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## The Sale of Small Firms: a multidimensional analysis\*

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**Summary.** This paper endogeneizes the security voting structure in an auction mechanism used to sell a small firm. The design of security voting structure allows the seller to choose between two objectives which are not mutually consistent. If the seller wants to maximize his revenue, he should retain some shares to benefit from the future dividends generated by the acquirer. At the opposite, if he wants to sell his firm to the most efficient candidate, he should sell all the shares.

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The small firms sale, i.e. those held and managed by a single owner, is not a central theme in corporate finance. However, it is not a marginal practice. The sale can take two forms: a negotiation between the seller and the acquirer or a tender offer which gives the firm to the highest bidder. This paper deals with the latter case. The seller may have two objectives: the maximization of his expected revenue and the allocative efficiency. This latter objective is reached when the firm is sold to the most efficient candidate, i.e. the one who will maximize the firm's value in the future.

Over the two last decades, a specific auction mechanism, the open bid, emerged as a central technic of small firms sales. The open bid is equivalent to a "classic" auction: the seller, as a monopolist, faces some potential acquirers. He has the bargaining power. He decides when to sell, what to sell (shares number) and who may participate in the auction. In practice, two types of auction can be used: ascending auction or first price auction. The ascending auction seems to be the most commonly used.

This paper considers the firm's sale as a transfer of corporate control. Corporation's securities provide the holder with particular claims on the firm's income stream and particular voting rights. These securities can be designed in various ways: one share of a particular class may have a claim to vote which is larger or smaller than its claim to income. The transfer takes place when the acquirer receives more than half of votes. Selling only the votes is prohibited. A potential acquirer must pay for the dividends rights which give voting rights (yielding private benefits). Thus, we have to distinguish between two items for sale: the dividends rights and the control of the firm through the voting rights. Nevertheless, these two items are interdependent because the income stream depends on the identity of the acquirer who has taken control.

In this paper, we show how a seller can take into account the respective impact of voting rights and dividends in bidders valuation. Actually, the choice of the optimal security voting structure reflects the interdependencies between voting rights/private benefits and dividends rights/public benefits. Hence, this optimal structure determines the willingness to pay of the bidder. It changes the relative weight of private and public benefits in the total valuation of the bundle of assets. We argue that from the seller point of view, the optimal structure may sometimes differ from the traditional one-share-one-vote structure.

The paper is organized as follows. Section 1 presents the model. Unlike Cornelli and Li (1997) who analyze privatization schemes in the context of optimal auction design, our aim is not to characterize the optimal selling mechanism. Actually, we study a standard auction process, in which we ex-

explicitly introduce security voting structure. Following Grossman and Hart (1988), we use it as a tool which allows the seller to screen the candidates. Our model is closed to the At and Morand (2000) approach. They study the role of security voting structure when a government wants to privatize a company and faces asymmetric bidders. We depart from these contributions considering that both private and public benefits are private information of the bidders. Section 2 derives the results. We identify the security voting structure which maximizes the expected revenue of the seller. We show that the one share-one vote structure is not always optimal from the seller's standpoint. However, it ensures that the most efficient candidate takes control, i.e. the candidate who will maximize the firm's value in the future. The literature on hostile takeovers has also discussed the role of security voting structure, notably Grossman and Hart (1988) and Harris and Raviv (1988). Our results mitigate the superiority of the one share-one vote structure shown by these authors. Section 3 concludes.

## 1 The model

The model considers a firm sold through an auction. It is owned by a single risk neutral shareholder. A candidate must obtain at least the fraction  $\alpha \in (0.5, 1]$  of votes to take control of the firm. Seller may keep a share in the firm, i.e. give up control but not all income claims. Hence, he sells a package  $\Omega$  of voting and income rights which brings the fraction  $\alpha$  of votes. The  $\Omega$ -security voting structure represents the amount of income rights attached to the voting rights majority. These income claims determine the cost of majority.

We consider two candidates which we shall refer as  $r$  and  $t$ . They are risk neutral. This model makes the traditional distinction between security benefits (discounted sum of dividends) denoted by  $y_r$  and  $y_t$  under the two managements and the private benefits of control, respectively  $b_r$  and  $b_t$ . The private benefits of control refer to benefits the acquirer obtain for himself (See Schleifer and Vishny (1997) for various examples of private benefits). The security benefits refer to the total market value of the corporation's securities. We assume that  $r$  is more efficient than  $t$  iff  $y_r > y_t$ . We have to mention that the highest bidder may not be the most efficient candidate because of private benefits. The willingness to pay of a buyer  $m$  for the package is:

$$w_m = \Omega y_m + b_m \quad \text{for } m = r, t \quad (1)$$

We may notice that under one share-one vote structure  $\Omega = \alpha$ . It implies

that under this security voting structure, the seller can not obtain a  $\Omega$ -structure such that  $\Omega < 0.5$ . In spirit of Grossman and Hart (1988), the seller may generate such a  $\Omega$ -structure by creating  $C$  classes of shares with  $C \geq 2$ . He specifies the fraction of dividends  $s^c$  and the fraction of total votes  $v^c$ , for  $c \in C$  to which class  $c$  is entitled. Let us remark that due to legal restrictions, unbundling votes, i.e. ( $s^c = 0; v^c > 0, \forall c \in C$ ), are ruled out. We assume that ( $v^c, s^c, \forall c \in C$ ) are common knowledge.

The seller obtains direct revenue from the auction  $E(S)$  and indirect revenue from the fraction  $(1 - \Omega)$  of shares kept. This fraction yields dividends generated by the winner of the auction. Let  $E(y_v)$  represents the expected value of dividends under the winner control. The total revenue of the seller is so:

$$E(S) + (1 - \Omega)E(y_v) \quad (2)$$

We suppose that both public and private benefits are private information. Nevertheless, even in this multidimensional setting, the only relevant private characteristics of the bidders are the total willingness to pay for the package. The two potential acquirers are assumed symmetrical ex-ante. Public benefits of bidder  $m = t, r$  are i.i.d over  $[y, \bar{y}]$  with a strictly positive density function  $f_y(\cdot)$ . Similarly, private benefits are drawn independently from the same cumulative distribution function  $F_B(\cdot)$  on the interval  $B = [\underline{b}, \bar{b}]$ , with density  $f_b(\cdot)$ . The probability density  $f_b(b_m)$  is non-negative. So, the probability density of the package value is given by the following convolution function with  $f_{\Omega y}(\cdot)$  the density function of  $\Omega y$ :

$$\begin{aligned} f_w(w) &= \int_{-\infty}^{+\infty} f_b(b)f_{\Omega y}(w - b)db \\ &= \frac{1}{\Omega} \int_{\underline{b}}^{\bar{b}} f_b(b)f_y\left(\frac{w - b}{\Omega}\right)db \end{aligned} \quad (3)$$

Without loss of generality, we consider that the firm, i.e. the package of securities, is sold through an ascending auction. In our settings, with risk neutral, symmetric bidders and independent private value, every traditional auction mechanism is revenue-equivalent. Therefore, the same results would apply with every auction mechanisms in which the package of securities always goes to the buyer with the highest valuation and any bidder with the lowest-feasible value expects zero surplus.

## 2 Firm's sale revenue

In ascending auction, the bidder optimal strategy is to remain active as long as the price is not greater than the willingness to pay and the opponent remains active. So, the expected revenue of the auction corresponds to the expected willingness to pay of the second bidder. The density function of the rank two order statistic is  $2f_w(\cdot)[1 - F_w(\cdot)]$ . We obtain:

$$E(S) = \int_{\Omega y + b}^{\Omega \bar{y} + \bar{b}} 2w f_w(w) [1 - F_w(w)] dw \quad (4)$$

Let us consider two pairs  $(y_m, b_m)$  for  $m = r, t$  and assume that  $\Omega y_r + b_r > \Omega y_t + b_t$ . The revenue of the auction is  $S(\Omega) = \Omega y_t + b_t$  i.e.e the willingness to pay of the second bidder. Consider that  $\Omega' > \Omega$ , then there are two cases. If  $\Omega' y_r + b_r > \Omega' y_t + b_t$  then  $S(\Omega') = \Omega' y_t + b_t > S(\Omega)$ . If  $\Omega' y_r + b_r < \Omega' y_t + b_t$  then  $S(\Omega') = \Omega' y_r + b_r > S(\Omega)$ . Hence, whatever the characteristics  $(y_m, b_m)$  of the two bidders are, we have  $S(\Omega') > S(\Omega)$ . The expected sale revenue  $E(S)$  is increasing with respect to  $\Omega$ . So, it is maximized when  $\Omega = 1$ . The seller sells all the shares.

### 2.1 The allocative efficiency

In this part, we shall derive the security voting structure which ensures the allocative efficiency. Let  $2F_w(w)f_w(w)$  be the density function of the better valuation between the two bidders. We can so derive the expected cash flows of the winner. Since  $w_v = \Omega y_v + b_v$ , we have  $y_v = \frac{w_v - b_v}{\Omega}$  and so:

$$E(y_v) = \frac{2}{\Omega^3} \int_{\Omega y + b}^{\Omega \bar{y} + \bar{b}} \left( \int_b^{\bar{b}} (w_v - b) f_b(b) f_y\left(\frac{w_v - b}{\Omega}\right) db \right) \left( \int_{\Omega y + b}^{w_v} \int_b^{\bar{b}} f_b(b) f_y\left(\frac{w - b}{\Omega}\right) db \right) dw_v \quad (5)$$

Let  $w_v = \Omega y_v + b_v$  be the willingness to pay of the winner. Let us consider two  $\Omega$ -structures  $\Omega' \in (0, 1]$  and  $\Omega'' \in (0, 1]$  such that  $\Omega'' > \Omega'$ . By assuming that  $\Omega' y_r + b_r > \Omega' y_t + b_t$ , the winner is  $(y_v | \Omega = \Omega') = y_r$ . Under the  $\Omega''$ -structure, there are two cases. If  $\Omega'' y_r + b_r > \Omega'' y_t + b_t$  then  $(y_v | \Omega = \Omega'') = y_r = (y_v | \Omega = \Omega')$ . If  $\Omega'' y_r + b_r < \Omega'' y_t + b_t$  then  $(y_v | \Omega = \Omega'') = y_t$ . From assumptions,  $\Omega' y_r + b_r > \Omega' y_t + b_t$  and  $\Omega'' > \Omega'$ , we deduce that  $y_t = (y_v | \Omega = \Omega'') > y_r = (y_v | \Omega = \Omega')$ . This result holds whatever the characteristics of the two bidders are, i.e.  $(y_m, b_m)$  for  $m = r, t$ .

We may conclude that  $E(y_v)$  is increasing with respect to  $\Omega$ . We derive the following proposition.

Allocative efficiency is maximized under  $\Omega = 1$ .

This result is straightforward. The relative weight of cash flows in total valuation of the shares bundle is increasing with respect to  $\Omega$ . This tends to minimize the probability of having a less efficient pretender with high private benefits, which defeats a more efficient one with low private benefits.

## 2.2 The maximization of expected total revenue

We have shown that the firm's sale efficiency (the probability of sale to the bidder with high public benefits) is obtained when the seller sells all the shares. This structure minimizes the relative weight of private benefits in the securities valuation and so maximizes efficiency of the sale. In symmetric auction, the bidder with the highest willingness to pay will obtain the firm. Nevertheless, the seller cannot obtain more than the second valuation of the bundle of share. Hence, if the winning bidder is also the most efficient acquirer, then the seller is better off retaining part of the shares. He will so obtain future high dividends. But, as the fraction of shares retained by the seller increases, the probability of selling the firm to the less efficient pretender increases because the relative weight of private benefits in the total valuation is increased. The choice of the security design maximizing the global revenue reflects this trade off.

Hence, in order to benefit from the dividends generated by the winner, the seller has to retain the fraction  $(1 - \Omega)$  of shares. This fraction implies a negative impact of  $\Omega$  in the total revenue of the seller. This total revenue may be written:

$$E(R) = E(S) + (1 - \Omega) E(y_v)$$

We cannot exhibit general results. To derive explicitly convolution function, we have to add a further assumption. Actually, the shape of the convolution function is closely related to the comparison of  $\|\underline{b}, \bar{b}\|$  and  $\|\underline{y}, \bar{y}\|$ . Recall that  $w = \Omega y + b$ . If  $y \in [\underline{y}, \bar{y}]$ ,  $\Omega y \in [\Omega \underline{y}, \Omega \bar{y}]$ , with  $f_{\Omega y}(s) = \frac{1}{\Omega} f_y(\frac{s}{\Omega})$ . If we suppose that  $\bar{y} - \underline{y} < \bar{b} - \underline{b}$ , then  $\Omega(\bar{y} - \underline{y}) < \bar{b} - \underline{b} \forall \Omega \in (0, 1]$ . Technically, this hypothesis involves a constant convolution density function, whatever the choice of  $\Omega$ . Let us remark that this hypothesis reflects that, even in an independent private value setting (i.e. where  $y_r$  is statistically independent of  $y_t$ ), public benefits cannot radically differ between the two potential acquirers. The difference between bidders efficiency has just a marginal effect on the benefits. Nevertheless, private benefits may highly differ. So, we can

derive the general density function of the willingness to pay for the bundle of assets:

$$\begin{aligned} \forall w &\in [\Omega \underline{y} + \underline{b}, \Omega \bar{y} + \underline{b}], \\ f_w(w) &= f_w^1(w) = \frac{1}{\Omega} \int_{\underline{b}}^{w - \Omega \underline{y}} f_b(b) f_y\left(\frac{w - b}{\Omega}\right) db \end{aligned} \quad (6)$$

$$\begin{aligned} \forall w &\in [\Omega \bar{y} + \underline{b}, \Omega \underline{y} + \bar{b}], \\ f_w(w) &= f_w^2(w) = \frac{1}{\Omega} \int_{w - \Omega \bar{y}}^{w - \Omega \underline{y}} f_b(b) f_y\left(\frac{w - b}{\Omega}\right) db \end{aligned} \quad (7)$$

$$\begin{aligned} \forall w &\in [\Omega \underline{y} + \bar{b}, \Omega \bar{y} + \bar{b}], \\ f_w(w) &= f_w^3(w) = \frac{1}{\Omega} \int_{w - \Omega \bar{y}}^{\bar{b}} f_b(b) f_y\left(\frac{w - b}{\Omega}\right) db \end{aligned} \quad (8)$$

With uniform distribution function and  $\underline{b} = 0$ , we have:

$$E(R) = \frac{20\bar{b}^3 + 30\bar{b}^2(\bar{y} - \underline{y}) + 5\bar{b}(\bar{y} - \underline{y})^2(2 - 3\Omega)\Omega + (\bar{y} - \underline{y})^3\Omega^2(4\Omega - 3)}{60\bar{b}^2}$$

Maximizing the total revenue gives  $\Omega^*$ , the optimal security structure. We have :

$$\frac{\partial E(R)}{\partial \Omega} = \frac{2(\bar{y} - \underline{y})^2 [\bar{b}(5 - 15\Omega) + 3(\bar{y} - \underline{y})\Omega(2\Omega - 1)]}{60\bar{b}^2}$$

and:

$$\frac{\partial^2 E(R)}{\partial \Omega^2} = \frac{(\bar{y} - \underline{y})^2 [-5\bar{b} + (\bar{y} - \underline{y})(4\Omega - 1)]}{10\bar{b}^2} \leq 0 \quad \forall \Omega \in (0, 1]$$

so, if  $\exists \Omega \in (0, 1]$  such that  $\frac{\partial E(R)}{\partial \Omega} = 0$  then this is the optimal  $\Omega$ -structure. We have:

$$\Omega^* = \frac{15\bar{b} + 3(\bar{y} - \underline{y}) - \sqrt{9(5\bar{b} + \bar{y} - \underline{y})^2 - 120\bar{b}(\bar{y} - \underline{y})}}{12(\bar{y} - \underline{y})}$$

Recall that if  $\Omega^* < 0.5$ , then the seller cannot use only one-share-one-vote structure. He has to issue two classes of shares characterized by different security voting structure. We can exhibit cases where the optimal structure departs from traditional one. Consider the following example:

Suppose  $\bar{y} = 1050$ ,  $\underline{y} = 1000$ ,  $\bar{b} = 100$ , then:

$$\Omega^* = \frac{33 - \sqrt{849}}{12} \simeq 0,32186$$



The optimal structure  $\Omega^*$  which maximizes total revenue, is strictly less than 0.5. It cannot be reached with traditional one share-one vote structure.

This last counterexample allows us to have the following proposition:

When the candidates are symmetrical and both private and public benefits are private informations, there exists some cases in which the seller should issue several classes of shares with one at least richer in votes. This structure differs from the efficiency-maximizing structure, which is reached under the one share-one vote structure and the vote rule  $\alpha = 1$ .

$\bar{y} = \underline{y}$ , i.e. no uncertainty on public benefits, yields  $\frac{\partial E(R)}{\partial \Omega} = 0$ . The global revenue of the seller is independent of the security design.

### 3 Conclusion

A small firm's seller may have two distinct objectives which are not mutually consistent. He may want to maximize the sale's revenue. But he may want to ensure himself that his firm will be well managed. This paper endogeneizes the security voting structure in an auction mechanism used to sell the firm. Thus, by designing a specific security voting structure, the seller may choose between the two objectives. If the seller wants to maximize his revenue, he should keep some shares. The security voting structure design trades off between sell all the shares and retain some shares to benefit from the future dividends generated by the winner. At the opposite, if he wants to sell his firm to the most efficient candidate, he should sell all the assets.

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