulates (at least of the ones on introspection) is in (i) *derivations* of factual from epistemic knowledge, or vice versa, and (ii) *higher-order* settings where you know that you know that you know that something is the case (or derive something else from that), or where such a setting is necessary to explain or analyse seemingly paradoxical (the Knower Paradox) or otherwise too complex phenomena of reasoning. None of this I found in Ibn Khaldūn’s writings. Is this therefore a failed enterprise? Not really, I presume to suggest. In the first place this small investigation might keep others from also fruitlessly repeating it, and it provides at least a record of all the logical fragments in the *Muqaddimah* (insofar as such a record is needed given the splendidly accessible translations for this masterpiece). Apart from that, there is the more general question whether the three postulates can be found as such in medieval logical arena. I close with a section referring to such matters. A surprising observation there will be that the works of Avicenna — a main source for Ibn Khaldūn’s schooling, and to which he refers in the Section *The Science of Logic* when discussing Aristotle — certainly contain such epistemic modal content.

5. Related sources and discussion

In this section we discuss some sources related to epistemic logic of Khaldūn’s contemporaries and predecessors. This overview is not exhaustive and may not even be typical for the era. First, we need to point out that the study of modalities as such was widely pursued in the early modern period, see e.g. the various sources mentioned in (Kneale & Kneale 1962), or in this volume Paul Thom’s *Logic and Metaphysics in Avicenna’s Modal Syllogistic* (Thom 2007). It should then be pointed out that this mainly concerns the general logic of reasoning about the necessary and the possible, where the typical understanding of ‘something is necessarily the case’ is that ‘something will be the case in all future developments of the world’. In other words, the modality is temporal, as in Aristotle’s seabattle argument. For the epistemic modality, a main source appears to be Avicenna and his Arab predecessors (Black 2007) — this we will present in some detail. We also report on an obvious relation apparent from the presentation of the *Knower Paradox* in Thomas Bradwardine’s 14th century writings, and a stipulated relation conjectured from the Scholastic medieval notion of *Obligatio*. (For further sources, see, again (Kneale & Kneale 1962), and (Boh 1993). In our explorations we focus on positive and negative introspection, as identified with awareness of knowledge and awareness of ignorance.

Avicenna, Al-Fārābī, and positive introspection. Deborah Black’s *Avicenna on Self-Awareness and Knowing that One Knows* (Black 2007) addresses epistemic aspects in Avicenna’s (Ibn Sīnā, 980-1037) work, in relation to relevant work of his predecessor Al-Fārābī (870-950). Avicenna’s experiment of the Flying Man is worth recounting: imagine yourself in a state where you have no sensory perception to distract you, you are, as it were, floating in the air in a suspended state. Are you then still aware of yourself? The answer to that is clearly: yes, we are. This proves that awareness of the self as a, so to speak, semantic object. One’s interpretation of this phenomenon depends on whether we see this as an observation about sensory perception or as an observation about intellectual reflection. In the second case we are more clearly not just talking about awareness of an (semantic) object but

about awareness of knowledge of factual information. Given our modern identification of awareness with knowledge (an identification that seems also questioned, in principle, in the original source as reported by (Black 2007) this clearly amounts to second-degree knowledge of factual information. Awareness of knowledge is then (somewhat surprisingly, to the modern mind) identified with certainty of knowledge (Black 2007). Positive introspection goes beyond that: it is awareness of any knowledge, also of epistemic proposition. This amounts to arbitrarily higher-order knowledge: to know that to know that to know that ... This problem of infinite regress 'seems not to worry Avicenna' although his justification — in our interpretation — is that otherwise certainty about knowledge would not be possible, which is undesirable (Black 2007). The modern justification arguing away infinite regress problems is that knowledge is interpreted on Kripke models where the accessibility relation satisfies the structural property of *transitivity*: this corresponds in an exact formal way to the postulate of positive introspection — we refrain from further details, see (Hintikka 1962). Black further mentions Avicenna's predecessor Al-Fārābī who also wrote on infinite regress of knowing that — it would be of clear interest to investigate that aspect of the work of Al-Fārābī. Black's *Knowledge and Certitude in Al-Fārābī's epistemology* (Black 2006) mentions six conditions for certain knowledge. The first three clearly (as she observes) seem to define knowledge as justified true belief, and they relate to the postulates of truth and positive introspection.

Bradwardine and epistemic paradox. In *Bradwardine's Revenge* (Read 2007) Stephen Read discusses the Knower Paradox (our source was his GPMR workshop presentation delivered in Bonn). We quote Read:

Thomas Bradwardine, writing in the early 1320s, developed a solution to the semantic paradoxes (insolubilia) based on a closure principle for signification: every proposition signifies whatever is implied by what it signifies. In ch. 9 of his treatise, he extends his account to deal with various epistemic paradoxes. Comparison of Fitch's paradox with one of these paradoxes, the Knower paradox ('You do not know this proposition') explains the puzzlement caused by Fitch's paradox. Bradwardine's argument shows that the Knower paradox signifies its own truth, and is false. (Read 2007a)

In epistemic logic, one way to model the Knower Paradox is to see 'You do not know this proposition' as the announcement of 'The proposition is true and you do not know that', where the proposition may as well — but does not have to — be a factual proposition. The example we already gave in Section 3 was 'it is raining in Bonn and you do not know that'. In dynamic epistemic logic (as mentioned in (van Ditmarsch 2005)) an announcement as 'it is raining in Bonn and you do not know that' is proposed to be processed as a Kripke model transforming operation: in case of this announcement, it restricts the current information state consisting of the two worlds (rainBonn, rainCairns) and (norainBonn, rainCairns) to a single state (rainBonn,rainCairns). This is then the only remaining possible state, in which you therefore know that it rains in Bonn. This explanation resolves the paradoxical character of the Knower Paradox. Now, not surprisingly of course, this 'modern' dynamic explanation is not found in Bradwardine's work (we consulted the translation in progress by Stephen Read (Read 2007c) with his kind permission) — it is not even found in G.E. Moore's work, one of the much more recent sources (early 1940s) that addresses this matter (for very detailed references to Moore's work on this paradox, see, yet again, (Hintikka 1962). Still, it is quite a surprise to see Bradwardine explain the paradoxical character of the 'Knower' in rather similar terms as the modern epistemic logician would do. His starting point is the phrase 'This proposition is not known by you' (or, in another manuscript, 'this proposition is not known by Socrates' — as observed by Read in work in progress) and the derivation towards a contradiction used distribution of knowledge over conjunction (if I know A and B, then I know A and I know B), and, indeed, the truth postulate (what I know is true) applied to a proposition of ignorance. Here, we come fairly close to negative introspection again.

Bradwardine's method to resolve the paradox seems quite different from our above dynamic approach. Bradwardine uses a certain treatment of self-reference that can in a different, unrelated, context (personal communication, Stephen Read), also be used to address the Liar's Paradox (see Read 2007, Rahman et al 2007).

Obligatio and negative introspection. The postulate of negative introspection concerns awareness of ignorance. It was suggested by Catarina Dutilh-Novaes at the GMPR Workshop Medieval Logic in Bonn that there is a link between negative introspection and the medieval concept of 'obligatio'. Obligatio is a philosophical method of dialogue (a game) between opponents, with the object of confirming or rejecting agreement, or to resolve inconsistencies (Dutilh-Novaes 2007). An important medieval source are the *Obligaciones Parisienses*, currently under translation by a group headed by Sara Uckelmans (Uckelmans et al. 2007). See also (Maat et al. 2007) — their source, and the status quo of their work, is:

Oxford MS Canon misc 281 contains a tract on *obligatio* which can be tentatively dated and placed in early 13th century France (the text was edited by de Rijk in *Vivarium*). The tract is divided into three sections, *positio*, *dubitatio*, and *depositio*. We are currently translating this text into English and will present work in progress concerning the middle section, on the obligation of doubting. (Maat et al. 2007)

The link with negative introspection is, that in the case of the *dubitatio* obligation, the uncertainty about information brings the obligation to question it, followed by a process of attempted justification. Insofar as this **obligation** can be identified with **awareness** and **uncertainty** with **ignorance**, what takes place here is indeed a step from 'I do not know this proposition' to 'I am aware that I do not know this proposition', in other words: I know that I do not know this proposition — negative introspection. On the other hand, it seems to us that one might as well interpret the doubt or uncertainty here as the absence of knowing that or knowing that not. That would be a somewhat stronger interpretation: in that case, *doubting* a proposition would mean 'I am aware that (I do not know this proposition and I do not know the negation of this proposition)'. Sara Uckelmans (personal communication) also suggests that apart from 'true' and 'false', 'doubtful' may well function as a third truth value. A *multi-valued* approach to reasoning would be fairly different from an *epistemic modal* approach, that we are trying to read into the obligation of doubt. We are uncertain at this stage which of the two is a more suitable modern re-interpretation.

Finally, one might observe, as Shahid Rahman with reason does, that negative introspection as a method is part of the general epistemological approach to logic, and in this form this brings us back to the Arabic tradition where the realization of ignorance is a condition to learn, and where the desire to learn was the original motivation for the Translation Project.

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AVICENNA ON THE QUANTIFICATION OF THE PREDICATE (WITH AN APPENDIX ON [IBN ZUR'A])

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Abstract. Avicenna (Ibn Sīnā, d. 1037) devotes two chapters of *al-'Ibāra* to the quantification of the predicate. *Al-'Ibāra* is the third book of the logical collection of his philosophical encyclopedia entitled *al-Shifā'* (The Cure). An English translation of these two chapters, the first in any language, is offered here. This translation is preceded by an analysis of the content of these chapters and is followed by an Appendix containing a translation of [Ibn Zur'a]'s treatment of the same topic. (The name of Ibn Zur'a, d. 1027, is bracketed to indicate a problem of authorship). The whole dossier is intended to pave the way for further studies of this subject. Avicenna's treatment of the quantification of the predicate has the following distinctive features: he deals systematically with singular and indefinite propositions; he states correctly the contradictories of the eighth proposition forms which he enumerates; he is aware of the equivalence between two of these forms but makes no attempt to reduce the number of these forms to more basic ones. A suggestion has been made that Avicenna thought of the logic of doubly quantified propositions on the model of propositions with an indefinite predicate (S is not-P). Contrary to his predecessors, Avicenna did not reject a priori these proposition forms and he countered arguments supporting such a rejection.

Long before William Hamilton (1788-1856) proposed his theory of the quantification of the predicate, engaging there by in controversy with Augustus de Morgan (1806-1871), the ancient and medieval logical tradition had already dealt with such a theory at length. Admittedly Hamilton was perfectly aware of the existence of this tradition, as shown by the informed historical notice he appended to his logical study.¹ Since then, modern scholarship has examined the treatment of this topic in the Greek and medieval Latin traditions.² But no study has been dedicated to the Arabic tradition. The present paper aims to pave the way for filling this lacuna.

The discussion of the quantification of the predicate by ancient and medieval Arab commentators is generally attached to *Peri Hermeneias* 7, 17b 12-16.

T1: It is not true to predicate a universal universally of a subject, for there cannot be an affirmation in which a universal is predicated universally of a subject, for instance 'every man is every animal'.³

These few lines in Aristotle's text will be the occasion for a seven and a half page development in the Busse edition of Ammonius's Commentary⁴ and for a development of about eleven pages in Avicenna's Cairo edition. Avicenna devoted to the quantification of the predicate two chapters of his *al-'Ibāra*, which is the third book of the logical part of his philosophical summa entitled *al-Shifā'* (The Cure). This treatise, which is an original and expanded exposition of the contents of Aristotle's *PH*, consists of two books; the developments on the quantification of the predicate are situated in chapters 8 and 9 of the first book.⁵

1. Singular propositions with quantified individual predicates

Avicenna is the only one among the commentators mentioned to consider systematically this class of propositions with a quantified predicate.⁶ He did it perhaps for the sake of exhaustiveness. He proceeds along the following lines. Given any proposition, let us first consider its subject. It can be either singular or universal, and in the latter case, it can be taken either universally, or particularly, or else indefinitely.⁷ Let us then look at the predicate. If the subject is singular, Avicenna considers here that the predicate can be either individual or universal.⁸ In the three other cases, that is in the cases where the subject is a universal taken indefinitely or universally or particularly, the predicate must be universal. Avicenna examines the behaviour of all these kinds of propositions when a quantifier is prefixed to their predicate. So he was not only the sole among ancient and Arab commentators to take into account the class of singular propositions with a quantified predicate, but he was also alone in doing the same with indefinite propositions with a quantified predicate.⁹

Singular propositions with quantified predicate have, if we take as an example the case where the quantifier joined to the predicate is universal affirmative, the following form:

Zayd is every this-individual

“Zayd” is a proper name or, as Avicenna says, a “singular term”, that is a term whose “signification is such that it is impossible for the mind to make it common to many items”. Such a term, when used normally, that is unambiguously, indicates “the self of that which is [ostensively] designated”, which self belongs uniquely to this designated object.¹⁰ Although the predicative expression “this individual” has the appearance of a description, the fact that it contains a demonstrative makes it equivalent to a singular term such as we have just characterized it. “Every” does not signify here the whole opposed to the part, but is actually a quantifier meaning “every instance”. This kind of proposition could be explicated, following Avicenna, this way:

Zayd is each of the things that are ones and that ‘Amr is [i.e. each of the individuals falling under ‘Amr]

Regarding these singular propositions with an individual quantified predicate, the most important point noticed by Avicenna is the *asymmetry between affirmation and negation*. The affirmation is said by him sometimes to be meaningless,¹¹ sometimes to be false; whereas the negation is held to be true. To make explicit the relation to truth of this last class of negative propositions, Avicenna distinguishes the proper *content* of a proposition [*al-mafhūmāt min anfusihā*] and what is *suggested* by it [*ihāmāt*].¹² The truth-value of a proposition is attached to its content, not to what it suggests. Avicenna illustrates this distinction by the example of a particular negative proposition whose terms are incompatible. So

Not-every S is P

is true, even though what it suggests, that is:

Some S is P

is false. For example, the particular negative “Not-every man is a stone” is true, though it suggests the particular affirmative: “Some man is a stone”, which is false. In the same way, one must say that the proposition “Zayd is not every this-individual” is a true proposition, though it suggests a falsehood, namely that “this individual” has several substrates or subjects [*maw*ū’āt*]. But, because “this individual” has not several substrates, it is true that Zayd cannot be each of them.

Avicenna mentions the following relations of contradiction between these kinds of propositions:

<i>Zayd is every this-individual</i>	F	<i>Zayd is not-every this-individual</i>	T
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<i>Zayd is some this-individual</i>	<i>F</i>	<i>Zayd is not some this-individual</i> ¹³	<i>T</i>
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2. The matter of propositions

When he comes to the enumeration of singular propositions whose predicate is a quantified universal term, Avicenna makes use, in order to determine the truth-value of doubly quantified propositions (henceforth DQP), of the notion of matter (*mādda*), which he characterizes in the following way:

T. 2 Ibn Sīnā, *al-Shifā'*, *al-'Ibāra*, I 7 (1970, 47.3-11)

You must know that the state of the predicate in itself in respect to the subject — not that state of which we make explicit in actuality the way it pertains to the [predicate]; neither the one which pertain to the predicate in any relation —, but that state which is that of the predicate in respect to the subject, in accordance with the *affirmative* relation, and which is of perpetuity or non-perpetuity of the truth or of the falsity, [that state] is called matter (*mādda*). For that state which consists either in the fact that the truth of the affirmation of the predicate is perpetual and necessary — it is then called, the matter of the necessity (*māddat al-wujūb*) — as is the state of the animal in respect to man; or in the fact that the falsity of the affirmation [of the predicate] is perpetual and necessary — as is the state of the stone in respect to man; or else [in the fact that] neither [this truth nor this falsity] are perpetual or necessary — this state is then called the matter of possibility (*māddat al-inkān*) — as is the state of writing in respect to man. This state is not different in the affirmation and in the negation, because this very same state belongs to the predicate of the negative proposition: this predicate is entitled to be in one of the situations just mentioned, even though it has not been the object of an affirmation.

From this rather tortuous text emerges the idea that the matter of a proposition is to be represented as its *implicit modal status*. That status arises from the kind of link existing between subject and predicate. This link is specified as soon as the *signification* of the proposition's terms is fixed, which is what Aristotle's commentators used to call the *matter* of a proposition as opposed to its *form*.¹⁴ The assignation of a signification to the terms of the proposition entails their being subsumed under one of the predicables (genus, species, property, accident). Although he does not say it explicitly in the above text, Avicenna evidently characterizes as a matter of the necessary the matter of the proposition in which the relation between predicate and subject is one of genus to its species, or even of property to its species, as is shown by the distinction, further introduced, between the “necessary which is more general” (*al-wājib al-a'amm*) and the necessary equal (*al-wājib al-musāwī*), the first characterizing the first kind of relation, and the second the second type of relation. Avicenna characterizes as the matter of the contingent the matter of those propositions whose terms have an accidental relation, for example “man” and “writing”, and as the matter of the impossible the matter of those propositions the terms of which are incompatible, for example “man” and “stone”. As we can see, this subsuming of the concrete terms of the proposition under the different predicables confers on the notion of matter a higher abstraction: the matters then offer an interpretation allowing the determination of the truth-value of a proposition without necessarily having recourse to an assignation of a concrete signification to its compounding terms.¹⁵

Avicenna emphasizes that in order to determine the matter of a proposition one must take into account the affirmative link. It is not that the matter of a proposition would be changed when the latter is transformed into a negative; for then, one has to put oneself in the counterfactual situation in which one would have made an affirmation, and this will be enough to determine the matter of the examined proposition. To fix the matter by considering the affirmative link allows one to follow a more *uniform* procedure: one has not to take account of truth or falsity,

perpetual or not, of the predicative link, now for the affirmation, now for the negation, but always in the first case.

As will be seen later, the propositions with a quantified predicate can be true or false in all three matters, in two of them or in only one. Thus, the proposition “Every S is every P” is false in every matter, whereas the proposition “Every S is no P” is true only in the matter of the impossible.

Appealing to the matter of propositions with quantified predicate when their truth-value is tested is not new. Ammonius,¹⁶ and, in a more allusive manner, the Anonymous commentator edited by Tarán,¹⁷ did the same.

A comment is needed regarding contingent matter when the subject of the proposition is singular. For this case is ruled by the principle of indetermination in the distribution of truth-values between two singular opposite propositions in contingent matter.

Avicenna states this principle when dealing with the truth-value of propositions of the kind “Zayd is no P”

T. 3. Ibn Sīnā, *al-Shifā': al-'Ibāra*, I 8 (1970, 56, 8-12)

But if the matter is contingent, no determinate (*bi-'aynih*) falsehood nor truth imposes itself; rather either Zayd may be, for example, writing and in that case it would be false that Zayd was no one of the writing, or Zayd may not be so, and in that case it would be true that Zayd was none of the writing. As for the proposition itself, that is its form, it does not impose anything. In sum, predicating contingents of individuals does not impose on their propositions the determination (*ta'yīn*) of truth or falsehood.

The idea as well as the vocabulary used by Avicenna in this passage remind us of his discussion of future contingents. At the beginning of a section that corresponds to Aristotle's *PH* 9, Avicenna writes:

T 4. Ibn Sīnā, *al-Shifā': al-'Ibāra*, I 10 (1970, 70, 11-16)

The situations of the contradictory [propositions] in their division of truth and falsehood among themselves [*fī iqtisāmihā al-ʿidq wa al-kadhib*] should not be the same in every case. For the truth of the quantified [*al-maʿlūmāt*] [propositions] is determined [*yata'ayyanu*] in virtue of the essence of the proposition and of the nature of the actual state of affairs. Similarly, for the singular temporal propositions which concern the past and the present, the time which obtained has of necessity made one of the two things [i.e. truth or falsehood] corresponding to the actual state of affairs.¹⁸ Now for the singular contradictory propositions in future states of affairs, there is no necessity on the side of the natures of the states of affairs, that truth or falsehood be determined for them.

So, while contradictory propositions about the past and the present are “determinately” true or false, contradictory propositions on future contingent matters “divide the truth and falsehood among themselves”, *i.e.* they have a truth-value, but *not determinately*. The pair of adverbs determinately/not determinately is a rendition of the pair of Greek adverbs (*aphôrismenôs/aoristôs*) used by Aristotle's ancient commentators to characterize the relation to truth and falsehood of contradictory propositions on future contingent matters.¹⁹

Once the notion of matter is introduced, the singular propositions whose predicate is a quantified universal term can be enumerated and their truth-value can be shown according to the matters. When the quantifier attached to the predicate is universal affirmative, the proposition is false in every matter; it is on the contrary true in every matter when the quantifier attached to the predicate is particular affirmative. When this quantifier is universal negative or particular affirmative, the proposition is sometimes true, sometimes false. In the first case, it is true in impossible matter, false in necessary matter, and it has an indeterminate truth-value in contingent matter; in the second case, it is true in necessary matter, false in impossible matter and has an indeterminate truth-value in contingent matter.

3. The indefinite proposition with a quantified predicate and the rule of use of the universal affirmative quantifier

Avicenna characterises the indefinite proposition as that in which “the subject is universal” and in which “the quality of the predication is revealed, but not its quantity”. The indefinite proposition is therefore classically characterized by the absence of a quantifier attached to the subject. But what logical strength must then be attributed to it: that of a universal proposition or of a particular one?

To understand Avicenna’s answer to this question, we should have in mind his famous doctrine of the treble status of the universal which came to be known in the Latin world as the *triplex respectus essentiae*.²⁰ A universal is either considered in itself as an essence or a nature, or as existing in the mind as a concept applicable to many things, or else as existing in extramental reality. Avicenna holds that an indefinite proposition in itself is neither actually universal nor particular, but is suitable to be one or the other. The subject-term of such a proposition signifies the nature in itself, which is not, as such, universal nor particular, but is suitable to be the two. The following passage from *al-‘Ibāra* I 7 shows this way of understanding the subject-term of an indefinite proposition.

T. 6. Ibn Sīnā, *al-Shifā’*: *al-‘Ibāra*, I 7 (1970, 48, 12-18)

It is not because your subject is universal that the judgment you pronounce on it becomes universal, as far as you do not judge that [the predicate] belongs or does not belong to the whole of [this subject]. And if you did not judge this way, then you have judged on the nature posed for the generality [and it] alone. But this nature, taken in itself, is something; taken as general, it is another thing; and taken as particular, it is again another thing. In itself, it is suited to be considered both ways, for if it were not suited for the particularity, it would not be suited to be for example a unique humanity in virtue of which Zayd is a unique man; and if it were not suited to be general in the mind, it would not be such as many associate in it.

Now, what is the import of this doctrine on the understanding of indefinite propositions with a quantified predicate? Consider the proposition:

Man is every laughing,

the subject-term “man” may signify either the nature of man (that is Avicenna’s understood doctrine), or else the “man as general” (Avicenna takes account of this possibility by way of concession). In the first case, the proposition will be, according to Avicenna, false. To show this, he argues by a *reductio* that if it was true, since what is true of the nature *man* is true of its instances, then a certain man would be every laughing, which is absurd. Now if the subject-term “man” is taken as general, that is man insofar as it is predicated of many differing numerically, the proposition would again be false, because the concept *man* as such cannot be every laughing. The relation of man as general or as a concept to its particulars has been stated by Avicenna in *al-‘Ibāra* I 7 as following:

T. 7. Ibn Sīnā, *al-Shifā’*: *al-‘Ibāra*, I 7 (1970, 49, 8-50)

As a general item, [humanity] is [...] like a unique thing, of which is true what is not passed on to its particulars, for as general, it is a universal, a species and so on. And these are affairs which pertain to it to the exclusion of what is under it.

Let us look now at the quantified predicate. “Every laughing” may be taken to mean the *class* of laughing entities. Against this point of view, Avicenna first reminds the reader of the *distributive* use of the universal affirmative quantifier, which excludes according to him the understanding of “every laughing” as the designation of a predicate-class. He then agrees to consider, for the sake of argument, this manner of understanding the quantified predicate. But then, either we take the subject-term “man” to signify the “general man” or else to signify “the nature of man without adding a condition of generality or particularity”. Avicenna discards the first possibility by merely asserting that the generality of man does not consist in

the fact that *man* would be identical with the class of laughing entities. And he rejects the second possibility by pointing out that while it is not the case that each instance of *laughing* is describable by the *class* of laughing entities, each instance of *man* is describable by the nature of man.

In the beginning of chapter 9, at p. 59, 7-8, Avicenna observes that one could be tempted to consider as true in necessary matter the proposition “Every man is every laughing”, but that, in so doing, one could fall into an error he has already exposed. This alludes to the passage in the previous chapter we just summarized.

4. The quantified propositions with a quantified predicate

Avicenna enumerates eight DQPs schemata and gives, for each schema, four examples covering the matters of the necessary (the examples of the general necessary and of the equal necessary are very often grouped together), of the contingent and of the impossible

1) Every S is every P

ex: Every man is every animal *or* every laughing

ex: Every man is every stone

ex: Every man is every writing

2) Every S is no P

ex: every man is no animal *or* no laughing

ex: Every man is no writing

ex: Every man is no stone

3) Every S is some P

ex: Every man is some animal *or* some laughing

ex: Every man is some writing

ex: Every man is some stone

4) Every S is not-every P

ex: Every man is not-every animal *or* not-every laughing

ex: Every man is not-every stone

ex: Every man is not-every writing

5) No S is every P

ex: No man is every animal *or* every laughing

ex: No man is every stone

ex: No man is every writing

6) No S is no P

ex: No man is no animal or no laughing

ex: No man is no writing

ex: No man is no stone

7) No S is some P

ex: No man is some animal or some laughing

ex: No man is no writing

ex: No man is some stone

8) No S is not-every P

ex: No man is not-every animal or not-every-laughing

ex: No man is not-every writing

ex: No man is not every stone

The order of the enumeration of these eight propositions is similar to the order we find in Ammonius.²¹ But, unlike Ammonius, Avicenna does not list the other eight propositions that can be found in the former. He proposes instead a rule to determine the truth-value of these

other eight propositions by taking account of the relation of *contradiction* to the eight already enumerated:

T 8. Ibn Sīnā, *al-Shifā'*: *al-'Ibāra*, I 9 (1970, 62, 2-6)

Now if the quantifier joined to the subject is particular affirmative, it will be false wherever [that proposition] is true to the subject of which is joined a negative universal quantifier—provided the [latter proposition] agrees with [the former] in all [other] circumstances; and it will be true wherever the [latter] is false. Test it yourself. Now if the quantifier joined to the subject is particular negative, then it will be true wherever that proposition is false to the subject of which is joined a universal affirmative quantifier —provided the [latter proposition] equals [the former] with regard to the predicate. Test it yourself.

One could be tempted to think that Avicenna’s procedure is somewhat empirical, as suggested by the sentence inviting the reader to test by himself the validity of the rule proposed. But, first, this testing follows a systematic procedure, since it amounts to verifying the truth-value of the particular propositions with a quantified predicate in the three matters and to ascertaining in each case that they are true where the corresponding universal propositions are false and *vice-versa*. And then Avicenna could have reached this result directly by having the negation act on what he considers as the main quantifier in a DQP.

Relations of *contradiction* between DQPs could be set out as follows, where C stands for “contradictory”:

- 1) Every S is every P C Not-every S is every P
- 2) Every S is no P C Not-every S is no P
- 3) Every S is some P C Not-every S is some P
- 4) Every S is not-every P C Not-every S is not-every P
- 5) No S is every P C Some S is every P
- 6) No S is no P C Some S is no P
- 7) No S is some P C Some S is some P
- 8) No S is not-every P C Some S is not-every P

The following table indicates the truth-value of the different types of DQPs: the list of Avicenna is completed by applying his rule for contradictories.

	∀/∀	∀/N	∀/∃	∀/¬∀	N/∀	N/N	N/∃	N/¬∀	∃/∃	∃/N	∃/∃	∃/¬∀	¬∀/¬∀	¬∀/N	¬∀/∃	¬∀/¬∀
Ne	F	F	T	T	T	T	F	F	F	F	T	T	T	T	F	T
I	F	T	F	T	T	F	T	F	F	T	F	T	T	F	T	T
C	F	F	F	T	T	F	F	F	F	T	T	T	T	T	T	T
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

∀=Every; ¬∀ = Not-every; ∃=Some; N=No.

Ne= Necessary matter (includes the necessary general and the necessary equal); I= Impossible matter; C=Contingent matter.

T: True; F=False.

Avicenna has not only given the relations of contradiction between the DQPs, he was also aware of the *equipollence* of at least two of these propositions, namely:

No S is no P ≡ Every S is some P.

Thus he writes about the proposition “No S is no P” in the contingent matter:

T. 9. Ibn Sīnā, *al-Shifā': al-'Ibāra*, I 9 (1970, 60, 15-61, 3)

It will be false in the contingent [matter], for it is false that no man is no writing, for the signification of this is that any man you take, it will be affirmed of him that he is some writing, since of no one of them is it true that he is no one of the writing. But this is manifestly false.

Avicenna of course was not the first to point out this equivalence between such propositions. This point came to the fore in Ammonius when he discussed the following exegetical question: Why, of all the types of DQPs, did Aristotle mention only the proposition with two universal affirmative quantifiers.²² So also [Ibn Zur'a], at the end of the text translated in the Appendix, mentions as a general rule the fact that a proposition with two negations is equivalent to an affirmative proposition and gives the false following example:

Not-every S is not-every P \equiv Every S is every P²³

Ammonius has the same false equivalence, together with the following:

Some S is every P \equiv No S is not-every P²⁴

Notwithstanding the mistakes which mar these examples, these latter are a testimony of the efforts to establish such equivalences. It is worthwhile noting that where Avicenna mentions such an equivalence, he avoids such mistakes. Had Avicenna pursued this line, he would have established the following equivalences between the following pairs of the eight propositions he enumerated:

Every S is every P \equiv No S is not-every P

Every S is no P \equiv No S is some P

Every S is some P \equiv No S is no P

Every S is not-every P \equiv No S is every P

He would then have reduced the number of DQPs enumerated.

The utility of propositions with a quantified predicate:

Traditionally, having enumerated the sixteen propositions with a quantified predicate and having ascertained their truth conditions, the commentators before Avicenna moved on to the topic of their utility. So did Ammonius. He distinguishes from this point of view the propositions which are always true or always false, which he considers useless, from those which are sometimes true and sometimes false, which he considers redundant since they are equivalent to and so reduce to normal propositions.

Avicenna's attitude towards this question of the utility is ambivalent: he prefaces his discussion and concludes it by protesting against those who introduced the topic of the quantification of the predicate. But when he comes to discuss the point of its utility, he protests against those who pretend that this kind of propositions should be rejected *en bloc*. He faces two objections raised by these people.

The first objection is that the truth of this kind of propositions is not a function of real states of affairs, since they could be true in different matters, that is either in the three matters of the necessary, the impossible and the contingent, or in the two matters of the necessary and the contingent. This objection is the main reason which lies behind the rejection by Ammonius of propositions with a quantified predicate which are always true:

T. 10. Ammonius, *in de int.* (1897, 106, 20-24)

For in general those who propose to examine assertions uttered without excessive variety (*poikilia*) must reject [propositions] which are always true no less than those which are always false, as neither signifying something different in the necessary or the impossible matter nor contributing to our ability to distinguish truth and falsity.

The same reason is alleged by [Ibn Zur'a] to reject the propositions with a quantified predicate:

T. 11. [Ibn Zur'a], *Kitāb Bārminyās* (1994, 46, 13-18)

We already said before that all these propositions should be rejected, because of their truth or their falsity in all the matters, or again because of their truth or falsity in two opposed matters, as in the

case of the necessary and the contingent matters. But such propositions do not fit with the syllogism; only those propositions whose truth is due to the states of things are suitable for the syllogism, and not those whose truth is due to the discourse, to its corrupted order and to additions [made] where there is no need for them.

To this objection Avicenna answers by asserting that the truth of a proposition is due to its correspondence to facts, disregarding whether this correspondence is in one matter or in more than one. By so doing, Avicenna blurs the distinction drawn by the commentators between two kinds of DQPs: those which are always true or always false on the one hand and those which are sometimes true and sometimes false. This distinction seems to correspond to a distinction between logical truths and contradictions on the one hand and contingent truths on the other. But one must observe that the commentators did not seem to be aware that by drawing this distinction they were isolating a class of propositions that should constitute the proper object of logic, since for them this class of propositions should be excluded from the field of logic as useless. We have here a good example of the *utilitarian* conception of logic which was so common in ancient philosophy.

The second objection puts forward the impurity of the quality of DQPs, that is of their affirmative or negative nature. This alludes to those propositions in which the two quantifiers differ in quality. Avicenna upholds here a radical point of view: according to him the predicate in DQPs is constituted by the quantifier plus the initial predicate, which form together a unit. So a proposition which has the form of a normal affirmative proposition will keep this quality even though a negative quantifier has been prefixed to its predicate. The quantifier of the predicate is conceived of as a predicate-forming operator on predicates: it generates new predicates from previous ones by attaching a quantifier to them. Indeed, the logic of the quantification of the predicate is modelled on the logic of propositions with an indefinite predicate, of the form: S is not-P. In this case too, “not-P” is conceived of as a unit, where the negation sign is a predicate-forming operator generating new predicates from previous ones. In this case too “S is not-P” is taken to be an affirmative proposition. There is a striking parallelism between the formulation of this idea in *al-'Ibāra* I 9 and in II 1 which is about metathetic²⁵ propositions or propositions with an indefinite predicate.²⁶

T. 12. Ibn Sīnā, *al-'Ibāra*, II 1 (1970, 78, 1-8)

When the proposition becomes ternary and a negation particle is joined to it, necessarily either this negation particle is prefixed to the copula, or it is the copula which is prefixed to the negation particle. An example of the first is our saying:

Zayd is-not just;²⁷

and an example of the second is our saying:

Zayd is not-just.²⁸

If the negation particle is prefixed to the copula, it will negate the tie [instituted by the copula between the subject and the predicate], and that will be a true negation. While if the copula is prefixed to the negation particle, it will make it part of the predicate, so that it will not be “just” taken alone which will be the predicate, but the total sum “not-just”. The word “is” will then make the total sum “not-just” affirmatively predicated of Zayd, as if one said:

Zayd is described as being not-just

and so that this [proposition] will be suited to be negated by a negation particle which will be, a second time, prefixed, [but] to the copula. One will say then:

Zayd is-not not-just.

This paradigm of metathetic propositions, on which Avicenna seems to model the logic of DQPs, allows him correctly to negate this kind of propositions, although it perhaps prevents him from recognising officially (as we saw, he recognised the fact practically) that apparently affirmative DQP have the force of negative ones and *vice-versa*.

Ibn Sīnā, *al-Shifāʾ, al-ʿIbāra* (1970, 54-65)

Book one, end of chapter seven: Making known the [different] sorts of determined, indefinite and particular propositions; the opposition which is by way of contradiction and that which is by way of contrariety, and the subalternation; and bringing the properties which pertain to propositions from this point of view

[52] The universal quantifier signifies the universality of the judgment with respect to the subject, not to the predicate. For even though the predicate is universal, the quantifier does not signify that the relation is to its universality, but rather that the relation is to the universality of the subject. So that if you say:

Every man is animal,

you do not mean that animal in its universality belongs to man, but rather that animal belongs to the universality of man. And if you need to signify that, [53] you will not signify it by this quantifier, but you need to bring in another word which signifies the quantity, as when you say:

Every man is every animal.

And if you take away this quantifier and thus say:

Man is every animal,

the mentioned word [of quantity] will be of no use to signify the universality of the judgment. This kind of propositions are called deviating (*munṭarīfāt*), and there is no great utility in enumerating them and studying them in depth. But it is the custom to mention them; so let us examine them and make known their states.

[54] Book one, chapter eight: On deviating singular [propositions]

[The meaning of the different quantifiers]

Let us consider these [propositions, first when they are] singular, that is with a singular subject, then when they are indefinite, and then when they are determined, that is with a mentioned quantifier. [The quantifier is] that²⁹ word which signifies quantity, either by a universal affirmation or universal negation, or by an affirmation in part, as when you say: “Some man is writing”,³⁰

or by a negation [denying] from the part, as when you say:

“Not-every man is writing” or “Not-some man is writing”.

(For your denying of the whole as a whole does not prevent from your affirming in part, as when you say:

Not-every man is writing, but rather some of [them];

it is not as when you say:

No [one] man is writing,

which prevents the part. Your saying: “not-every” thus necessitates only that the generality is not, but not that the particularity is not either – this is not included in it.)

Thus we say: When we say: “Zayd” and then join to its predicate the word of quantification, it will be either the word “every” or “none” or “some” or “not-every”, and the predicate will be either a universal notion or a singular notion. Now if [the predicate] is a singular notion, it is evident that prefixing to it the whole or the part in affirmation would be nonsense; unless it is meant by “whole” the total sum and by “part” the part [of a whole], so that one might say for example:

This arm is the whole of these fingers, forearm and arm,

or:

This arm [55] is part of the body,

and not the whole and the part which are the quantifiers, and with which we are dealing the way we do. In using the words “every” and “some” [as] quantifiers, we in no way think of them in that way; rather we mean by “every” not the sum but every one, and by “some” not the part, but some of what is described by the subject and shares its definition³¹. So when we say “some man” we

mean but a part of the sum of men which, besides being a part, is also a man. He is therefore one of all those who are called “man” and are defined by its definition.

[Singular propositions with a quantified singular predicate]

So, if we use “every” and “some”, the two quantifiers, in a singular predicate and thus say:

Zayd is every this-individual,

that is every one of that individual, it will be false; for that individual is not predicated of ones, every one of which *is* that individual. And since that is meaningless and [since] predicating it by an affirmation is not correct, its contradictory which is:

Zayd is not-every this-individual,

will be true. But if we say:

Zayd is some this-individual,

it will be false, and then its contradictory which is that:

Zayd is not some this-individual,

will be true. And if we say:

Zayd is not any this-individual,

it will be actually true although it suggests falsehood. It suggests a falsehood, because it suggests that this-individual is general, that it has many subjects and that this is not one of them. However, one should not pay attention to the suggestions [induced by] propositions, but only to what is grasped from [the propositions] themselves. That is why our statement:

Not-every man is a stone,

will not, by suggesting that some man is a stone, become false. Likewise, if the singular is made particular negative, so that it will be said that:

Zayd is not-every this-individual,

that is, not every one of those of whom [56] this-individual is predicated, it will be true, even though it suggests a falsehood, namely that this-individual has many subjects. It is true just because, if this individual has not many subjects of which it would be predicated, it is then manifest that Zayd is not every one of them which are not, for the non-existent is denied of every existent without the latter being a non-existent thing or things. And if it is not possible for Zayd to be every one of what is ‘Amr, that is of what is not, then it will be true that Zayd is not every one of what is ‘Amr.

Now if the predicate is universal, as in:

Zayd is every man *or* every animal *or* every writing,

it will be undoubtedly false. And if we say:

Zayd is no one of so-and-so

that will be true when the matter is impossible and false when the matter is necessary. But if the matter is contingent, neither determinate falsehood nor truth imposes itself; rather either Zayd may be, for example, writing and in that case it would be false that Zayd was no one of the writing, or Zayd may not be so, and in that case it would be true that Zayd was none³² of the writing. As for the proposition itself, that is its form, it does not impose anything. In sum, predicating contingents of individuals does not impose in their propositions the determination of truth or falsehood.

Now if the quantifier is particular affirmative, that will be true in the matter of the necessary, as when we say:

Zayd is some man,

false in the matter of the impossible, and suspended in the matter of the contingent.

Now if the quantifier is particular negative, as when we say:

Zayd is not-every so-and-so,

it will be true in every matter. It will be true for us to say:

Zayd is not-every animal *or* is not-every stone *or* is not-every writing.

How indeed would an individual be every instance of a universal notion?

[57] [Indefinite propositions with quantified predicate]

As for the indefinite [propositions], one could opine that those in which the quantifier of the universal affirmation is joined to their predicate would be true in some occurrences as in the statement of him who states that man is every laughing. But this is a false opinion, because by “man” we mean the nature of man, while by “every laughing” we mean every one of what is laughing. But the nature of man is not describable as being every one of laughing people, for otherwise a certain man could be every one of the laughing. Similarly too, if *man* is taken inasmuch as it is general, it will not be any one of the laughing, but rather it will be the general which is predicated of each of them.³³ Now if one means by “every laughing” all the laughing, that is their sum, that will not be the way we think of quantifiers when we use them. But in spite of that, we consider it (this meaning) and then say: the generality of the general *man* does not consist in his being the sum of the laughing and all of them (let us accept this, for the place to elucidate it is another place), nor is the nature of man, without the addition of a condition of generality or particularity, that (sum). How could it be, given that each one is not describable by the sum of the laughing while each one is describable by the nature of man? Now, if by “every laughing” one means the general laughing inasmuch as it is general, this is *not* what we want and what we think of when we use the phrase “every laughing”. However, it may be true to say that the general man *is* the general laughing by way of predication. But this will not be true of the nature of man, for the nature of man is *not* the general laughing; otherwise every man would be a general laughing, for the nature of man belongs to every individual. So it is for the necessary matter. As for the impossible and the contingent [matters], the falsehood [of indefinite propositions with a universal affirmative quantifier joined to the predicate] is manifest, as when we say:

Man is every stone,

or

Man is every writing,

whatever the way [these statements] are taken.

Now, if the universal quantifier is negative, it will be false in the more general necessary [matter], for if you say:

Man is no animal,

the statement will be false. [58] As for the necessary equal [matter], if you say that

Man is no laughing,

you may mean by “man” the general man and by the phrase “no [one] of the laughing” a denying from each of the individuals [under] “laughing”. If that is what you mean, no one of the individuals posited under “laughing” will be the general man and conversely, and so the proposition will be true. But if it does not come out this way, it will be false, and that is the case where what is meant by “one of the laughing” is all that is said laughing, be it individual³⁴ or universal. And this is what should be grasped first from the wording of this proposition. As for the impossible [matter], it will be true in it, as when you say:

Man is no stone.

As for the contingent [matter], it will be true if you want the subject to be the general inasmuch as it is general, as when we say:

The general man, inasmuch as it is general, is no one of the writing.

But if you mean the nature, it will be false, as when you say:

Man is no one of the writing.

Now if the quantifier is taken as particular affirmative, it will be true in the necessary general, as when you say:

Man is some animal;

but its truth is not necessary in the necessary equal, as when you say:

Man is some laughing.

For if you take the nature of man or its generality, the truth [of the proposition] will not be necessary, while if you mean a certain man — since he will also be a man — [the proposition] will be true. As for the impossible [matter], [the proposition] will be false in it, when you say:

Man is some stone.

Now if the quantifier is particular negative, it will be true in the necessary [matter], as when you say:

Man is not every animal, *or* is not every laughing,

on the account of what has been previously said. It will be also true in the impossible [matter], for man is not-every stone; and it will be also true in the contingent [matter], for man is not-every writing, as it was false that man is every writing.

Let us now deal with determined [propositions], for it was customary to deal with them, to the exclusion of the other [types of propositions].

[59] Chapter nine: On the truth and falsehood of determined propositions

[I. Universal affirmative propositions with a quantified predicate]

[I 1. The quantifier of the predicate is universal affirmative]

Now, if the subject is quantified by a universal quantifier and so is the predicate, its affirmative is not true in any matter, as when you say:

Every man is every animal *or* is every laughing *or* every man is every stone *or* every writing.

But some people took for true our saying:

All men are all laughing,

that is the totality of men is the totality of laughing. But you already knew what error and laps lie in this.

[I. 2. The quantifier of the predicate is universal negative]

Now if the quantifier of the predicate is universal negative, as when you say:

Every man is no so-and-so,

that will be false in necessary [matter], as when you say:

Every man is no animal *or* no laughing.

As for contingent [matter], according to what on the face of it has been previously stated on the contingent, its particular should be necessarily true.³⁵ So, your statement:

Every man is no one of the writing,

should be false too. For it is not every man who is so, but some-men-who-are-not-writing, it is those who-are-no-one-of-the-writing; as for those-who-are-writing, they indeed are not no-one-of-the-writing, and man includes them. Unless it happens that the matter of the proposition be as we previously alluded to, if **[60]** [such a thing] is possible.³⁶ In this case, one has to suspend his judgment and so not judge that the statement is true or that it is false, except in determined matters.³⁷ But ascertaining the truth on this point pertains to a discipline different from logic.³⁸

[This universal affirmative proposition with a universal negative quantifier joined to the predicate] will be true in the impossible matter, as when you say:

Every man is no stone.

[I. 3. The quantifier of the predicate is particular affirmative]

Now if the quantifier of the predicate is taken particular affirmative, as when you say:

Every so-and-so is some so-and-so,

this will be true in the necessary general [matter] and³⁹ in the [necessary] equal [matter], as when we say:

Every man is some animal *or* some laughing.

But it will be false in the contingent and the impossible [matters], as when we say:

Every man is some writing,

or

Every man is some stone.

[I. 4. The quantifier of the predicate is particular negative]

Now if the quantifier is taken particular negative, as when you say:

Every man is not-every so-and-so,

this will be true in the necessary [matter], as when you say:

Every man is not-every animal *or* not-every laughing,

in the impossible [matter], as when you say:

Every man is not-every stone,

and in the contingent [matter], as when you say:

Every man is not-every writing.

[II. Universal negative propositions with a quantified predicate]

[II. 1. The quantifier of the predicate is universal affirmative]

Now, if the subject is quantified by a negative universal [quantifier], and then a universal affirmative quantifier is joined to the predicate, as when you say:

No man is every so-and-so,

this will be true in the necessary [matter], as when you say:

No man is every animal *or* every laughing,

in the impossible [matter] as when you say:

No man is every stone,

and in the contingent [matter], as when you say:

No man is every writing.

[II. 2. The quantifier of the predicate is universal negative]

Now if the quantifier joined to the predicate is taken negative universal, as when you say:

No man is no so-and-so,

this will be true in the necessary [matter], for no man is no animal or no laughing; but it will be false in the contingent [matter], for it is false that⁴⁰ no man is **[61]** no writing, for the meaning of this is that any man you take, it will be affirmed of him that he is some writing, since of no one of them is it true that he is no one of the writing. But this is manifestly false. Nevertheless, the later commentator on whom these people rely has mentioned that this [proposition] is true. As for the matter of the impossible, it will be false, as when you say:

No man is no stone,

this is false.

[II. 3. The quantifier of the predicate is particular affirmative]

Now, if the quantifier joined to the predicate is taken particular affirmative, as when you say:

No man is some so-and-so,

it will be false in the necessary [matter], as when you say:

No man is some animal *or* some laughing,
and it will be false [too] in the contingent [matter], as when you say:

No man is some writing,
except on the consideration you are aware of. But it will be true in the impossible matter, as when you say:

No man is some stone.
[II. 4. The quantifier of the predicate is particular negative]

Now if the quantifier joined to the predicate is particular negative, as when you say:

No man is not-every so-and-so,
it will be false in the necessary [matter], as when you say:

No man is not-every animal *or* laughing,
And [it will be false] in the contingent [matter] too, as when you say:

No man is not-every writing,
And it will also be false in the impossible [matter], as when you say:

No man is not-every stone.
[III./IV. Particular affirmative and particular negative propositions with a quantified predicate]

[62, 2-4]⁴¹ Now if the quantifier joined to the subject is particular affirmative, it will be false wherever [that proposition] is true to the subject of which is joined a negative universal quantifier— provided the [latter proposition] agrees with [the former] in all [other] circumstances; and it will be true wherever the [latter] is false. Test it yourself.

[61,16 - 62,2] But the aforementioned commentator opined that when one says:

Some man is no writing,
that will be false. But this is the result of his inadvertence. For this is true, because the illiterate is no writing and he is some man.

[62, 4] Now if the quantifier joined to the subject is particular negative, then it will be true wherever that proposition is false to the subject of which is joined a universal affirmative quantifier— provided the [latter proposition] equals [the former] with regard to the predicate. Test it yourself.

[Utility of doubly quantified propositions]

Pay no attention to what is said about these [doubly quantified propositions], to wit that they should be rejected and thus should not be used at all. True, that is the case for those of them which are false; as for those which are true, the quantifier is a part of the predicate in them: the quantifier and what is with it are as a single one thing which is predicated, affirmatively or negatively, of the subject. So if you find some of [these true propositions] useful somewhere, use them just as you use the other propositions in the predicate of which there is no quantifier at all.

And he who says that these [true doubly quantified propositions] are not true in virtue of the *ma'ānī* because some of them are true in the three matters and some of them are true in the necessary and the contingent⁴² matters, and that they are not pure affirmatives nor pure negatives, utters nonsense. Because, first, if the predicates are divided in parts, these will have, one to another, relations which are different from the relation which is that of the proposition itself. And in this case, propositions will have, with regard to their parts, features which are different from those that belong to the predicate as a whole [related] to the subject, so that there could be a negative [relation] in [the parts] while the proposition [itself] would be affirmative. But [these features] will not change anything to the properties which belong to the proposition inasmuch as [the predicate and the subject] are predicated and subjected in it, even though they will necessitate properties more specific and secondary [with regard to the former properties].

Regarding propositions and their use, attention should be paid to nothing but to the truth. [63] If they are true, use them inasmuch truth enters in them,⁴³ and do not pay attention to the fact that

their truth is in virtue of this or that, for the true, in virtue of whatever it is, if you have to use it, will lead you to the intended aim. As for the statement of this man according to which these propositions are not true in virtue of the *ma'ānī*, if by *ma'nā* he means what is intelligible from the affirmation or the negation which are contained in the proposition, then he tells a falsehood, for the affirmation in the true among [those propositions] is true and false in the false; and if he means by *ma'nā* the form of the proposition, he [also] tells a falsehood, for the truth which occurs in this [type of proposition] depends always upon their form. As for his arguing in support of the truth of his claim by a syllogism that he constructs, it is thus: these [propositions] are true in the three matters or in two contrary matters, and what is true in this way is not true in virtue of the *ma'nā*. But the second premise is not conceded, for the true is not true at all, except for the truth of the *ma'nā*. And the true is not true and the false is not false because its truth includes its truth in the matters or not, but because it has, or on the contrary has not, a conformity and a correspondence to existence, be it in one matter or in more.

And his statement according to which [these propositions] are not pure affirmatives nor pure negatives is [also] a false statement. For the affirmation and the negation do not admit adulteration or purity, because whatever the item which you take as predicate and about which you then judge that it belongs to the subject, that will be equally an affirmation and whatever the item which you take as predicate and about which you then judge that it does not belong to the subject, that will be equally a negation. So if we take as a single item our saying: “every animal”, “some animal”, “no animal” or “not-every animal”, it will be possible to consider it as a whole predicate—not as if the predicate was that part of it which is “animal” nor that which is the quantifier, but [that which is] the whole. Then, if we affirm it, it will be a true affirmation and if we deny it, it will be [64] a true negation, and we have besides that to make the affirmation and the negation universal or particular. Further, one should not opine that these matters are the matters of the propositions, rather they are the matters of the parts of the predicate. So, when we say:

Every man is no animal,

the matter of this predicate is the impossible, even though the matter of a part of it, to wit “animal”, is the necessary. It is not “animal” which is the predicate, so that consideration should be taken of its matter, so that when a proposition would be, for example, true in matters which are not the matters of the proposition, but the matters of its parts, its truth would be sinful and would deserve to be rejected. To such things as these no attention should be paid.

[Quantified propositions with a pseudo-quantified predicate]

As for he who says that the universal quantifier, when joined to the predicate, will be also true, as when we say:

Every man is receptive of every art,

[he commits] a mistake too. For when one says: “in deviating propositions the quantifier is joined to the predicate”, he will not be making a true statement; for the true statement about [these propositions] is that it is the quantifier with another thing that is made predicate [in them]; this other thing, had it been⁴⁴ taken alone as a predicate without a quantifier prefixed to it, would have had a property [of its own]; but when the quantifier is prefixed to it and when that thing is joined to the quantifier and if the whole is taken as a single thing, it is then this sum which will be the predicate. It is not this thing taken apart alone which is the predicate in these propositions; but rather, this part is said to be the predicate, because the initial inquiry was about the universality of a subject and a predicate, so that it was then said that one should not look after the universality of the predicate. For the aim is not to signify that the predicate is belonging in its particularity or in its generality to the thing, but that its nature, however it is, is belonging to the thing. If you try then to add a quantifier, the proposition will be deviated: the predicate will no longer be a predicate, but rather it will become part of the predicate. The consideration of the truth will thus be transferred to the relation [65] which occurs between this sum and the subject. That is why these propositions were called deviating and why the First Teacher did not concern himself with them; this was rather what those who came after him did, who were fond of long discourses and who forced upon others to go into insignificant [inquiries] — forced as they are to agree with what [these people] nevertheless embrace in those long discourses.

As for your statement:

Every man is receptive of every art,

the quantifier is here joined to “art”; but “art” is not the predicate which, had there not been a quantifier, would have been the predicate [of the proposition]; rather, it is a part of that predicate. And that predicate, taken completely, is your phrase “receptive of an art”. If one said:

Every man is every receptive of an art *or* of every art,

it would be a deviating [proposition]. However, the statement:

Man is receptive of every art

does not belong to the deviating [propositions] since the quantifier is not joined to what, had there not been a quantifier, would have been a predicate, this joining being without an addition made to [the predicate].

APPENDIX I

In this Appendix, I translate a passage on the quantification of the predicate from [Ibn Zur‘a]’s Epitome of the *De Interpretatione*. This passage is interesting as another testimony, different from Avicenna’s, of the treatment of this question in Arabic logical treatises. It is characterized by being more dependent on Ammonius’s treatment of the subject than Avicenna’s chapters. It constitutes besides a testimony of the tradition criticised in some places by Avicenna. But before briefly discussing these points, a preliminary remark is in order regarding the authorship of this Epitome.

The editors have attributed the treatises they published under the title *Ibn Zur‘a’s Logic*⁴⁵ to Abū ‘Alī ‘4sā b. Is1āq b. Zur‘a (942-1008), a Christian Jacobite philosopher and a pupil of Ya1yā b. ‘Ad3 (d. 974), the head of the so-called Baghdad school in philosophy. These treatises are epitomes⁴⁶ of the following Aristotelian books: *Peri Hermeneias*, *Prior* and *Posterior Analytics*. They seem to form parts of a more complete collection of logical treatises which includes also paraphrases of Porphyrius’ *Isagoge* and Aristotle’s *Categories*. This collection is extant, in a more or less complete form, in many manuscripts throughout the world. The editors of *Ibn Zur‘a’s Logic* settled the problem of authorship of these treatises uncritically.⁴⁷ Fortunately, we have another description of this collection by M. T. Dāneshpazhūh.⁴⁸ According to him, this collection should be attributed to Ibn Zur‘a. His judgment is based on the colophon of one of the manuscripts which he did not further identify. This colophon at the end of the Epitome of the *Categories* reads as follows: “At this point, Aristotle finishes his discourse, and thereby are perfected The Book on the Ideas of the *Isagoge* and the Explanation of the Purpose of Aristotle in the *Categories*, [which form parts] of the Book of the Purposes of Aristotle[’s Books] on Logic, composed by the wise man Abū ‘Alī ‘4sā b. Zur‘a, the Christian, born in Dhū al-2ijja 331 of the Hijra and dead seven days before the end of Sha‘bān 398 of this Hijra.”⁴⁹

But it happens that this collection of logical treatises has been already “pre-empted” by modern scholarship who has attributed it to a different author⁵⁰. Contrary to the editors of *Ibn Zur‘a’s Logic*, M. T. Dāneshpazhūh was aware of this situation and he considered that the scholars who described manuscripts containing some or all of these treatises and attributed them to a different author were wrong. Indeed, two of these manuscripts, the British Library, Or. 1561, which contains the Epitome of the *Isagoge*, and the London, India Office, Or. 3832, which contains the Epitomes of the *Isagoge*, the *Categories* and the *Posterior Analytics*, have been examined fifty years ago, by S. Stern.⁵¹ In these two manuscripts, the beginning of the Epitome of the *Isagoge* is missing and so they show no authorship indication. But S. Stern, comparing the introductory material of this Epitome with the extant Large Commentary on the *Isagoge* by the Christian Nestorian philosopher and later representative of the Baghdad “school”, Abū al-Faraj b. al-□ayyib (d. 1043), concluded “that the abbreviations of the *Isagoge*, as well as those of the *Categories* and of the *Posterior Analytics*, are epitomes of Ibn al-□ayyib’s commentaries to these books, or epitomes of the commentaries which also served as source for Ibn al-□ayyib”.⁵² Subsequently, in a paper published in 1974, ‘A. Badawī described the contents of a manuscript he discovered in India, which contains the entire collection of our treatises, but which once more bears no author name. Comparing here again the introductory material to the Epitome of the *Categories* with that of the Large Commentary of by Abū al-Faraj b. al-□ayyib on this same book, ‘A. Badawī concluded that the former was an abbreviation of the latter⁵³.

But what should we conclude ourselves? The precise information contained in the colophon cited by M. T. Dāneshpazhūh constitutes, *prima facie*, a compelling argument in favour of Ibn Zur‘a’s authorship. The eminent scholar gives reasons to accept it and seems to criticise S. Stern and ‘A. Badawī for having attributed the treatises we are dealing with to Abū al-Faraj b. al-□ayyib.

However, the similarities underlined by these two scholars, in both content and style, between the treatises on the *Isagoge* and the *Categories* on the one hand and the extant Large Commentaries on these same books by Ibn al-Ḥayyib on the other, seem to give strong support to their conjecture.

It is not possible to settle definitely the question in the limits of this paper. It is enough for us to be aware of the difficulty; only a careful study of the treatises will allow us to reach a final conclusion.

[Ibn Zur'a]'s treatment of the quantification of the predicate is more dependent than Avicenna's on Ammonius. The following are some points which allow us to verify this statement:

1) [Ibn Zur'a], like Ammonius and unlike Avicenna, enumerates all sixteen DQPs. But this statement should be qualified. The eight propositions listed by Avicenna are listed in exactly the same order as by Ammonius.⁵⁴ Their method of enumeration is the same: They fix the quantifier of the subject and then vary the quantifier of the predicate, according to the order: universal affirmative, universal negative, particular affirmative, particular negative (the same order is adopted to vary the quantifier of the subject once all the possible types of quantifiers of the predicate have been listed for a given quantifier of the subject). Whereas [Ibn Zur'a], although he announces the same enumeration method, adopts in practice a different one consisting of fixing the quantifier of the predicate and then varying that of the subject⁵⁵.

2) Like Ammonius, [Ibn Zur'a] isolates the propositions which are true (respectively false) in every matter from those which are sometimes true, sometimes false.

3) [Ibn Zur'a] has the same justification for the falsehood of the universal affirmative proposition with a universal affirmative quantifier attached to the predicate. If every man were every animal, [Ibn Zur'a] claims, Socrates, for instance, would be every species falling under the genus *animal*: a bird, a fox and so on. Ammonius says the same, even if his examples of species of animals that every individual would differ: "horse, cow and all the rest."⁵⁶

4) Ammonius asserts that propositions with a quantified predicate are useless or redundant and that even apparently legitimate cases can be disposed of by a correct interpretation of what will then appear as a pseudo-quantifier. Examples of these cases are taken by those who put them forward from Plato and Aristotle, the two philosophical authorities for the ancient commentators. The example taken from the first is the following:

Rhetoric is some experience (*empeiria tis*)⁵⁷

And the example taken from the second is the following:

The soul is some actuality (*entelekheia tis*)⁵⁸

For Ammonius "*tis*" is there to show "that the predicate is not convertible with the subject but is its genus and requires the addition of some *differentiae*, in order to make the definition of the subject".⁵⁹ The whole argument of Ammonius here is reproduced by [Ibn Zur'a], including the examples from Plato and Aristotle.

In two places in *al-'Ibāra* I 9,⁶⁰ Avicenna, using a description as is his usual practice in the *Shifā'*, alludes to "the later commentator on whom these people rely" or "to the aforementioned commentator". Then when discussing the utility of DQPs, he mentions someone "who says that these [kind of propositions] are not true in virtue of the *ma'ān*3, because some of them are true in the three matters and some of them are true in the necessary and the contingent matters, and that they are not pure affirmatives nor pure negatives."⁶¹ Are the mentioned commentator and the author of the two arguments against propositions with a quantified predicate the same person? And if this is the case, is there a way to discover the identity of this person? Unfortunately the description used is not unequivocally identifying. The formula 'later commentator' may designate an Alexandrian as well as an Arab commentator. The fact that he is described as an authority for Avicenna's contemporaries and perhaps also for his immediate predecessors seems to restrict the range of possible candidates: Ammonius for the Alexandrians and some head of the Baghdad "school" for the Arabs. Here again, as in the case of the authorship of the logical treatises attributed to Ibn Zur'a, the data at our disposal are underdetermined and do not allow us to reach a definitive conclusion. All the same it is striking to find in the following text of [Ibn Zur'a] all the theses Avicenna alludes to. We can safely conclude that this text is at least representative of the commentary tradition countered by Avicenna.

1) Avicenna criticises the "later commentator" for claiming that the propositions of the form:

No S is no P

are true in the contingent matter, that is in the case of propositions of the type:

No man is no writing.

This claim appears in [Ibn Zur'a]’s text, at p. 46,7 and 9. It does not appear in Ammonius.

2) Avicenna criticises the same commentator for claiming that the propositions of the form:

Some S is no P

are false in the contingent matter, that is in the case of propositions of the type:

Some man is no writing.

This claim appears in [Ibn Zur'a]’s text, at p. 46, 11 and 13, It does not appear in Ammonius.

3) When Avicenna advises his reader to pay no attention to those who claim that propositions with a quantified predicate should be rejected, his wording matches up with [Ibn Zur'a]’s formulation when making this very same claim:

[Ibn Zur'a], p. 46, 13-14	Avicenna, p. 62, 6-7
<p>Wa-qad kunnā qaddamnā al-qawla bi-anna hādhihi al-muqaddamāti mardhūlatun bi-asrihā ...</p> <p>We already said before that all these propositions should be rejected</p>	<p>Thumma lā taltafit ilā mā yuqālu min anna hādhihi kullahā mardhūlatun, fa-lā tusta'malu al-battata.</p> <p>Then, pay no attention to what is said about these [doubly quantified propositions], to wit that they should be rejected and thus should not be used at all.</p>

A parallel can also be drawn between the report by Avicenna of the first argument for rejecting propositions with a quantified predicate and the formulation of this argument by [Ibn Zur'a] at p. 46, 14-18.

[Ibn Zur'a], p. 46, 14-18	Avicenna, p. 62, 10-11 and 63,7-8
<p>... min qibali {idqihā fī jamī' al-mawādd wa-kadhibihā fī jamī'ihā, aw {idqihā fī māddatayni mutaḡābilatayni bi-manzilat al-*arūrī wa-al-mumkin aw kadhibihā fī māddatayni mutaḡābilatayni bi-manzilat al-*arūrī wa-al-mumkin. Wa-al-qiyāsu fa-lā ya{lūlu lahu muqaddamātun shabīhatun bi-hādhihi al-{ifa, lākin innamā ya{lūlu lahu min al-muqaddamāti mā kāna {idquhu bi-sabab al-umūrī.</p> <p>... because of their truth or their falsity in all the matters, or again because of their truth or falsity in two opposed matters, as in the case of the necessary and the contingent matters. But such propositions do not fit with the syllogism; only those</p>	<p>Wa-alladhī qāla inna hādhihi laysat {ādiqatan li-ajl al-ma'ānī, li-anna ba'*ahā ya{duqu fī al-mawādd al-thalāth wa ba'*ahā ya{duqu fī al-wājibi wa al-mumkin.</p> <p>... inna hādhihi ta{duqu fī al-mawwād al-talāthati aw fī māddatayni muta*āddatayni, wa mā ya{duqu kadhālika fa-laysa {ādiqan fī al-ma'nā.</p> <p>And he who says that these [true doubly quantified propositions] are not true in virtue of the <i>ma'ānī</i> because some of them are true in the three matters and some of them are true in the necessary and the contingent matters</p> <p>... these [propositions] are true in the three matters or in two</p>

propositions whose truth is due to the states of affairs are suitable for the syllogism.	contrary matters, and what is true in this way is not true in virtue of the <i>ma'nā</i> .
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The idea, expressed by [Ibn Zur'a], that propositions that are true in the three matters, or in two opposed matters, are useless for the construction of syllogisms, because their truth is not due to their correspondence with real states of affairs, appeared already in Ammonius, at least implicitly. For 'states of affairs', [Ibn Zur'a] uses the Arabic *amr*, pl. *umūr*. The word used by Avicenna instead is *ma'nā*, pl. *ma'ānī*. The two words have been used by Is1āq b. 2unayn (d. 910), who translated in Arabic the *Peri Hermeneais*, to render the Greek word *pragma*. So, at *PH* 1, 16a7, *pragmata* is rendered in Arabic as *ma'ānī*,⁶² whereas, when the same Greek word occurs in *PH* 9, 18b38 and 19a33, it is translated by *umūr*. This last occurrence is interesting because it appears in a sentence which expresses the idea that the truth of propositions depends on states of affairs. Aristotle says that "*hoi logoi aletheis hōsper ta pragmata*",⁶³ what Is1āq translates as "*kānat al-aqāwīlu al-ḥādiqatu innamā tajrī 'alā lasabi mā 'alayhi al-umūru*".⁶⁴ It is probable that Avicenna read in the author he criticizes *ma'ānī* instead of *umūr*. It is worth noticing that he interprets it, not as states of affairs or facts, but as an ambiguous term which can mean either the intelligible content⁶⁵ of a proposition or its form.⁶⁶

One could be tempted to consider that we have assembled the pieces of the puzzle and that the solution to our two problems is at hand. It is a well-known fact indeed that Avicenna was critical of his contemporary Abū al-Faraj b. al-Ḥayyib. So we would have here an example of this attitude and this would back up the attribution of the logical treatises to Ibn al-Ḥayyib rather than to Ibn Zur'a. Surely this possibility should be kept in mind. But one should also be alert to arguments which militate against it.

First I am not sure that Avicenna would be willing to call Ibn al-Ḥayyib "the commentator on whom these people rely", even if he is not supposed to endorse the judgment, but attributed it to others. Avicenna had a low opinion of Ibn al-Ḥayyib, especially as a philosopher, which includes his expertise in logic. He writes at the beginning of a small treatise entitled: *al-Radd 'alā Kitāb Abī al-Faraj b. al-Ḥayyib* (Refutation of a book of Abū al-Faraj b al-Ḥayyib):

It has already happened that we came across books composed by al-Shaykh Abū al-Faraj b. al-Ḥayyib, may God keep his high rank in medicine, and we found them, contrary to his compositions in logic, in physics and similar disciplines, sound and satisfactory, then we came across a discourse on natural faculties...⁶⁷

Second, there are two details in Avicenna which do not match up with [Ibn Zur'a]'s text. The first is his already mentioned use of the word *ma'ānī* instead of *umūr*, and more important, the fact that he needs to interpret it. That means that he probably had this word in the text he criticises. The second detail is the following: Avicenna reports that the author he criticises constructs a syllogism in order to validate his claim that truth of DQPs does not depend on the *ma'ānī*. But even though it would be possible to extract such a syllogism from [Ibn Zur'a]'s text, it does not appear explicitly in it.

The three facts just mentioned prevent us from identifying, without further hesitation, the commentator cited by Avicenna as [Ibn Zur'a]. One should be content with the statement that the latter is a representative of the commentary tradition from which Avicenna distances himself.

[Ibn Zur'a], *Kitāb Bārminyās* (1994, 44, 13-47, 8)

After that, Aristotle undertakes to show that one cannot join the quantifier to the predicate, but [only] to the subject. (From there the commentators drew the description of the quantifier saying that it is a word which is joined to the subject.) He shows this by a *reductio*, saying: if a quantifier⁶⁸ were added to the predicate, it would follow that, among propositions the most worthy regarding quantity, quality and matter would be destroyed. And if the most worthy among propositions were destroyed, those which are of a lesser value would be *a fortiori* destroyed. But then if the propositions are all destroyed and are no more in a position to be true, no proposition appropriate to the syllogism will remain.

The worthy proposition which would be so destroyed, is the universal affirmative [proposition] in the necessary matter. As to knowing how it would be destroyed when a quantifier is added to [its] predicate, that can be demonstrated in the following way. Indeed, when a quantifier is added to the

predicate in⁶⁹ the proposition stating: “Every man is every animal”, it follows that, any one we suppose among men, he will be every animal, so that Socrates will be a winged creature, a fox, and so on. But this is unacceptable, and this unacceptable [consequence] is due to⁷⁰ the doctrine that the quantifier must be added to the predicate. Therefore, one must not add the quantifier to the predicate; it remains that it is added to the subject.

As for the commentators, they open this chapter and examine it in detail saying: if it is possible to add the quantifier to the predicate after it has been joined to the subject, let us do it. The quantifiers being four in number, one must address a given proposition which is, of course, made up of a predicate and a subject, and join to its subject one of the quantifiers, and then join [successively] to the predicate the four quantifiers, so that the proposition have two quantifiers. Sixteen propositions are thus generated by this way:

- [1] Every man is every animal
- [2] Some man is every animal
- [3] Not-every man is every animal
- [4] No man is every animal
- [5] Every man is some animal
- [6] Some man is some animal
- [7] Not-every man is some animal⁷¹
- [8] No man is some animal
- [9] Every man is no animal
- [10] Some man is no animal
- [11] Not-every man is no animal
- [12] No man is no animal
- [13] Every man is not-every animal⁷²
- [14] Some man is not-every animal
- [15] Not-every man is not-every animal
- [16] No man is not-every animal

These are the sixteen propositions generated by adding a quantifier to the predicate so that the proposition have two quantifiers. However, unacceptable [consequences] follow from all these sixteen [propositions]. Indeed, four of them are always true, in every matter; four are false in every matter; four are true in necessary and contingent matters, and false in impossible matter; and four are false in necessary and contingent matters, and true in impossible matter. From such propositions behaving this way follow absolutely unacceptable consequences, because they are true in one thing and in its contrary; then it would result that the judgment be true about one thing and about its contrary.

As for propositions which are false in all matters, they are four:

the first is:

- [1] Every man is every animal [1];

the second is:

- [2] Some man is every animal [2];

the third is:

- [3] Not-every man is not-every⁷³ animal [15];

the fourth is:

- [4] No man is not-every animal [16]

Among the four propositions true in all matters, the first is

[1] Not-every man is every animal [3];

[then we have]:

[2] Every man is not-every animal [13];

[3] No man is every animal [4]

[4] Some man is not-every animal [14]

As for the four [propositions] which are true in necessary and contingent matters, false in impossible matter, the first of them is:

[1] Every man is some animal [5];

[then we have]:

[2] Some man is some animal [6];

[3] No man is no animal [12];

[4] Every man is not-every animal [11]

As for the four [propositions] which are false in necessary and contingent matters, true in impossible matter, they are:

[1] Not-every man is some animal [7];

[2] No man is some animal [8];

[3] Every man is no animal [9];

[4] Some man is no animal [10].

We already said before that all these propositions should be rejected, because of their truth or their falsity in all the matters, or again because of their truth or falsity in two opposed matters, as in the case of the necessary and the contingent matters. But such propositions do not fit with the syllogism; only those propositions whose truth is due to the states of affairs are suitable for the syllogism, and not those whose truth is due to the discourse, to its corrupted order and to additions [made] where there is no need for them. One should know, even if talking about this we overstep the limits of our subject, that every proposition in which we find two negations, is an affirmative⁷⁴ proposition and not a negative one. So when we say: "Not-every man is not-every animal", this proposition is equivalent to the following: "Every man is every animal".

Some people have argued this way: even when a quantifier is added to the predicate, the proposition should be true, for Aristotle and Plato have joined it to the predicate and their statements were true. So Aristotle said in the treatise *On the Soul*: "The soul is a certain *entelecheia*⁷⁵", that is a certain perfection, and he means by a *certain* perfection, some perfection, and "perfection" is predicated of the soul. As to Plato, he said that *rhetorikê*, that is the rhetoric, is a certain faculty. Thus they claimed that the phrase "a certain" is a quantifier. But this is not so. But the phrase "a certain" is to be assimilated in this occurrence to a difference, because in many occurrences this phrase serves as a difference. Thus, if we join it to "animal", it will be a substitute for "rational" or for "mortal". This is enough to remove this objection.

APPENDIX II

[Ibn Zur'a]'s and Avicenna's discussions of the quantification of the predicate are the longest ones I am aware of in the Arabic logical tradition. Usually, the other Arab authors on logic dealt briefly with this topic and generally to dismiss it. Following are listed some of the passages I know of, recorded according to the chronological order.

– 'Abdallah Ibn al Muqaffa' (d. 756): In the Compendium of the *PH* ascribed to him, he devotes about a page to the topic of the quantification of the predicate.⁷⁶

– Al-Fārābī (d. 950): His Large Commentary on the *PH* devotes about half a page (13 lines) to this topic.⁷⁷

– Ibn 2azm, (d. 1064): He devotes few lines of his *al-Taqrīb li-1add al-man}iq* (Clarification of the definitions of Logic) to this topic.⁷⁸

– Abū al-Barakāt al-Baghdādī (d. after 1164): In his *al-Kitāb al-mu'tabar* (The Pondered Book; Part I on Logic, Book II, chap. 2) he devotes about half a page to this topic.⁷⁹

– Averroès (d. 1198) has three lines on this subject in his *Talkhīṣ al-'Ibāra* or Middle Commentary on the *PH*.⁸⁰

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Notes

¹ See Hamilton (1860, 509-589); for the Historical Notice of Doctrine of Quantified Predicate see esp. p. 546-555; for De Morgan's criticisms, see *On the Syllogism: II*, in Heath (1966, 41-68) and *Logic (ibid., 256-270)*. The first is a reprint of "On the Symbols of Logic, the Theory of the Syllogism, and in particular of the Copula and the Application of the Theory of Probabilities to Some Questions of Evidence", which appeared in *The Transactions of the Cambridge Philosophical Society*, IX (1856), part i, pp. 79-127 (Heath omits the part of this paper dealing with probabilities); the second is a "shortened and adapted" reprint of an article of *The English Cyclopaedia*, Arts and Sciences Division, v (1860), pp. 340-354. For an account of the controversy between Hamilton and De Morgan, see Prior (1955, 146-156), and also Heath, Introduction, pp. xi-xxx.

² For the Greek tradition, see especially the remarkable paper, curiously little quoted, of the late M. Mignucci (1983); for the Latin tradition, see (Parry 1966) and (Weidemann 1980).

³ Aristotle, *Peri Hermeneias* (henceforth *PH*) 7, 17b 12-16 (1963, 48).

⁴ Ammonius (1897, 101, 14-108). D. Blank (1996) has given an English translation of this part of the commentary which covers the eight first chapters of Aristotle's *De Interpretatione*, and of that which covers the ninth chapter (Ammonius 1998). Two other Greek commentaries on this treatise will be mentioned further, that of Stephanos (1885), of which there is also an English translation by W. Charlton (2000); and an anonymous commentary edited by L. Tarán (1978). On the Greek tradition of the commentary on the *De Interpretatione* in general, see C. Hasnaoui (2003).

⁵ Ibn Sīnā, *al-Shifā': al-'Ibāra* (1970, 64-65). To designate propositions with a quantified predicate Avicenna uses a seemingly technical term which is, as far as I know, unique to him in this context. Such propositions are said to be *mun'arīfāt*, which I translate as "deviating". Avicenna uses in other contexts words deriving from the root 2RF to signify 1) that the illocutionary force of a proposition is changed or 2) that the truth-value of a proposition is suspended by the introduction of a "hypothetical particle". He describes the first situation as follows: "The signification which is desired for itself (and not to provoke a reaction of the interlocutor) is [that of] the assertions (*akhbār*, or perhaps *ikhbār*, that is the act of asserting), used either normally (*'alā wajhīhā*), or deviating (*mu'arrafah*) as is the case with wish and astonishment, for they reduce to the assertion(s)" (*al-'Ibāra*: 31,10-11). He describes the second situation as following: "The unity of hypothetical propositions is due to the hypothetical link (*ribā' al-shar'*), which, when joined to the antecedent [...], renders it deviating (*arrafahu*), by making it neither true nor false." (*al-'Ibāra*, 33,16-34,1). Whether 1) is reducible to 2) is not explicitly stated by Avicenna. The same semantic core, namely that a clause added to a proposition or to a part of it, makes the proposition deviate from its normal functioning, is present in the description Avicenna gives of propositions with a quantified predicate as *mun'arīfāt*: "If you try then to add a quantifier, the proposition will be deviate (*in'arrafat*): the predicate will no longer be a predicate, but rather it will become part of the predicate. The consideration of the truth will thus be transferred to the relation [65] which occurs between this sum and the subject. That is why these propositions were called deviating" (*al-'Ibāra*: 64, 17- 65, 1).

⁶ Ammonius mentions, *en passant*, this class of propositions. See Ammonius (1897, 106, 15-20).

⁷ Avicenna is of course inspired here by Aristotle's *PH* 7, 17a38-17b3 for the distinction between universal and singular subjects, and 17b5-12 for the distinction between a universal taken universally and a universal not taken universally. Avicenna extends these distinctions to the predicate and combines the two sets of properties of the subject and of the predicate.

⁸ For the use of individual predicates in the peripatetic tradition see the remarks of Barnes in Porphyry (2003, 325-327).

⁹ Ammonius also mentions this class of propositions, in the same passing way as the singular propositions with a quantified predicate. See Ammonius (1897, 106, 10-15).

¹⁰ See for this characterization of the singular, Ibn Sīnā, *al-Shifā': al-Madkhal* I 5 (1952, 26, 18-27, 7).

¹¹ The word *hadhr*, which literally signifies “babble” or “idle talk”, must be so understood. An utterance is a *hadhr* if it makes no difference as to the information content: it is tautological or meaningless.

¹² That principle, which could be called principle of *semantic closure* of the proposition, is asserted elsewhere in *al-'Ibāra*, for example on p. 50, 4-8 and on p. 103. In the first passage, Avicenna aims to show that an indefinite proposition is neuter with respect to universality and particularity. He concludes this way: “Our aim was to establish what we have shown, to wit, that the judgement on a universal [subject] without the stipulation of a generality or a particularity does not necessitate in any way generality, nor is there in it a literal indication of particularity. Rather the indication of particularity is consequent, from without, to the signification of the judgment, it is not the signification of the judgment about [the subject]. Likewise, every proposition has consequents such as [propositions resulting from] conversion and others among those you will know [later], [but all these consequents] are not the significates themselves [indicated] by the proposition”.

¹³ The contradictory of “Zayd is some this-individual” should be “Zayd is not any this-individual”. The latter is mentioned by Avicenna who gives also its truth-value; see Ibn Sīnā, *al-'Ibāra* (1970, 55, 11-13).

¹⁴ Barnes (1990, see esp. 39-53) has useful comments on the different logical contexts where this distinction appears in the ancient Aristotelian commentary tradition. The notion of the matter of a proposition as opposed to its form (*{ūra*) or composition (*ta'l3f*) appears in Avicenna's *al-'Ibāra* most explicitly in II 1 at p. 82,6-7. Moreover, in the two chapters dedicated to the quantification of the predicate, Avicenna mentions the form of a proposition (*{urat al-qa'iyya*) at two occasions, at p. 56, 11 (see T3 below) and at p. 63, 5-6. The notion of the matter of a proposition, understood as its implicit modal status and opposed to its explicit modality (*jiha*) is found also in Avicenna's *al-'Ibāra* II 4, p. 112, 10-15.

¹⁵ It is tempting to see in the three or four matters indicated here by Avicenna an equivalent of at least four of Euler's diagrams: the matter of the necessary general corresponding to the case of the diagram in which the subject-class is included in the predicate-class; the matter of the necessary equal corresponding to the case of the diagram in which subject and predicate are coextensive; the matter of the contingent corresponding to the case of the diagram in which subject-class and predicate-class overlap; the matter of the impossible corresponding to the case of the diagram in which subject-class and the predicate-class are mutually exclusive. Avicenna does not contemplate here the case in which the subject-class includes the predicate-class.

¹⁶ The notion of matter is introduced by Ammonius (1897, 88, 12-28); it underlies the whole development by Ammonius of the quantification of the predicate, and is expressly mentioned pp. 103, 11, 104, 1-12; 30; 33-34.

¹⁷ Tarán (1978, 41, 6-12 and 43,7).

¹⁸ Reading *al-amr* for *al-ākhar*.

¹⁹ On this pair of adverbs and its signification, see now Gaskin (1995, 147-184); for its Arabic equivalent in al-Fārābī who uses *'alā al-ta1{3l / lā 'alā al-ta1{3l ('alā ghayr al-ta1{3l)*, see al-Fārābī (1960, 81, 11-15; this pair occurs frequently in the pages 81-98 where al-Fārābī is commenting the *PH* 9). See also Zimmermann (1981, lxxvii-lxxviii and the passages cited in the General Index, s.v. “true: definitely”). Avicenna prefers to use the verb *ta'ayyana* and its negation to signify this opposition.

²⁰ The *loci classici* for this doctrine in Avicenna are *al-Shifā': al-Madkhal* I 12 (1952, 65-69); *al-Shifā': al-Ilāhiyyāt* V 1 (1960, 195-206).

²¹ See Ammonius (1897, 102, 33-103, 24) for the eight enumerated propositions corresponding to those enumerated by Avicenna, and see 103, 24-104, 12 for the eight other ones, not directly enumerated by Avicenna.

²² Ammonius (1897, 105, 1-106,9, for the whole discussion).

²³ *Kitāb Bārminyās*, in Ibn Zur'a (1994, 46.18-21). The equivalence, referred to the proposition with two particular negative quantifiers, should be:

Not-every S is not-every P ≡ Some S is every P;

and, referred to the proposition with two universal affirmative quantifiers, it should be:

Every S is every P ≡ No S is not-every P

²⁴ Ammonius (1897, 105.11-15) for the first equivalence, and 15-19 for the second. M. Mignucci (1983, 29-30) proposed to amend these examples to make them logically correct. The fact that the first false equivalence is found in [Ibn Zur'a] shows that this at least has survived to Ammonius.

²⁵ For this denomination which is ascribed to Theophrastus, see Fortenbaugh, Huby, Sharples, and Gutas (1992, 87 A-F, 148-153).

²⁶ The exact title of this chapter is: “On binary and ternary propositions; on metathetic, plain and privative propositions and on the relations that occur between the contradictories in these [three types of propositions] in [the cases where these propositions are] singular or indefinite.”

²⁷ In Arabic, the negation particle “laysa” precedes the copula which is expressed in the technical language of the philosophers by the verb “yūjadu”. The example in Arabic has the following form:

Zayd laysa yūjadu 'ādilan.

²⁸ The negation particle attached to the initial predicate is now in Arabic “lā”. The example in Arabic is:
Zayd yūjadu lā-'ādilan.

²⁹ 54, 5: Reading, with three mss., *wa-huwa* for *wa-hādhā*.

³⁰ Avicenna says at *al-'Ibāra* (1970, 46,12), that the proposition “every man is writing” should be understood as “every man is actually (*bi-al-fi'l*) writing”.

³¹ In this paragraph, Avicenna uses the same words “kull” and “ba'd” for the quantifiers, resp. “every” and “some” on the one hand, and for the whole and part resp. on the other hand.

³² P. 56, 10: Reading, with two mss, *laysa wa-lā*, for *aw lā*.

³³ P. 57, 6: *Minhā*, one would expect *minhum* in which the pl. masc. pronoun *hum* would have referred to “laughing”. But this is not satisfactory either.

³⁴ P. 56, 6: Deleting *shakhfan*.

³⁵ This is an allusion to the common view reported in *al-'Ibāra*, p. 47, 14-17. “As to the particular [affirmative and negative propositions], their status in the necessary and in the impossible matters is that of the two universal [propositions] (that is the affirmative is true in the necessary matter and the negative in the impossible matter). As to the contingent matter, what is commonly admitted is that they *should* both be true; but what is evident about them is that they *may* be true in the contingent matter. [...] Now, that this should be necessarily is not by itself evident for the beginner”. Avicenna seems to mean that although *metaphysically* there are no unrealised possibilities, this truth is not manifest for the beginner who studies logic and who has not yet reached the metaphysical truths.

³⁶ This seems to allude to the passage which follows that quoted in the previous note, see *al-'Ibāra*, 47,17-48, 3: “For it is not necessary, for the beginner, that the predicate which is from the matter of the contingent, be existent in some [instance] of the subject and not-existent in some of it. The beginner indeed does not disapprove of the fact that a predicate be among the remote and odd contingents, then that it happen that it does absolutely not exist, in any time, in any of the individuals of a species.” Or, alternatively, it may allude to another passage of *al-'Ibāra* I 8, p. 56, 8-11, where is stated that to predicate contingents of individuals does not necessitate definiteness of truth or falsehood of the propositions in which this predication is made.

³⁷ Matters here should not be understood as the implicit modal status of a proposition, but specific situations in which truth or falsehood can be assigned to the proposition.

³⁸ That is, metaphysics.

³⁹ P. 60,5: Adding, with all mss except one, *wa* between *al-'āmm* and *al-musāw3*.

⁴⁰ P. 61, 15: Delete *wa* before *laysa*.

⁴¹ I delete 61, 14-16, which is corrupted and which, once corrected, seems to duplicate 62, 2-4. Here is a translation of the corrected text of 61, 14-16: “Now if the quantifier joined to the subject (reading *bi-al-maw*ū'* for *bi-al-maImūl*) is affirmative particular, it will be true where the [proposition] to the subject of which is joined a universal negative (reading *kulliyyan sāliban* for *juz'iyyan mūjiban*) quantifier is false, and it will be false where the latter is true — provided that the two propositions are equal in other circumstances. Test it yourself.” The passage at 61, 16-62, 2 should come immediately after any of the two passages, either at 61,14-16 or at 62, 2-4, because it deals with a special case of the class of propositions mentioned in these passages. I have no satisfactory explanation of this paleographic accident.

⁴² P. 62,11: Reading *al-mumkin* instead of *al-mumtani'*.

⁴³ P. 63, 1: Reading *f3hā* for *f3hi*.

⁴⁴ P. 64, 11: Reading with some mss. *law* for *aw*.

⁴⁵ (Ibn Zur'a 1994).

⁴⁶ The treatises on *Prior* and *Posterior Analytics* are called *jawāmi'*, see Ibn Zur'a (1994, 93, 4).

⁴⁷ The editors mention that they used three manuscripts from Tehran, but we are not told which ones.

⁴⁸ See Ibn al-Muqaffa' (1978, 36-37) of the Persian Introduction. (I am grateful to my colleague Hossein Masoumi of the Sharif University of Technology–Tehran, for his help in reading this passage.)

⁴⁹ The title *The Book on the Ideas of the Isagoge* and also the title *The Book on the Purposes of Aristotle's Books on Logic* appear in the notice dedicated to Ibn Zur'a in *al-Fihrist* of Ibn al-Nad3m, a bibliographical work written in 987. See Ibn al-Nad3m (1988, 323, 4-5).

⁵⁰ This duality creates strange situations. Thus J. Lameer *Ibn al-Muqaffa's Logic*, enumerates the manuscripts of what he takes to be the Epitomes of Ibn al-Ḥayyib's commentaries on the *Isagoge*, *Categories*, *de Interpretatione*, and *Prior and Posterior Analytics*. But, as he is not aware of the fact that Dānespazhūh's list of manuscripts of Ibn Zur'a's treatises concerns the same collection, he misses many of these latter, practically all those which are in Iranian libraries. See J. Lameer (1996, 90-98, esp. p. 96).

⁵¹ (Stern, 1957).

⁵² Stern (1957, 425).

⁵³ Badawī (1974, 74).

⁵⁴ Another point on which Avicenna sides with Ammonius: both of them discuss the case of what they consider as propositions with a pseudo-quantified predicate, like the following: “Every man is receptive of every science”. We may call Theophrastian this type of propositions, for Theophrastus put them forward as

propositions with a quantified predicate. Then an objector alleged them as a counterexample to the thesis that propositions which have the form “Every S is every P” are always false. Both Ammonius and Avicenna dispose of this kind of propositions by showing that the second quantifier in them is not attached to the predicate, but to a *part* of it. For ascribing to Theophrastus this kind of propositions, see Fortenbaugh, Huby, Sharples and Gutas (1992, 84, 144-146); for Ammonius’s handling this issue, see Ammonius (1897, 107, 7-108, 6); for these two points, cf. Mignucci (1983, 38-40); for Avicenna’s handling of the same issue, see *al-‘Ibāra* (1970, 64, 8-65,8).

⁵⁵ This procedure was followed by the Tarān’s Anonymous in his first list of the sixteen DQPs. This list is then duplicated. Fixing the quantifier of the predicate, Anonymous, in his first list, varies the quantifier of the subject according to the order: universal affirmative, particular negative, particular affirmative, universal negative. Anonymous adopts this order because his purpose is to list pairs of contradictories. He duplicates this list because he seems to think, like Ammonius seems to, that the contradictory of a DQP may be obtained by negating either the quantifier attached to the subject or that attached to the predicate. For these two lists, see Tarān, Anonymous in *de int.* (1978, 41-43). For the occurrence of the thesis just mentioned in Ammonius, see Ammonius (1897, 105, 21-26).

⁵⁶ Ammonius (1897, 101, 17-20).

⁵⁷ Gorgias, 462C. *Empeiria* is replaced in [Ibn Zur’a] by capacity or faculty (*quwwa*).

⁵⁸ *De anima*, II 2, 414a27.

⁵⁹ Ammonius (1897, 106, 32-107), I quote Ammonius’s sentence in Blank’s translation.

⁶⁰ Ibn Sīnā, *al-‘Ibāra* (1970, 61, 3 and 17).

⁶¹ *Ibid.*, 62,10-12. Avicenna mentions again the same author at 63,6-8 and 11.

⁶² See Aristotle, *Kitāb al-‘Ibāra* (1980, 99, 11). It is again *ma’nā* which translates *pragma* at *PH* 3, 16b23 (1980, 102,14), and *ma’ān3* which translates *pragmata* at *PH* 7, 17a38 and 12, 21b28 (1980, 105, 6).

⁶³ In Ackrill’s translation: “statements are true according to how the actual things are.”

⁶⁴ See Aristotle, K. *al-‘Ibāra* (1980, 113, 1-2).

⁶⁵ Ibn Sīnā, *al-‘Ibāra* (1970, 63, 3-4): *al-ma’nā al-ma’qūl....*

⁶⁶ *Ibid.*, 63, 5: *ḥūrat al-qa’iyya*.

⁶⁷ The passage is quoted by Y. Mahdavi (1954, 116). This treatise of Avicenna has been published in H. Z. Ülken (1953, 66-71) (I have not seen this volume). The same low opinion Avicenna had of Ibn al-‘Ayyib is displayed in a report by a disciple of the former, see Ibn Sīnā (1992, 80-85). This passage has been translated and analysed by D. Gutas (1988, 64-72).

⁶⁸ P. 44, 16: reading *al-sūr*. For *tarf al-salb*.

⁶⁹ P. 45, 1: reading *f3* for *wa*.

⁷⁰ P. 45, 3: reading *jarrahā* (?) for *idd-hā*.

⁷¹ P. 45, 13: reading *wālidun min* for *laysa kull*.

⁷² P. 45, 15-17: these ll. repeat ll. 13-15, they should be read as follow.

[13] *kullu insānin laysa kulla layawānin.*

[14] *wālidun min al-nās laysa kulla layawānin.*

[15] *laysa kullu insānin laysa kulla layawānin.*

[16] *wa-lā wālidun min al-nās laysa kulla layawānin.*

⁷³ P. 46, 4: adding *laysa* before *kull*.

⁷⁴ P. 46, 20: reading with the mss. *mūjiba* for *muwajjaha*.

⁷⁵ P. 47, 3: reading *an}ālākhyā* for *lInā*.

⁷⁶ Ibn al-Muqaffa’ (1978, 45-46, §§. 85-86).

⁷⁷ Al-Fārāb3 (1960, 70, 11-24) = Zimmermann (1981, 64-65).

⁷⁸ Ibn 2azm (1983, 195, 5-10).

⁷⁹ Abū al-Barakāt al-Baghdād3 (1938, 70,12-24).

⁸⁰ Ibn Rushd (1981, 72, 6-9).

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Name (*ism*), Derived Name (*ism mushtaqq*) and Description (*wa{f}*) in Arabic Grammar, Muslim Dialectical Theology and Arabic Logic

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Abstract. Recent studies on Avicenna's modal syllogistic have pointed out the significance of his distinction between the understanding of predications 'with regard to essence/essentially' (*dh#ī*) and 'with regard to description/descriptionally' (*wa{fī}*) (Street 2000a, 2000b, 2005a, 2005b). In this paper I investigate the grammatical, theological and metaphysical context of Avicenna's understanding of that what is 'derived' (*mushtaqq*) either with regard to essence/essentially or with regard to description/descriptionally. I argue that this distinction is based on two different kinds of understanding 'derivation' (*ishtiq#q*). The Arabic grammarian Sībawayh distinguished two classes of the 'derived': [a.] "[the name of] the agent" (*ism al-f#'il*) and [b.] "the description/attribute which is similar to [the name of] the agent" (*al-ifa al-mushabbaha bi-l-f#'il*). These terms can be understood as derived either logically or grammatically. I argue that Avicenna's *dh#ī*-reading is based on the logical derivation of the 'name of an agent' or the 'description/attribute' from a noun which signifies an abstracted essence, and that Avicenna's *wa{fī}*-reading is based on their grammatical derivation from a verb/acting (*fi'l*) which indicates the occurring (*Iudūth*) and the happening (*Iu{ūl}*) of an acting (*fi'l*) or of an affection by a quality (*ifa*). Thus, Avicenna's *dh#ī/wa{fī}* distinction is a typical product of the mutual rapprochement between Neoplatonic and Peripatetic metaphysics and logic on one hand and Arabic grammar on the other hand. I further argue that the *dh#ī/wa{fī}* distinction is not only basic for Avicenna's syllogistic, but also for al-Ghaz#lī's semantical-logical explanation of the names of God.

1. Introduction

One of the most disputed issues among logicians and scholars of the history of logic has been the explanation of what has been called Aristotle's multiplicity of approaches to modal logic and their integration in one consistent system.¹ In the Arabic tradition Aristotle's modal syllogistic was superseded by a system of modal logic in which the distinction between the understanding of predications 'with regard to essence/essentially' (*dh#ī*) and 'with regard to description/descriptionally' (*wa{fī}*) plays an important role.² Predication 'with regard to essence/essentially' is obviously derived from a technical term of Neoplatonic and Peripatetic metaphysics and logic, namely the term 'essence' (*dh#*) in the sense of 'Form' (*{ūra*) and 'secondary substance', that is to say, what is predicated of a thing in response to the question 'what is it?' (cf. Aristotle, *Cat.* 5, 29-32; *Met.* VII, 4). Earlier Arabic logicians and theologians used the term '*ayn*' (cf. Endress 1977, 79-80; Schöck 2006, 121-3, 129, 283-4) instead of the term *dh#*. Predication 'with regard to description/descriptionally', however, is not derived from a term of metaphysics and logic, but from a technical term of Arabic grammar. As a technical term of Arabic grammar 'description' (*wa{f}*) denotes what is called in English grammar a 'participle', and what is signified in Arabic grammar by 'attribute' (*{ifa/na't*) or 'the name of the agent' (*ism al-f#'il*). But the Arabic term 'description' is broader in meaning. It can signify any act of describing and any description of someone or something, not only by an attribute or a name of an agent, but also by verbal description. That is to say, the 'description' (*wa{f}{ifa/na't*) by which the 'described' (*maw{ūf/man'ūt*) is

explained in language might be a verb (*fi'l*) or an expression which is 'derived from a verb' (*mushtaqq min fi'l*), namely an 'attribute' (*{ifa/na't}*) or a 'name of an agent' (*ism f#'il*). In any case the description characterizes the 'described', qualifies it, praises or blames it, explains and specifies it by (*bi-*) something. In this broader sense the term 'description' is used throughout the Arabic literal tradition.³ Thus the Arabic grammatical term 'description' denotes a semantical function, namely the function of describing something — which is the 'described' — by something else, namely by a quality (*{ifa}*) which corresponds to the grammatical category 'attribute' (*{ifa}*) or by an action (*fi'l*) which corresponds to the grammatical category 'verb' (*fi'l*). Both, 'with regard to essence/essentially' and 'with regard to description/descriptionally' are semantical terms, inasmuch they refer to the way in which an expression (*lafz**) is used in language. However, they are also logical terms, inasmuch they refer to the way in which two things combined (*mu'allaf*) and connected (*muqtarin*) in language are combined and connected logically.

The use of an originally grammatical term side by side with a logical term and their integration in one system of understanding sentences is a typical product of the appropriation of the Aristotelian logic in the Arabic world. From the very beginning of the adoption of antique logic 'Greek' logic and Arabic grammar were rivals. As the early Arab grammarians saw it, the rules of the Arabic language guarantee an immediate understanding of the evident (*z*#hir*) meaning of a sentence. For them 'Greek' logic was not only superfluous, but it could not serve to understand an Arabic sentence, since it is based on the language of the Greeks. In opposition to them the logicians, whatever language they speak, hold logic to be based on reason which is common to all human beings. This basic conflict led to a reflection on the relation of Aristotle's logic with Arabic language and to an increasing influence not only of Neoplatonic and Peripatetic logic on Arabic grammar but also of Arabic grammar on Arabic logic. The first Arabic writing commentator whose brief paraphrase of the Aristotelian Organon is preserved, Ibn al-Muqaffa' (first half 2nd/8th century), identified the grammatical categories of name/noun (*ism*) and attribute (*na't*) with the logical categories of substance/essence (*'ayn*) and accident (*'ara**) (Schöck 2006, 121-3). For al-F#r#bī (d. 339/950) the reflection on the relationship of grammatical function with logical function was a key-element of the integration of Aristotle's logic in Arabic thought. This increasing mutual influence is reflected in the report of the reciprocal teaching of the grammarian Ibn as-Sarr#j (d. 316/928) and the logician al-F#r#bī. The report of this interdisciplinary joint-venture might be only legendary. But the cross-fertilisation between grammar and logic is documented in the fact that Ibn as-Sarr#j systematized the different parts of speech according to rational definitions while al-F#r#bī compared and synthesized the meanings conveyed by the correct use of the Arabic language and by reasoning (Endress 1986, 2001). One of the most significant products of this process of mutual rapprochement between grammar and logic is the synthesis of the Aristotelian accidental predication with the Arabic 'description' (*wa{f}*). The identification of that which is signified by the different grammatical categories 'name/noun' and 'description' with the logical distinction between substance/essence and accident provided the basis for Ibn Sīn#'s distinction between an understanding of predications 'with regard to essence/essentially' (*dh#ī*) or 'with regard to description/descriptionally' (*wa{fi}*).

Before Ibn Sīn# made use of this distinction in his syllogistic the term 'description' had already gone through a long history of dispute between Arab grammarians and Muslim dialectical theologians (*mutakallimūn*). In this article I will seek to shed some light on the history of this dispute helping to understand what Ibn Sīn# had in mind when he spoke of 'with regard to essence/essentially' and 'with regard to description/descriptionally'. To point out the broader significance of this distinction within the intellectual history of Arab-Muslim

thought I shall begin with an attack of the famous twelfth-century Jewish philosopher Maimonides on his no less famous Muslim counterpart Abū al-Ghazālī, both of them mile-stones of the intellectual history of medieval thought.

2. Maimonides' attack on the name-description distinction of the *mutakallimūn*

In his *Guide for the Perplexed* Maimonides (d. 1204) criticizes the Muslim dialectical theologians (*mutakallimūn*) for naming God 'agent' (*f#il*) while they avoid the denomination (*tasmiya*)⁴ 'first cause' (*al-'illa al-ūl#*) and 'first ground' (*al-sabab al-awwal*) (*Dal#at* I, 88r-88v; transl. Pines I, 166). Maimonides reports:

[They] think that there is a great difference between our saying (*qawl*) 'cause' and 'ground' and our saying 'agent'. For they say that if we say that He is a cause ('*illa*), the existence of that which is caused/effected (*ma'lūl*) follows necessarily (*lazima*), and that this leads to the doctrine of the pre-eternity of the world and of the world necessarily following from God. If, however, we say that He is an agent/enactor (*f#il*), it does not necessarily follow that that which is enacted (*maf'ūl*) exists together with Him. For the agent (*f#il*) sometimes precedes his act (*qad yataqaddamu fi'lahū*). Indeed, they only form the idea (*ma'n#*) of the agent as an agent as preceding his act (*ill# an yataqaddama fi'lahū*). This is the saying of those who do not distinguish between what is in *potentia* (*bi-l-quwwa*) and what is in *actu* (*bi-l-fi'l*).

But you know that, regarding this subject, there is no difference between your saying a cause ('*illa*) and your saying an agent (*f#il*). For if you regard the cause ('*illa*) as being likewise in *potentia*, it precedes its effect (*ma'lūl*) in time. If, on the other hand, it is a cause in *actu*, its effect exists necessarily in virtue of the existence of the cause as a cause in *actu*. Similarly if you regard an agent/enactor (*f#il*) as an agent/enactor in *actu*, the existence of that which is enacted (*maf'ūl*) by him follows necessarily. For before he builds a house, a builder (*bann#*) is not a builder in *actu*, but a builder in *potentia*; just as the matter of a particular house, before it is built, is matter in a state of potentiality. However, when a builder builds, he is a builder in *actu*, and then the existence of a built thing follows necessarily. Thus we have gained nothing by preferring the naming/denomination (*tasmiya*)⁵ 'doer/agent' to the naming/denomination (*tasmiya*)⁶ 'cause' ('*illa*) and 'ground' (*sabab*) (*Dal#at* I, 88v; cf. transl. Pines I, 166-7).⁷

Maimonides does not name openly which of the *mutakallimūn* he has in mind in his critique. But since he says "this is the saying of those who do not distinguish between what is in *potentia* (*bi-l-quwwa*) and what is in *actu* (*bi-l-fi'l*)" he is referring to the well-known theory of occasionalism, which became the 'orthodox' Muslim Sunnī doctrine from the time of al-Ash'arī (d. 324/935).⁸

However, in the passage quoted above Maimonides is not arguing against occasionalism. He is rather arguing that "there is no difference between your saying a cause and your saying an agent... Thus we have gained nothing by preferring the denomination 'agent' to the denomination 'cause' and 'ground'." Obviously Maimonides is reacting against opponents who refused to call God 'the first cause'.

Two centuries before Maimonides the Sunnī scholar al-2alīmī (d. 403/1012) who is a disciple of two disciples of the *mutakallim* al-Ash'arī (Gimaret 1988, 31f.) mentions as one of five articles of faith the doctrine of God's creation out of nothing (*ex nihilo*) with the following words: "[The affirmation] that the existence of everything other than Himself comes into being because He originated and created it for the first time, to dissociate oneself from those who hold the doctrine (*qawl*) of the cause and the caused (*al-'illa wa-l-ma'lūl*)" (*Minh#j* I, 183ult.-184,1; cf. Gimaret 1988, 101).

Although the Muslim dialectical theologians used the term 'cause' ('*illa*) always linked to the term 'caused' (*ma'lūl*) (cf. Frank 2000, 9 n. 21) the early *mutakallimūn* did not necessarily treat these terms as correlatives (*mu*##*). Their disputes focussed on the question whether

the ‘cause’ (*‘illa*) exists before (*qabla*) the ‘caused’ (*ma‘lūl*), together with (*ma‘a*) the caused, and/or after (*ba‘da*) it. Depending on their answer on this question some of them used the term ‘cause’ (*‘illa*) in the sense of a necessary condition of the ‘caused’ (*ma‘lūl*), namely a potency (*quwwa/qudra/isti*}{#‘a) which precedes the ‘caused’ in time. Others used it in the sense of the ground and the reason of doing something and therefore also hold that it precedes the ‘caused’ (*ma‘lūl*). The Mu‘tazilite scholar Abū l-Hudhayl (d. about 227/841) had already explained the term *‘illa* as the ‘reason’ of an inference corresponding to the middle term of a syllogism (cf. Schöck 2006, 182-4). But ‘cause’ (*‘illa*) could also be used for the final cause (*ghara**) which is after the ‘caused’ (*ma‘lūl*) or in the sense of the sufficient cause which exists together (*ma‘a*), that is to say simultaneous with the ‘caused/effect’ (*ma‘lūl*) (cf. al-Ash‘arī, *Maq#l* 389-91). This latter sense wins through in Muslim thought. Like the theologians Ibn Sīn# (d. 428/1037) hold that a cause (*‘illa*) in the real sense (*fī l-‘aqqā*), that is to say a sufficient cause, must exist simultaneous (*ma‘a*) with the caused/effect (*ma‘lūl*). Otherwise it would be possible that a cause is not cause what is contradictory (cf. Ibn Sīn#, *al-Shif#*: *al-Il#hiyy#*, book 4.1, I, 165,15 - 166,17; cf. also Lizzini 2004, 181; Schöck 2004). Maimonides used the term *‘illa* as interchangeable with the term *sabab*. This is possible from a position which does not deny causal efficacy in this world. But from an occasionalistic point of view *‘illa* and *sabab* are not synonymous. From this point of view the *mutakallimūn* use *sabab*, often together with *#la* (“tool, instrument”), in the sense of “means”, namely for those factors which are necessary conditions and occasions by (*bi-*) which an act and an effect comes to existence, but not for their sufficient ground and efficient cause (cf. al-M#turīdī, *al-Tawīd* 410,10f.; al-Ghaz#lī, *Musta#f* I, 59f.; al-Ghaz#lī, *al-Maq#ad* 100,9; 145,11; cf. Frank 1992, 27f.).⁹

In the passages quoted from al-2alīmī and Maimonides, ‘the cause and the caused’ is used as an abbreviation for the Neoplatonic theory of the emanation of the world from the first being. In this context the *mutakallimūn* understood ‘cause’ and ‘caused’ as correlatives in the sense that from the existence of the eternal cause, namely God, necessarily follows the existence of the eternal caused, namely the world, which was inconsistent with their faith.

Maimonides intended to prove wrong the inconsistency of Neoplatonic philosophy with monotheism. His argumentation which follows the passage cited above is based on the Neoplatonic identification of efficient cause, form and final cause.¹⁰ Therefore he felt need to react against the *mutakallimūn*.

The argument of the *mutakallimūn* Maimonides refers to runs through the first part of Abū 2#mid al-Ghaz#lī’s (d. 505/1111) *Incoherence of the Philosophers* (*al-Tah#fut al-Fal#sifa*) (cf. Wisnovsky 2004, 130f.; Druart 2004, 344), the most prominent Muslim ‘refutation’ of Neoplatonic philosophy. Here, al-Ghaz#lī explains at great length why God should be called ‘agent’ rather than ‘first cause’, and obviously until the time of Maimonides al-Ghaz#lī’s argument had become a key-element in the dispute between the defenders and the opponents of the compatibility of Neoplatonic philosophy and monotheism.

Maimonides’ arguments for calling God ‘cause’ fall in the domain of physics and metaphysics. Since he cites opponents not present, he seems to have an easy victory. The opponents have no chance to reply. As we shall see, they would have had pretty good counter-arguments if they had had the chance to answer. Their argumentation would fall in the domain of grammar and logic. They would have argued that ‘cause’ (*‘illa*) is a primitive noun, that is to say, an underived name (*ism*) which signifies an essence (*dh#t*), the reality (*‘aqqā*) of a named/denoted thing (*musamm#*) and ‘what it is’ (*m# huwa*). Therefore the term ‘cause’ can only be used to denote something ‘with regard to essence/essentially’ (*dh#t*). ‘Agent’ (*f#‘il*) and ‘builder’ (*bann#*) on the other hand are names of agents/*nomina agentium* (*asm# al-f#‘ilīn*) which are derived from verbs/actions (*mushtaqqā min af#l*) (cf. Wright 1981, I, 106).

They do not signify the reality of the named/denoted nor ‘what it is’. They are rather attributes (*{if#}*) of an essence which indicate a relation (*i*#fa*) of an essence and a substance with an action. Therefore they can be used either ‘with regard to essence/essentially’ (*dh#ī*) or ‘with regard to description/descriptionally’ (*wa{fī}*) (cf. below §§ 6-8).

Maimonides knows the difference between a name and a name derived from a verb/action, since he uses this distinction in his *Guide*. According to him all the names of God are derived from verbs/actions (*mushtaqa min al-af#l*), except the name Y-H-W-H (*Dal#at* I, 77v; transl. Pines I, 147). But in the context of calling God ‘first cause’ and ‘agent/enactor’ he does not apply this distinction.

In the following I am concerned with the logical and semantical key-elements of the argumentation of the *mutakallimūn* mentioned above. I shall begin with al-Ghaz#lī’s logical arguments for preferring the term ‘agent’ to the term ‘cause’ (§ 3), then focus on the grammatical background of the Arabic ‘name of the agent’ (§ 4) and the debates on its meaning between the early *mutakallimūn* (§ 5), then give a brief outline on al-F#r#bī’s synthesis between the grammatical and the logical use of the ‘name of the agent’ (§ 6) which leads to Ibn Sīn#’s distinction between the understanding of a derived name ‘with regard to essence/essentially’ and ‘with regard to description/descriptionally’ (§ 7) and finally turn to al-Ghaz#lī’s semantical-logical treatment of the distinction of name on the one hand and derived name and description on the other hand (§ 8). Finally it will become clear what was gained by preferring the naming/denomination ‘agent’ to the naming/denomination ‘cause’ and ‘ground’ in the sight of the Arabic-Muslim *mutakallimūn*.

3. Al-Ghaz#lī’s argument for calling God ‘agent’ (*f#’il*) instead of calling him ‘cause’ (*’illa*)

It is important to note the way in which Maimonides describes the doctrine of the *mutakallimūn*:

They say [...] the agent (*f#’il*) sometimes precedes his act (*qad yataqaddamu fi’lahū*). Indeed, they only form the idea (*bal l# yata{awwarūna ma’n#*) of the agent as an agent as preceding his act (*ill# an yataqaddama fi’lahū*) (*Dal#at* I, 88v; cf. above § 2).

Maimonides stops after the first sentence and seems to correct himself. However, what on first sight seems to be a correction is a rhetorical trick to catch the attention of the reader and to focus the main point of the issue.

The dispute between the elder Mu’tazilite scholars on one hand and the Sunnī scholars al-Ash’arī and al-M#turīdī on the other hand whether man might be called ‘agent/enactor’ (*f#’il*) was already based on Aristotle’s distinction between two-sided potency/power/faculty (Arab. *quwwa/qudra/isti}{#’a*) and one-sided potency. The *mutakallimūn* agreed that the term ‘agent’ (*f#’il*) can only be used in case of two-sided power/faculty, but they disagreed on the question whether man’s power/faculty is two-sided. The Mu’tazilite scholars hold that man is an agent in so far as he has a two-sided power/faculty to two contraries (*qudra/isti}{#’a ’al# *iddayn*) which he can determine by an act of will (*ir#da*) and a choice (*ikhtiy#r*) of one of the possible contraries by which he brings about the change from a possible to an actual action. That is to say, they called man ‘agent’ in the sense of the ‘enactor’ of his actions. Al-Ash’arī, however, held that man’s power/faculty is only one-sided, that is to say, it does not exist prior to the action but only simultaneous “together with the action for the [particular] action (*ma’a l-fi’l li-l-fi’l*)” (al-Ash’arī, *Luma’* 56,17, §128). Man’s faculty, or more precisely man’s particular faculties for his particular actions, are only necessary conditions but not sufficient grounds for his particular actions. This is why al-Ash’arī refused to call man ‘agent/enactor’ (*f#’il*), but called him ‘acquirer’ (*muktasib*) of his actions (*Luma’* 39,10-20, §§87-8). Al-M#turīdī agreed

with the Mu'tazilite position in so far as he held that man's power/faculty is two-sided and thus preceding the action and that man can determine his faculty for a particular action by an act of will and a choice. Therefore al-M#turīdī held that not only God but also that man might be called 'agent'. However, he agreed with al-Ash'arī that at the moment man intends a particular action and chooses it, his power/faculty for this particular action still is only a necessary condition for the actuality of the action. Although man's intention, his act of will and his choice is in accordance with his particular act, man's power/faculty does not bring about this action. Therefore man cannot be called 'agent' in the sense of bringing about and enacting the action, but only in the sense of a voluntary acquisition (*kasb*) of the action, whereas God is called 'agent' in the sense of enacting, that is to say, in the sense of creating man's actions (al-M#turīdī, *al-Tawīd* 364,3f.).¹¹

A key-element of al-Ash'arī's doctrine as well as that of al-M#turīdī is the assumption that the 'agent' (*f#il*) in the sense of the 'enactor' is the one who "is enacting (*f#il*) the action (*fi'l*) as it really is" ('*al# 1aqīqatihī*) (*Luma'* 39,15-8) and that this is only possible if the intention (*qa{d}*) and the act of will (*ir#da/mashī'a*) of the agent is in conformity with the reality (*1aqīqa*) of the action. Since it is man's experience to intend something he holds to be good but which is not really good but bad, man's knowledge and intention is not in accordance with the reality of the action. Therefore the reality of the action does not depend on man's will and thus man cannot be called the 'agent/enactor' of the 'act'. The 'agent' in the sense of the 'enactor' rather is the one who brings [the action] in existence (*mu'ldith*) as it really is by his intention and his act of will, namely God (*Luma'* 38,9-19, §85; cf. al-M#turīdī, *al-Tawīd* 366,1 — 367,1).

This is what al-Ghaz#lī had in mind, when he criticized the Neoplatonic philosophers for calling God 'cause' instead of 'agent/enactor'. Here is his statement of the argument:

'Agent/enactor' (*f#il*) is an expression [referring] to one from whom the act proceeds, together (*ma'a*) with the will (*ir#da*) to act by way of choice (*ikhtiy#r*) and the knowledge ('*ilm*) of what is willed. But, according to you [Neoplatonic philosophers], the world follows from God as a necessary consequence (*yalzamu luzūman #arūriyyan*) as the caused/effect from the cause (*ka-l-ma'lūl min al-'illa*), inconceivable for God to prevent, in the way the shadow is the necessary consequence (*luzūm*) of the individual and the light [the necessary consequence] of the sun. And this does not pertain to action in anything. Indeed, whoever says the lamp enacts (*yaf'alu*) the light and the individual enacts the shadow has ventured excessively into metaphor and stretched it beyond [its] bound... The agent, however, is not called 'a making agent' (*f#ilan {#ni'an}*) by simply being a ground (*sabab*), but by being a ground in a special way — namely, by way of [an act of] will (*ir#da*) and choice (*ikhtiy#r*) — so that if one were to say, "The wall is not an agent; the stone is not an agent; the inanimate is not an agent, action being confined to animals," this would not be denied and the statement would not be false. But [according to the philosophers] the stone has an action — namely falling due to heaviness and an inclination toward [the earth's] center — just as fire has an action, which is heating, and the wall has an action — namely the inclination toward the center and the occurrence of the shadow — for all [these latter things] proceed from it. And this is absurd (*al-Tah#fut* 89,22 — 90,14; transl. Marmura 56).¹²

Al-Ghaz#lī denies that a one-sided nature (*jab'*) might be called 'enacting' (*f#il*) and declares that only who has a two-sided power/faculty determined by an act of will and a choice can be called 'agent/enacting'. As already for al-Ash'arī and al-M#turīdī also for al-Ghaz#lī 'enacting' by an act of will and choice presupposes knowledge ('*ilm*) (cf. al-Ghaz#lī, *al-Iqti{#d* 97,2; transl. Marmura 312) since will is an intentional act. Therefore sentences as "He acted by choice" and "He willed, knowing what he willed" are repetitious. This repetition intends only to remove the possibility of taking the expressions 'he acted' and 'he willed' metaphorically. Thus 'acting by choice' and 'willing by knowing what is willed' are not to be taken as a specification (*takh{īf}*) of a special kind of acting and a special kind of willing to distinguish these kinds from other kinds of acting and willing, namely acting without choice

by nature and willing without knowing what is willed (*al-Tah#fut* 91,13 - 92,5; transl. Marmura 57f.).

Hence, regarding God's enacting the world the main point of al-Ghaz#lī's opposition to the Neoplatonic philosophers is their denial of God's will. Here is al-Ghaz#lī's exposition of the argument of the Neoplatonic philosophers:

Even though we did not say that the First wills origination (*i1d#h*) nor that the whole [world] is temporally originated (*1#dith 1udūthan*), we [nonetheless] say that [the world] is His act (*fi'l*) and has come to existence from Him, except that He continues to have the attribute of the agents (*{ifat al-f#'ilīn*) and, hence, is ever enacting (*fa-lam yazal f#'ilan*)... (*al-Tah#fut* 158,9-11; transl. Marmura 128).

This is opposed to the doctrine of the *mutakallimūn* as reported by Maimonides in so far as they “only form the idea (*ma'n#*) of the agent as an agent as preceding his act (*ill# an yataqaddama fi'lahū*).” Here is al-Ghaz#lī's answer to the Neoplatonists:

The first is that [according to the philosophers] action divides into two [kinds]: voluntary (*ir#dī*), like the action of the animal and of man/human, and natural (*{abī'ī*), like the action of the sun in shedding light, fire in heating, and water in cooling. Knowledge of the act is only necessary in the voluntary act, as in the human arts. As regards natural action, [the answer is,] “No.” [Now,] according to you [Neoplatonic philosophers], God enacted the world by way of following (*luzūm*) from His essence (*dh#*) by nature (*{ab'}*) and necessity (*i*}ir#r*), not by way of will (*ir#da*) and choice (*ikhtiy#r*). Indeed, [according to you Neoplatonic philosophers] the whole [of the world] follows from His essence as the light follows from the sun. And just as the sun has no power (*qudra*) to stop light and fire [has no power] to stop heating, the First has no power to stop his acts... (*al-Tah#fut* 158,16-22; transl. Marmura 128).¹³

Like the early *mutakallimūn*, al-Ghaz#lī distinguishes between two-sided potency, signified as ‘the power/faculty to two contraries’ (*al-qudra 'al# l-*iddayn*) (cf. *al-Tah#fut* 57,9f.; transl. Marmura 22), and one-sided potency, signified as ‘nature’ (cf. Schöck 2004). What proceeds from essence, proceeds by nature and therefore always proceeds from essence and exists together with the essence, as the light from the sun and the heating from the fire. Therefore it is

false to say that it [viz. God's act (*fi'l*)] proceeds from his essence (*dh#*). If it were like that, it were eternal (*qadīm*) together (*ma'a*) with the essence (al-Ghaz#lī, *al-Iqti{#d* 81,4).

In contrast to ‘nature’, ‘power/faculty’ (*qudra*) is two-sided. Therefore

the one who is powerful/capable (*q#dir*) is the one who acts if he wills and does not act if he wills (al-Ghaz#lī, *al-Maq{ad* 145,3; 176,6f.).

Since only the two-sided ‘power/faculty’ is rational and therefore presupposes knowledge whereas ‘nature’ is irrational (cf. Aristotle, *Met.* IX, 2, 1046b 4-9; *De int.* 13, 22b 36 - 23a 3) al-Ghaz#lī goes on to explain:

The second way [of answering the philosophers] is to concede that the proceeding of something from the agent also requires knowledge of what proceeds. [Now,] according to them [viz. the Neoplatonic philosophers], the act (*fi'l*) of God is one — namely, the first caused/effect (*ma'lūl*), which is a simple intellect. [From this follows] that He must know only it... (*al-Tah#fut* 159,13-5; transl. Marmura 128f.).

To sum up, ‘power/faculty’ (*qudra*) according to the *mutakallimūn* is power to possible — not yet actual — contraries and therefore presupposes the power to act by an act of will and a choice which presupposes knowledge. By the act of will and a choice of one of the possible alternatives, the two-sided power/faculty becomes determined (*mutaqaddir*) (cf. al-Ghaz#lī, *Maq{ad* 145,2) to this formerly possible, now actual alternative. Since at the moment of an act of will and a choice one of the possible alternatives is determined and has become actual,

the existence of its contrary is impossible, because the two contraries cannot exist together at one and the same time. Consequently the powerful (*q#dir*) agent/enactor (*f#il*) must precede his act. Otherwise one of the possible alternatives would be actual together with him and he would not have had the possibility and the power to enact its contrary.

On the other side, it follows from the priority of the ‘agent’ to the ‘events’ (*law#dith*, sing. *l#dith*) brought to existence by his act of will that every existent except himself exists contingently, that is to say, necessary in so far as it exists by an act of will of its enactor, but not necessary by itself. Thus by claiming the precedence of the agent before his act God’s will is established as the only reason of every existent other than God himself, that it is and what it is.

al-Ghaz#lī’s argumentation is based on the logical relation of condition and consequence. From the assumption — *a priori* and by the revelation of the Qur’#n — that God is powerful follows that he is acting by an act of will and a choice and from this follows that he is knowing. It must be concluded therefore, that God himself, that is to say, the divine essence precedes its act (cf. Marmura 2004, 141f.).

Maimonides does not challenge this argument. He rather calls into question the assumption that the expressions ‘agent’ and ‘cause’ cannot be used in the same ways. By this Maimonides neglects the difference between a derived name (*ism mushtaqq*) and a primitive name (*ism*). This topic falls in the realm of semantics. Al-Ghaz#lī treats it in the opening of his *The Loftiest Intention Concerning the Explanation of the Meanings of God’s Most Beautiful Names* (*al-Maq#ad al-asm# fī shar# ma’#nī asm#’ All#h al-1usn#*). Before we turn to this, I want to give a brief outline on the grammatical, theological and logical background of al-Ghaz#lī’s explanation of what is ‘derived from a verb’.

4. What is ‘derived from a verb’ (*mushtaqq min fi’l*) in early Arabic grammar

The term ‘agent’ (*f#il*) is formed from the radical letters of the trilateral Arabic verb. In Arabic grammar it is used paradigmatically for the pattern and form of a part of speech which signifies an action and an agent. Its function in speech corresponds to the English participle, but in contrast to the English the Arabic does not distinguish between the continuous and progressive form on one side and the noun on the other side, that is to say, between “acting/doing/making” and “agent/doer/maker” or, for example, not between “writing” (*k#tib*) and “writer” (*k#tib*) (cf. Wright 1981, I, 131, §§ 229-30).

Some early Kūfian grammarians hold that the *f#il* is a verb, distinguishing that which they called a continuous verb (*fi’l d#im*) and a verb of the state (*fi’l al-1#l*) (Troupeau 1993, 914a; Versteegh 1995, 66). In opposition to this view, the Ba{rian grammarian Sībawayh (d. 180/796) and the Kūfian grammarian al-Farr# (d. 207/822) claimed that it is not a verb, but the name of the agent/*nomen agentis* (*ism al-f#il*). This name is derived from a verb (*ism mushtaqq min fi’l*) (Kinberg 1996, 359-60; cf. Sībawayh, *Kit#b* II, 224-30, § 432; Mosel 1975, I, 127-8), and verbs, in the words of Sībawayh, “are actions” (*hiya a’m#l*)” (*Kit#b* II, 224,14, § 432; cf. Carter 2004, 74).

Sībawayh makes use of the term ‘*f#il*’ in different ways. First *f#il* stands for the form ‘*f#il*’, and secondly it stands for the agent (*al-f#il*) and subject of an action, which is “concealed” (*mu*mar*) in the *f#il* [-form] (*al-Kit#b* I, 80,3, § 40). Hence, Sībawayh does not draw a clear distinction between signifier and signified, that is to say, between word [-form] and thing, namely between the *f#il* [-form] and the agent (*f#il*) (cf. Mosel 1975, I, 246f.). Also the meaning conveyed by the *f#il*-form is ambiguous in several ways.

In §§ 32, 37 and 39 of his *Kit#b* Sībawayh tries to find grammatical rules to decide in which

cases the name of the agent (*ism al-f#’il*) stands for an imperfect action and in which cases it stands for a finished, perfect action (cf. Mosel 1975, I, 128-35). He claims that the name of the agent without the article only stands for an imperfect action which either takes place at the time of the sentence or in the future. For example: “You say ‘this one is hitting/a hitter (*h#dh# *#ribun*)...’ in the sense of ‘this one hits’ (*h#dh# ya*ribu*), and he acts at the time of your message (*wa-huwa ya’malu fī 1#li 1adīthika*).” But “this one is hitting/a hitter (*h#dh# *#ribun*)” may also stand in the sense of “this one will hit (*h#dh# saya*ribu*)” (I, 54,8-10, § 32).

However, a little later in his *Kit#b* Sībawayh explains that in the sentence “this one is the hitter of ‘Abdall#h and his brother (*h#dh# *#ribu ‘Abdill#hi wa-akhīhi*)” the name of the agent stands for the perfect and finished action (*al-fi’la qad waqa’a wa-nqa’a*) (I, 73,6-10, § 37).

Also the name of the agent with the article can stand for an imperfect action and for a perfect action. Sībawayh explains, that “if you say ‘this hitter’ (*h#dh# l-#ribu*), then you determine him in the sense of ‘the one who hits/is hitting’ (*alladhī ya*ribu*)” (I, 54,10, § 32). On the other hand the sentence “this is the hitter of Zayd” (*h#dh# l-#ribu Zaydan*) has the meaning of “this is the one who hit Zayd (*h#dh# lladhī *araba Zaydan*)” (I, 77,8, § 39).

It would appear, then, that there is no rule with regard to the use of the name of the agent with or without the article in relation to either an imperfect or a perfect action (cf. Mosel 1975, I, 134-5).

In §§ 39-41 of his *al-Kit#b* (I, 77-88) Sībawayh accounts for the difference between “[the name of] the agent” (*ism al-f#’il*) and “the description/attribute which is similar to [the name of] the agent” (*al-ifa al-mushabbaha bi-l-f#’il*). Both are derived from verbs, as, for example, the name of the agent (*ism al-f#’il*) ‘*q#il*’ from the verb *qatala* and the description/attribute (*ifa*) ‘*lasan*’ from the verb *lasana*. However, since only [the names of] agents are derived from verbs which are actions (*hiya a’m#l*), only the *f#’il* [-form] can indicate an imperfect or a perfect action, while the description/attribute (*ifa*) which is similar to it can only stand as a description which is not a state of becoming, but is already a perfect state of being (I, 82,18f., § 41; cf. Mosel 1975, I, 128-35). For example ‘*q#il*’ can be used in the sense of ‘murdering’ and in the sense of ‘murderer’, and ‘*k#ib*’ can be used in the sense of ‘writing’ and in the sense of ‘writer’, but ‘beautiful’ (*lasan*) can only be used in the sense of ‘being [already] beautiful’, and ‘ill’ (*marī**) can only be used in the sense of ‘being [already] ill’. Whereas ‘murdering/murderer’ and ‘writing/writer’ stand for an action (*fi’l*) and its agent (*f#’il*), ‘beautiful’ and ‘ill’ stand for a description/quality (*ifa*) and the one described/qualified (*mawfūf*).

In §§ 432-6 of his *al-Kit#b* (II, 224-39) Sībawayh tries to assign the verbs and their corresponding names (*asm#*) of the agents and descriptions/qualities (*if#*) to semantical classes and grammatical forms (cf. Mosel 1975, I, 138-45). According to him, the first and second class are actions, the other classes are descriptions/qualities.

The first class (§ 432) are “the verbs which are actions (*hiya a’m#l*) which pass from you to someone [or: something] else” (II, 224,14) — in other words transitive actions. The second class (§ 432) are “the actions which do not pass to an accusative [object] (*mansūb*)” (II, 226,9) — in other words intransitive actions. However, in some cases from these verbs one may also form descriptions, namely if one does not want to indicate an action (II, 225,9-11).

The third, fourth and fifth class (§§ 433-4) are verbs and descriptions which signify an affliction (*bal#*) of the heart (*qalb, fu’#d*), body (*badan*) or soul (*nafs*) (II, 230, 11-3; 232,3; 233,11), as disease, hunger, thirst, fear, grief, etc, as well as their contraries, and colours.

The sixth class (§ 436) are “the qualities which are in the things” (*al-khi{# allatī takūnu fī l-*

ashy#'), as beautiful and ugly, tall and short, many and little, strong and weak, reasonable and ignorant, etc.

It is obvious that these semantical classes represent logical rather than grammatical categories. To sum up, there are three kinds of dissent or ambiguity in regard to the term '*f#il*' in Arabic grammar:

- [1.] It can stand for the word or the thing.
- [2.] It is either a verb or a name.
- [3.] It can stand for an agent and his imperfect action or for an agent and his finished, perfect action.

The term *{ifa* is as ambiguous as the term *f#il*. It can not only stand for "the description which is similar to [the name of] the agent", but also for the "quality" itself. And it can stand for the function of describing/qualifying a name (Mosel 1975, I, 141-5). When this function of describing/qualifying is meant, the Arab grammarians and the Muslim dialectical theologians rather use the verbal noun "describing/description" (*waff*) to signify the act of describing. The term '*{ifa*' on the other hand tended to be used to signify the word and the thing itself by (*bi-*) which the name (*ism*) is described/qualified (*maw{ūf*), namely an attribute and a descriptive predicate as well as the affliction (*bal#*), the colour or the quality (*kha{la*) which is supposed to be in the described thing (cf. Frank 2004).

5. The controversies on the derived name 'wicked' (*f#siq*) in early Muslim dialectical theology (*kal#m*)

According to Sibawayh the verb "to deny someone or something, not to believe" (*kafara*) with its noun 'unbelief/unbelieving' (*kufir*) belongs to the transitive actions (*al-Kit#b* § 432, II, 226,1). From this it follows that 'unbelief/unbelieving' is an action, and 'unbeliever' (*k#fir*) is the name of an agent who denies someone or something. The verb "to depart from [an obligation or law], to act wickedly" (*fasaqa*) with its nouns *fusūq* and *fisq* belongs to the intransitive actions (§ 432, II, 226,20). 'Wickedness/acting wickedly' (*fisq*) is an action and 'wicked' (*f#siq*) is the name of an agent which can stand for an imperfect or a perfect action and the agent, namely for an agent who is acting wickedly or who has acted wickedly.

This interpretation was the basis of the doctrine of the Ba{rian theologian W#{il b. 'A}# (d. 131/748-9). He was not only considered the founder of the theological school of the Mu'tazila (van Ess 1992, 234-5), but also the founder of dialectical theology (*kal#m*) ('Abdaljabb#r, *Fa*l* 234,14). His dogma of the 'wicked' was subject of controversies over several centuries.

This dissent over the use of the name of an agent arose when W#{il interchanged the categories of the 'described' (*maw{ūf*) which is the 'name' (*ism*) and its 'description' (*waff*) by an attribute (*{ifa*). W#{il argued that the great sinner (*{#1ib al-kabīra*) from the Muslim community who was called by four different Muslim dogmatical parties 'wicked polytheist' (*mushrik f#siq*), 'wicked ungrateful' (*k#fir ni'ma f#siq*), 'wicked hypocrite' (*mun#fiq f#siq*) or 'wicked believer' (*mu'min f#siq*) should be named 'wicked' (*f#siq*). Grammatically all four denominations the different parties used to signify the great sinner consist [1.] of a derived name (*ism mushtaqq*) which stands in function of the described (*maw{ūf*) and [2.] the description (*waff*) by the attribute (*{ifa*) 'wicked' (*f#siq*). In Arabic the description/attribute follows the name, since it has the function of describing (*waff*) the name (cf. Mosel 1975, I, 325-7). W#{il, however, argued that because all parties agree on 'wicked' (*li-ttif#q... 'alayhi*), this is the right 'naming' (*tasmiya*) of the great sinner instead of the different denominations 'polytheist', 'ungrateful', 'hypocrite', 'believer'. This resembles the Aristotelian method of finding the 'common' (*koinon*) in different things and setting it over the different things as a genus. But grammatically W#{il interchanged and converted the

described (*maw{ūf*) which is a name and its description (*wa{ff}{ifa*).¹⁴

By introducing ‘wicked’ (*f{#siq*) as the denomination (*tasmiya*) of the great sinner W#{il tried to solve the question of the ‘status’ (*manzila*) of those who transgressed and departed from religious obligations and laws in Muslim society. W#{il claimed that ‘wicked’ is a third status between ‘believer’ and ‘unbeliever’. The wicked in Muslim society should not be treated as an unbeliever who cannot be member of the Muslim community. In the afterlife, however, he would be in hell like the unbeliever (cf. van Ess 1992, 260-7).

Thus, it is only in this world that ‘wicked’ is a third status, while in the afterlife there are only two statuses, namely ‘inhabitant of paradise’ and ‘inhabitant of hell’. Being a believer and a future inhabitant of hell is impossible. And being an unbeliever or wicked and a future inhabitant of paradise is impossible. Therefore being an unbeliever and wicked is possible. But being a believer and wicked is impossible. This doctrine provided the starting point of a long dispute among dialectical theologians. Logically and grammatically it was linked to two major problems:

First, if believing (*īm{#n*) and unbelieving (*kuf{r*) are contradictory, because ‘belief/believing’ is “to ascribe truth” (*ta{dīq*) [to someone] and “to confirm” (*iqr{#r*), and ‘unbelief/unbelieving’ is “to ascribe falsehood” (*takdhīb*) [to someone] and “to deny” (*ink{#r*), then ‘believing’ neither consists of parts nor can it increase or decrease. How then ‘wicked’ can be a middle or a third between ‘believer’ and ‘unbeliever’? There is no middle between two contradictories (Aristotle, *Met.* X, 4, 1055b 2). In contrast to the former interpretation of ‘belief/believing’, which was held by Abū Zanīfa (d. 150/767) and his followers (cf. Schöck 2006, 104-11), W#{il and his colleagues held that ‘believing’ is a sum of actions, and if this sum of actions is incomplete, then ‘believing’ is abolished. Therefore both ‘unbelieving’ and ‘wickedness/wicked acting’ (*fisq/fusūq*) must be understood as a privation of ‘believing’. However, in this case it is impossible that ‘unbelieving’ and ‘wicked acting’ are both the same kind of privation of ‘believing’ (cf. *Met.* X, 4, 1055b 21-23). And from this it follows that it is impossible that ‘unbelieving’ and ‘wicked doing’ are both contraries of ‘believing’ (cf. *Met.* X, 5, 1056a 11).

This brings us to the second problem. According to W#{il, ‘unbelieving/unbeliever’ (*k{#fir*) and ‘wickedly acting/wicked’ (*f{#siq*) are both names of agents which are derived from verbs which ‘are’ or signify actions (cf. above § 4). But, if an ‘unbelieving/unbeliever’ is someone who denies someone or something, namely God, the prophets and their messages, then he remains an ‘unbelieving/unbeliever’ only as long as he denies them. Because if he stops ‘denying’, he stops ‘unbelieving’ and therefore stops being an ‘unbeliever’ (cf. Fakhraddīn al-R#zī, *Tafsīr, K. al-awwal fī qawlihī a‘ūdhu bi-ll{#h... b{#b al-kh{#mis*, I, 47,19-21). On the other side, according to W#{il’s doctrine someone ‘wickedly acting/wicked’ stays ‘wicked’ after he has finished his wicked action.

The Mu‘tazila never succeeded in finding a satisfactory solution to these problems. However, the oppositional arguments made them rethink and modify their doctrine. During this process the meaning and use of the derived name was further clarified and extended.¹⁵

W#{il’s early opponents focused on the restriction (*taqyīd*) of ‘wicked acting’ (*fisq*) according to the categorical questions ‘at which time?’ and ‘in what respect?’ They argued that ‘wicked acting’ can only be a privation of believing at some time and in some respect, while ‘unbelieving’ is an absolute (*mu}laq*), unrestricted and complete privation of ‘believing’.

The Ba{rian theologian Abū Shamir was probably a younger contemporary of Sībawayh (cf. van Ess 1992, 174). He held the following opinion:

I do not say ‘absolute wicked’ (*f{#siq mu}laq*) in regard to the wicked from the Muslim community (*al-f{#siq al-millī*), without me restricting (*dūna an uqayyida*) and saying: ‘wicked in regard to such a thing’ (*f{#siq fī kadh{#*) (al-Ash‘arī, *al-M{#q{#l{#* 134,12f.).

Similar to this, from an anonymous opponent of the Mu'tazilite dogma is reported as saying:

I do not say in an absolute sense 'wicked' (*f#siq 'al# l-i}l#q*) to someone who commits great sins without saying: 'wicked in regard to such a thing' (*f#siq fī kadh#*) (al-Ash'arī, *al-Maq#/#* 141,12f.).

While Abū Shamir restricted 'wicked' (*f#siq*) to a particular action, Abū Mu'adh at-Tūmanī restricted 'wicked acting' (*fisq*) to a particular time. He also was probably a contemporary of the Ba'rian grammarian Sībawayh and perhaps also lived in Ba'ra (cf. van Ess 1992, 735). It is reported that he maintained:

... Every act of obedience (*}#a*) in regard to which the Muslims do not agree on the unbelief of the one who omits it (*al-t#rik*) is an ordinance of belief (*sharī'a min shar#'i' al-īm#n*). If it is a duty, then he [who leaves it undone] will be described/qualified with 'wicked acting' (*yū{afu bi-l-fisq*), and one says of him 'he acts/acted wickedly' (*innahū fasaqa*), but one does not name him with 'wickedness/wicked acting' (*l#yusamm# bi-l-fisq*) and one does not say of him 'wicked' (*wa-l# yuq#lu f#siq*). The great sins do not exclude someone from believing, if they are not unbelieving... (al-Ash'arī, *al-Maq#/#* 139,14-140,3).

The verb 'he acts/acted wickedly' (*fasaqa*) is *verbum finitum* which signifies a finished act, not a particular time (cf. Wright 1974, I, 51). A finished act may be an act completed at some past time, an act which has been already completed and remains in a state of completion, an act which is just completed or an act, the occurrence of which is so certain, that it may be described as having already taken place (Wright 1981, II, 1f.). But in any case the verb refers to a time and to that extent is restricted (*muqayyad*).

According to Abū Mu'adh the name of an agent as 'wicked' (*f#siq*) cannot stand in a restricted sense. If 'wicked acting' (*fisq*) is not meant absolutely (*mu}laq*), that is to say, unrestricted to a particular action which takes place at a particular time, then one must use a verb which signifies the agent together with the time. Abū Mu'adh uses the verb "to describe/to qualify" (*wa{afa*) to indicate the meaning which is restricted to a time, and the verb "to name" (*samm#*) to indicate the meaning which is not restricted to a time.

Another anonymous opponent of the Mu'tazilite doctrine argued:

One does not name (*l#yusamm#*) 'the wicked' (*al-f#siq*) among the people who pray in the direction of the Ka'ba (*ahl al-qibla*) as 'wicked' (*f#siqan*) after his [wicked] action has come to an end (al-Ash'arī, *al-Maq#/#* 141,10f.).

That is to say, the name of an agent 'wicked' cannot stand for a past action which is fully completed and does not stay in a state of completion.

By these arguments the Mu'tazilite scholars were forced onto the defensive. To avoid refutation they used one of the oldest dialectical tactics. They distinguished two different aspects of the matter in dispute. In regard to one of the aspects they admitted that their antagonists were right. In regard of the other aspect they contradicted them by turning the tables.

The Mu'tazilite 'Abb#d b. Sulaym#n (d. after 260/874) claimed:

One says to him [viz. the wicked (*al-f#siq*)], 'he believes/believed [in God]', and one does not say to him 'believer/believing' (*yuq#lu lahū [innahū] #mana [bi-ll#h] wa-l# yuq#lu lahū mu'min*) (al-Ash'arī, *al-Maq#/#* 274,9f.).

ʿmana is *verbum finitum* which in the Qur'#n often stands as antecedent of a conditional sentence, for example in verse 2,62: "who [ever] believes in God... and does what is good..." (*man #mana bi-ll#h... wa-'amila {#i1an...*). 'Abb#d distinguished between 'to believe in [God]' (*#mana bi-*), and 'to believe' in the sense of 'to obey [God]' (*#mana li-*), namely to do what is good. The first kind of belief/believing corresponds to the above-cited definition of Abū Zanīfa and his followers that 'belief/believing' is "to ascribe truth" (*ta{dīq*). The second

kind of ‘belief/believing’ corresponds to the old Mu‘tazilite teaching that ‘belief/believing’ means to act in accordance with religious obligations and duties, that is, to obey (*a}#’a*). ‘Abb#d maintained that he who believes only in the first sense without also believing in the second sense does not believe in the full sense, but only in a restricted sense. Therefore one must use the verb ‘he believes/believed’, which restricts his believing to a particular time. This meant that someone does not believe while he acts wickedly. His belief is restricted to the particular time he does not act wickedly. However, this is a weak counterargument, since it is possible to ascribe truth to an obligation while acting against it.

Abū ‘Alī al-Jubb#’ī (d. 303/916) followed ‘Abb#d and maintained:

One says ‘he believes/believed’ (*#mana*) [in the sense] of the descriptions of the language (*aw{#f al-lugha*), and one says ‘believer/believing’ [in the sense] of the names of the language (*asm#’ al-lugha*) (al-Ash‘arī, *al-Maq#/# 274,12f.*).

Al-Jubb#’ī used the term ‘description’ (*wa{f*, pl. *aw{#f*) for the meaning which is restricted to a particular time, and he used the term ‘name’ (*ism*, pl. *asm#’*) for the meaning which is not restricted to a particular time. But later he changed his categories and, instead of two different grammatical categories, distinguished between a grammatical category and a socio-religious category:

He maintained, that there are two kinds of names: names of the language (*asm#’ al-lugha*) and names of the religion (*asm#’ al-dīn*). The names of language, which are derived from actions, come to an end together with the end of the actions. And by the names of religion man/human is named (*yusamm#*) after his action has come to an end and while he is in the state (*1#la*) of [doing] his action. The wicked from the Muslim community (*al-f#siq al-millī*) is a believer/believing [in the sense] of the names of language. The name [‘believer/believing’] comes to an end together with the end of his act of obedience (*fī’lihī li-l-īm#n*). And he is not named by ‘belief/believing’ (*īm#n*) [in the sense] of the names of religion (al-Ash‘arī, *al-Maq#/# 269,9-14*).

‘The wicked from the Muslim community’ (*al-f#siq al-millī*) is named ‘wicked’ in the socio-religious sense, that is to say in the sense of ‘the name of religion’. This name is not restricted to a particular action and time. In contrast to this ‘the name of language’ is restricted to the time of a particular action.

Finally, the Mu‘tazilite scholar Abū l-Q#sim al-Ka‘bī (d. 319/931) distinguished between [1.] a name which is derived from an action and which is restricted to a particular action, and [2.] an absolute (*mu}laq*), unrestricted name of the agent which is derived from an action and which has the function of a sign (*sima*) to distinguish different classes of people:

Our word ‘believer/believing’ is not only derived from the verb/action [‘to believe in [God]’ (*#mana bi-*), and ‘to believe’ in the sense of ‘to obey [God]’ and to be submissive to him (*#mana li-*), since not everyone who ascribes truth to someone (*{addaqa a1adan*) and obeys him (*a}#’ahū*) and is submissive to him (*kha*a’a lahū*) is named with it in the sense of an absolute name (*ism mu}laq*). And it also is not only a sign (*sima*)¹⁶, since, if it were [only] a sign, it would be possible to name with it someone who is not so [viz. who does not believe in God, does not obey him and is not submissive to him]; similarly if one names the beauty (*al-Iasn#’*) ‘ugly’ (*qabīla*). Because this is not the case, it has been settled that it is a name which is derived from an action and a praise in respect to religion (*mad1 fī l-dīn*) and a sign to distinguish [between ‘believer’, ‘unbeliever’ and ‘wicked’] (al-M#turīdī, *al-Tawīd 551,12-5*).

To sum up, according to the Muslim dialectical theologians up to the time of al-F#r#bī the names of agents (*asm#’ al-f#’ilīn*) ‘believer’ and ‘wicked’ can stand for three different meanings:

- [1.] They can stand in a restricted meaning, namely in regard to a particular action.
- [2.] They can stand for the bearer of the name (*{#lib al-ism*) while he is in the state (*1#la*) of [doing] his action.

[3.] They can stand restricted to a particular action and time *and* as a sign to distinguish the bearer of the name (*{#1ib al-ism*) from other subjects. In this third sense the derived name (*ism mushtaqq*) is used as a class name which is linked to some action (*fi'l*) or quality (*{ifa*) of the bearer of the name, but not linked to the time at which he performs the action and not linked to the time at which he is described/qualified (*wu{ifa*) by the quality. To this extent the derived name can be used in an absolute sense (*'al# l-i}l#q*) which is not restricted to a particular action and time, in other words as a paronym like brave (*shuj#'*) and grammarian (*fa{ī1*) (Aristotle, *Cat.* 1, 1a 12-15; Ed. Badawī I, 3; Ed. Jabre I, 25).

6. The different meanings of the 'derived' (*mushtaqq*) according to al-F#r#bī

The two meanings mentioned above — namely what al-Jubb#ī called 'the names of the language' (*asm# al-lughā*) and 'the names of the religion' (*asm# al-dīn*) and what al-Ka'bī described as first 'the name which is only derived from a verb/action' and second 'the name which is derived from a verb/action *and* is a sign to distinguish' — correspond to two different meanings of the derived name which al-F#r#bī explains in his commentaries on Aristotle's *On interpretation*.

Al-F#r#bī (d. 339/950) knows very well the old dispute on the question, whether the '*f#il*' is a verb (*kalima*) or a derived name (*ism mushtaqq*). He reports that many of the ancients (*qudam#'*) held that it is a verb (*Fu{ū1 70,5-9*). From this we can conclude that at al-F#r#bī's time Sībawayh's opinion¹⁷ had been generally accepted. Al-F#r#bī also follows Sībawayh. However, he clearly identifies Sībawayh's "[name of] the agent" (*[ism] al-f#il*) and Sībawayh's "description which is similar to [the name of] the agent" (*al-{ifa al-mushabbaha bi-l-f#il*)¹⁸ with Aristotle's 'derived name', since he gives the examples: the white (*al-abya**), the black (*al-aswad*), the hitter (*al-#rib*), the moving (*al-muta1arrik*), the brave (*al-shuj#'*) and the eloquent/the grammarian (*al-fa{ī1*) (*Fu{ū1 69ult.-70,1*; cf. *'Ib#ra 135,10*). These examples represent Sībawayh's categories of 'agent' and 'description' together with Aristotle's examples for paronyms. According to Sībawayh 'the white' and 'the black' are descriptions which are derived from colours, 'the hitter' is a name of the agent, 'the moving' is a description which is derived from an affliction (*bal#'*) of the body and 'the brave' and 'the eloquent' are "qualities (*khi{#l*) which are in the things" — and 'brave' (*shuj#'*) and 'eloquent/grammarian' (*fa{ī1*) are Is1#q b. 2unayn's (d. 298/910) translations of Aristotle's examples (*Cat.* 1, 1a 12-15; Ed. Badawī I, 3; Ed. Jabre I, 25).¹⁹

The difficulty al-F#r#bī deals with is that there exists no grammatical pattern and form (*shakl*) to distinguish between derived names and descriptions which are restricted to particular actions on the one hand and a potency (*quwwa*) and specific difference (*fa{l*) of a subject on the other hand. In language both are formed by derivation (*ishtiqaq*). Al-F#r#bī solves this problem by the following explanation:

For example the name/noun 'standing' (*qiy#m*) signifies the essence 'standing' as [an] abstracted [entity] (*dh# al-qiy#m mujarradan*) without the thing in which is 'standing'. Then it is changed by replacing the order of some of its consonants and vowels, so that its form (*shakl*) is replaced. So from [the name/noun] 'standing' becomes the word '[the one who is] standing' (*q#im*). It signifies that the [essence] 'standing' is connected (*muqtarin*) with a subject not articulated (*mau#ū' lam yu{arra1*) (*al-'Ib#ra 143,10-13*).

Al-F#r#bī deviates here from the Arab grammarians in so far as he claims that the name 'standing' (*qiy#m*) signifies the abstracted (*mujarrad*) 'self/essence' (*dh#*) 'standing', that is to say the quality (*kayfiyya*)/cf. Greek *poiotēs* 'standing' itself. Thus, the name of the agent

(*ism al-f#’il*) ‘standing’ (*q#’im*) is not — as the Arab grammarians say — derived from the verb (*fi’l*) which signifies a time of being standing, nor from the infinitive and verbal noun (*ma{dar}*)²⁰ ‘to stand/standing’ (*qiy#m*) which signifies the happening (*lu{ūl}*) of ‘to stand/standing’ and therefore also signifies ‘standing’ temporally. It is rather derived from the name/noun ‘standing’ in the sense of an abstracted atemporal essence and quality. Only on this basis al-F#r#bī can claim that for example the name of an agent ‘*n#}iq*’ can stand both for [1.] someone who is ‘rational’, that is to say who has the potency/faculty of ‘speech/reason’ in the sense of rationality, for [2.a] someone actually ‘reasonably thinking’ and for [2.b] the ‘speaker/speaking/talking’, that is to say, someone actually ‘speaking/talking’. This is a new way of understanding which is foreign to the understanding of the Arab grammarians. It is al-F#r#bī’s aim to reconcile the new logical understanding with the old grammatical understanding. He argues:

In Arabic the name ‘speech’ (*nu}q*) can signify the [specific] difference itself, namely the potency/faculty (*quwwa*) [‘rationality’ (*nu}q/’aqI*)] by which man/human thinks reasonably (*ya’qilu*). And it can signify the action/act (*fi’l*) of [applying] this potency/faculty. And it can also signify [the happening of] speaking/talking with the tongue. When we say ‘he will speak/he will think reasonably’ (*yan}uqu*), then this does not signify that man/human will have this potency/faculty at a future time. It signifies the same as when we say ‘he will think reasonably’ (*ya’qilu*), ‘he will say’ or ‘he will talk’... (*Shar1* 34, 15-18; cf. transl. Zimmermann 1981, 23f.).²¹

From this it follows that ‘the name of an agent’ can stand not only for an agent and his imperfect or perfect action and thus for an agent while he is in the state (*1#l/1#a*) of performing an action, after his action has come to an end, or for someone who will perform an action in the future (cf. above § 4), but also without relation to an action.

To sum up, according to al-F#r#bī the name of an agent can stand for:

[1.] an agent who has a potency/faculty, as for example the ‘rational’ (*n#}iq/’#qil*);

[2.a] an agent who is applying a potency/faculty, as for example ‘reasonably thinking/understanding’ (*n#}iq/’#qil*);

[2.b] an agent who is performing an action, as for example ‘standing’ (*q#’im*) and ‘speaking/talking’ [with the tongue] (*n#}iq*).

The second two ways of understanding the name of the agent (*ism al-f#’il*) are connected and restricted to the time of the happening (*lu{ūl}*) of the action. The first kind of understanding is not connected and not restricted to a time of an action. This distinction leads to Ibn Sīn#’s distinction between the *wa{fti-* and the *dh#tī-*readings of propositions.

7. The different meanings of the ‘derived’ (*mushtaqq*) according to Ibn Sīn#

In his *Pointers and Reminders* (*al-Ish#r# wa-l-tanbīh#*) Ibn Sīn# (d. 428/1037) distinguishes two kinds of necessary relation of a subject-term with a predicate-term: absolute/unrestricted (*’al# l-i}l#q*) necessity (**arūra*) and necessity dependent on conditions, with other words, restricted (*muqayyad*) necessity. He explains two kinds of restriction. These two kinds of restricted necessity are based on the distinction between a name and what is ‘derived’ and the distinction between two meanings of the ‘derived’:

Necessity may be [1.] absolute (*’al# l-i}l#q*), as in ‘God exists/is existent’; or [2.] it may be connected to a condition. The condition may be either [2.a] the duration of the existence of the essence (*dh#*), as in ‘man/human is necessarily (*bi-l-’arūra*) a rational (*n#}iq*) body’. By this we do not mean to say that man/human has always been and always will be a speaking/talking/reasonably thinking (*n#}iq*) body without beginning and without ending, because that would be false for each human individual. Rather, we mean to assert that he is a rational (*n#}iq*) body while/as long as the essence exists as a man/human. [...] Or [2.b] [the condition may be] the duration of the subject’s being described (*maw{ūf*) by (*bi-*) what is set down together with

it, as in ‘every moving is changing’. This is not to be taken to assert that this is the case absolutely (*‘al-#l-i#l#q*), nor for [the time of] the duration of the existence of the essence, but rather as long as the essence of the moving [thing] is moving. There is a distinction between this condition and the first condition, because in the first is set down the root/origin of the essence’ (*a#l al-dh#*) which is ‘man/human’ (*al-ins#n*), whereas here the essence is set down by an attribute (*bi-#ifa*) that attaches to the essence which [viz. the essence] is the moving [thing]. To ‘moving’ belongs an essence and a substance (*lahū dh# wa-jawhar*) to which attach that it is moving or²² that it is not moving; but ‘man/human’ (*al-ins#n*) and ‘blackness’ (*al-saw#d*) are not like that (*al-Ish#r#* I, 310; cf. transl. Street 2000a, 213; id. 2005b, 259-60).²³

Ibn Sīn# distinguishes here with regard to the combination (*ta’līf*) of a subject-term with a predicate-term three kinds of truth-condition:

- [1.] pure and simple actuality (*fi’l#energeia*),
- [2.a] the actuality (*fi’l#energeia*) of an essence,
- [2.b] the actuality (*fi’l#energeia*) of the attachment of an attribute to an essence and a substance.

In book 5 of the *Il#hiyy#* Ibn Sīn# explains the “how-ness” (*kayfiyya*) of the existence (cf. Greek *tropos tēs hyparxeōs*) of common things (*al-umūr al-#mma*) (Ibn Sīn#, *al-Shif#*: *al-Il#hiyy#*, book 5.1, I, 195,3). Existence may either belong to quiddities (*m#hiyy#*) *qua* quiddities and universals (*kulliyy#*) *qua* universals, or to quiddities and universals in so far as they are the quiddities and essences of individuals (*ashkh#*) existing outside the mind.

In light of this metaphysical background the three kinds of logical necessity explained by Ibn Sīn# in the *Ish#r# wa-l-tanbīh#* are equivalent to three modes (lit. “how-nesses”) of existence: That whose existence is actual necessarily exists either [1.] because it is existent by itself, or [2.] because it is existent by something else, namely either [2.a] by the universality which is attached to it, or [2.b] by accidents which are attached to it.

[1.] That which is not existent by something attached to it but by itself (*bi-dh#ihī/per se*) is God. [2.a] That which is existent by universality attached to it are the quiddities and universals in so far as they exist as abstracted quiddities and universals in the mind. Since universality does not belong to the common things (*al-umūr al-#mma*) as such, existence belongs to them by accident (*bi-l-#ara*/per accidens*). [2.b] That which is existent by accidents which are attached to it is an aggregate (*jumla*) (book 5.4, I, 226,7 and 15) of an essence/substance and its accidents. Since accidents do not belong to essences as such, individuals (*ashkh#*) also exist by accident.

Hence, [1.] what is pure and simply actual is necessary by itself, whereas [2.a] the actuality of an essence, and [2.b] the actuality of the belonging of an attribute to an essence and a substance are necessary by accident.

Therefore [1.] the first kind of logical necessity explained in the cited above passage is atemporal, whereas [2.] the second two kinds are temporal:

The [1.] first kind of necessity is ‘absolute’, that is to say, the predicate is affirmed of the subject without any restriction (*taqyīd*), namely not restricted (*muqayyad*) to one of the conditions of the two other kinds of necessity explained in the following. In so far as these two other kinds of necessity are restricted either by the duration of the existence of an essence or by the duration of a description of an essence, ‘absolute’ here means without relation to duration and change and consequently without change from possible existence to actual existence (cf. Aristotle, *Phys.* III, 1, 201b 4-5). Thus, the absolute necessity of the proposition ‘God exists/is existent’ means that God’s existence is *in actu* without beginning or ceasing to exist and therefore existing without having been possible before being actual (cf. Aristotle, *Met.* IX, 8, 1050b 6 - 1051a 2). ‘Absolutely necessary’ means without change and therefore without any relation to time (cf. Aristotle, *Phys.* VIII, 1, 251b 10-11). Hence, ‘absolute necessity’ is atemporal necessity. This is the kind of necessity which is opposed to “necessary

when it exists” (Aristotle, *De int.* 9, 19a 23-26).²⁴ That is to say, God’s existence does not depend on the condition that he is existent in the mind, nor on the condition that he is existent physically outside the mind.

The [2.] second two kinds of necessity are restricted with regard to the time of the duration of the existence of either essence or description, that is to say, with regard to the time either [2.a] an essence or [2.b] a description of an essence is existent. Hence, necessity here means temporal necessity. This is the kind of necessity Aristotle explains *De int.* 9, 19a 24-6 as the necessity of the existence of something *when* (*idh#*) it exists actually and the impossibility of its non-existence when and in so far as it exists (cf. Street 2000a, 214).

In [2.a] the first case necessity is restricted to the time of the duration of the existence of the essence (*daw#m wujūd al-dh#t*) signified by the subject-term, for example as long as existence is attached to the quiddity (*m#hiyya*) ‘humanness/humanity’ (*ins#niyya*) by which the universal ‘man/human’ exists in the mind. This is the time when (*idh#*) the essence ‘humanness/humanity’ is *in actu* and thus this is the time when the essence necessarily exists in so far as it exists (cf. Aristotle, *De int.* 9, 19a 23-26). The name ‘man/human’ may either signify the universal ‘man/human’ existing in the mind or denote a concrete man/human existing outside the mind. If it is used to signify the universal which according to Ibn Sīn# is existing only in the mind, then the statement ‘man/human is a rational body’ is necessarily true as long as the quiddity ‘humanness/humanity’ is existing in the mind. Since existence is not essential to the quiddity ‘humanness/humanity’ *qua* quiddity, but is inseparable from the universal ‘man/human’ *qua* universal, the proposition ‘man/human is necessarily a rational body’ is omnitemporally true, that is to say for all times *when* ‘man/human’ exists in the mind. Thus, there is no ‘absolute’ logical necessity with regard to the relations between quiddities abstracted from things existing outside the mind. The logical necessity of a proposition as ‘man/human is necessarily a rational body’ rather depends on the condition of the existence of the universal ‘man/human’ in the mind.

If the name ‘man/human’ is used to denote men/humans existing physically outside the mind, then the statement ‘man/human is necessarily a rational body’ is true with regard to the time from the particular generation to the particular corruption of each particular substance denoted as ‘man/human’. Also in this case the proposition ‘man is necessarily a rational body’ is omnitemporally true, namely for each particular time *when* ‘man/human’ exists physically outside the mind.

However, whereas the term ‘man/human’ in the first case is used as *signification* (cf. Arab. *dal#la*) of the meaning of the abstracted quiddity ‘humanness/humanity’ and the universal ‘man/human’, in the second case it is used as *appellation* (cf. Arab. *tasmiya*), that is to say ‘to name’ all human individuals. Therefore, in the first case the predication is intensional, and in the second case the predication is extensional.

The change by generation and corruption might be understood as a change from one thing to another thing, that is to say, from one substance to another substance as for example the change from metal to statue (Aristotle, *Phys.* III, 1, 201a 29-30), or — to take Maimonides’ example in the passage cited above (cf. § 2) — from building material to house (cf. Aristotle, *Phys.* III, 1, 201a 16-18), or — to take Ibn Sīn#’s example ‘man/human’ — from sperm to man/human (cf. Aristotle, *Met.* IX, 7, 1049a 2; Qur’#n 16,4) and from man/human to an inanimate body. But generation and corruption might also be understood as the change from nothing to something and from something to nothing (cf. al-F#r#bī, *al-Qiy#s al-‘aghīr*, ed. Türker 270,7-9; ed. ‘Ajam 49,6-8). Therefore the generation and corruption of a substance might be understood as the change from the possible to the actual and in this respect necessary, whether generation is understood in Aristotle’s sense as generation from something, namely from matter, or in the sense understood by the *mutakallimūn* as creation from nothing. In any case the statement ‘man/human is necessarily a rational body’ is only

true when the term ‘man/human’ signifies the universal ‘man/human’ existent in the mind and/or denotes a substance ‘man/human’ existing outside the mind. ‘Rational’ and ‘body’ belong to every concrete man’s ‘reality’ (*Iaqīqa*) denoted (*musamm#*) by ‘man/human’ (cf. Lizzini 2004, 178). However, when a man dies, the substance ‘man/human’ has been corrupted and the new substance which has been generated when the substance ‘man/human’ ceases to exist is an inanimate body which is not rational.

The subject-term ‘man/human’ is grammatically a primitive name/noun (*ism*) which is not derived from a root, but is itself a ‘root/origin’ (*a{l*), as ‘grammar’ is the root and origin from which is derived ‘grammarians’ (cf. Aristotle, *Cat.* 1, 1a 12-15; 8, 10a 30). The predicate-term ‘rational’ (*n#}iq*) is — as Ibn Sīn# claims — derived from [the root/origin] ‘speech/reason’ (*nu}q*) in the sense of the abstraction ‘rationality’ (Ibn Sīn#, *al-Shif#*: *al-Il#hiyy#*, book 5.6, I, 230,7-9). Like al-F#r#bī Ibn Sīn# also must deviate here from the Arab grammarians (cf. above § 6). If, as the Arab grammarians say, *n#}iq* is derived from the verb (*fi'l*) or from the verbal noun (*ma{dar*) which both indicate the performance and the happening (*Iu{ūl*) of the action ‘to speak/speaking/to think reasonably/reasonably thinking’, then *n#}iq* can only be predicated as temporally restricted. Only under the condition that *n#}iq* is derived from the [the root/origin] ‘*nu}q*’ in the sense of the abstracted essence ‘speech/rationality’ which is the quiddity and reality of ‘man/human’, can Ibn Sīn# hold that the derived name ‘rational’ is predicated univocally (*bi-l-taw#}u*) of the universal ‘man/human’, of the species ‘man/human’ and of the individual ‘man/human’ (cf. Ibn Sīn#, *al-Shif#*: *al-Il#hiyy#*, book 5.6, I, 230,12-13). It is only on this basis that the statement ‘man/human is necessarily a rational (*n#}iq*) body’ is a logically necessary statement. If, however, in this statement the term *n#}iq* were derived from the verb or the verbal noun ‘to speak/speaking’ and therefore were used to indicate the temporal application of the potency/faculty ‘rational’, namely ‘reasonably thinking/understanding’, or in the sense of the temporal description (*wa{f*) ‘speaking/talking’ with the tongue (cf. above § 6), then the proposition would have the sense ‘man/human is necessarily a reasonably thinking/understanding body’ or ‘man/human is necessarily a speaking/talking body’ which is false, whether the term ‘man/human’ is used to signify the universal and the species ‘man/human’ or to denote concrete individual men/humans.

In [2.b] the second case necessity is restricted to the time of the duration of the attachment of an attribute to the essence and substance denoted by the subject-term. This is the time *when* the essence and substance is described as either being in a certain state (*1#l*) or as performing an action (*fi'l/amal*). Hence, the statement ‘every moving is changing [*when* it is moving]’ is also omnitemporally true, namely for each time when movement is attached to an essence and a substance, whether in the mind or in physical existence outside the mind. Thus, the logical necessity of the proposition depends on the condition of the existence of the attachment of an attribute to an essence in the mind, but it does not depend on the condition of the existence of concrete states or actions existing outside the mind.

The subject-term ‘moving’ is grammatically a description/attribute (*wa{f/ifa*) which is similar to [the name of] the agent (*ism al-f#il*). Therefore — similar to “the bearer of the name” (*{#lib al-ism*) which is “concealed” (*mu*mar*) in the grammatical *f#il* [-form] (cf. above § 4) — the bearer of the attribute ‘moving’ is concealed in the grammatical form of the attribute (*ifa*) ‘moving’ (*mutalarrik*). That is to say — from Ibn Sīn#’s logical point of view — that by a grammatical attribute (*ifa*) as for example ‘moving’ is set down an essence and a substance to which the attribute ‘moving’ is attached. Grammatically the attribute (*ifa*) ‘*mutalarrik*’ is similar to the name of an agent ‘*n#}iq*’, however, they differ logically. ‘Rationality’ is essential and thus constitutive (*muqawwim*) for that of which it is predicated. Therefore ‘rational’ is not ‘attached’ to the essence and substance which is rational.

‘Movement’, however, is a quality (*kayfiyya*) of everything to which it belongs. Therefore ‘moving’ is an accident (*‘ara**) of the essence and the substance to which it is attached. Ontologically speaking the essence and quiddity (*m#hiyya*) to which the quality (*kayfiyya*) ‘movement’ is attached and the substance (*jawhar*) to which the accident (*‘ara**) ‘moving’ is attached is “the bearer of the potency” (*1#mil al-quwwa*) (Ibn Sīn#, *al-Shif#*: *al-Il#hiyy#*, book 4.2, I, 184,8) ‘movement’. Therefore the grammatical attribute ‘moving’ may be used in language either [1.] ‘with regard to the essence’ (*dh#ī*) which might be moving or not moving. Under this condition the statement ‘all moving are resting’ is not false but possibly true, since an essence and a substance which is in the state of moving may at another time not be moving but resting (cf. Aristotle, *Met.* IX, 1048b 1-3). Hence, the logical necessity of the *dh#ī*-reading of the proposition ‘all moving are resting’ is restricted to the duration of the attachment of the potency of moving or not moving to an essence. It is omnitemporally true: Whenever the potency of moving or not moving is attached to an essence and a substance [whether in the mind or outside the mind in physical existence], the potency of changing or not changing is attached to the essence.

Or the grammatical attribute ‘moving’ may be used in language [2.] ‘with regard to the description’ (*wa{fī}*) ‘moving’ which describes a state (*1#l*) of being of an essence and a substance. Under this condition the logical necessity of the *wa{fī}*-reading of the proposition ‘every moving is changing’ is restricted to the duration of the attachment of the quality ‘movement’ to an essence and of the accident ‘moving’ to a substance. It is omnitemporally true: Whenever ‘movement’ is attached to an essence and ‘moving’ is attached to a substance [whether in the mind or outside the mind in physical existence], ‘change’ is attached to the essence and ‘changing’ is attached to the substance.

To sum up, according to Ibn Sīn# “the derived” (*al-mushtaqq*) — namely “[the name of] the agent” (*[ism] al-f#’il*) and “the description/attribute which is similar to [the name of] the agent” (*al-{ifa al-mushabbaha bi-l-f#’il*) (cf. above § 4) — can be used in language to indicate five different meanings:

[1.] It can stand ‘with regard to essence/essentially’ (*dh#ī*) to indicate:

[1.a] an essence and a quiddity to which is attributed an essential potency and quality, as for example ‘rational’ (*n#’iq*) in the statement ‘All rational have the power of volition’;

[1.b] an essence and a quiddity to which is attributed a passive-potency (*quwwa*) to be in a state (*1#l*) of being and to be in a contrary state of being, as for example ‘moving’ (*muta larrik*) in the statement ‘All moving are resting’;

[1.c] an essence and a quiddity to which is attributed an active-potency (*quwwa/qudra*) for an action (*fī’l’amal*) and for a contrary action, as for example ‘speaking’ (*n#’iq*) in the statement ‘all speaking are keeping quiet’ or as for example ‘standing’ (*q#’im*) in the statement ‘all standing are sitting’.

[2.] It can stand ‘with regard to description/descriptionally’ (*wa{fī}*) to indicate:

[2.a] an essence and a quiddity to which is attributed a quality (*kayfiyya*) by which the substance is in a state (*1#l*) of being, as for example ‘moving’ (*muta larrik*) in the statement ‘All moving are changing [when moving]’;

[2.b] an essence and a quiddity to which is attributed a quality (*kayfiyya*) by which the substance is connected (*muqtarin*) (cf. above § 6) and related (*mu*#f*) (cf. below § 8) to an acting/doing (*fī’l’fa’l’amal*), as for example ‘walking’ (*m#shin*) in the statement ‘All walking are changing [when walking]’.

In [1.] the first case “the derived” is derived from names/nouns which signify the abstractions, that is to say, the essences (*dhaw#*) ‘rationality’ (*nu}q*), ‘movement’ (*laraka*), ‘standing’ (*qiy#m*). In [2.] the second case “the derived” is derived from the verbs (*af’#l*) or from the verbal nouns (*ma{#dir*) ‘to move/moving’ (*laraka*) and ‘to walk/walking’ (*mashy*). Both, the

verb and the verbal noun signify the temporal happening (*lu{ūl}*) of the actions ‘to move/moving’ and ‘to walk/walking’.

Whereas al-F#r#bī had identified the logical derivaton of a name or an attribute from an abstracted meaning and the grammatical derivation from a verb or a verbal noun (cf. above § 6), Ibn Sīn# distinguishes the two kinds of understanding derivation with regard to their meaning: the logical derivation indicates the relation of a subject with a quality or an action; the grammatical derivation indicates the relation of a subject with the happening (*lu{ūl}*) of the affection by a quality or with the happening of an acting.

The use of *n#{iq}* in the sense of [1.a] ‘rational’ corresponds to what Abū ‘Alī al-Jubb#’ī first called “the name of the language” (*ism al-lugha*) and then “the name of the religion” (*ism al-dīn*), and of what Abū l-Q#sim al-Ka’bī said that it is “not only [grammatically] derived from the verb/action” but that it is also used as “a sign (*sima*) to distinguish” and as “a praise (*madī*)”. Hence, from Abū ‘Alī al-Jubb#’ī’s and Abū l-Q#sim al-Ka’bī’s nominalistic point of view ‘rational’ (*n#{iq}*) had to be explained as a grammatically derived name which is used with regard to the subject of an action and unrestricted (*mu{laq}*) to the time of the action in the sense of a class name. However, in contrast to the essential name ‘rational’ the class name can only signify a sum of individuals and therefore can only be predicated extensionally, but not intensionally. The use of for example ‘walking’ [2.b] as a description (*wa{ff}*) corresponds to what Abū ‘Alī al-Jubb#’ī first called “the description of the language” (*wa{ff al-lugha*) and then “the name of the language” (*ism al-lugha*), and what Abū l-Q#sim al-Ka’bī explained as an attribute which is “[only gramatically] derived from the verb/action” (cf. above § 5).

With regard to Maimonides’ attack on the *mutakallimūn* (cf. above § 2) the most crucial sentence in the passage quoted above from Ibn Sīn# is the last sentence: “but ‘man/human’ and ‘blackness’ are not like that”. ‘Man/human’ (*al-ins#n*) and ‘blackness’ (*al-saw#d*) both are primitive names/nouns which can only be used to signify the universal or the substance ‘man/human’ and the quality ‘blackness’. According to Ibn Sīn# the generation of a substance is a non-gradual substantial change which occurs all at once. That is to say, substantial change occurs with the appearance of a Form (*{ūra*) that replaces the Form which is corrupted and takes its place (McGinnis 2004). Therefore from the semantical as well as from the logical point of view it is not true to say ‘every man/human is an irrational body [at some given time]’ and it is not true to say ‘every sperm is possibly man’, since there is nothing underlying an essence and a substance which endures when an essence and a substance is corrupted and another essence and substance is generated. When a man/human dies his concrete reality (*laqīqa*) and the essence ‘man/human’ has ceased to exist and the dead irrational body is not denoted as ‘man/human’. When sperm has [been]²⁵ changed to man, the essence ‘sperm’ does not exist any more and the essence which has generated from — or, instead of — sperm is denoted as ‘man/human’ but not as ‘sperm’.²⁶ This point of view coincides with the Sunnī doctrine that there is no natural potency in things by which they change from being something to being another thing, that the will of God is the only reason why things exist as they do and “that the existence of everything other than Himself comes into being because He originated and created it for the first time” (al-2alīmī, *Minh#* I, 183ult.-184,1; cf. above § 2). This semantical-logical aspect is also the basis of al-Ghaz#lī’s argument for calling God ‘agent’ rather than ‘first cause’ as shall be explained in the following.

8. Al-Ghaz#lī’s semantical-logical distinction between ‘name’ and that what is ‘derived’ (*mushtaqq*)

In the passages from the *Incoherence of the Philosophers* discussed above in § 2 al-Ghaz#lī dealt with the term ‘agent’ (*f#’il*) from a logical point of view without touching the semantical

aspect. In the opening of his *The Loftiest Intention Concerning the Explanation of the Meanings of God's Most Beautiful Names* (*al-Maq{ad al-asn# fī shar1 ma' #nī asm# All#h al-1usn#*) he provides a semantical-logical explanation of the difference between name (*ism*) and that what is 'derived' (*mushtaqq*) and their semantical functions:

[1.] What is understood (*mafhūm*) from the name (*ism*) may be the essence of the named/denoted (*dh# al-musamm#*), its reality (*laqīqatuhū*) and its quiddity (*m#hiyya*) [viz. the reality and the quiddity of the named/denoted]. These are the names of the species (*asm# al-anw#*) which are not derived, as when you say 'man/human' (*ins#nun*), 'knowledge' (*'ilmun*), 'whiteness' (*bay#*un*) (*al-Maq{ad 25,13-15*).

From the [primitive] name/noun (*ism*) has to be distinguished

[2.] what is derived (*mushtaqq*) and what does not signify the reality of the named/denoted, but leaves its reality indefinite (*mubhama*) and signifies an attribute that belongs to it (*{ifa lahū*) [viz. to the named/denoted], as when you say 'knower/knowing' (*#imun*) and 'writer/writing' (*k#tibun*). Then the derived is divided in [2.a] what signifies the description of a state of the named/denoted (*wa{f 1#l fī l-musamm#*) as 'the knowing' (*al- #lim*) and 'the white' (*al-abya**), and in [2.b] what signifies a relation (*i*#fa*) of it [viz. the named/denoted] with something inseparable [which cannot exist independently, apart from the named/denoted] as 'the creator/the creating' (*al-kh#liq*) and 'the writer/the writing' (*al-k#tib*) (*al-Maq{ad 25,15-19*).

What is understood from [2.a] 'the knowing' (*al- #lim*) is something indefinite to which belongs the description/attribute 'knowledge' (*lahū wa{f al-'ilm*), and what is understood from [2.b] 'the writer/the writing' (*al-k#tib*) is something indefinite to which belongs the action 'writing' (*lahū fī'l al-kit#ba*) (*al-Maq{ad 26,7-9*).

Like Ibn Sīn#'s distinction between a predication whose necessity depends either on the actuality of an essence or on the actuality of a description (*wa{f*) al-Ghaz#lī's distinction between [1.] what signifies the reality of the named/denoted and [2.] what does not signify the reality of the named/denoted is based on Aristotle's distinction between [1.] the secondary substances and [2.] the accidents explained in *Cat. 5*. This distinction orders things [1.] in being by itself and [2.] in being with regard to something else and in [1.] vertical predication of 'what it is' (*m# huwa*) and [2.] horizontal predication of a relation of something with something else (cf. Zimmermann 1981, xxv).

By passing over what is [grammatically] derived and what *does* signify the reality of the named/denoted al-Ghaz#lī synthesizes the grammatical functions of [1.] name (*ism*), [2.a] description/attribute (*{ifa*) and [2.b] '[the name of the] agent' (*[ism al-] f#'il*) with Aristotle's basic distinction between substance and accident. [1.] A name/noun (*ism*) signifies and denotes the denoted (*musamm#*), [2.a] an attribute/description (*{ifa*) signifies a state (*1#l*) of the described (*maw{ūf*) and [2.b] a [name of the] agent (*f#'il*) signifies an action (*fī'l*) of an agent which is "concealed" in the grammatical *f#'il*-form (cf. above § 4). From the logical point of view both, a state of being of a substance and an acting of a substance can be subsumed under the horizontal predication of a relation of something with something else, since they both are expressed in language by derivation (*ishtiq#q*) from a verb. A name, however denotes by vertical predication. Therefore

[1.] the term 'essence' (*dh#t*) stands for the Aristotelian *eidos* in the sense of the Form (*{ūra*), which is the principle by which a thing is an object of imagination, whereas 'reality' (*laqīqa*) signifies what a concrete particular denoted thing (*musamm#*) is by its Form (cf. al-Ghaz#lī, *Musta{f# II*, 12,16-18; cf. also Lizzini 2004, 178). The term 'quiddity' (*m#hiyya*) is the abstract noun for 'what-it-was-to-be' (cf. Aristotle, *Top. I*, 5, 101b 38), that is to say, the answer to the question 'what is it?'. The term *nau'* stands for the Aristotelian *eidos* in the sense of the species. In the above-cited passage 'the name of the species' — which corresponds to what is called in Arabic grammar more usually 'the name of the genus' (*ism*

al-jins) — is the expression (*lafz**) in language which signifies the essence.²⁷

And [2.] what is derived and what does not signify the reality of the named/denoted, but signifies an attribute (*{ifa}*) of the named/denoted corresponds to Aristotle's accidents.

In accordance with Arabic grammar al-Ghaz#lī distinguishes two kinds of that what is 'derived':

[2.a] The description/attribute (*{ifa}*) which is similar to the [name of the] agent (*f#'il*) signifies a state (*1#/1#a*) of the described (*maw{ūf}*) which is 'concealed' in the grammatical pattern of the attribute (*{ifa}*).

[2.b] The [name of the] agent (*f#'il*) which signifies a relation (*i*#fa*) of an agent which is 'concealed' in the grammatical *f#'il*-form with an action (*fi'l*) (cf. above § 4).

By identifying Aristotle's distinction of the ways in which secondary substances and accidents are predicated with the Arabic grammatical distinction of names which are not derived and descriptions which are derived, al-Ghaz#lī meets the same difficulty already al-F#r#bī had dealt with. Not only accidents (*a'r#**) in the Aristotelian sense, but also differentia (*fu{ūl}*) are signified by derivation as for example 'rational' (*n#}iq*). However, according to *Cat.* 5 differentia signify 'what it is', that is to say, they signify the quiddity (*m#hiyya*) of the named, while accidents signify qualities of the named. Al-Ghaz#lī solves this problem in the same way as al-F#r#bī had done (cf. above § 6). He subsumes both differentia and accidents under the derived name in the sense of *Cat.* 1 and treats them as appellations which may either be used with regard to a potency/faculty (*quwwa*) of an essence as for example the derived name *n#}iq* may be used in the sense of 'rational'. Or they may be used as appellation of someone who applies this potency/faculty, as for example the derived name *n#}iq* may be used in the sense of someone speaking/talking with the tongue and reasonably thinking. By this distinction al-Ghaz#lī is able to explain why not only God's essential attributes but also attributes which signify God's actions can be attributed to him without a beginning:

With regard to the names which go back to the action (*tarji'u il# l-fi'l*) as 'the creator/creating' (*kh#liq*), 'the former/forming' (*mu{awwir}*) and 'the giver/giving' (*wahh#b*) some people say: "He [viz. God] is described as being creator/creating without a beginning (*bi-annahū kh#liq fi l-azal*)". And others say: "He is not described [as being creator/creating without a beginning]". [However,] there is no foundation for this disagreement. 'The creator/creating' is applied to [indicate] two meanings: The first of them is certain definitely without a beginning. The second of them is denied definitely. And there is no kind of disagreement between them, since the sword is named/denoted 'cutter/cutting' (*q#}i*) while it is in the scabbard and it is called 'cutter/cutting' when it is in the state (*1#a*) of incising into the neck. In the scabbard it is cutter/cutting *in potentia* (*bi-l-quwwa*) and at ('*inda*) the incision it is named/denoted 'cutter/cutting' *in actu* (*bi-l-fi'l*). And the water in the jug is thirst-satisfying (*murwin*), however, *in potentia*, and in the stomach it is thirst-satisfying *in actu*. The meaning of the water's being thirst-satisfying in the jug is that it is by the attribute/quality (*bi-{-ifa}*) that the thirst-satisfying (*irw#'*) happens at ('*inda*) the encounter with the stomach. And this is the attribute/quality of the waterhood²⁸ (*{ifat al-m#}iyya*). And the sword in the scabbard is 'cutter/cutting', that is to say, that it is by the attribute/quality (*bi-{-ifa}*) that the [act of] cutting (*qa}*) happens when (*idh#*) it [viz. the act of cutting (*qa}*)] meets the place [of the cutting]. And this is the [attribute/quality] of the sharpness. [...]

The creator (*b#ri*) is creator/creating without a beginning (*fi l-azal kh#liq*) in the meaning in which the water in the jug is said to be thirst-satisfying. And this [meaning] is, that it is by the attribute/quality (*bi-{-ifa}*) ['actorness' and 'creatoriness'] that the acting (*fa'l*) and the creating/creation (*khalq*) is possible. And in the second meaning He is not the creator, that is to say, the creation does not proceed from Him [without a beginning] (*al-Maq{ad 31,14-32,6}*).

This is nothing else than Ibn Sīn#'s distinction of understanding a derived name with regard to essence (*dh#tī*) or with regard to description (*wa{fi*) (cf. above § 7). If taken with regard to essence (*dh#tī*) the attribute 'creating/creator' means: *When* (*idh#*) the divine essence exists, the divine power to create exists. 'Creator/creating' here is understood with regard to the

divine essence, which has the power (*qudra*) to create. Being ‘creator/creating’ here is taken as an attribute of the divine essence as being ‘rational’ is an attribute of man/human as long as the essence ‘man/human’ exists as man/human and has not changed to an inanimate body and as being ‘thirst-satisfying’ is an attribute of water as long as the essence ‘water’ exists as water and has not evaporated and changed to the new essence ‘air’.²⁹ However, since in contrast to man and water the divine essence exists absolutely, that is to say, without generation and corruption and thus purely actual, the divine essence has the power to create without a beginning and without an end and consequently the attribute ‘creating/creation’ (*khalq*) belongs to the divine essence without a beginning and without an end. Thus the sentence ‘God is creator/creating’ if taken with regard to the divine essence is an absolutely necessary statement in the same sense as ‘God exists/is existent’ is an absolutely necessary statement.

Taken with regard to description (*waʿfi*) the attribute ‘creator/creating’ means God is creating *when* he is creating. “When (*idh#*) it [viz. the act of cutting (*qaʿ*)] meets the place [of the cutting]” is the moment/time (*waqt*) of the change from possible cutting to actual cutting. Similarly, when (*idh#*) the act ‘creating/creation’ (*khalq*) meets the place of the creating/creation (*khalq*) is the moment/time of the change from possible creating/creation to actual creating/creation. Consequently, when the act of creating actually proceeds from the divine essence the divine essence is described as being creating. However, since there was no time before the act of creation, there is no time *when* God is being described as being not-creating.

Hence, in the first sense ‘creator/creating’ is purely actual, whereas in the second sense ‘creator/creating’ is omnitemporally actual, namely as long as time is brought into existence by God’s act of creation.

9. Conclusion

Aristotle explains in *Physics* III, 1, 201a 29 - 201b 5 that there is no change in metal from being metal *in potentia* to being metal *in actu*. And also a statue is not a statue *in potentia* before changing to a statue *in actu*. In so far as the change of a substance and an essence is the change from something to something else metal rather is metal *in actu* and a statue *in potentia*. The change from metal to statue is a change from potentiality to actuality. Therefore Maimonides was wrong when he maintained that a cause (*illa*) might be named/denoted a cause *in potentia* before it is a cause *in actu*. And his example in proof this statement, namely that “the matter of a particular house, before it is built, is matter in a state of potentiality” (cf. above § 2) was misleading. Matter in a state of potentiality is not opposed to matter *in actu* but to house *in actu*. There is no change from being a cause in a state of potentiality to being actually a cause as there is no change from being matter in a state of potentiality to being actually a house without the change from one substance and essence to another substance and essence. One might say ‘sperm’ is potentially ‘man/human’. However, the saying ‘man/human is potentially ‘man/human’ is self-contradictory. Therefore, against Maimonides can be argued from a logical and from a semantical point of view. From the logical point of view it can be argued that if the divine essence changes from being cause *in potentia* to being cause *in actu* the divine essence itself would change from being *in potentia* to being *in actu* and thus would not be eternal. From the semantical point of view it can be argued that if the divine essence changes from being cause *in potentia* to being cause *in actu* the name ‘cause’ would be used equivocally for two different essences. From this it becomes clear what the *mutakallimūn* had gained “by preferring the naming/denomination (*tasmiya*) ‘doer/agent’ to the naming/denomination (*tasmiya*) ‘cause’ (*illa*) and ‘ground’ (*sabab*)” (cf. above § 2). The terms ‘cause’ and ‘ground’ are primitive names which can only denote a substance and an

essence. The term ‘agent’ (*f#’il*) however is a derived name which can signify either with regard to essence (*dh#tī*) or with regard to description (*wa{ftī*). In case it is used with regard to essence it signifies an agent who has the potency/faculty to enact by an act of will and a choice to enact or not to enact (cf. above § 3). In case it is used with regard to description the term ‘agent’ (*f#’il*) signifies an agent when (*idh#*) he is in the state of being (*l#*) enacting. One can only wonder whether Maimonides himself has fallen victim of a fallacy or whether he consciously used an eristic argument to overcome his opponents.

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Notes

¹An exposition of the problem is given in Lagerlund 2000, ch. 1, esp. pp. 12-14.

²See Street 2005a and 2005b, 256-262.

³On this broader sense of ‘description’ (*wa{ftī*) see Sumi 2004.

⁴I read *tasmiya* (“denomination”) instead of *ismiyya* (lit. “nounhood, nounness”).

⁵Cf. the preceding note.

⁶Cf. the preceding note.

⁷I have slightly modified Pines’ translation.

⁸On the genesis of this theory see Rudolph 2000; Schöck 2004.

⁹Frank mixed up the relation of condition and consequence with the relation of cause and effect and hold the *asb#b* to be “causal conditions” which have “effects” (see esp. Frank 1992, 38 and 40). Al-Ghaz#lī, however, — as already al-Ash’arī (*Luma’* 56,17-20, § 128; cf. Schöck 2004, 119-21) and al-M#turīdī (cf. Schöck 2004, 121-3) — holds that man’s power/faculty (*qudra*) is a necessary ‘means’ and ‘ground’ (*sabab*), that is to say, a necessary condition by (*bi-*) which the consequence, namely the act, follows. This does not mean that man’s power/faculty effects his act (cf. Marmura 1995).

¹⁰On this problem see Wisnovsky 2003, 61-98.

¹¹See in detail Schöck 2004.

¹²I have very slightly modified Marmura’s translations.

¹³I have very slightly modified Marmura’s translation.

¹⁴See in detail Schöck 2006, 43-53.

¹⁵See in detail Schöck 2006, 152-79.

¹⁶On *simā* see Schöck 2006, 171f., n. 94.

¹⁷Cf. above § 2.

¹⁸See above § 2.

¹⁹Zimmermann (1981, xxx) comments that of these examples only **#rib* and *mutalarrīk* are “participles; the others are original nouns”. However, this is not the Arabic understanding. Zimmermann (loc.cit.) further comments that al-F#r#bī’s understanding that *lāyy* (“living”) is derivative in pattern (*shakl*) (al-F#r#bī, *Shar1* 35,3f.) “makes morphological nonsense” and that al-F#r#bī “here has fallen victim of multiple confusion”. As shown above § 4, al-F#r#bī is in complete agreement with the Arabic grammarians.

²⁰On the verbal noun, lit. the “origin” (*ma{dar*) cf. A. Bäck 2007, § 3.

²¹I have slightly modified Zimmermann’s translation.

²²Lit.: and.

²³I have slightly modified Street’s translation.

²⁴Is1#q b. 2unayn has translated *De int.* 9, 19a 23-26 as follows: “The existence (*wujūd*) of a thing is necessary when it exists (*idh# k#na mawjūdan*). And when it does not exist then the negation of its existence is necessary. Not all what is existent has a necessary (**arūrī*) existence. And not all what does not exist has a necessary non-existence. That is to say that when we say ‘the existence of all what is existent is necessary when it exists’ this is not the same as when we say that its existence is absolutely necessary (*bi-anna wujūdahū *arūratan ‘al# l-*

i}l#q). Aristotle 1999, I, 128.

²⁵From the Sunnī point of view one has to say ‘has been changed’, namely has been changed by God’s immediate creation of man/human from sperm (cf. Qur’#n 16,4) and not ‘has changed’, namely has changed by virtue of its potency (*quwwa*) to change to man.

²⁶See on this problem Street 2000b, 134-35 with n. 11.

²⁷ Thus, in contrast to Ibn al-Muqaffa’ (cf. above § 1) al-Ghaz#lī does not identify [primitive] name and second substance, but distinguishes between ‘the name of the species’ and what is understood (*maf#hūm*) from it, this is the signified (*madlūl*).

²⁸The term does not have an equivalent in the English language. It could also be translated as ‘waterity’ (cf. humanity).

²⁹ On the change from water to air or, as we would say, steam see McGinnis 2004, 57 n. 23.

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Logic and Metaphysics in Avicenna's Modal Syllogistic

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Abstract. Recent discussions of Avicenna's modal syllogistic by Street (2000), Street (2002) and Thom (2003) have adopted a simple *de re* reading of Avicenna's *dhātī* propositions, and either ignored or rejected the possibility of metaphysical applications for his modal theory. In this paper I seek to supplement these interpretations by exploring an interpretation of Avicenna's *dhātī* propositions that incorporates a *de dicto* element. I argue that, given such a reading, his absolute and modal propositions have application to Aristotelian metaphysical theory.

1. Introduction

A logical theory comprises a set of theorems each of which states a logical truth. If the theory is presented in a deductive manner then certain basic elements, functioning as axioms, together with transformation rules, generate theorems from these axioms. The theory includes all theorems that are derivable from the axioms by means of the transformation rules. In the presentation or discussion of a logical theory it is customary to distinguish between the theory's syntax and semantics. Both these are distinguished from the theory's application. The syntax comprises rules for generating the theory's well-formed formulae, its basic theorems, its rules of transformation, and its theorems. The semantics gives the conditions under which the theory's well-formed formulae are true or valid. The application applies the theory to some section of language or thought.

A syllogistic logic is a logical theory whose theorems are syllogisms, i.e. inferences from two or more premises to a conclusion, where the premises and conclusion of an individual syllogism are two-term propositions, any two of which share a term. A modal syllogistic is a syllogistic logic in which the notions of necessity (“*L*”), one-sided possibility (“*M*”) and two-sided possibility or contingency (“*Q*”) play a role. These notions are known as the alethic modalities.

The first author to present a modal syllogistic was Aristotle, in his *Analytica Priora* Book 1 Chapters 3 and 8 to 22. Avicenna's modal syllogistic is presented in three of his works – *The Book of Salvation* [*al-Naj#*], *Pointers and Reminders* [*al-Ish#r#t*], and *The Book of the Syllogism from The Cure* [*al-Shif#*].¹ His presentation of the subject-matter in these works shows a knowledge of Aristotle's treatment as well as familiarity with the work of some of the commentators. Avicenna's modal syllogistic differs from Aristotle's by including an explicit semantics and in dealing with the temporal modalities “always”, “sometimes” and “sometimes and sometimes not”, along with the alethic modalities.² The propositions that figure in Avicenna's modal syllogistic are either absolute or modal. An absolute proposition connects a predicate with a subject but does not include any overt indication of temporal or alethic modality. A modal proposition is one that does include such an indication.

2. The simple *de re* analysis

Unlike Aristotle, Avicenna states truth-conditions for the propositions that figure in his modal syllogistic. He takes the subject-term of an absolute or modal proposition to apply to whatever falls under the term, “be it so qualified in a mental assumption or in external existence, and be it so qualified always or not always, i.e., in just any manner”.³ This formulation self-consciously rejects the idea that the subject-term of an absolute or modal proposition applies just to what actually exists. It recalls Avicenna’s distinction between what a subject requires for the realization of its quiddity and what it requires for the realization of its existence, “such as the fact that a human being is begotten”.⁴ (He notes that “in conceiving the body as body, we can strip creaturehood from it”.) Avicenna’s formulation suggests a simple *de re* reading of absolute and modal propositions, according to which the subject-term is amplified to cover whatever *can* fall under that term. Thus an absolute or modal proposition with grammatical subject “*j*” would have for its logical subject “ j_M ”. (The subscript acts as a term-forming operator on terms. In the present instance the resultant term stands for whatever is possibly *j*.) On the simple *de re* reading, the logical predicate of a modal proposition consists of its grammatical predicate, qualified by the proposition’s modality. Thus “Every *j* is necessarily *b*” means that every possible *j* is a necessary *b*, and “Every *j* is possibly *b*” means that every possible *j* is a possible *b*. There are two types of absolutes – general and special. The predicate of an affirmative general-absolute proposition is taken to apply *at some time* to the subject. Thus the general-absolute proposition “Every *j* is *b*”, though it contains no express modality, means that every possible *j* is *sometimes b*. This is clear from what Avicenna says about the contradictory of universal general-absolute propositions:

... it is necessary that the contradictory of the statement ‘Every C is B’, taken in the most general absolute sense, is ‘Some C is always not B’. And the contradictory of the statement, ‘Nothing of C is B’, which is in the sense of ‘B is denied of every C’, without addition, is the statement ‘Some C is always B’.⁵

If the contradictory of the universal affirmative general-absolute states that some (possible) *C* is always not *B*, then the universal affirmative general-absolute states that every (possible) *C* is sometimes *B*. Similarly, the universal negative general-absolute states that every (possible) *C* is sometimes not *B* or (equivalently) that no (possible) *C* is always *B*. The special-absolute means that every possible *j* is sometimes but not always *b*. In this paper in order to simplify the discussion we will leave the special-absolute, the contingency proposition and their contradictories out of consideration.

Using subscript “*m*” for “sometimes”, subscript “*l*” for “always”, subscript “*L*” for “necessarily”, subscript “*M*” for “possibly”, and “*C*” for inclusion, the truth-conditions for the universal affirmative absolute and modal propositions, on the simple *de re* reading, are:

1. X, the universal affirmative general-absolute “Every *j* is *b*”: $j_M \subset b_m$
2. P, the universal affirmative perpetual “Every *j* is always *b*”: $j_M \subset b_l$
3. L, the universal affirmative necessity-proposition “Every *j* is necessarily *b*”:

$$j_M \subset b_L$$

4. M, the universal affirmative possibility-proposition “Every *j* is possibly *b*”:

$$j_M \subset b_M$$

Truth-conditions for negative and particular propositions can be worked out along similar lines.

Temporal and alethic modalities stand in various logical relations to one another, as indicated in the following principles.

P1. What is sometimes *b* is possibly *b*: $b_m \subset b_M$

P2. What is always *b* is sometimes *b*: $b_l \subset b_m$

P3. What is necessarily b is always b : $b_L \subset b_I$

When discussing modal syllogisms, Avicenna follows the Aristotelian division of all syllogisms into the three Figures.⁶ In Figure 1, Avicenna says that if every j is b and every b is a , “by necessity or otherwise”, it follows that every j is a , with the conclusion having the same modality as the major.⁷ This means that if the minor is an absolute, the conclusion has the same modality as the major, so that LXL, MXM, PXP and XXX are all valid. On the simple *de re* reading favoured by Thom and Street, these moods are in fact valid.

Barbara LXL: if $j_M \subset b_m$ and $b_M \subset a_L$ then $j_M \subset a_L$

Barbara PXP: if $j_M \subset b_m$ and $b_M \subset a_I$ then $j_M \subset a_I$

Barbara XXX: if $j_M \subset b_m$ and $b_M \subset a_m$ then $j_M \subset a_m$

Barbara MXM: if $j_M \subset b_m$ and $b_M \subset a_M$ then $j_M \subset a_M$.

These syllogisms all hold by virtue of P1. Celarent, Darii and Ferio in these moods are equally valid.⁸ Avicenna simply says these syllogisms are “evident”, not acknowledging that they rely on the extra assumption that what sometimes happens is possible. The syllogisms would be perfect only if temporal and alethic modalities were equivalent.

Avicenna also implies that the corresponding syllogisms with necessity-minors are valid, when he says “Thus the syllogistic conjunctions of this first figure are these four: that is, if every C is, in some manner of being, B in actuality”.⁹ If we take the “manners of being in actuality” to include necessary being, perpetual being and absolute being, then this group includes (i) the LLL, PLP, XLX and MLM syllogisms of Figure 1, along with (ii) the LPL, PPP, XPX and MPM syllogisms, and also (iii) the LXL, PXP, XXX and MXM syllogisms of the same Figure. Regarding these, one would again have to insist that they are not perfect since they rely on the extra assumptions, (i) that what belongs with necessity, possibly belongs, (ii) that what always belongs, possibly belongs, and (iii) that what sometimes belongs, possibly belongs. These assumptions all follow from P1-P3.

Avicenna next considers first-figure syllogisms with possibility-premises. He considers three sub-cases – where the major is, respectively, a possibility-proposition, an absolute, or a necessity-proposition. He takes the MMM moods in Figure 1 to be valid. And so they are, on the simple *de re* reading.

Barbara MMM: if $j_M \subset b_M$ and $b_M \subset a_M$ then $j_M \subset a_M$

In addition to being valid, these syllogisms appear to satisfy the condition that Avicenna lays down for being perfect, in that they are “evident and not in need of proof”.¹⁰

Similar comments apply to the PMP first-figure moods.

Barbara PMP: if $j_M \subset b_M$ and $b_M \subset a_I$ then $j_M \subset a_I$

Also valid (and perfect) are the first-figure XMx syllogisms.

Barbara XMx: if $j_M \subset b_M$ and $b_M \subset a_m$ then $j_M \subset a_m$

This syllogism is valid, and perfect, on the simple *de re* reading; but it's not clear whether Avicenna thinks it is valid. He does mention a Barbara with absolute major and possibility-minor – but he specifies that the minor expresses a “real and proper possibility” (i.e. a contingency). And even here it's not clear whether he thinks that an absolute conclusion follows:

If every C is B according to the real and proper possibility, and if every B is A absolutely, then it is permissible that every C is A in actuality, and it is permissible that it is so in potentiality. And what is common to both must be the possible, in the general sense.¹¹

The last sentence asserts that it is valid to infer a possibility-conclusion. But what does Avicenna mean when he says that an absolute conclusion is permissible? Does he mean that an absolute conclusion follows from the premises, or merely that it is compatible with the premises? On the simple *de re* reading, the possibility-conclusion does indeed follow.

Barbara XMM: if $j_M \subset b_M$ and $b_M \subset a_m$ then $j_M \subset a_m$.

This conclusion is warranted because of P1. Now, Street remarks that “Avicenna defends certain inferences (such as Barbara *XMM*) in ways which seem to blur the distinction between the possible proposition and the absolute”.¹² But no such blurring is needed in order to validate this inference.

On the other hand, if Avicenna is merely saying that a possibility-conclusion is compatible with the premises, then what he says is right; but if his silence on the validity of inferring a possibility-conclusion implies a belief that Barbara *XM*X is invalid, then the simple *de re* analysis diverges from his thinking on this point.

Where the major is a necessity-proposition, Avicenna infers a necessity-conclusion.¹³

Barbara LML: if $j_M \subset b_M$ and $b_M \subset a_L$ then $j_M \subset a_L$

To these syllogisms we may add PMP and XM X in Figure 1. All these syllogisms are perfect. The perfect syllogisms in Figure 1 are thus LML, MMM, XM X and PMP; but Avicenna explicitly accepts only the LML and MMM syllogisms. The corresponding syllogisms with absolute minors or necessity-minors are valid as a consequence of these, by virtue of P1-P3.

In Figure 2, Avicenna says that the mixture of a necessity-premise with a possibility-premise gives a perfect syllogism.¹⁴ Thus he takes any mixture of a possibility-premise with a necessity-premise in the second figure to yield a syllogistic conclusion. Now, it is true that all LML-2 and MLL-2 syllogisms are valid. The former reduce to LML-1, the latter to MMM-1. But are they perfect? On the simple *de re* reading, Cesare LML states that if every possible j is a possible a and no possible b is a possible a then no possible j is a possible b , and this is a substitution in non-modal Cesare – but non-modal Cesare is not perfect. I cannot see that Avicenna’s judgment about the perfection of these syllogisms is warranted. The syllogisms in question are not perfect, though they do reduce to perfect syllogisms in Figure 1. Also valid in Figure 2 are the XPL and PXL syllogisms (these are equivalent to XM X -1 and PMP-1).

In Figure 3, Avicenna tells us that “the conclusion retains the mode of the major premise only, as has been determined in the first figure, along the lines described”.¹⁵ The simple *de re* analysis gives Avicenna’s results in Figure 3. So for example Darapti XM X is valid, because if every possible j is possibly b and is sometimes a , then some possible b is sometimes a . The LML and MMM syllogisms of Figure 3 reduce to the MMM and LML syllogisms of Figure 1. The PMP and XM X syllogisms reduce to first-figure XM X and PMP.

In sum, the simple *de re* analysis makes valid all the syllogisms Avicenna explicitly says are valid.¹⁶ In addition to those, it renders valid the XM X syllogisms in Figure 1, about which there is a doubt as to Avicenna’s opinion.

3. A combined *de dicto/de re* analysis

To take *dhātī* propositions in the simple *de re* fashion is to take them as categorical propositions, albeit ones having a modal content by virtue of an inner structure in their predicates. What I now wish to explore is the idea of taking them as having a more complex syntax, one in which a *de re* proposition is embedded within the scope of a propositional modal operator. Avicenna’s characterization of the subject of these propositions as standing for whatever it applies to, “be it so qualified in a mental assumption or in external existence, and be it so qualified always or not always”, leaves open two ways to construe the propositions. They could be read as *de re* predications with amplified subjects. However, Avicenna’s way of thinking of modal and absolute propositions is not entirely consistent with that reading. For example, when he is describing affirmations in general, he writes:

A predicative affirmation is something like the statement, ‘Human being is an animal.’ The meaning of this is that the thing which we suppose in the mind to be a human being, be that in concrete existence or not, we must suppose to be an animal.¹⁷

And the *de re* reading sits ill with this, since it suggests that there is something in existence answering to the subject. Alternatively, the effect of extending the application to all *js*, even those in mental supposition, and those that are only sometimes *j*, can be achieved by reading the proposition not *de re* but *de dicto*, as stating, in the case of the absolute proposition, that it is necessary that every *j* is sometimes *b*; or in the case of possibility- and necessity-propositions, that it is necessary that every *j* is a possible-*b* or a necessary-*b*. This captures the intent of extending the application of the subject “*j*” to past and future, and to possible, *js*, since that is the effect of placing the subject-term within the scope of a necessity-operator.

All of Avicenna’s modal syllogisms can be proved on this reading too. I proceed to show this in the case of the MMM and LML syllogisms in Figure 1. I use a notation in which the propositional operators ‘*L*’ and ‘*M*’ stand for *de dicto* necessity and possibility.

The proof of Barbara MMM can be set out as follows.

$$\begin{array}{c}
 \text{Barbara} \frac{j \subset b_M \quad b_M \subset (a_M)_M \quad (a_M)_M \subset a_M}{j \subset a_M} \\
 \xrightarrow{\text{Rule 1}} \frac{(1)L[j \subset b_M] \quad \text{P4} \frac{(2)L[b \subset a_M]}{(3)L[b_M \subset (a_M)_M]} \quad \text{P5} \frac{(4)L[(a_M)_M \subset a_M]}}{(5)L[j \subset a_M]}
 \end{array}$$

This proof depends on a rule and two principles:

Rule 1. When premises entail a conclusion the premises’ necessity entails the necessity of the conclusion.

P4. If it’s necessary that whatever is *X* is *Y*, then it’s necessary that whatever is possibly *X* is possibly *Y*.

P5. It’s necessary that whatever is possibly possibly *X* is possibly *X*. Since this is a theorem of the modal system S4, I will call it the S4 Principle.¹⁸

The above proof shows Barbara MMM to be valid by supposing (1) it’s necessary that whatever is *j* is possibly *b*, and (2) it’s necessary that whatever is *b* is possibly *a*. (2) implies (3) it’s necessary that whatever is possibly *b* is possibly possibly *a* (by P4). Finally we suppose (4) it’s necessary that whatever is possibly possibly *a* is possibly *a* (P5). Now, (1), (3) and (4) entail (5) it’s necessary that whatever is *j* is possibly *a*. Why does this entailment hold? Because it follows from a simple Barbara by Rule 1. So the syllogism is valid. But it is far from being perfect.

Without the S4 principle, Barbara MMM appears to be invalid on the proposed readings. A counter-example might be found by using a term that implicitly involves the notion of possibility, for example the notion of an embryo. It is necessary that all human embryos are potentially human, and it is necessary that all humans are possible-walkers, but it is not necessary that all human embryos are possible-walkers. If we favour the proposed reading and wish to save Barbara MMM’s validity, we need to reject this counter-example. This can be done by denying that there is a single concept of possibility in the claims that embryos are potentially human and that humans can walk. This could be done as follows. The possibility of being human, for an embryo, consists in the fact that it will become human given certain conditions. The possibility of walking, for a human, consists in the fact that it will walk given certain other conditions. Let us call these two sets of conditions the enabling conditions for becoming human, and for walking, respectively. Because the enabling conditions are different, we can say that there are two different notions of possibility here. Were we to suppose a single set of enabling conditions, the counter-example would fail. For example, if we take the enabling conditions in both cases to be those that enable the embryo to develop into a human being, then the second premise is false. If we take the other set of enabling

conditions, the first premise is false. If we take the enabling conditions to consist of the conjunction of both sets (i.e. whatever is necessary to enable the embryo to develop into a human being plus whatever is necessary to enable the human being to learn to walk), then both premises are true but so is the conclusion. Given all this, we can consistently maintain the validity of Barbara MMM.

Barbara LML:

$$\begin{array}{c} \text{Barbara} \frac{j \subset b_M \quad b_M \subset (a_L)_M \quad (a_L)_M \subset a_L}{j \subset a_L} \\ \xrightarrow{\text{Rule 1}} \frac{(1)L[j \subset b_M] \quad P4 \frac{(2)L[b \subset a_L]}{(3)L[b_M \subset (a_L)_M]} \quad P6 \frac{(4)L[(a_L)_M \subset a_L]}}{(5)L[j \subset a_L]} \end{array}$$

The extra principle here states:

P6. It's necessary that whatever is possibly necessarily X is necessarily X . Since this is a theorem of S5, I will call it the S5 Principle.¹⁹

The proof supposes (1) it's necessary that whatever is j is possibly b , and (2) it's necessary that whatever is b is necessarily a . By P4, (2) entails (3) it's necessary that whatever is possibly b is possibly necessarily a . Finally the proof supposes (4) it's necessary that whatever is possibly necessarily a is necessarily a . Now, (1), (3) and (4) entail (5) it's necessary that whatever is j is necessarily a . This entailment is generated by applying Rule 1 to a simple Barbara.

As mentioned earlier, Avicenna does not explicitly endorse Barbara XM X ; but its validity can be shown as follows.

$$\begin{array}{c} \text{Barbara} \frac{j \subset b_M \quad b_M \subset (a_m)_M \quad (a_m)_M \subset a_m}{j \subset a_m} \\ \xrightarrow{\text{Rule 1}} \frac{L[j \subset b_M] \quad P4 \frac{L[b \subset a_m]}{L[b_M \subset (a_m)_M]} \quad P7 \frac{L[(a_m)_M \subset a_m]}}{L[j \subset a_m]} \end{array}$$

The extra assumption on which this proof relies is as follows.

P7. It's necessary that what is possibly sometimes X is sometimes X .

Since what can occur, can occur at some time, P7 implies that what can occur does occur at some time. So it is tantamount to identifying the possible with what sometimes occurs. It is noteworthy that on the simple *de re* analysis Barbara XM X was valid, but its validity did not depend on this identification of the possible with the sometimes actual.

Similarly, Barbara PMP can be shown valid, given

P8. It's necessary that what is possibly always X is always X .

The combined *de dicto/de re* analysis thus implies the validity of all the syllogisms that Avicenna accepts. But, whereas the simple *de re* analysis commits Avicenna to the validity of XM X -1 and PMP-1, the combined *de dicto/de re* analysis commits him to their validity only if we attribute P7 and P8 to him. Thus, the combined *de dicto / de re* analysis is the subtler of the two. It enables us to make a choice as to whether to attribute XM X -1 and PMP-1 to Avicenna. The choice depends on whether or not we attribute P7 and P8 to him. The comparative subtlety of this analysis is one reason for preferring it over the simple *de re* analysis.

A second reason for preferring this reading is that it gives a better fit with Avicenna's remarks about Barbara MMM than does the simple *de re* reading. Here is what Avicenna says:

But if every C is B in possibility, then the judgment must not be carried over from B to C in an evident manner. However, if the judgment about B is in possibility, then there is a possibility of a possibility which is close to being known by the mind as a possibility. For it is within reach of our nature to judge that the possible of a possible is possible.²⁰

In saying that Barbara MMM is not evident, Avicenna makes it clear that he thinks it is not a perfect syllogism. Nonetheless, in saying that it is within the reach of our nature to see the conclusion as following from the premises, he makes it clear that he regards this mood as valid. In this respect Barbara MMM is like the syllogisms of the second and third figures, which while not perfect are "within the reach of our nature".²¹ He indicates that we can grasp its validity if we bear in mind that what is possibly possible is possible. Now, the simple *de re* analysis makes Barbara MMM perfect, since it says that if every possible *j* is a possible *b*, and every possible *b* is a possible *a*, then every possible *j* is a possible *a*. On this reading, Barbara MMM is just a special case of ordinary Barbara, with minor term "possible *j*", middle "possible *b*" and major "possible *a*". By contrast, the combined *de dicto/de re* analysis makes this syllogism valid but not perfect, at the same time making its validity depend of the principle that the possibly possible is possible (P5).

A third reason in favour of our proposed reading of Avicenna's modals is that, unlike the simple *de re* reading, it provides logically valid representations of his proofs by the procedure known as "upgrading". These are proofs that reduce a given inference to a second inference, containing a possibility-premise, and proceed to validate the second inference by reference to an inference containing the corresponding non-modal sentence. This last step is supposed to follow by virtue of a rule to the effect (as Avicenna puts it) that "if something is possible, its consequent is possible too."²² Street (2002) articulates the problem, "how can we understand the move or moves Avicenna makes in these proofs when he supposes a possible to be actual, and have them work for divided propositions?"²³ It is true that Avicenna's rule seems to have no application to divided (i.e. *de re*) propositions. However, things change if we adopt the complex *de dicto/de re* reading. It enables us to work out solutions for Street's problem.²⁴

Street mentions Barbara LML as a syllogism that Avicenna proves by upgrading.²⁵ Suppose Barbara LML's premises and the opposite of its conclusion. On our reading, this is to suppose that while (1) it's necessary that every *j* is a possible *b*, and (2) it's necessary that every *b* is a necessary *a*, it's not necessary that every *j* is a necessary *a*, i.e. (3) it's possible that not every *j* is a necessary *a*. What we want to show is that from (2) and (3) we may infer (4) it's possible that not every *j* is a possible *b*. Now, this inference is indeed valid, because if we suppose (5) not every *j* is a possible *a*, and (6) every possible *b* is possibly a necessary *a*, then it follows that (7) not every *j* is a possible *b* – given that we can assume that whatever is possibly necessarily *a* is possibly *a*. Given this inference, our desired inference must be valid – by virtue of Rule 2.

Rule 2: If *r* follows from *p* and *q* then *possibly r* follows from *necessarily p* and *possibly q*.

$$\begin{array}{c}
\text{Baroco} \frac{(5) j \not\subset a_M}{\text{Barbara} \frac{(6) b_M \subset (a_L)_M \quad (a_L)_M \subset a_M}{b_M \subset a_M}} \\
\text{Rule 2} \rightarrow \frac{(3) M[j \not\subset a_L] \quad L[b_M \subset (a_L)_M] \quad L[(a_L)_M \subset a_M]}{(4) M[j \not\subset b_M]} \\
\text{reductio} \rightarrow \frac{(1) L[j \subset b_M] \quad \text{P4} \frac{(2) L[b \subset a_L]}{L[b_M \subset (a_L)_M]} \quad \text{P6} \frac{}{L[(a_L)_M \subset a_L]}}{L[j \subset a_L]}
\end{array}$$

If this is to be taken as a representation of Avicenna's proof, then it implies that Avicenna makes tacit use of P4 and P6, and thus assumes alethic modalities obeying S5.

Now consider Barbara XMM, the mood upon which Aristotle first used the upgrading procedure.²⁶ Suppose its premises and the opposite of its conclusion. On our reading, this is to suppose that while (1) it's necessary that every j is a possible b and (2) it's necessary that every b is sometimes a , it's not necessary that every j is a possible a , i.e. (3) it's possible that some j is not a possible a . What we want to show is that from (2) and (3) we may infer (4) it's possible that some j is not a possible b . Now, this inference is indeed valid, because if we suppose (5) some j is not a possible a and (6) every possible b is a possible possible a , then it follows (by Baroco and Barbara) that (7) some j is not a possible b – given that it's necessary that what is possibly possible a is possibly a . Given this Baroco, our desired inference must be valid – by virtue of the rule that if two premises imply a conclusion then the necessity of one together with the possibility of the other implies the possibility of the conclusion (Rule 2).

$$\begin{array}{c}
\text{Baroco} \frac{(5) j \not\subset a_M}{\text{Barbara} \frac{(6) b_M \subset (a_M)_M \quad (a_M)_M \subset a_M}{b_M \subset a_M}} \\
\text{Rule 2} \rightarrow \frac{(3) M[j \not\subset a_M] \quad L[b_M \subset (a_M)_M] \quad L[(a_M)_M \subset a_M]}{(4) M[j \not\subset b_M]} \\
\text{reductio} \rightarrow \frac{(1) L[j \subset b_M] \quad \text{Barbara LLL} \frac{(2) L[b \subset a_m] \quad \text{P1} \frac{}{L[a_m \subset a_M]}}{L[b \subset a_M]} \quad \text{P4} \frac{}{L[b_M \subset (a_M)_M]} \quad \text{P5} \frac{}{L[(a_M)_M \subset a_M]}}{L[j \subset a_M]}
\end{array}$$

A fourth advantage of the combined *de dicto/de re* analysis is that it implies a generalization of an observation of Avicenna's about Barbara MMM. Recall that Avicenna observed that this syllogism depended on a principle to the effect that what is possibly possible is actually possible. A general theorem about modal syllogistics (relative to the combined *de dicto/de re* analysis of modal sentences) can be stated as follows.

Theorem. A Barbara syllogism with α as the modality of the major and β as the modality of the minor and γ as the modality of the conclusion is valid provided that (i) if $L[X \subset Y]$ then $L[X_\beta \subset Y_\beta]$, and (ii) $L[(a_\alpha)_\beta \subset a_\gamma]$.

Proof. The syllogism in question states that if it's necessary that whatever is j is b_β and it's necessary that whatever is b is a_α then it's necessary that whatever is j is a_γ . This syllogism's validity can be proved as follows, given (i) and (ii).

$$\begin{array}{c}
 \text{Barbara} \frac{j \subset b_\beta \quad b_\beta \subset (a_\alpha)_\beta \quad (a_\alpha)_\beta \subset a_\gamma}{j \subset a_\gamma} \\
 \xrightarrow{\text{Rule1}} \frac{L(j \subset b_\beta) \quad \frac{L[b \subset a_\alpha]}{L[b_\beta \subset (a_\alpha)_\beta]}^{i)} \quad L[(a_\alpha)_\beta \subset a_\gamma]^{ii)}}{L[j \subset a_\gamma]}
 \end{array}$$

In other words, just as MMM syllogisms depend on the principle $L[(a_M)_M \subset a_M]$, so LLL syllogisms depend on the principle $L[(a_L)_L \subset a_L]$, and LML syllogisms depend on the principle $L[(a_M)_L \subset a_L]$. Condition (ii) applies to different trios of modalities in different modal systems. In *S3*, it applies to LLL and LLM. In *S4*, it additionally applies to MMM, LMM, MLM. In *S5*, it additionally applies to LML. This result is not expressible in the simple *de re* analysis.

To sum up, I have shown that the combined *de dicto/de re* analysis gives just as accurate a formal representation of Avicenna's modal syllogistic as does the simple *de re* analysis. Further, I have presented four reasons for preferring it over the simple *de re* analysis. First, its greater subtlety enables us to articulate semantic conditions under which XMX and PMP syllogisms would be acceptable, and thus to reduce the question whether Avicenna accepts these syllogisms to the question whether he subscribes to those semantic conditions. Second, it accords better with Avicenna's remarks about the imperfection of Barbara MMM. Third, it is capable of representing proofs by "upgrading". And fourth, it implies a generalization of Avicenna's remark on the relation between modal syllogisms and iterated modalities. We move now to another area in which the new analysis can show its superiority.

4. Metaphysical applications of the modal syllogistic

According to Street (forthcoming) "Avicenna's syllogistic is not set up to express all the propositions which make up his metaphysics", and is "philosophically something of a disappointment".²⁷ On the other hand, Street points out that Avicenna's term "*dhātīyya*" literally means "essential,"²⁸ and this suggests a metaphysical application for Avicenna's *dhātī* modals.

Now, it may seem that the simple *de re* analysis provides a metaphysical application for Avicenna's modal syllogistic, because *de re* propositional forms are instantiated by metaphysical propositions. Are there not metaphysical propositions of the form "Every possible *j* is necessarily *b*"? One thinks of propositions whose predicate is constitutive of their subject – propositions like "A human being is corporeal" or "A triangle is a figure". Avicenna devotes some attention to such propositions;²⁹ and surely they are of the form "Every possible *j* is necessarily *b*". Every possible human being is necessarily corporeal, and every possible triangle is necessarily a figure. So, it seems that despite Street's negative comments the propositional forms studied in Avicenna's modal syllogistic are after all capable of metaphysical application.

However, Avicenna himself draws a distinction that calls this line of reasoning into question. He distinguishes between modality or absoluteness as applied to a predication and as applied to a mode. He states:

Thus if at a certain time it is assumed, for example, that there is no color except white ..., the statement 'Every color is white ...' is then true in an absolute sense by virtue of the absoluteness of the mode; before that, it was possible. But this possibility is not true if linked to the predicate. For it is not by proper possibility that every color is white. Rather, there are colors that are by necessity not white. Similarly, if we assume that at a certain time there is no animal except the human being, then 'Every animal is a human being' is true at that time in accordance with the

absolute sense of the mode. Before that [this was] in possibility. But this cannot be in possibility if possibility is made to belong to the predicate.³⁰

Avicenna's point is that statements like "There are colors that are by necessity not white" or "There are animals that by necessity are not human beings" are false in one sense and true in another. The first sense he describes as involving consideration of the mode, the second as involving consideration of the predication. Whatever this means precisely, it is worth noting that the same point can be made in relation to the simple *de re* sense of modals and absoluteness, as against the combined *de dicto/de re* sense. Simple *de re* statements apply at a certain time; combined *de dicto/de re* statements are omnitemporal. In the simple *de re* sense it is not true that there are colors that are necessarily not white, or that there are animals that are necessarily not human, if at a certain time those other colours or animals are not found in existence. By contrast, in the complex *de dicto/de re* sense it is indeed true that there are such other colors and animals. It is necessary, for example, that some possible animal is not human; and this would still be necessary if at a certain time it happened that only human animals existed.

What emerges from these reflections is that metaphysical propositions in which the predicate is constitutive of the subject do not after all exhibit a simple *de re* form. Rather their form involves both *de dicto* and *de re* elements. The metaphysical statement that humans are necessarily corporeal is before all else a statement of *de dicto* necessity. It is supposed to hold under all imaginable circumstances. True, it has a *de re* predicate: each possible human is supposed to have a necessary property, that of being corporeal. But the logical form of the whole statement is different from that of an accidental *de re* predication such as "All (actually existing) possible animals are (as it happens) necessarily human", which is true merely under the supposition that for a time no other animals exist. What it states is that it's necessary that every human is necessarily corporeal. We may conclude that necessity-propositions on the combined *de dicto/de re* analysis (but not on the simple *de re* analysis) have an application to metaphysical propositions in which the predicate is put forward as being constitutive of the subject.

Similarly, possibility-propositions on the combined *de dicto/de re* reading state that it's necessary that every *j* is a possible-*b*; and statements to this effect within an essentialist metaphysics state a natural or essential capacity or potentiality of the subject-term. So we may also conclude that possibility-propositions on the combined *de dicto/de re* analysis have an application to metaphysical propositions in which the predicate is put forward as expressing a potentiality of the subject.

What about absolute propositions? At first sight it appears that the absolute proposition in Avicenna corresponds to what Aristotle calls the *haplōs* predication. Aristotle introduces the idea of a predication taken without qualification [*haplōs*] saying:

We must understand 'that which belongs to all' with no limitation in respect of time, e.g. to the present or to a particular period, but simply without qualification. For it is by the help of such premisses that we make syllogisms, since if the premiss is understood with reference to the present moment, there cannot be a syllogism.³¹

Goichon endorses the view that Avicenna's absolute proposition corresponds to Aristotle's *haplōs* predication.³² In favour of that view it can be said that Aristotle's remarks occur in a context of discussing the mood Barbara XQM, which he regards as being invalid. Aristotle notes parenthetically that this mood would be valid if the major premise were to be taken *haplōs*. And as we have seen Barbara XMM (and thus Barbara XQM) is indeed valid when the major premise is an absolute proposition taken in the combined *de dicto/de re* sense.

But beyond this, there is no evidence to link Avicenna's absolute with Aristotle's *haplōs* predications. Aristotle gives no examples of affirmative *haplōs* predications and he does not return to the topic outside the passage quoted. He does, however, give one example of a

supposedly true negative *haplōs* proposition: nothing intelligent is a raven.³³ This proposition is certainly true without any temporal restriction; but it would not be a very satisfactory example of an Avicennan negative absolute. On Avicenna's reading of negative absolutes they state that nothing falling under the subject falls under the predicate always. Avicenna's example of such a proposition is "No men are laughing", which is true in the sense that each possible man is sometimes not laughing. (Incidentally, it should also be true, on Avicenna's understanding of the universal negative absolute proposition, that nothing laughing is laughing – since it is necessary that whatever laughs will stop laughing.) Thus, if "Nothing intelligent is a raven" were put forward as an example of a negative absolute, it would have to mean that each possible intelligent being is sometimes not a raven. This, while not false, could hardly function as a useful example of a true negative absolute, since intelligent beings are never ravens. So I do not see Goichon's view as very enlightening. It's not clear what Aristotle understands by a *haplōs* predication. It's not clear that such predications are metaphysical in nature. Nor is it clear that they correspond at all closely with Avicenna's absolute propositions. Let us start again.

Avicenna's examples of true absolute propositions include "All who sleep wake".³⁴ Now, similar propositions occur in Aristotle's short work *De Somno et Vigilia*, where we find him saying:

Likewise it is clear that [of those which either sleep or wake] there is no animal which is always awake or always asleep, but that both these affections belong [alternately] to the same animals.... But it is equally impossible also that either of these two affections should perpetually attach itself to the same animal, e.g. that some species of animal should be always asleep or always awake, without intermission; for all organs which have a natural function must lose power when they work beyond the natural time-limit of their working period; for instance, the eyes [must lose power] from too long continued seeing, and must give it up; and so it is with the hand and every other member which has a function.³⁵

For Aristotle, the proposition "The sleeping wake" is thus not just an observation-statement but is a theorem, derived from metaphysical doctrines about biological species and the natural functions of their organs. It would be a theorem of the science of any genus for which sleeping and waking are essential accidents. As a theorem the proposition is supposed to express a necessary truth. The necessity in question is natural or metaphysical necessity. Avicenna notes that in every science there are essential accidents of its subject that are investigated by the science;³⁶ his examples are "proportion and equality which belong to measurements or their genus, evenness and oddness which belong to number, and health and disease which belong to animal".³⁷ These examples make it clear that it is not characteristic of essential accidents in general that they are linked by true absolute propositions. It is not true, for example, that whatever is possibly even is at some time odd. Two essential accidents are linked by a true absolute proposition only when those accidents are naturally alternating states. Celestial phenomena such as the movements of the heavenly bodies would seem to provide further clear examples of naturally alternating states. The absolute propositions that apply to naturally alternating states may be either general or special, the special absolute stating that it's necessary that what is possibly in one of the alternating states will (alternately) be in either of them.

Another class of metaphysical propositions that fit the specifications of Avicenna's absolute is made up of statements of natural contingency. Aristotle's example is the statement that a human being will go grey.³⁸ Greying happens in the natural course of human life, though it is not inevitable. Thus, the necessity-operator that governs the statement "A human being will go grey" is a deontic rather than an alethic one, in the sense that "It's necessary that *p*" doesn't imply that *p*.

Another type of case is found in statements of final causality. For Avicenna, these include the statement that a being will exercise its "second perfection" or function, e.g. that a human

being will exercise the capacity for distinction-making.³⁹ A statement like “Human beings make distinctions” carries a kind of necessity; and again it is a deontic rather than an alethic necessity, since what is necessary in this sense may not occur in fact. For Avicenna it is necessary that this second perfection occurs – necessary in the sense that it holds in all worlds where beings attain their final causes. The absolute propositions that apply to statements of natural contingency and final causality are general, not special, absolutes. A statement of natural contingency, while implying that one state of affairs will naturally occur, does not imply this about the opposite state of affairs – though it does imply that that opposite is possible. And a statement of final causality, while implying that one state of affairs is bound to occur if beings attain their final causes, does not carry any such implication about the opposite state of affairs.

Avicenna’s notion of an absolute proposition therefore applies to at least three classes of metaphysical statement – statements linking naturally alternating states, statements of natural contingency and statements of final causality.

Notes

¹ Street (2002) p. 129.

² Thom (2003) ch.4 reduces the temporal to the alethic modalities – or at any rate fails to distinguish between them.

³ Inati (1984) 281. Inati translation.

⁴ Inati (1984) 153. Inati translation.

⁵ Inati (1984) 308. Inati translation.

⁶ Inati (1984) 386.

⁷ Inati (1984) 389.

⁸ Inati (1984) 390.

⁹ Inati (1984) 390.

¹⁰ Inati (1984) 385. Inati translation p.134.

¹¹ Inati (1984) 392. Inati translation p.136.

¹² Street (2005) p.5.

¹³ Inati (1984) 393.

¹⁴ Inati (1984) 421. Inati translates “complete”.

¹⁵ Inati (1984) 425. Inati translation.

¹⁶ Thom (2003) chapter 4.

¹⁷ Inati (1984) 226. Inati translation.

¹⁸ Zeeman (1973) p.179.

¹⁹ Zeeman (1973) p.181.

²⁰ Inati (1984) 391. Inati translation p.136.

²¹ Inati (1984) 386. Inati translation p.134.

²² Street (2002) p.142.

²³ Street (2002) p.142.

²⁴ Street (2002) p.144 also gives Avicenna’s proof of the convertibility of universal negative necessity-propositions as an instance of upgrading. But I fail to see that it can be, since upgrading seems to apply only to syllogisms.

²⁵ Street (2002) p.152.

²⁶ Aristotle (1928), Book I Chapter 15, 34a34ff.

²⁷ Street (2005) p.4.

²⁸ Street (2000) p. 55.

²⁹ Inati (1984) 153.

³⁰ Inati (1984) 291. Inati translation.

³¹ Aristotle (1928), Book 1 Chapter 15, 34b7ff. Jenkinson translation.

³² Goichon (1951) p.134 n.2.

³³ Aristotle (1928), Book 1 Chapter 15, 34b35.

³⁴ Street (2005) p.4.

³⁵ Aristotle (1931), ch.1, 454a18-30. Beare translation.

³⁶ Inati (1984) 475. Inati translation p.152.

³⁷ Inati (1984) 169-170. Inati translation p.58.

³⁸ Aristotle (1928), Book I Chapter 13, 32b7.

³⁹ Wisnovsky (2003) p.121. Wisnovsky's translation.

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