



HAL
open science

Freezeout, Compensation Rules, and Voting Equilibria

Christian At, Sylvain Béal, Pierre-Henri Morand

► **To cite this version:**

Christian At, Sylvain Béal, Pierre-Henri Morand. Freezeout, Compensation Rules, and Voting Equilibria. *International Review of Law and Economics*, 2015, 41, pp.91-102. <10.1016/j.irle.2014.10.006>. <halshs-01308314>

HAL Id: halshs-01308314

<https://shs.hal.science/halshs-01308314v1>

Submitted on 30 Aug 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Freezeout, Compensation Rules, and Voting Equilibria*

CHRISTIAN AT[†], SYLVAIN BÉAL[‡], PIERRE-HENRI MORAND[§]

2015

Abstract

A single proposer has the opportunity to generate a surplus by buying out the assets of a group of individuals. These individuals vote to accept or reject the monetary offer made to them by the proposer, who needs the agreement of a qualified majority. The voters rejecting the offer while the qualified majority is met are frozen out but they can claim compensation in exchange for their asset. This article analyses how compensation rules influence both the votes and the offer made by the proposer. The existence of a symmetric subgame perfect Nash equilibrium is ensured, and multiple equilibrium outcomes with the same offer can arise when compensation depends on the probability of acceptance by the voters. We show that increasing the offer does not always increase the probability of acceptance, in sharp contrast to many similar models. We identify the equilibrium offer when compensation does not depend on the proposal. Increasing compensation always reduces the expected social surplus and the proposer's expected profit, but does not always benefit the voters. Reinforcing the qualified majority always increases the proposer's expected profit, and can lower both the expected social surplus and the voters' expected utility.

Keywords: Voting games, Compensation, Fairness, Freezeout.

JEL Classification number: D72, K2.

1 Introduction

How are controlling shareholders seeking to acquire large corporations, developers assembling land shares, debtors restructuring debts, and attorneys entering into aggregate settlements for a class-action alike? They all have to obtain the consent of many owners of individual rights, and consequently they face the “holdout” problem. Such an agreement requires using at least some coercion and abandoning the unanimity rule.¹ While it undermines property rights, the natural question is therefore to specify what happens to the voters rejecting the offer if the qualified majority is attained. Although these voters cannot prevent the proposal from being implemented, they can still legitimately claim a payoff or compensation in exchange for their rights. In the specific case of freezeouts, Fischel (1983) notes that “the appraisal remedy is typically analyzed as an adjustment in the right of the minority necessitated by abandonment of the unanimity rule” (p.877) but that “one

*Financial support from research programs “DynaMITE: Dynamic Matching and Interactions: Theory and Experiments”, contract ANR-13-BSHS1-0010, and MODMAD is gratefully acknowledged by the second author. An earlier draft of the article circulated under the title “On the effect of compensation rules on strategic voting equilibria”.

[†]Université de Franche-Comté, CRESE (EA 3190), 30 Avenue de l’Observatoire, 25009 Besançon, France. E-mail: christain.at@univ-fcomte.fr. Tel: (+33)(0)3.81.66.68.91. Fax: (+33)(0)3.81.66.65.76.

[‡]Corresponding author. Université de Franche-Comté, CRESE (EA 3190), 30 Avenue de l’Observatoire, 25009 Besançon, France. E-mail: sylvain.beal@univ-fcomte.fr. Tel: (+33)(0)3.81.66.68.26. Fax: (+33)(0)3.81.66.65.76.

[§]LBNC – Université d’Avignon et des Pays de Vaucluse and GREQAM, 74 rue Louis Pasteur, 84029 Avignon cedex 1, France. E-mail: pierre-henri.morand@univ-avignon.fr. Tel: (+33)(0)4.90.16.28.39. Fax: (+33)(0)4.90.16.27.44.

¹The Mailath and Postlewaite (1990) Theorem implies that no private (voluntary and self-financing) transaction procedure achieves assembly with positive probability as the number of sellers grows large.

of the most important issues is the measure of compensation that dissenters are entitled to. The usual answer — ‘fair value’ — provides little guidance because this standard has several meanings” (p.886). Properly understood, the fair compensation a court may award to dissenters has profound consequences on the probability of the transaction occurring, on the terms on any transaction, and thus on the welfare of all agents, and not only on the ex post situation of the minority voters. This issue is the main concern of the present article.

We propose a model which is general enough to grasp some aspects of the various situations mentioned. A single proposer has the opportunity to generate a surplus by buying out the assets (property or rights) of a group of individuals. These individuals vote to accept or reject the monetary offer made to them by the proposer, who needs the agreement of a qualified majority of voters. A voter will receive this offer as a payment in exchange for his asset provided that he votes in favor and that the qualified majority is reached. Such situations are common in real life as pointed out below and a legal recourse is available to dissatisfied minority voters. The determination of fair compensation for dissenters is at the heart of the decision making process since it will influence both the votes and the offer made by the proposer.

The difficulty raised by the above-mentioned question of law is to determine what would constitute “fair requirements”. This question is one of the two main issues addressed in this article. Firstly, we provide various formulations of what might be considered fair compensation. In this first main issue, we ignore strategic considerations. On the contrary, these formulations rely on natural equity principles such as treating two minority shareholders equally if they own the same number of shares and if they vote the same way. Strategic considerations are incorporated into the second main issue that we address. More specifically, we analyze the equilibrium of the strategic voting game. In particular, we examine the impact of the suggested fair compensation on the two sides of the decision making process, *i.e.* on the equilibrium offer made by the proposer as well as on the equilibrium probability of acceptance.

Various aspects of the holdout problem have been carefully analyzed in the takeover literature. Since Grossman and Hart (1980)’s seminal paper pointing out the extreme “free-rider” problem when atomistic shareholders perceive themselves as non-pivotal, alternative articles such as Bagnoli and Lipman (1988) and Holmström and Nalebuff (1992) have considered the case of a finite number of shareholders who must take into account the probability that a bid will fail without their shares. Further developments in this literature incorporate multiple bidders (Burkart (1995) and Bulow et al. (1999) among others), multiple rounds of bidding (Dewatripont (1993)), and asymmetric information (Hirshleifer and Titman (1990) and Yilmaz (2000)). But the question of the fair compensation of the minority shareholders lay beyond the scope of this literature until now. Even so, this question of law has been widely disputed in legal studies. As an example, Subramanian (2007) decries the current standard as doing little to protect minority claimants from tender offer freezeouts by controlling shareholders. Conversely, Carney and Heimendinger (2003) suggest that minority shareholders often receive appraisal windfalls for which they did not pay and had no reasonable ex ante expectation of receiving. Based on Norwegian data, Bøhren and Krosvik (2013) provide empirical evidence of how courts deal with minority freezeout compensation.

The question of explicitly incorporating fair criteria into compensation determined for dissenters by a court remains open. As a result, the influence of this compensation on the votes and the proposal is an unresolved theoretical puzzle. In order to focus the study on fairness considerations, we will consider a deliberately simple framework in which the voters are symmetric and in which information is complete and symmetric. The fact that a majority of the voters accept the proposal while others dissent is sufficient to create ex post heterogeneity among the ex ante symmetric rights owners. It

should also be clear that we are not primarily interested in the design of optimal mechanisms or efficient procedures, since such achievements may be conflicting or incompatible with the fairness considerations that we are investigating.

A first general result establishes the existence of mixed-strategy subgame perfect Nash equilibria and shows that multiple equilibrium outcomes with the same offer can exist if and only if the compensation set by the court is influenced by the voters' decisions (Proposition 1). This result reveals that the relationship between the equilibrium offer and the equilibrium probability of acceptance is not always a one-to-one mapping. In particular, Corollary 1 shows that increasing the offer does not always increase the probability of acceptance, in sharp contrast to many similar models. The other results are then split into two categories. Firstly, we focus on exogenous compensation, *i.e.* compensation that depends neither on the proposer's offer nor on the voters' decisions. Proposition 3 characterizes the associated equilibrium probability of acceptance. Based on this result, various relevant effects are pointed out. Increasing the compensation always reduces the expected social surplus and the proposer's expected profit, and does not always benefit the voters (Proposition 4). Reinforcing the qualified majority always increases the proposer's expected profit, and may decrease both the expected social surplus and the voters' expected utility (Proposition 5). The equilibrium probability of acceptance increases when the voters' status quo utility increases if and only if this probability is large enough to guarantee that the proposal is implemented on average, and otherwise it is decreasing (Proposition 6). Secondly, we analyze compensation rules that may be influenced by the proposer's offer. If the compensation is equal to the proposer's offer, then there exists a dominant pure-strategy such that the proposal is accepted with probability 1 and the equilibrium offer coincides with the voter's status quo utility, so that all the surplus accrues to the proposer (Proposition 7). Furthermore, Proposition 8 examines, for a fixed level of compensation, the relationships between the equilibrium probabilities of acceptance when the compensation is exogenous and when it is endogenous in the sense that it depends on the strategic variables of the model. It is worth noting that the phenomena studied in this article rely on the simplest forms of compensation rules. Our results are therefore best regarded as suggestive and other plausible but more sophisticated types of compensation are also discussed.

The remainder of the article is organized as follows. The general model is described in section 2 while section 3 returns to its most significant interpretations. Section 4 contains the equilibrium analysis. The design of fair compensation and the study of its impact on equilibrium decisions are discussed in section 5. Section 6 concludes. All proofs are relegated to the appendix.

2 The model

We consider a single proposer and a finite even number $n > 2$ of symmetric voters. We denote by $N = \{1, \dots, n\}$ the set of n voters, each of whom holds exactly one vote. The timing can be described as follows.

- $t = 0$: the proposer makes a proposal $b \in \mathbb{R}_+$ that generates a revenue $\Delta > 0$.
- $t = 1$: the voters observe the proposal b and simultaneously and non-cooperatively cast ballots to accept or reject it.
- $t = 2$: if at least $\sigma > n/2$ voters accept, the proposal is implemented and each accepting voter enjoys a utility which we normalize to b . Otherwise the status quo prevails and every voter perceives the utility $U_0 \geq 0$. Note that σ , or equivalently σ/n , represents the qualified majority.

$t = 3$: if at least σ voters have accepted the proposal, the remaining voters, who have rejected it, are expropriated and each of them receives compensation $c(\zeta) \geq 0$ determined by a court, where ζ is the set of parameters on which the court's decision is based.

The situation described above is called a strategic qualified majority voting problem with compensation, or simply a voting problem.

3 Interpretations

Our framework enables us to model essential aspects of the following economic situations.

- *Freezeouts*: A freezeout is a transaction in which a controlling shareholder buys out the minority shareholders in a publicly traded corporation. Freezeouts often occur when a bidder has accumulated most but not all of the shares in a preceding tender offer and wants to acquire the remainder. In our model, the proposer is the controlling shareholder and the voters are the minority shareholders in a publicly traded corporation. In this context, the proposal is called a freezeout merger. The offer b made by the proposer is thus interpreted as the tender offer proposed by the controlling shareholder for buying any single share owned by the minority shareholders. This problem is examined by Maug (2006) and Subramanian (2005) among others. A typical problem arises from the valuation of freezeouts for the dissenting shareholders (valuation of $c(\cdot)$ in our notation). The compensation comes up when a court has to evaluate the rights of the dissenting shareholders. Minority freezeouts have been scrutinized by Delaware courts since they involve an inherent conflict of interest between the controlling and minority shareholders. Under Delaware law:

“Shareholders controlling at least 90% of a target’s stock can utilize a short-form merger under Del. Corp Code 253, which obviates the fairness requirements applied to freezeout bids but grants appraisal rights to minority shareholders regardless of the consideration granted.”

A first possibility for the court is to choose the market price, which does not distinguish among the voters on the basis of their decision. Another option is to select compensation that relies on previous purchases or a valuation by appraisal where a court-appointed appraiser seeks fairness. The reader is referred to Subramanian (2005) for numerous examples and an analysis of US doctrine.

- *Land assembly projects*: An assembler or developer is willing to buy adjacent land parcels belonging to different owners in order to undertake an urban project. Land assembly projects are frequently delayed or blocked by holdout landowners attempting to capture a greater share of the gains from trade. Hence, assemblers pressure local governments to condemn land on their behalf. As Heller and Hills (2008) note, eminent domain overcomes the holdout problem, but only at the expense of introducing other fairness and efficiency concerns, especially because the compensation is evaluated by the court. In our model, the proposer is the developer and the voters are the landowners. The offer is the per acre price proposed for each land plot. Various views of what can be considered as fair compensation for the unwilling sellers can be found in Miceli and Segerson (2007), which provides a comprehensive survey of the closely connected problem of the government’s right to regulate private property. Our interpretation in terms

of the land assembly problem is necessarily imperfect. For instance, it does not incorporate sequential bargaining as in Eckart (1985), O’Flaherty (1994), Strange (1995), and Miceli and Segerson (2007). In these articles, once the assembly has begun, each remaining landowner realizes that its parcel has become more important for the completion of the project. The resulting bargaining delay can favor late sellers. On the contrary, in our model, the developer purchases land properties contingent upon acquisition of adjacent parcels necessary to complete its project, which is another important aspect of the land assembly problem. This contingency also confers a bargaining advantage on each landowner, which has a similar flavor to the advantage induced by a possible negotiation delay in the aforementioned articles.

- *Debt restructuring with a prepackaged bankruptcy plan:* A debtor seeks to restructure debt claims held by n identical creditors. If a prepackaged plan b is accepted by the requisite percentages σ/n of creditors, the plan will become binding on all parties, including parties that did not vote or voted against the plan (this eliminates any economic advantage in non-participation). The regulation of debt restructuring differs from country to country. Under the U.S. Bankruptcy Code, two-thirds of the creditors who vote must accept the plan before it can be implemented (only a simple majority is required in Australia, and Canada reduced the required proportion from three-quarters to two-thirds in 1997). Using our notation, this is equivalent to setting $\sigma/n = 2/3$. Usually, we observe that $c(\zeta) = b$ is very often chosen as compensation, but other values may be chosen. An empirical investigation of this question for U.S. firms is made in Tashjian et al. (1996).
- *Aggregate settlements in class-actions:* In an aggregate settlement, all plaintiffs must consent after consultation before the agreement can be final and binding. Otherwise, a majority vote ($\sigma/n = 1/2$) can be organized to decide whether to settle (in New Jersey for example), but cannot bind all to a settlement. In other words, the dissenting plaintiffs receive $c(\zeta)$ from the trial result. For a recent study of this problem, we refer readers to Erichson and Zipursky (2011) and the references therein.

4 Equilibria

We begin with a basic equilibrium analysis of voting problems in which the compensation is not yet specified. The formal framework shares similarities with that in Bagnoli and Lipman (1988). As in the latter article, we look for symmetric mixed-strategy subgame perfect Nash equilibria for the following reasons, even if few results involve degenerated mixed-strategy subgame perfect Nash equilibria. Looking for symmetric pure-strategy subgame perfect Nash equilibria obviously leads to limited results in which no compensation is ever paid. Looking for asymmetric equilibria is generically a complex task due to the combinatorial nature of the problem. We will also concentrate on equilibria without coordination problems as in Dalkır and Dalkır (2014).²

Note that it is suboptimal for the proposer to choose b greater than the compensation, which implies that we can restrict attention to $b \in [0, c(\zeta)]$ without any loss of generality. In the remainder,

²Here, a coordination problem refers to an equilibrium situation in which the voters coordinate on a certain probability p whereas each of them could obtain a strictly better utility by coordinating on another probability. Dalkır and Dalkır (2014) circumvent coordination problems by using the so-called responsive equilibrium.

we denote by $p \in [0, 1]$ the probability with which each voter accepts the proposal.³ More specifically, each voter randomizes with probability p in response to a proposal $b \in [0, c(\zeta)]$. At any mixed strategy equilibrium, for a voter to be randomizing he must be indifferent between accepting and rejecting the proposal. If v is the number of voters who have accepted the proposal and, for each $i \in N$, v_{-i} the number of positive votes among the set $N \setminus \{i\}$ of voters other than i , then the latter equilibrium condition can be expressed as follows:

$$b\mathbb{P}_i^s + U_0(1 - \mathbb{P}_i^s) = c(\zeta)\mathbb{P}_{-i}^s + U_0(1 - \mathbb{P}_{-i}^s) \quad (1)$$

where

$$\mathbb{P}_i^s := \mathbb{P}(v_{-i} \geq \sigma - 1) = \sum_{j=\sigma-1}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j}$$

is the probability that the proposal will succeed when a given voter decides to accept and

$$\mathbb{P}_{-i}^s := \mathbb{P}(v_{-i} \geq \sigma) = \sum_{j=\sigma}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j}$$

is the probability that the proposal will succeed when a given voter decides to reject it. The equilibrium proposal must be such that the expected benefit to a voter from accepting (left-hand side of 1) equals the expected benefit from rejecting (right-hand side of 1).

It is natural to ask whether or not the proposal is efficient. In our setting, the following preliminary lemma sheds light on the relation between detrimental compensation and the possibility of equilibrium allocative inefficiency. It states that inefficient proposals can be successful at equilibrium if and only if the compensation falls below the status quo utility of a voter.

Lemma 1 *Inefficient proposals ($\Delta < nU_0$) can be implemented at equilibrium if and only if a court can punish a voter for rejecting the offer ($U_0 \geq c(\zeta)$).*

This lemma admits the following practical interpretation. For the sake of illustration, consider a freezout situation. Bøhren and Krosvik (2013) find that minority stockholders who take their case to court often face years of waiting but they also report that litigation costs are almost never paid by minority stockholders. In this case, litigation and delay costs are eventually incorporated into the compensation paid to the non-tendering shareholders. This example indicates that in order to determine compensation a court must take into account all additional costs a dissenter may bear. An implication of our lemma is thus that allocative inefficiencies can be avoided if the court makes the compensation greater than the status quo utility so as to reflect such extra costs. The actual value of the compensation can fall below the status quo utility because of long delays with the legal process. However, Lemma 1 cannot be used to draw conclusions about the relationships between allocative efficiency and the detrimental nature of the compensation. It turns out that efficient proposals can be implemented in presence of very low compensation.

In the rest of the article, and in order to concentrate on the main concerns of our paper, we make the following assumptions:

³It would be more correct to define a strategy for a voter as a function $p : [0, c(\zeta)] \rightarrow [0, 1]$ which assigns to each possible offer $b \in [0, c(\zeta)]$ available to the proposer a probability of $p(b) \in [0, 1]$ of accepting b . However, when there is no risk of confusion, we will abuse notations by writing p to refer to the (equilibrium) probability along the (equilibrium) path of play. For instance, for an equilibrium (b^*, p^*) , on top of referring to the equilibrium strategy of each voter p^* will also stand for $p^*(b^*)$.

Assumption 1 *Implementing the proposal yields a positive social surplus: $\Delta > nU_0$.*

Assumption 2 *A court cannot punish a voter for rejecting the offer: $U_0 \leq c(\zeta)$.*

Assumption 1 means that we rule out the situations in which implementing the proposal is socially sub-optimal compared to the initial situation. Altogether, these assumptions basically state that each compensated voter should get some share of the surplus generated. Since we are primarily interested in fair compensation, it only excludes situations that are not very appealing. The first proposition deals with the existence and multiplicity of symmetric mixed-strategy subgame perfect Nash equilibria.

Proposition 1 *Every voting problem has at least one symmetric mixed-strategy subgame perfect Nash equilibrium. Multiple symmetric mixed-strategy Nash equilibrium outcomes with the same equilibrium proposal $b \in [0, c(\zeta)]$ can exist if and only if the compensation depends on p .*

Proposition 1 provides a first insight into the sensitivity of the equilibrium to the form of compensation, which can have substantial consequences for the players' behavior. Two comments are in order.

Firstly, as observed in Bagnoli and Lipman (1988), Holmström and Nalebuff (1992), Ferguson (1994), and Dalkır and Dalkır (2014), the equilibrium condition (1) leads to a simple relation between the proposal b and the probability of acceptance p , that is increasing and monotonic in p . Figure 1 provides an illustration of this aspect, in which $\mathcal{P}(p) := \mathbb{P}_{-i}^s / \mathbb{P}_i^s$ (compared to our framework, $c(\zeta)$ is a constant in the aforementioned articles).

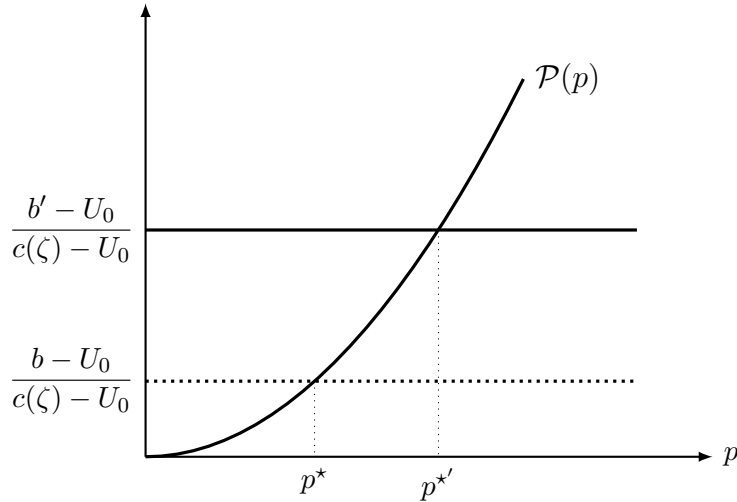


Figure 1: Uniqueness of equilibrium.

Hence, it is equivalent to consider either that the proposer chooses b to maximize his expected profit, or that the proposer targets the probability p so that the expected number of positive votes is just the required number, thereby determining the equilibrium offer b . However, such a relationship does not always hold in our model depending on the form of compensation. Choosing compensation that relies on the expected number of accepting voters can lead to multiple equilibria. Such compensation is plausible as explained in section 5.2. As a consequence, the proposer cannot always induce

a given level of p with his offer, because there may be more than one optimal level of p , at least for some levels of b . This aspect is illustrated in the next example.

NUMERICAL EXAMPLE 1 Consider the voting problem given by $n = 10$, $\sigma = 7$, $U_0 = 0$ and the compensation $c(\zeta) = 1/[p(2p - p^2)]$. For a proposal $b = 1$, the equilibrium condition (1) is satisfied for $p \in \{0.38, 0.7\}$. Besides proving the existence of multiple equilibrium outcomes featuring the same offer, this example also reveals that these equilibria can be very different. In the first equilibrium, the proposal is rejected on average since $p = 0.38$ implies that the expected number of accepting voters is 3.8, far fewer than the required threshold of 7. This situation can be explained by the high value of the compensation, which is more than 4 times larger than the offer. On the contrary, in the second equilibrium with $p = 0.7$, the proposal is accepted on average while the compensation is only 1.58 times larger than the offer. \square

Secondly, even with compensation leading to a one to one mapping from p to b , *i.e.* compensation based only on the proposal b , the equilibrium relation between the proposal and the probability of acceptance may be counterintuitive. This is summarized by the following corollary.

Corollary 1 *Increasing the offer does not always increase the probability of acceptance.*

Now, let us turn to the proposer's objective, *i.e.* maximizing his expected surplus. In order to determine this surplus, let us first consider the expected social surplus, that is:

$$E(S) = \mathbb{P}^s(\Delta - nU_0)$$

where

$$\mathbb{P}^s = \mathbb{P}(v \geq \sigma) = \sum_{j=\sigma}^n \binom{n}{j} p^j (1-p)^{n-j}$$

is the probability that the proposal will be successful. The voters' cumulated expected surplus is defined by:

$$E(U) = \mathbb{P}_i^s(b - U_0)pn + \mathbb{P}_{-i}^s(c(\zeta) - U_0)(1-p)n. \quad (2)$$

The first term is the expected number of positive votes times the expected gain of a positive vote if the proposal succeeds. The second term is the expected number of negative votes times the expected gain of a negative vote followed by compensation if the proposal succeeds. Given the equilibrium relation (1), we can replace b by its value in $E(U)$, which gives:

$$E(U) = \mathbb{P}_{-i}^s(c(\zeta) - U_0)n.$$

Therefore, the proposer's expected surplus $E(\pi)$ is defined as the difference between the expected social surplus and the expected surplus of the voters:

$$E(\pi) = E(S) - E(U) = \mathbb{P}^s(\Delta - nU_0) - \mathbb{P}_{-i}^s(c(\zeta) - U_0)n. \quad (3)$$

We conclude this section by considering the particular case when unanimity is required as a qualified majority, that is when $\sigma = n$. Under unanimity, each voter is pivotal, meaning that any voter has the power to prevent the implementation of the proposal by voting against it. Each voter's decision depends on the comparison between his status quo utility U_0 and the offer b . Hence, all voters are willing to accept any offer at least as large as U_0 . It is also clear that the expected social surplus derived from implementing the proposal can only be achieved if all voters vote in favor. These facts prove the following result.

Proposition 2 *Under the unanimity rule, the proposal is accepted with probability 1 and the equilibrium offer is $b^* = U_0$. The expected social surplus is maximal and goes to the proposer.*

The unanimity voting rule unveils the traditional trade-off between efficiency and equity. On the one hand, this rule ensures the success of any proposal that yields a positive social surplus. On the other hand, this efficient outcome can clearly be considered as unfair since the proposer captures all the generated surplus.

5 Compensation rules

The equilibrium analysis provided in the previous section leaves the problem of finding equitable or fair compensation unresolved. This section addresses this problem. Several types of compensation are proposed in order to evaluate their impact on the equilibrium probability of acceptance for the voters and on the agents' surpluses. To that end, we envisage the compensation as a function of both the strategic variables b and p (for instance through the expected number of accepting voters $v = pn$), and the parameters of the model. The model parameters are the threshold σ , the surplus Δ , the status quo utility U_0 , and the total number n of voters. Thus, we have $\zeta = \{b, p, \sigma, \Delta, U_0, n\}$. These various possibilities are grouped into two categories depending on the impact of b and p on c .

To begin with, compensation which does not rely on the strategic variables is studied. Then, the other cases, in which the compensation is influenced by the offer and/or the probability of acceptance (as well as the other parameters of the model), are examined. The former type of compensation is called exogenous, the latter endogenous. One advantage of exogenous compensation over endogenous compensation is that it cannot be manipulated by the proposer and the voters. The associated drawback is that exogenous compensation offers fewer levers for implementing central planning.

5.1 Exogenous compensations

To begin with, we consider compensation rules that do not depend on b and p . In order to save on notations, we shall write \bar{c} instead of $c(\zeta)$ when these strategic variables are not involved. The proposition below provides the equilibrium probability of voting in favor of the proposal as a function of the level of exogenous compensation.

Proposition 3 *If the compensation is exogenous, then the equilibrium probability of acceptance (which maximizes the proposer's profit along the equilibrium path) is given by*

$$p^* = \begin{cases} 0 & \text{if } \bar{c} \geq \frac{\Delta - \sigma U_0}{n - \sigma}, \\ 1 - \frac{(n - \sigma)(\bar{c} - U_0)}{\Delta - nU_0} & \text{if } \bar{c} < \frac{\Delta - \sigma U_0}{n - \sigma}, \\ 1 & \text{if } \bar{c} = U_0. \end{cases}$$

It is interesting to dwell upon the threshold which gives rise to the different cases in Proposition 3. Recall that Assumption 2 implies that the participation constraint of the accepting voters requires an offer not less than U_0 . As a consequence, the cheapest (total) cost at which the proposer can induce the implementation of its proposal is σU_0 (before compensating the other voters), *i.e.* paying

the smallest individually rational compensation U_0 to the smallest number σ of votes required. This implies that the net value remaining to the proposer in such a situation is $\Delta - \sigma U_0$, which is precisely the numerator in Proposition 3's threshold. Next, imagine that the proposer calculates what could be the greatest level of compensation each dissenting voter can credibly expect in that context. For sure, the proposer cannot afford more than a total compensation of $\Delta - \sigma U_0$. From the proposer's perspective, the $n - \sigma$ dissenting voters are identical, so that the maximal compensation each can expect is exactly $(\Delta - \sigma U_0)/(n - \sigma)$. According to this interpretation, the threshold is therefore the greatest compensation each dissenting voter can expect in the situation where the qualified majority is reached at a minimal cost by the proposer.

Another way to interpret this threshold is to rewrite the inequality when the compensation is above the threshold. We obtain

$$\sigma U_0 + (n - \sigma)\bar{c} > \Delta$$

which means that the minimal cost of acquiring the required number of votes plus the costs of compensating all other voters is greater than what the proposer can pay without incurring losses.

It is also interesting to understand the behavior of p^* when n goes to infinity. This analysis can only be carried out if at the same time σ grows sufficiently to maintain $\sigma > n/2$. Therefore, it is better to replace σ by a given proportion of the total number of voters, which we denote by $\alpha \in (1/2, 1]$. Proceeding in this fashion, we obtain

$$\lim_{n \rightarrow +\infty} \frac{\Delta - \alpha n U_0}{n - \alpha n} = -\frac{\alpha}{1 - \alpha} U_0.$$

Hence, we always have $\bar{c} > (\Delta - \alpha n U_0)/(n - \alpha n)$ giving $p^* = 0$ as shown in Proposition 3. In such a situation, the proposer's surplus also tends to zero. This is an illustration of extreme free-riding endured by the proposer: a single voter's choice has no impact on whether the proposal is accepted or not. The proposer anticipates that the voters do not perceive themselves as pivotal. As a consequence, each of them has an incentive to wait for a compensation, but in turn the proposal eventually fails to be implemented.

The legislator or the central planner has two tools for regulating freezeout: the qualified majority σ and the compensation \bar{c} . Therefore, it can be useful to analyze the influence of these tools on the results of the vote and on the distribution of the expected surplus. Each tool has a complex impact on the proposer and voters' choices. This is why they deserve two separate paragraphs. We complete the analysis by measuring the impact of the voters' status quo utility on the resulting equilibria in a third paragraph.

5.1.1 Impact of c

While the probability of acceptance is decreasing in compensation, the relation between the equilibrium proposal $b(p^*(\bar{c}), \bar{c})$ and the compensation is non monotonic. From equation (1), we have:

$$\frac{db}{d\bar{c}} = \underbrace{\mathcal{P}(p)' \frac{\partial p^*}{\partial \bar{c}} (\bar{c} - U_0)}_{<0} + \underbrace{\mathcal{P}(p)}_{>0}.$$

It also holds that

$$\left. \frac{db}{d\bar{c}} \right|_{\bar{c}=U_0} = \mathcal{P}(1) > 0 \quad \text{and} \quad \left. \frac{db}{d\bar{c}} \right|_{\bar{c} \rightarrow \frac{\Delta - \sigma U_0}{n - \sigma}} < 0.$$

For low compensation, this quantity is increasing but for high compensation it becomes decreasing. In other words, when the compensation is high, too many voters prefer to reject the offer to benefit from high compensation and the proposer prefers to bear the risk of failing to achieve a qualified majority (on average) than paying too much to induce the voters to accept. The impact of the compensation on the surplus distribution is summarized in the following proposition.

Proposition 4 *Increasing exogenous compensation always reduces the expected social surplus and the expected profit of the proposer, and does not always benefit the voters.*

If the proposal is implemented, increasing \bar{c} does not affect the social surplus since it is just a monetary transfer between the proposer and some voters, even if this influences the number of compensated voters. Nevertheless, this change reduces the probability of a success. The same negative effect is incurred by the proposer in addition to the higher compensation he has to pay. For the voters, an increase in the compensation has two opposing effects: an obvious straightforward positive effect, and an indirect negative effect since an increase in the compensation reduces the probability of the proposal succeeding and so the possibility of claiming the compensation. A direct consequence is that if a central planner wishes to give more to the voters by reducing the expected social surplus, he can increase the compensation, but his action may produce the opposite effect. This aspect is illustrated by the following example.

NUMERICAL EXAMPLE 2 Suppose that $n = 10$, $\sigma = 7$, $U_0 = 1$ and $\Delta = 15$. By choosing the compensation $\bar{c} = U_0$ we have $p^* = 1$ and $E(S) = E(\pi) = 5$ and $E(U) = 0$. Now, assume that the central planner wants to increase the voters' expected utility by increasing the compensation. For a 40% increase, the compensation becomes $\bar{c} = 1.4$, and it holds that $p^* = 0.76$, $E(S) = 4$, $E(\pi) = 1.5$, and $E(U) = 2.5$. For a 50% increase, the compensation becomes $\bar{c} = 1.5$, and this yields a reduced probability $p^* = 0.7$, and $E(S) = 3.25$, $E(\pi) = 0.94$, and $E(U) = 2.31$. Therefore, while the first 40% increase benefits for the voters, the extra 7.14% raising \bar{c} from 1.4 to 1.5 is damaging to the voters. \square

REMARK. The special case $\bar{c} = \Delta/n$, in which each individual who votes against the proposal gets an equal share of the surplus, enables us to recover a well-known result in the associated literature. In such a situation, we get $p^* = \sigma/n$ as in Bagnoli and Lipman (1988) and Holmström and Nalebuff (1992) within the framework of a takeover.

5.1.2 Impact of σ

The equilibrium probability of acceptance is increasing in the qualified majority. As σ increases, each voter is more likely to be pivotal and tends to accept the proposal more frequently (recall that in the limit case under unanimity, the probability of acceptance is equal to one). However, the relation between the equilibrium proposal $b^*(\sigma)$ and the qualified majority is non monotonic. A change in σ has significant consequences that are summarized in the following proposition.

Proposition 5 *At any equilibrium of a voting problem with exogenous compensation, reinforcing the qualified majority always raises the proposer's expected profit, and may reduce both the expected social surplus and the expected utility of the voters.*

Two opposing effects are observed: a higher σ increases the equilibrium probability of acceptance but at the same time this makes it more difficult to obtain enough positive votes. Hence, we observe

ambiguous consequences for both the expected social surplus and the expected utility of voters. However, we obtain the counterintuitive result on the proposer's expected profit found by Holmström and Nalebuff (1992): raising the qualified majority increases the proposer's expected profit. The reason is that when a greater number of votes is required for success, this raises the probability with which each voter is pivotal.

5.1.3 Impact of U_0

The impact of the status quo utility U_0 on the equilibrium probability of acceptance is detailed in the following result.

Proposition 6 *The equilibrium probability of acceptance increases when the voters' status quo utility increases (a larger U_0) if and only if this probability is large enough to guarantee that the proposal is implemented on average. Otherwise it is decreasing.*

The phenomena pointed out by Proposition 6 can be represented in Figure 2. We consider two situations, which only differ in that U'_0 is greater than U_0 . Obviously, the impact of the status quo utility U_0 on the equilibrium probability of acceptance is not monotonic. Let us distinguish two cases.

Firstly, consider case 1 in which $p^* > \sigma/n$ (or equivalently $\bar{c} < \Delta/n$ as shown in the proof of Proposition 6 and as represented in the picture), which means that on average the proposal is implemented. If a voter rejects the offer, then the probability of obtaining the compensation \bar{c} is larger than the probability of obtaining the status quo utility U_0 . The expected payoff of such a rejection therefore decreases if the voters are richer in the initial situation ($U'_0 > U_0$). In a sense, a voter has more to gain by rejecting the offer when he is poor (U_0) than when he is rich (U'_0). This stems from the fact that when a voter becomes richer, the compensation becomes less valuable, which in turn implies that it is less in the interest of the voter to reject the offer, *i.e.* the equilibrium probability of acceptance is increasing in U_0 .

Secondly, consider case 2 in which $p^* < \sigma/n$, which means that on average the proposal is not implemented. The reasoning is reversed. If a voter rejects the offer, then the probability of obtaining the compensation \bar{c} is lower than the probability of obtaining the status quo utility U_0 . The expected payoff of rejecting the offer therefore increases if the voters are richer in the initial situation ($U'_0 > U_0$). As such, a voter has more to lose by rejecting the offer when he is poor (U_0) than when he is rich (U'_0), *i.e.* the equilibrium probability of acceptance is decreasing in U_0 .

To sum up, rich voters are more inclined to accept the proposal than poor voters when the compensation is small, while opposing incentives arise when the compensation is large. As shown by the slope of the two functions in Figure 2, the equilibrium probability of acceptance is more and more decreasing in compensation as the voters become richer in the status quo situation.

We can supplement Proposition 6 by statements about the level of the equilibrium offer. Differentiating the equilibrium relation (1) using the interior solution of p yields:

$$\frac{db(p^*(U_0), U_0)}{dU_0} = \underbrace{\frac{\partial b(p^*(U_0), U_0)}{\partial U_0}}_{>0} + \underbrace{\frac{\partial b(p^*(U_0), U_0)}{\partial p} \frac{\partial p^*(U_0)}{\partial U_0}}_{\leq 0} \quad (4)$$

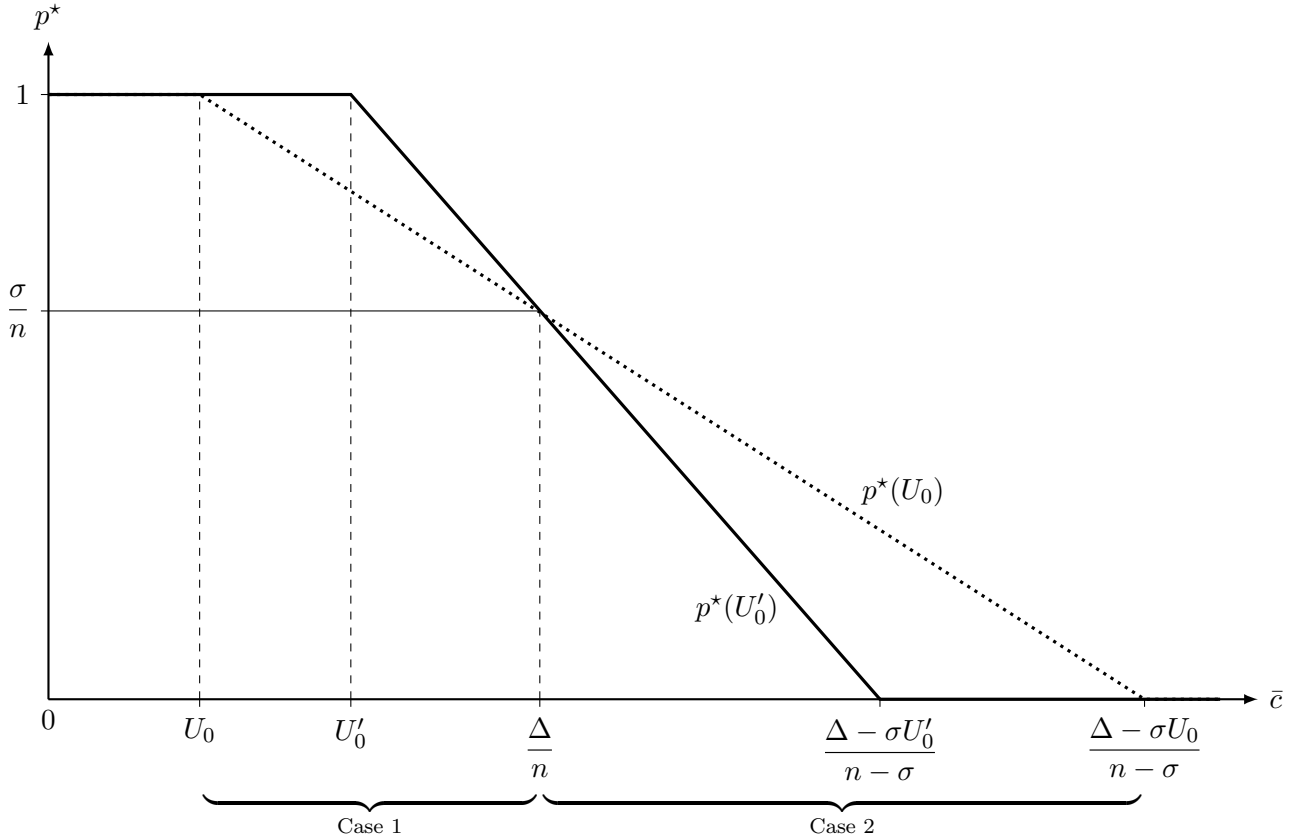


Figure 2: Evolution of the equilibrium probability of acceptance with the compensation levels for two values $U'_0 > U_0$ of the status quo utility.

The first term on the right hand side of (4) is a direct effect, which is always positive: increasing the status quo utility encourages each voter to reject the offer and so, the proposer must increase his offer.

The second term of the right hand side of (4) is an indirect effect since a variation in U_0 impacts the equilibrium probability of acceptance, which in turn influences the equilibrium offer. This indirect effect is positive when the equilibrium probability of acceptance is large enough to guarantee that the proposal is implemented on average (from proposition 6 we have $\partial p^*(U_0)/\partial U_0 > 0$ in this case), which implies that the equilibrium offer increases when the voters' status quo utility increases (a larger U_0). Otherwise, this indirect effect is negative mitigating the positive direct effect.

5.2 Endogenous compensation

This section examines endogenous compensation rules, which are based on at least one of the two strategic variables of the model: the offer b and the symmetric probability of acceptance p .

5.2.1 Compensations depending on the offer but not on the probability of acceptance

A court can base its choice of compensation on the proposer's offer. For instance, in a takeover environment, Amihud et al. (2004) argue that the current practice in the US corresponds to a freezeout price that is the larger value between the market price before the tender offer and the price paid in

the tender offer (b in our framework).

The most simple endogenous compensation rule relying on the offer is to give the proposer's offer to the minority voters, $c(\zeta) = b$. It is worth noting that setting $c(\zeta) = b$ does not amount to an absence of compensation.

Proposition 7 *For any voting problem with compensation $c(\zeta) = b$, there exists a unique (in the absence of coordination failure) symmetric subgame perfect Nash equilibrium $(b^*, p^*(b))$ with the following properties: it is weakly dominant, $b^* = U_0$ and $p^*(b^*) = 1$, so that all the surplus accrues to the proposer.*

Two comments on the choice of such a compensation can be made. Firstly, it is not harmful for the voters since none of them is worse off after implementing the proposal than before. Secondly, it is not very equitable since the whole created surplus $\Delta - nU_0$ goes to the proposer. The same result can be obtained by setting the compensation equal to U_0 . The difference is that when $c(\zeta) = b$, the compensation is equal to $\bar{c} = U_0$ as a result of the strategic choice of the proposer and not arbitrarily fixed at U_0 right from the beginning.

Under more general compensation rules, we may establish the following result:

Proposition 8 *Fix a level of compensation $c(\zeta)$. Denote by p_{exo} and p_{endo} the corresponding equilibrium probabilities of acceptance under exogenous and endogenous compensations respectively. It holds that:*

- if $\partial c(\zeta)/\partial b < 0$, then $p_{exo} < p_{endo}$,
- if $\partial c(\zeta)/\partial b \in (0, 1]$, then $p_{exo} > p_{endo}$,
- if $\partial c(\zeta)/\partial b \geq 1$, then the comparison is ambiguous.

The first (second) case can be interpreted by the fact that having $\partial c(\zeta)/\partial b < 0$ (> 0) creates an additional marginal benefit (cost) from increasing the offer of the proposer.

Both situations considered in Proposition 8 are interesting in terms of economics and fairness. Firstly, assume that the compensation is increasing in the offer made by the proposer, which means that the higher the proposer's offer, the greater the compensation a rejecting voter can expect. In such a situation, Proposition 8 states that a voter is less likely to accept the proposal than in a situation where the compensation is exogenous. The intuitive explanation is that a rejecting voter has a greater incentive to reject the proposal in order to claim the compensation. Secondly, assume that the compensation is decreasing in the offer made by the proposer. This second type of situation also makes sense because one can consider that rejecting a larger offer should not be stimulated by an increasing compensation. Proposition 8 then states that a voter has less incentive to reject the proposal.

Proposition 8 also states that for a given level of compensation, the equilibrium probability of acceptance is greater when the compensation is decreasing in the offer than when the offer is weakly increasing, *i.e.* increases but less than the offer. Since the expected social surplus is increasing in the probability of acceptance, a court seeks to induce as high a probability as possible. According to Proposition 8, the form of the compensation is just as crucial as its amount. If a social planner selects compensation that depends on the offer, he must carefully calibrate endogenous compensation

depending on b and exogenous compensation.

Consider the voting problem $n = 10$, $\sigma = 7$, $U_0 = 0$ and $\Delta = 100$. Suppose that a court decides to set the compensation level to the average between the offer and the maximal per-capita surplus each voter may obtain if the proposal succeeds. Formally, $c(\zeta) = (b + \Delta/n)/2 = 0.5b + 5$. Facing such a compensation rule, the voters determine their probability of accepting each possible offer, which is represented in the left part of Figure 3. It induces the expected profit for the proposer's function represented in the right part of Figure 3, as a function of his offer.

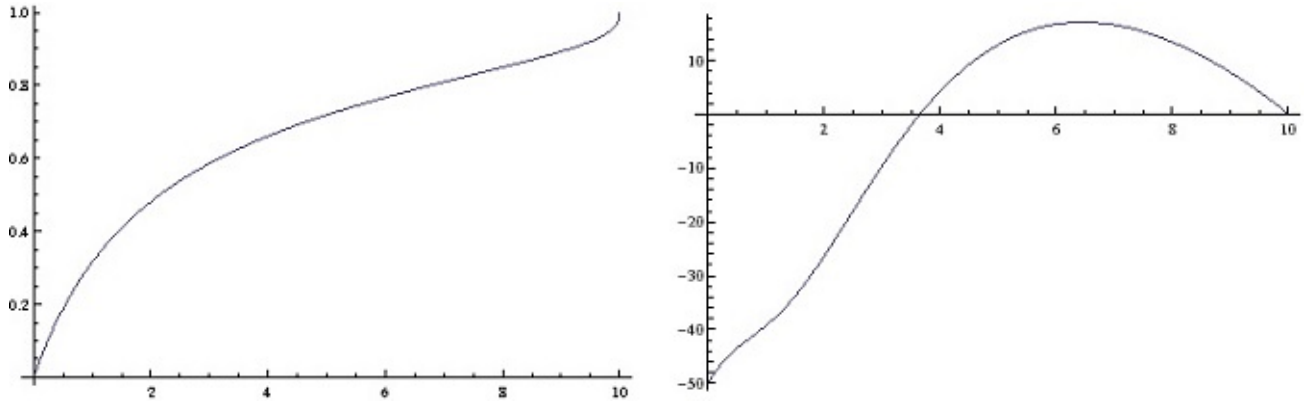


Figure 3: Optimal probability of acceptance and profit with endogenous compensation

Hence, the proposer's equilibrium offer is $b = 6.45$. Each voter will accept the offer with probability $p(6.45) = 0.78$ so that the proposal is implemented with probability 0.85. On the equilibrium outcome, each dissenter obtains a compensation of 8.22.

Now consider the following alternative compensation policy: the compensation is (exogenously) set to 8.22, whatever the offer made by the proposer. This amount is the compensation level on the equilibrium outcome of the previous case. Of course the compensation level in the two cases differs whenever the offer fails to coincide with the equilibrium offer 6.45 found in the first case. Facing such a compensation rule, the optimal strategy $p(b)$ corresponds to the left-hand part of Figure 4, and induces the profit function for the proposer given by the right-hand part of Figure 4.

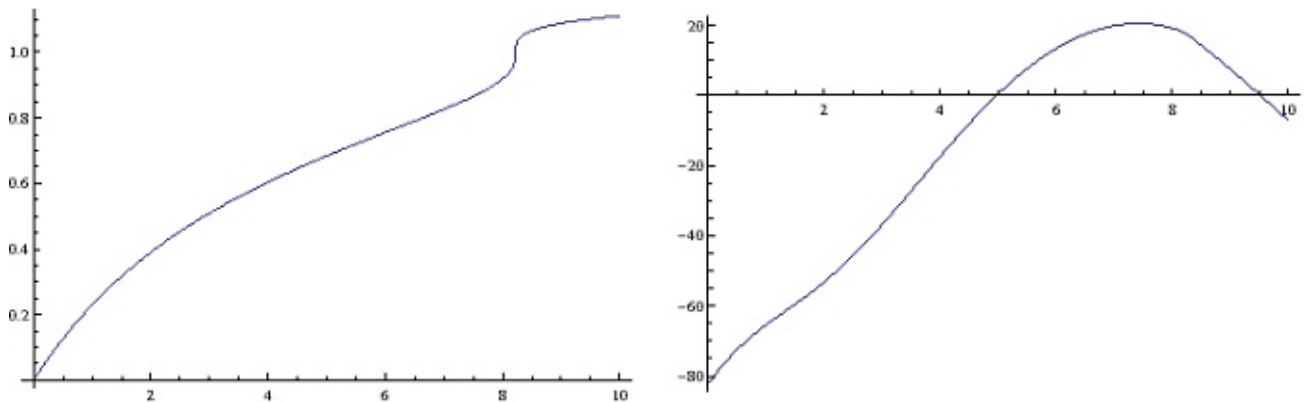


Figure 4: Optimal probability of acceptance and profit with exogenous compensation

As a consequence the equilibrium offer is $b = 7.42$. Each voter will accept this offer with probability $p(7.42) = 0.85$, and the proposal is implemented with probability 0.96. Obviously all dissenters obtain a compensation of 8.22. This illustration sheds light on the sensitivity of the equilibrium to the court's policy. Even if the compensation obtained by the dissenting voters is identical on both equilibria, the form given by the court to the compensation rule has a crucial impact on the strategic behavior of the voters and the proposer and in turn on the equilibrium offer and probability of acceptance.

5.2.2 Discussion of other types of endogenous compensation

The results provided in this section are incomplete, mainly for technical reasons. In particular, we have restricted the analysis to compensation that is not influenced by the number of accepting voters. In the context of a minority freezeout, Bøhren and Krosvik (2013, p. 49) show that

“[...] the litigation premium tends to be higher the larger the case. This finding suggests that even though both the majority and the court may consider minority stockholder rights in general, the minority's ability to insist on these rights in court is greater the larger their aggregate claim.”

In our model, each voter owns exactly one vote/share. Thus, the above-mentioned aggregate claim is proportional to the number of non-tendering voters $n - v = n(1 - p)$, which means that the court's decision may depend on the equilibrium probability of accepting the offer. Example 1 features compensation which is (non-linearly) increasing in the expected number of non-tendering voters.

It would be interesting to extend our current work to this kind of compensation rule. Nonetheless, making the compensation rule rely on p almost always prevents any analytical solution from being deduced. An alternative way to construct more sophisticated compensation rules is to determine a finer evaluation of the real surplus which is achieved when the proposal is implemented, but before distributing the compensation to the unwilling voters. The success of the proposal generates a value Δ , which accrues to the proposer. However, the proposer's final payoff is not equal to this entire value. In fact, the $v \geq \sigma$ voters who have accepted the proposal are each paid b by the proposer, meaning that only $\Delta - vb$ is still available to the proposer. It makes sense to think that the level of compensation should be evaluated in light of the loss $-vb$ borne by the proposer to ensure the implementation of its proposal. Since the v accepting voters have left the situation with their payoffs, we only have to study the smaller $n - v - 1$ problem in which the $n - v$ remaining voters bargain with the proposer. Since each remaining voter enjoys a utility U_0 for owning his single share, the total surplus still available can therefore be measured by $\Delta - vb - (n - v)U_0$. Recall that in this problem, the proposer already owns v shares. If we accept the principle that each bargaining entity has to get a portion of the above-mentioned surplus which is proportional to its number of shares, then it is reasonable to give to each remaining voter a compensation that consists of his status quo utility U_0 plus a fraction $1/n$ of the available surplus. Formally, we get:

$$c(\zeta) = U_0 + \frac{\Delta - vb - (n - v)U_0}{n}.$$

Other similar compensation rules can be constructed by using different fair requirements, for instance by assuming that the available surplus is divided equally among the $n - v - 1$ bargainers instead of proportionally to their respective number of shares. If such sophisticated compensation rules are

incorporated into the model, then it is likely that the analysis would become intractable, preventing us from stating general results. Nevertheless, numerical simulations can help us to understand their impact on the decision makers' choices. The discussion of these aspects is left for future work.

6 Conclusion

Voting problems display many important facets that deserve to be analyzed. These include acquiring information in an incomplete information setting, measuring the impact of sequential voting, taking the heterogeneity of the voters into account, and understanding the role of compensation. To the best of our knowledge, no article in the associated literature has managed to examine all these aspects simultaneously. Our work, which provides a seminal formal examination of the influence of compensation on voting equilibria but leaves out the other above-mentioned aspects, is no exception to the rule. While being aware of the limitations with this approach, it seems appropriate to conclude this article by revisiting some key messages that our study conveys in terms of public policy.

- A central planner can easily avoid the successful implementation of an inefficient proposal by setting compensation not lower than the value of a dissenting voter's asset.
- While a central planner can always target an equilibrium level for the proposer's offer by calibrating the compensation (in the absence of coordination failure), there can be no certainty about whether or not the corresponding proposal will be implemented unless the compensation is independent of the voters' choices.
- A central planner will fail to reach an equitable equilibrium outcome by imposing a unanimity voting rule or by matching the level of the compensation with those of the proposer's offer.
- The central planner has to be careful when adjusting the level on an exogenous compensation or the qualified majority since the effect on the dissenting minority is ambiguous.

Appendix

Proof of Lemma 1

Since the cases where the equilibrium probability of acceptance is either 0 or 1 are obvious, we shall only detail the proof for interior equilibrium probabilities of acceptance. So denote by $D(b, p)$ the difference between the expected benefit to a voter from accepting equals the expected benefit from rejecting. In a mixed strategy subgame perfect Nash equilibrium, we must have $D(b, p) = 0$. This is equivalent to:

$$\frac{b - U_0}{c(\zeta) - U_0} = \mathcal{P}(p) \tag{5}$$

where $\mathcal{P}(p) := \mathbb{P}_{-i}^s / \mathbb{P}_i^s$. Firstly, let us study $\mathcal{P}(p)$. It is straightforward that $\mathcal{P}(p)$ is continuous in p and that $\mathcal{P}(p) \in [0, 1]$. When $X \sim \mathcal{B}(n, p)$, we know that the cumulative distribution function can be represented in terms of the regularized incomplete beta function:

$$\mathbb{P}(X \geq x) = I_p(n - x + 1, x) = x \binom{n}{n - x} \int_0^p (1 - \epsilon)^{n-x} \epsilon^{x-1} d\epsilon.$$

Hence, we can write

$$\mathcal{P}(p) = \frac{I_p(n - \sigma, \sigma)}{I_p(n - \sigma + 1, \sigma - 1)}.$$

A little calculation gives

$$\mathcal{P}(p)' = \frac{(n - \sigma)(n + 1 - \sigma) \int_0^p (1 - \epsilon)^{n - \sigma - 1} \epsilon^{\sigma - 2} (p - \epsilon) d\epsilon}{n(\sigma - 1) \left(\int_0^p (1 - \epsilon)^{n - \sigma} \epsilon^{\sigma - 2} d\epsilon \right)^2} > 0.$$

Consider first the case where $c(\zeta) < U_0$. Then, if $b > U_0$, the left hand side of (5) is negative so that with $\mathcal{P}(p)$ positive and increasing, there is no p that satisfies $D(b, p) = 0$.

The expected amount paid by the proposer corresponds to $n(pb + (1 - p)c(\zeta))$. The proposer will make an offer which implements his proposal if and only if $n(pb + (1 - p)c(\zeta)) < \Delta$, and inefficiency may occur if $\Delta < nU_0$. Notice that $n(pb + (1 - p)c(\zeta)) \geq nb$. But $nb < nU_0$ only for $b < U_0$, which means that $c(\zeta) < U_0$ is a necessary condition for a symmetric mixed equilibrium to exist. Hence, allocative inefficiency may occur only in presence of detrimental compensation, *i.e.* $c(\zeta) < U_0$.

Proof of Proposition 1

Under assumptions A1 and A2, the left hand side of (5) lies within the interval $[0, 1]$. We have to study two cases:

1. if $c(\zeta)$ does not depend on p then, by the intermediate value theorem, there exists a unique p that satisfies $D(b, p) = 0$.
2. if $c(\zeta)$ depends on p (e.g. through the expected number of voters who have accepted the proposal $v = pn$), then there may exist more than one p that satisfy $D(b, p) = 0$ for a given proposal b . An illustration is given by the numerical example 1.

Proof of Corollary 1

Consider compensation depending only on b (note that considering compensation also based on p would not affect the nature of the result). From the Proposition 1, for each $b \in [0, c(\zeta)]$, there exists a unique p that satisfies $D(b, p) = 0$. The implicit function theorem establishes that:

$$\frac{dp}{db} = \frac{1 - \frac{\partial c(\zeta)}{\partial b} \mathcal{P}(p)}{(c(\zeta) - U_0) \mathcal{P}(p)'} \quad (6)$$

The inequality $(c(\zeta) - U_0) \mathcal{P}(p)' \geq 0$ holds whatever the compensation. Hence, $dp/db < 0$ if the marginal compensation $\partial c(\zeta)/\partial b$ is greater than the ratio $1/\mathcal{P}(p)$.

Proof of Proposition 3

Since the compensation is exogenous, we have both $\partial c(\zeta)/\partial b = 0$ and $\partial c(\zeta)/\partial p = 0$. From the implicit function theorem, we know that b is increasing in p . Thus, it is possible to maximize the proposer's expected surplus according to p . We get the following first order condition:

$$\begin{aligned}
\frac{dE(\pi)}{dp} &= \frac{(1-p)^{n-\sigma} p^{\sigma-1} (\Delta - nU_0)}{\text{Beta}(n+1-\sigma, \sigma)} - \frac{n(1-p)^{n-1-\sigma} p^{\sigma-1} (\bar{c} - U_0)}{\text{Beta}(n-\sigma, \sigma)} \\
&= (1-p)^{n-\sigma-1} p^{\sigma-1} \sigma \left((\Delta - nU_0)(1-p) \binom{n}{n-\sigma} - n \binom{n-1}{n-\sigma-1} (\bar{c} - U_0) \right) \\
&= (1-p)^{n-\sigma-1} p^{\sigma-1} n\sigma \binom{n-1}{\sigma} \left[(1-p) \frac{\Delta - nU_0}{n-\sigma} - \bar{c} + U_0 \right] = 0. \tag{7}
\end{aligned}$$

This yields three candidates for the equilibrium probability: 0, 1 and

$$p_0 := 1 - \frac{(n-\sigma)(\bar{c} - U_0)}{\Delta - nU_0}.$$

The second derivative valued at $p = 0$ and $p = 1$ gives

$$\left. \frac{d^2 E(\pi(p))}{dp^2} \right|_{p=0} = \left. \frac{d^2 E(\pi(p))}{dp^2} \right|_{p=1} = 0,$$

so that only p_0 remains. We have:

$$\left. \frac{d^2 E(\pi(p))}{dp^2} \right|_{p=p_0} = -n\sigma(\bar{c} - U_0) \binom{n-1}{\sigma} \left(1 - \frac{(n-\sigma)(\bar{c} - U_0)}{\Delta - nU_0} \right)^{\sigma-1} \left(\frac{(n-\sigma)(\bar{c} - U_0)}{\Delta - nU_0} \right)^{n-2-\sigma} < 0.$$

There exists an interior solution $0 < p_0 < 1$ if and only if $U_0 < \bar{c} < (\Delta - \sigma U_0)/(n - \sigma)$. The candidate p_0 is a local maximum and also a global one since it is unique.

Proof of Proposition 4

Some comparative statics give:

- $\left. \frac{\partial E(S)}{\partial \bar{c}} \right|_{p=p^*} = \frac{\partial E(S)}{\partial p} \frac{\partial p}{\partial \bar{c}}$. From (4), we have $\frac{\partial E(S)}{\partial p} = \frac{d\mathbb{P}^s(p)}{dp} (\Delta - nU_0) > 0$. We know that $\frac{\partial p^*}{\partial \bar{c}} > 0$, hence $\left. \frac{\partial E(S)}{\partial \bar{c}} \right|_{p=p^*} > 0$.
- Let us write the expected surplus of all voters as

$$E(U) = \mathbb{P}_i^s(b(p(\bar{c}), \bar{c}) - U_0)pn + \mathbb{P}_{-i}^s(c(\zeta) - U_0)(1-p)n,$$

where $b(p(\bar{c}), \bar{c})$ is given by the equilibrium relation (1). We have

$$\left. \frac{\partial E(U)}{\partial \bar{c}} \right|_{p=p^*} = \frac{\partial E(U)}{\partial p} \frac{\partial p}{\partial \bar{c}} + \frac{\partial E(U)}{\partial b} \left(\frac{\partial b}{\partial \bar{c}} + \frac{\partial b}{\partial p} \frac{\partial p}{\partial \bar{c}} \right) + \frac{\partial E(U)}{\partial \bar{c}}.$$

Since we know that $\frac{\partial b(p^*(\bar{c}), \bar{c})}{\partial \bar{c}}$ has an indeterminate sign, this is also the case for $\frac{\partial E(U)}{\partial \bar{c}}$.

- As before, let us write the proposer's expected profit as

$$\begin{aligned}
E(\pi) &= E(S) - E(U) \\
&= \mathbb{P}^s(\Delta - nU_0) - (\mathbb{P}_i^s(b(p(\bar{c}), \bar{c}) - U_0)pn + \mathbb{P}_{-i}^s(c(\zeta) - U_0)(1-p)n).
\end{aligned}$$

Since $E(\pi)$ is maximized for p^* and so for $b(p^*(\bar{c}), \bar{c})$, the envelope theorem gives

$$\left. \frac{\partial E(\pi)}{\partial \bar{c}} \right|_{p=p^*} = \frac{\partial E(\pi)}{\partial \bar{c}} = -\mathbb{P}_{-i}^s < 0.$$

Proof of Proposition 5

In this proof, we shall use subscripts σ and $\sigma+1$ in order to distinguish among situations in which the qualified majorities require σ and $\sigma+1$ votes in favor respectively. For instance $E(\pi(p_\sigma))_{\sigma+1}$ will stand for the proposer's expected surplus with qualified majority $\sigma+1$ and when the voters choose the equilibrium probability with qualified majority σ .

Firstly, we prove that raising the majority increases the proposer's expected surplus. So suppose that the qualified majority increases from σ to $\sigma+1$. In the new situation, the proposer's expected profit is thus maximized for

$$p_{\sigma+1} = 1 - \frac{(n - \sigma - 1)(\bar{c} - U_0)}{\Delta - nU_0},$$

which yields the following maximum:

$$E(\pi(p_{\sigma+1}))_{\sigma+1} = \sum_{j=\sigma+1}^n \binom{n}{j} (p_{\sigma+1})^j (1 - p_{\sigma+1})^{n-j} A - \sum_{j=\sigma+1}^{n-1} \binom{n-1}{j} (p_{\sigma+1})^j (1 - p_{\sigma+1})^{n-1-j} B,$$

where $A = \Delta - nU_0$ and $B = (\bar{c} - U_0)n$. It holds that:

$$\begin{aligned} E(\pi(p_{\sigma+1}))_{\sigma+1} &> E(\pi(p_\sigma))_{\sigma+1} \\ &= \sum_{j=\sigma+1}^n \binom{n}{j} (p_\sigma)^j (1 - p_\sigma)^{n-j} A - \sum_{j=\sigma+1}^{n-1} \binom{n-1}{j} (p_\sigma)^j (1 - p_\sigma)^{n-1-j} B \\ &= \sum_{j=\sigma}^n \binom{n}{j} (p_\sigma)^j (1 - p_\sigma)^{n-j} A - \sum_{j=\sigma}^{n-1} \binom{n-1}{j} (p_\sigma)^j (1 - p_\sigma)^{n-1-j} B \\ &\quad - \binom{n}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-\sigma} A + \binom{n-1}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-1-\sigma} B \\ &= E(\pi(p_\sigma))_\sigma - \binom{n}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-\sigma} A + \binom{n-1}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-1-\sigma} B \\ &= E(\pi(p_\sigma))_\sigma + \binom{n}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-1-\sigma} \left(-(1 - p_\sigma)A + \frac{n - \sigma}{n} B \right). \end{aligned}$$

From Proposition 3, we know that for a majority σ , the equilibrium probability is

$$p_\sigma = 1 - \frac{(n - \sigma)(\bar{c} - U_0)}{\Delta - nU_0}.$$

Using the latter expression in the previous inequality gives the desired result $E(\pi(p_{\sigma+1}))_{\sigma+1} > E(\pi(p_\sigma))_\sigma$.

Secondly, we know that both $E(U)$ and $E(S)$ are increasing in p since p is increasing in σ . As a consequence:

$$E(U(p_{\sigma+1}))_{\sigma+1} > E(U(p_\sigma))_{\sigma+1} = E(U(p_\sigma))_\sigma - \binom{n-1}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-1-\sigma}$$

and

$$E(S(p_{\sigma+1}))_{\sigma+1} > E(S(p_\sigma))_{\sigma+1} = E(S(p_\sigma))_\sigma - \binom{n}{\sigma} (p_\sigma)^\sigma (1 - p_\sigma)^{n-\sigma}.$$

We can conclude that the impact of σ is not monotonic. The following numerical example shows that both the expected social surplus and utility of the voters can decrease when σ increases. Suppose that $n = 10$, $U_0 = 0.1$, $\Delta = 20$ and $\bar{c} = 1.2$. Then, it holds that $E(S)_{\sigma=6} = 17.89 > E(S)_{\sigma=7} = 17.5$ and $E(U)_{\sigma=6} = 9.54 > E(U)_{\sigma=7} = 8.86$.

Proof of Proposition 6

By Proposition 3, we can focus on the situations which satisfy $U_0 \leq \bar{c} \leq (\Delta - \sigma U_0)/(n - \sigma)$ since the equilibrium probability of acceptance is constant otherwise. In such situations, recall that the equilibrium probability of acceptance is given by

$$p^* = 1 - \frac{(n - \sigma)(\bar{c} - U_0)}{\Delta - nU_0}$$

The partial derivative of p^* according to U_0 is positive if and only if $\bar{c} < \Delta/n$. Next, let us look for an equilibrium probability of acceptance which is large enough to ensure that the proposal is implemented on average. In other words, since all voters are identical, we need p^* to satisfy $np^* \geq \sigma$. Replacing p^* by its above-mentioned expression, it can easily be checked that the inequality $np^* \geq \sigma$ is indeed equivalent to $\bar{c} \leq \Delta/n$. As a consequence, we conclude that p^* is increasing in U_0 if and only if $np^* \geq \sigma$.

Proof of Proposition 7

Consider a voting problem in which $c(\zeta) = b$. Therefore, a voter's expected surplus reduces to

$$p(\mathbb{P}_i^s - \mathbb{P}_{-i}^s)(b - U_0),$$

which implies that, in the absence of coordination failure, the best-response correspondence of each voter is

$$p(b) = \begin{cases} 0 & \text{if } b < U_0, \\ [0,1] & \text{if } b = U_0, \\ 1 & \text{if } b > U_0. \end{cases}$$

Fix any $q \in [0, 1]$, and construct the optimal strategy p^q for each voter defined as

$$p^q(b) = \begin{cases} 0 & \text{if } b < U_0, \\ q & \text{if } b = U_0, \\ 1 & \text{if } b > U_0. \end{cases}$$

Anticipating p^q , the expected profit of the proposer from offering b is

$$E(\pi) = \begin{cases} 0 & \text{if } b < U_0, \\ nq(\Delta - nU_0) & \text{if } b = U_0, \\ (\Delta - nb) & \text{if } b > U_0. \end{cases}$$

It is easy to check that the pair (U_0, p^1) is a subgame perfect Nash equilibrium with the desired properties. In particular, it is weakly dominant since each voter is indifferent between p^1 and any other p^q , $q \neq 1$. In order to complete the proof, consider any alternative optimal strategy p^q with $q \neq 1$ for each voter. Then, from assumption 1, we know that $b < U_0$ cannot be a best response for the proposer since $\Delta - nU_0 > 0$. The offer $b = U_0$ is not a best response either since the

associated expected profit $nq(\Delta - nU_0)$ is strictly less than what the proposer can obtain by offering $b = U_0 + \varepsilon$ for any $\varepsilon \in (0, (1 - qn)(\Delta - nU_0)/n)$. By a similar argument, it is easy to conclude that the proposer has indeed no best response against p^q , $q \neq 1$, since any offer $b > U_0$ yields a smaller expected surplus than an offer $b - \varepsilon$ for a sufficiently small and positive ε . This means that the above-mentioned subgame perfect Nash equilibrium is unique.

Proof of Proposition 8

We have $\partial c(\zeta)/\partial p = 0$. From Corollary 1, it holds that:

$$\frac{db}{dp} = \frac{(c(\zeta) - U_0)\mathcal{P}(p)'}{1 - \frac{\partial c(\zeta)}{\partial b}\mathcal{P}(p)}. \quad (8)$$

Since $\text{sign}(db^*/dp) = \text{sign}(1 - (\partial c(\zeta)/\partial b)\mathcal{P}(p))$ and $\mathcal{P}(p) \in [0, 1]$, this implies three cases:

- if $\partial c(\zeta)/\partial b < 0$, then $db^*/dp > 0$,
- if $\partial c(\zeta)/\partial b \in (0, 1]$, then $db^*/dp > 0$,
- if $\partial c(\zeta)/\partial b \geq 1$, then the sign of db^*/dp rests on the level of $\mathcal{P}(p)$ and there is no one-to-one mapping from p to b .

Maximizing the proposer's expected surplus gives the following first order condition:

$$(1 - p)^{n-\sigma-1} p^{\sigma-1} n\sigma \binom{n-1}{\sigma} \left[(1 - p) \frac{\Delta - nU_0}{n - \sigma} - c(\zeta) + U_0 \right] - n\mathbb{P}_{-i}^s \frac{\partial c(\zeta)}{\partial b} \frac{db}{dp} = 0 \quad (9)$$

We know that:

$$\frac{dE(\pi)}{dp} \Big|_{p=p_{\text{exo}}} = -n\mathbb{P}_{-i}^s \frac{\partial c(\zeta)}{\partial b} \frac{db}{dp} \Big|_{p=p_{\text{exo}}}, \quad (10)$$

from which we conclude that:

- if $\partial c(\zeta)/\partial b < 0$, then $p_{\text{exo}} < p_{\text{endo}}$,
- if $\partial c(\zeta)/\partial b \in (0, 1]$, then $p_{\text{exo}} > p_{\text{endo}}$,
- if $\partial c(\zeta)/\partial b \geq 1$, then result is ambiguous,

as desired.

References

- Amihud, Y., Kahan, M., and Sundaram, R. K. (2004). The foundations of freezeout laws in takeovers. *Journal of Finance*, 59(3):1325–1344.
- Bagnoli, M. and Lipman, B. (1988). Successful takeovers without exclusion. *Review of Financial Studies*, 1(1):89–110.
- Bøhren, O. and Krosvik, N. E. (2013). The economics of minority freezeouts: Evidence from the courtroom. *International Review of Law and Economics*, 36:48–58.

- Bulow, J., Huang, M., and Klemperer, P. (1999). Toeholds and takeovers. *Journal of Political Economy*, 107(3):427–454.
- Burkart, M. (1995). Initial shareholdings and overbidding in takeover contests. *The Journal of Finance*, 50(5):1491–1515.
- Carney, W. J. and Heimendinger, M. (2003). Appraising the nonexistent: The Delaware courts’ struggle with control premiums. *University of Pennsylvania Law Review*, 152(2):845–880.
- Dalkır, E. and Dalkır, M. (2014). On the optimality of partial tender offers. *Journal of Economic Theory*, 151:561–570.
- Dewatripont, M. (1993). The ‘leading shareholder’ strategy, takeover contests and stock price dynamics. *European Economic Review*, 37(5):983–1004.
- Eckart, W. (1985). On the land assembly problem. *Journal of Urban Economics*, 18:364–378.
- Erichson, H. M. and Zipursky, B. C. (2011). Consent versus closure. *Cornell Law Review*, 96:265–321.
- Ferguson, M. (1994). Ownership structure, potential competition, and the free-rider problem in tender offers. *Journal of Law, Economics and Organization*, 10(1):35–62.
- Fischel, D. R. (1983). The appraisal remedy in corporate law. *Law and Social Inquiry*, 8(4):875–902.
- Grossman, S. J. and Hart, O. D. (1980). Takeover bids, the free-rider problem, and the theory of the corporation. *The Bell Journal of Economics*, 11(1):42–64.
- Heller, M. and Hills, R. (2008). Land assembly districts. *Harvard Law Review*, 121(6):1465–1527.
- Hirshleifer, D. and Titman, S. (1990). Share tendering strategies and the success of hostile takeover bids. *Journal of Political Economy*, pages 295–324.
- Holmström, B. and Nalebuff, B. (1992). To the raider goes the surplus? a reexamination of the free-rider problem. *Journal of Economics and Management Strategy*, 1(1):37–62.
- Mailath, G. J. and Postlewaite, A. (1990). Asymmetric information bargaining problems with many agents. *The Review of Economic Studies*, 57(3):351–367.
- Maug, E. (2006). Efficiency and fairness in minority freezeouts: Takeovers, overbidding, and the freeze-in problem. *International Review of Law and Economics*, 26(3):355–379.
- Miceli, T. J. and Segerson, K. (2007). The economics of eminent domain: Private property, public use, and just compensation. *Foundations and Trends in Microeconomics*, 3:275–329.
- O’Flaherty, B. (1994). Land assembly and urban renewal. *Regional Science and Urban Economics*, 24:287–300.
- Strange, W. C. (1995). Information, holdouts, and land assembly. *Journal of Urban Economics*, 38:317–332.
- Subramanian, G. (2005). Fixing freezeouts. *Yale Law Journal*, 115(1).

- Subramanian, G. (2007). Post-siliconix freeze-outs: Theory and evidence. *The Journal of Legal Studies*, 36(1):1–26.
- Tashjian, E., Lease, R. C., and McConnell, J. J. (1996). An empirical analysis of prepackaged bankruptcies. *Journal of Financial Economics*, 40:135–162.
- Yilmaz, B. (2000). A theory of takeover bidding. *Rodney L White Center for Financial Research-Working Paper*.