Managing relational contracts

Marta Troya-Martinez, Liam Wren-Lewis

WORKING PAPER N° 2016 – 20

Managing relational contracts

Marta Troya-Martinez
Liam Wren-Lewis

JEL Codes: D73, D86, L14.
Keywords:

Funded by a French government subsidy managed by the ANR under the framework of the Investissements d'avenir programme reference ANR-17-EURE-001

Managing relational contracts

Marta Troya-Martinez and Liam Wren-Lewis*

Abstract

Relational contracts are typically modeled as being between a principal and an agent, such as a firm owner and a supplier. Yet, in a variety of organizations, relationships are overseen by an intermediary such as a manager. Such arrangements open the door for collusion between the manager and the agent. This paper develops a theory of such managed relational contracts. We show that managed relational contracts differ from principal-agent ones in important ways. First, kickbacks from the agent can help solve the manager's commitment problem. When commitment is difficult, this can result in higher agent effort than the principal could incentivize directly. Second, making relationships more valuable enables more collusion and hence can reduce effort. We also analyze the principal's delegation problem and show that she may or may not benefit from entrusting the relationship to a manager.

JEL classifications: D73, D86, L14.

*Troya-Martinez: New Economic School and CEPR (email: mtroya@nes.ru); Wren-Lewis: Paris School of Economics (email: liam.wren-lewis@psemail.eu). We are grateful to Simon Board, Mikhail Drugov, Georgy Egorov, Matthew Ellman, Florian Englmaier, Ruben Enikolopov, Matthias Fahn, Guido Friebel, Willie Fuchs, Juanjo Ganuza, Bob Gibbons, Marina Halac, Jonathan Levin, Jin Li, Michael Katz, Paul Klemperer, Jim Malcomson, David Martimort, Patrick Rey, Andrew Rhodes, Orie Shelef, Giancarlo Spagnolo and the participants of AEA, ASSET, BGSE Summer Forum, CSAE, CMPO, EEA, ES World Congress, GREThA, SIOE and NGO conferences, the Workshop on Relational Contracts, Advances in IO and the Org. Econ. NBER meeting, and the seminars at Copenhagen, Cunef, HSE, LMU Munich, Lund, NES, Oxford, Paris I, PSE, SSE, TSE, UAB and UC Berkeley for many insightful comments. Any remaining errors are our own. We thank Danil Fedchenko for excellent research assistance. Marta Troya-Martinez thanks the Russian Science Foundation for financial support of the research, project No. 15-18-30081.

In 2006, the American retailer Aéropostale accused its chief merchandising manager Christopher Finazzo of receiving more than $25 million in kickbacks from a supplier, South Bay. Aéropostale argued that Finazzo had paid inflated prices to South Bay in exchange. Finazzo responded that he had favored South Bay since they provided higher quality and a willingness to adapt to Aéropostale's procurement needs. He argued that Aéropostale often remained "*loyal*" and "*committed*" to long-time vendors "*even when those vendors charged higher prices*" (Droney, 2017, p.77). In 2013, Finazzo and South Bay were found guilty of fraud. They appealed the restitution amount and in 2017 the Court of Appeals for the Second Circuit demanded a recalculation. Judge Droney argued that it was possible that Aéropostale did not lose money as a result of the kickback scheme. He argued that instead Finazzo's "*conduct may have reduced transactions costs for South Bay*" and the relationship may have made it profitable for South Bay to pay kickbacks even at non-inflated prices (Droney, 2017, p.77).

Relational contracts between organizations are ubiquitous and crucial for enforcing promises. Indeed, "*lack of trust and commitment*" is behind most supplier collaboration failures in private and public settings (Webb (2017) and Desrieux, Chong and Saussier (2013)). The task of maintaining these relationships is often delegated to a manager like Finazzo. As illustrated by Aéropostale's case, the firm can never guarantee that the manager will exclusively act in the firm's best interest. Managers can exploit the (otherwise very valuable) trust relationship with their suppliers to collude with them. Indeed, a recent report shows that 20% of surveyed firms had been victims of procurement fraud in the past year, with most fraud incidents involving firm insiders (Kroll, 2018). This paper aims to understand how relationships behave when they are managed on the principal's behalf. In particular, we focus on the following main questions: First, is having a manager open to collusion with the agent always worse for the principal? Second, with the threat of collusion, do stronger manager-agent relationships achieve more quality?

To answer these questions, we develop a theory of managed self-enforcing relational contracts. A manager and an agent can exchange side payments, which represent kickbacks, bribes or other favors. If the manager does not care about producing quality as much as the principal, then she will have an incentive to collude with the agent by receiving side payments in return for paying the agent higher prices. Collusion can disincentivize quality if the manager pays a discretionary price premium regardless of quality. In particular, she may do so when she trusts that the agent will respond by making a side payment.

The first main contribution of this paper is to show that having a manager with an incentive to collude can nonetheless increase the quality sustained by the relational contract. Side payments can enhance a manager's ability to commit, and hence allow higher quality. This is because the supplier will renege on paying side payments if the manager reneges on the promised price. This is consistent with evidence that side payments can help contract enforcement. Cole and Tran (2011) analyze informal payments in an Asian country and find that, when contract payments are dependent on whether a non-contractible quality has been delivered, *"the kickback is paid only after all contract payments have been made"*.[1] We show that side payments are thus not detrimental for the principal when commitment is scarce. Hence, this theory provides an instance of the *"reduced transaction costs"* mentioned by Judge Droney.

The second main contribution of this paper is then to understand how quality varies as a function of the strength of the relationship between the manager and the agent. In the standard principal-agent model of relational contracts, more trustworthy relationships (i.e. those with higher discounted value) produce higher quality. In managed relational contacts, we show that the opposite may happen. More valuable relationships produce higher effort, and hence higher quality, only up to a point. Once the relationship is sufficiently valuable, extra value facilitates collusion, which reduces effort. In particular, it allows the manager to pay the agent a high price in exchange for a side payment even when quality is low. This non-monotonicity result is consistent with evidence on firms' use of guanxi, a

[1] In a similar case, Paine (2004) describes how *"a purchasing official called about an overdue payment for items already received, [explaining] 'we can get you a check by next week if you can give us a discount – in cash so we can distribute it to employees'"*. Likewise Freeman, Hernadi and Tolbert (2000) describe how a supplier insisted the purchasing manager accept kickbacks as otherwise *"the supplier's invoices would not be paid on time"*, arguing that was the way that business was done in the country.

system of trust-based "*informal social relationship*" in China which is used, amongst other purposes, to ensure "*that a contract is honored*" (Chow, 1997). Vanhonacker (2004) observes that "*it would be naive to think—as many Western executives do—that the more guanxi you have on the front lines in China, the better*". Instead, he argues too much guanxi can "*divide the loyalties of the sales and procurement people*" and suggests "*rotating the front line*" to disrupt "*unduly powerful guanxi connections*".

Our model features a manager and an agent who have a bilateral relational contract over an infinite number of periods. To model that the relationship is managed on behalf of a third party, we assume that profits generated by the relationship are shared between the manager and a principal who is inactive during the relational contract. Every period, the agent privately exerts costly effort to produce a quality which cannot be formally contracted on. To motivate effort, the manager promises to reward high quality with a price premium which is bounded above. The manager and agent can also make side payments after the quality has been realized and the price has been paid. The payment of both the price and side payments needs to be self-enforced.

In Section 2, we begin by considering the principal as completely passive and characterize the surplus maximizing stationary contract between the manager and agent. The manager can incentivize effort through variation in prices or side payments. Variation in prices is more credible than variation in side payments. This is because the manager effectively pays only a fraction of the price, and can receive side payments from the agent in return. When the discounted value of the relationship is low, the manager cannot trust the agent to return much of the price paid as a side payment. She thus prefers to use variation in prices to incentivize the agent. When the discounted value of the relationship is high, the manager can trust that the agent will return a large side payment if quality is low. She can thus pay a price premium regardless of quality and motivate effort through variation in side payments. This increases the manager-agent surplus, since part of the price is paid for by the principal. Paying a premium price regardless of quality, however, means effort no longer has an impact on the price level. This decreases the value of quality to the manager and agent, and they prefer a contract with less effort. Hence effort and quality can decrease in the value of the relationship.

The Aéropostale case provides an example of how hierarchical collusion can result in prices becoming insensitive to quality. In 2005 and 2006, South Bay had delays "*that cost Aéropostale approximately $1.8 million in lost sales*". A product manager suggested that they should ask South Bay for discounts "*to compensate for the delays*" (Droney, 2017). Yet Finazzo, presumably knowing that this would reduce future kickbacks, refused.

Section 3 analyzes an environment where the principal can choose the profit share and the maximum price at the beginning of the relationship. We show that she should choose these parameters to prevent the manager colluding to pay the agent a premium price for low quality. An interesting feature of the optimal contract is that, when the discounted value in the manager-agent relationship is high, the principal limits the manager's discretion and the manager varies side payments to motivate effort. This is consistent with evidence documented by Ledeneva (2013) that, in Russian government procurement, kickbacks were "*linked to performance and facilitated the quality of service*". In this case, potential collusion is still costly for the principal because it forces her to decrease the discretion available to the manager, lowering quality. Hence, if the principal has the option to manage the agent herself, then she would rather do that when relational contracts are relatively easy to enforce. If relational contracts are difficult to enforce, however, then the principal prefers to employ a manager, because she can achieve a higher quality.

Finally, Section 4 considers a number of alternative specifications and extensions to understand the robustness of our results and how the model could be applied to other contexts. We show that if the principal could sell the firm entirely to the corrupt manager - and thereby avoid collusion - she may not always want to do so. The principal can benefit from having a corrupt manager and, in these circumstances, the principal would not want to sell the entire venture. We also discuss how the model could be adapted to consider relational contracts between government bureaucracies and private actors, as well as employment relationships within organizations. We also extend the model to understand how results are likely to change depending on the cost of making side payments. In contexts where side payments are less likely to be severely punished, we find that delegation is likely to be more costly for the principal.

To the best of our knowledge, this is the first paper that studies the impact of hierarchical collusion on relational contracts. It bridges two large theoretical literatures - that on relational contracts and that on collusion in hierarchies. Models in the relational contracting literature have typically not considered hierarchies with intermediary layers (see Malcomson, 2013, for a survey). A recent exception is the paper by Fong and Li (2017*a*) whose model includes a non-strategic supervisor carrying out subjective performance evaluations of the agent. They show that by garbling the evaluations intertemporally, the supervisor can relax the self-enforceability problem, thereby benefiting the principal. The possibility of collusion, however, is not the focus of their analysis, and hence they conclude that "*formal study of how collusion affects relational contracts is an interesting line of future research*".

Collusion between supervisors and agents has been the focus of a large literature including seminal works by Tirole (1986), Milgrom (1988) and Kofman and Lawarree (1993).[2] A few papers have studied how the need for self-enforcement impacts such collusion - see, for instance, Martimort (1999), Martimort and Verdier (2004) and Dufwenberg and Spagnolo (2015) - but these do not consider any interaction with other commitment problems.

Our model also relates to a literature investigating how delegation can solve commitment problems. Early papers in this literature include Vickers (1985) and Katz (1991).[3] Our paper adds to this literature by showing that commitment benefits can arise as a result of side payments from the agent. These side payments have the advantage that they can be conditional on manager-agent cooperation, something principals often cannot observe. In this sense, our paper shares the spirit of Hermalin (2015), who builds a

[2]Within this literature, Olsen and Torsvik (1998) find that potential supervisor-agent collusion can mitigate a commitment problem in a two-period setting with adverse selection. In a competitive procurement setting, Calzolari and Spagnolo (2009) consider links between long-term relationships and horizontal collusion among suppliers while Burguet and Che (2004) allow suppliers to bribe the supervisor in a setting without long-term relationships but with competition from heterogeneous suppliers.

[3]Spagnolo (2005) and Spagnolo (2000) use a repeated game framework to show how delegation and stock-related compensation can enhance the enforcement of horizontal collusion between firms.

model whereby *“wining and dining”* helps sustain a productive relationship between two firms’ managers. He also considers managers colluding against their principals, but this collusion is not sustained through relational contracting, only occurs when side payments are costless, and does not risk distorting other incentive schemes. As a result, in his model, allowing cross-firm managerial rewards always benefits the principals, and it is therefore unclear to what extent these benefits would carry through to a context where side payments can be potentially damaging for the principals.

1 The model

Our model focuses on a manager and an agent who interact repeatedly over an infinite horizon of discrete periods. The manager works on behalf of a principal who is inactive during the game. We refer to the manager and principal as female and the agent as male.

The timeline for each period is shown in Figure 1. The manager first proposes a pricing scheme and a set of side payments to the agent (to be defined below). The agent either rejects or accepts - let $d_t \in \{0, 1\}$ denote the agent’s decision. If the agent accepts, then he chooses an effort $e_t \in [0, 1]$ incurring a cost $c(e_t)$, where $c(0) = c'(0) = 0$, $c''(\cdot) > 0$ and $c'''(\cdot) \geq 0$. The agent’s effort generates a binary stochastic quality, which is high (h) with probability e_t and low (l) otherwise. High quality always produces a revenue Y_t of $y > 0$, while low quality always produces a revenue of 0. If the agent rejects, then for the rest of the game the manager and the principal receive low quality goods that yield zero profit and the agent receives a per-period payoff of $\underline{u} \leq \max_{e_\tau} \mathbb{E}\left[Y_\tau - c(e_\tau)\right]$.

The agent is compensated with a discretionary payment p_t that can depend on quality. The price the manager can pay to the agent is limited by a maximum price $\overline{p}$ that does not change over time, such that $p_t \leq \overline{p}$. This represents the limit of the manager’s discretion.

In addition to the pricing scheme, the manager also suggests a package of side payments that will be paid by the agent to the manager. The side payment is paid in two parts: the first part, s_t^F, is paid before quality is

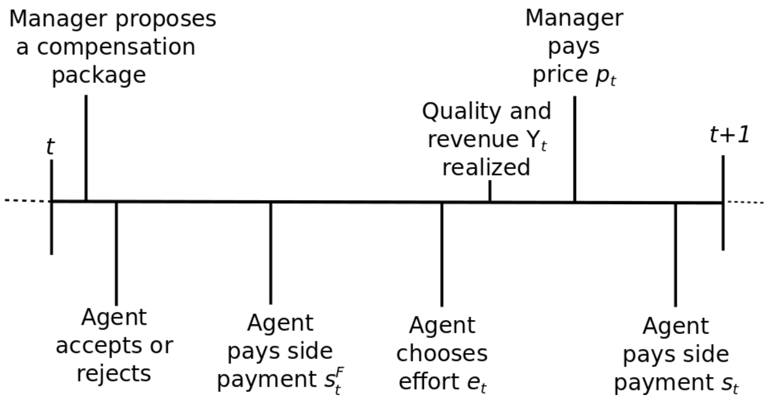


Figure 1: Timeline of period t in manager-agent game

realized, while the second part, s_t, is paid after quality is realized.[4] Let $S_t = s_t^F + s_t$ denote the total side payment made. The payment of both p_t and s_t need to be self-enforced.

The total profit $Y_t - p_t$ is shared between the principal and the manager, who receive shares $1-\alpha$ and α respectively, where $\alpha \in [0,1]$. All players have the same discount factor $\delta \in (0,1)$. The expected payoff functions of the principal, the manager and the agent can thus be written as follows:[5]

$$\pi_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau \left[(1-\alpha)\left(Y_\tau - p_\tau\right)\right]\right]$$
$$v_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau [\alpha(Y_\tau - p_\tau) + S_\tau]\right]$$
$$u_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{d_\tau [p_\tau - S_\tau - c(e_\tau)] + (1-d_\tau)\,\underline{u}\right\}\right]$$

The information structure is one of moral hazard. Effort is the agent's private information, while the quality and agent's compensation are observed by both the manager and the agent. The principal does not observe

[4]We split the side payment into two parts to allow for purely stationary contracts to be optimal (Levin, 2003). If we were to have only the ex-post part, optimal contracts would have to either differ in the first period, akin to the initial payment in Malcomson (2016), or depend on the prior period's quality.

[5]We assume that the principal and manager receive an equivalent to zero effort as their outside option, as in Levin (2002). This ensures our results are not driven by differences between the principal and manager in the relative valuations of the outside option.

anything. This assumption captures the idea that *"shareholders cannot directly observe whether their managers are behaving cooperatively"* in inter-firm relationships (Hermalin, 2015).[6] This lack of information precludes the use of more sophisticated contracts with the manager. In the Online Appendix, we develop a multi-agent model where we distinguish between non-contractible individual agent outputs and a contractible aggregate output. One way of considering the information structure in the main model is as a reduced form version of this more extensive model where the principal observes the sum of the outputs but not the individual components of each agent.

The assumption that the manager, but not the agent, receives a fraction α of the total profit is important. One way of rationalizing such an assumption is to think of the agent as representing a group of individuals, such as a firm's workforce or suppliers. In this way, the profit provides information on the effort exerted by the group, but cannot be used to motivate a single agent (Rauh, 2020). This is especially true when the agents are independent competing suppliers. In the case of Aéropostale, for instance, the manager Finazzo received bonuses and stock options linked to the firm's profit, but no such incentives were provided to the firm's suppliers (Department of Justice, 2013).[7] In the multi-agent model of the Online Appendix, we microfound this argument. We show that if the number of agents is above a threshold it is not optimal for the manager to share her profit with the agents because of the free-riding problem.

One way to think about the maximum price $\overline{p}$ is as a function of the market price set by other suppliers, above which a manager might attract suspicion. Indeed, one of the triggers of the Aéropostale case was Finazzo's reluctance to provide "*a price breakdown of South Bay's T–shirts by component to compare to another vendor*" (Droney, 2017). In other settings,

[6]Even within a firm, the principal may not be able to observe if two agents in a team are cooperating or not, as pointed out by Che and Yoo (2001).

[7]Similarly, in some sectors, firm owners employ other firms through management contracts, which frequently use profit shares to incentivize good management. One sector where such contracts are commonly used is the hotel sector (Lafontaine, Perrigot and Wilson, 2017). According to one source, in this context *"hotel management companies sometimes fraudulently profit at the owners' expense"* with, for instance, hotel suppliers providing *"a hotel management company with millions of dollars in undisclosed rebates"* (KPM, 2018).

$\overline{p}$ can be thought as a reservation price above which the manager needs to seek the principal's approval for the transaction to go through.[8] Using simple limits, such as a price cap, on intermediaries' discretion is common in models of procurement (see, for instance, Burguet and Che, 2004; Compte, Lambert-Mogiliansky and Verdier, 2005) . We can think of these limits as resulting from an organizational design that minimizes costly communication or information processing so that they can be enforced by third parties such as banks or other workers (Garicano and Prat, 2013).

In Section 2, we solve for the optimal manager-agent relational contract, and assume that the principal takes no actions. This analysis provides useful insights for contexts where principals do not have any control over managers' incentives or discretion. For instance, the manager could be a controlling shareholder who deals with suppliers and employees. In this case, the principal represents the remaining dispersed shareholders, and α is determined by the pre-existing equity structure and $\overline{p}$ is constrained by the corporate governance in place. Similarly, this analysis is relevant in contexts where these variables cannot be changed frequently enough to adapt to changes in the manager-agent relational context. For instance, this would be the case if there were many such agents (or managers) as we discuss in the Online Appendix.

In Section 3, we then consider the case where the principal can set $\overline{p}$ and α at the beginning of the game, and then takes no further action. This allows us to draw inferences as to how principals should incentivize and constrain managers as a function of the potential value of the manager-agent relationship.

Managed relational contracts occur in many contexts beyond procurement, such as employment relationships. We use a static profit-sharing rule for our main analysis since it is a simple and widely used rule which is sufficient to demonstrate the key insights of the paper.[9] In some cases the principal might have a richer set of instruments with which to contract with

[8] Wells (2007, p. 124) notes that a common anti-fraud policy is to set "*strict limits on the authorized size of each transaction.*"

[9] Bhattacharyya and Lafontaine (1995, p. 763) survey the empirical literature on contracts and find that *"(a) payment rules tend to be simple and often linear, (b) in many cases, the same contract terms are used across numerous principal-agent pairs or across all agents by a given principal, and (c) contract terms are relatively stable over time or as the number of agents increases"*.

the manager and agent, while in other cases she may have a more restricted set. We consider some alternative contracts in Section 4. In general, more instruments allow the principal to constrain collusion and approach first best, while fewer will make managed relational contracting less attractive.

2 Managed relational contracts

We begin this section by considering the case where $\alpha = 1$, i.e. when the manager receives the total benefit of the agent's effort. This serves as a benchmark as it is equivalent to a standard principal-agent model. We then consider the model with $\alpha < 1$, when the manager has an incentive to collude, and solve for the optimal manager-agent contracts.

2.1 Managed contracts with $\alpha = 1$

When $\alpha = 1$, the manager receives the total benefit of the agent's effort and pays the full cost of any payments made to the agent (i.e. price and side payments are substitutable), making the model equivalent to that of Levin (2003). We can thus treat the results of this case as the 'no delegation' benchmark. Then, if the manager and agent could write a third-party enforced contract on Y_t, it would be optimal to induce the value of effort e_t that maximizes the surplus, $ye_t - c(e_t)$. Defining this first-best effort as e^{FB}, we then have $c'(e^{FB}) = y$.

When the manager and agent cannot contract on Y_t, a self-enforcing contract is needed. We follow the definition of a self-enforcing contract given by Levin (2003) and similarly define a self-enforcing contract as optimal if no other self-enforcing contract generates higher expected surplus for the manager and agent. Levin (2003) shows that, if we are concerned with optimal contracts, then there is no loss of generality in focusing on stationary optimal contracts (our side-payment s_t^F plays the same role as the fixed wage in Levin (2003)). Let p_h be the price and s_h be the side payment when quality is high. Similarly, let p_l be the price and s_l be the side payment when quality is low. Then effort will be determined by the following binding incentive compatibility constraint:

$$c'(e) = p_h - p_l - s_h + s_l \tag{IC}$$

where we have dropped the t subscript. This payment variation is constrained by the following inequality:

$$c'(e) \leq \frac{\delta}{1-\delta}(ey - c(e) - \underline{u}) \tag{1}$$

This inequality states that effort incentives (the left hand side) are limited by the future gains from the relationship and it is known as the dynamic enforcement constraint. The tightness of this constraint depends on the value of the future relationship. When it is not valuable enough, the manager cannot credibly pay enough to implement the first-best effort. Instead, the effective reward for high quality will be the largest that can be credibly promised. Effort will therefore be increasing in the future discounted surplus.

2.2 Managed contracts with $\alpha < 1$

In this section, we outline the key constraints that will potentially bind in any optimal manager-agent contract. This allows us to derive the main proposition in this section, which characterizes the optimal contract as a function of the future discounted surplus. In particular, it details the varying ways in which effort is motivated and how the relationship between effort and future discounted surplus is non-monotonic.

A first notable point is that the surplus depends directly on the price paid. This is because the manager only pays for part of the price p_t that the agent receives in full. If contracting on Y_t were possible, the manager and the agent would maximize their joint surplus by setting the price at the maximum $\overline{p}$ regardless of quality and then use side payments to induce an effort level e_{MA}^{FB}, where $c'(e_{MA}^{FB}) = \alpha y$.

Given that side payments can be used to divide surplus between the manager and the agent, we can focus on relational contracts that generate the largest possible surplus. We follow Levin (2003) in defining a self-enforcing contract as strongly optimal if the continuation contract is optimal for all potential histories, even those off-equilibrium. We then obtain the following lemma:

Lemma 1. *If an optimal contract exists, there are stationary contracts that are strongly optimal.*

The intuition behind this stationarity result is that any variation in promised continuation values can be transferred into side payments in the same way that, in the principal-agent case studied by Levin (2003), any variation can be transferred to bonus payments.

We therefore focus on stationary contracts and drop the t subscripts on our variables. Note that, within stationary contracts, effort is incentivized via variation in prices and side transfers as established by (IC).

We define $g(e, p_h, p_l)$ as the expected surplus in any contract that has prices p_h and p_l and induces effort e. It is given by:

$$g(e, p_h, p_l) = \alpha e y + (1 - \alpha)(e p_h + (1 - e)p_l) - c(e) - \underline{u}$$

The following lemma shows that the price will never be negative and, if the price or side payments vary as a function of quality, then they will do so in a way that encourages effort.

Lemma 2. *In any optimal contract, the price is always non-negative, i.e.* $p_h \geq 0$ *and* $p_l \geq 0$*. Moreover, the price is weakly higher when quality is high* ($p_h \geq p_l$) *and side payments are weakly lower* ($s_h \leq s_l$)*.*

If the manager wants to take surplus from the agent, then she prefers to do so using side payments rather than the price. This is because the two are equivalent for the agent, but the manager captures the whole value of any side payments given.

The need for the contract to be self-enforcing can be expressed in terms of dynamic enforcement constraints. Lemma 2 pins down the dynamic enforcement constraints that potentially bind. Since the price is never negative, only the manager has a reason to deviate when it comes to paying the price. This temptation will be greatest when quality is high, as this is when the price is greatest. On the other hand, only the agent may wish to deviate from paying the agreed side payments, because if the manager does not wish to pay the side payment, she would have already deviated by not paying the price. The agent will be most tempted to renege when quality is low, as this is when the side payment is greatest. We therefore need to concentrate on the two following dynamic enforcement constraints:

$$(1-\delta)\left(-\alpha p_h + s_h\right) + \delta v \geq 0 \tag{DE_M}$$

$$-(1-\delta) s_l + \delta u \geq \delta \underline{u} \tag{DE_A}$$

From these constraints, we can see that variation in prices is easier to sustain than variation in side payments. Increasing p_h by 1 only tightens (DE_M) by an amount α, but increasing s_l or decreasing s_h by 1 tightens the constraints by 1.

It is useful to compare the manager's dynamic enforcement constraint here with that in Section 2.1. When $\alpha = 1$, the manager's dynamic enforcement constraint is $(1-\delta)(-p_h + s_h) + \delta v \geq 0$. Contrasting this with (DE_M), we see that an $\alpha < 1$ makes the manager's dynamic enforcement constraint easier to satisfy. Intuitively, by paying the promised price, the manager can capture some surplus from the principal via the side payments. In other words, the manager will be more willing to pay a high price if she knows she will get part of this back from the agent as a 'kickback'. A lower value of α increases the relative amount of surplus that can be captured from the principal in this way, making a high price more credible.[10]

Summing (DE_M) and (DE_A) together and substituting in (IC) gives us the following constraint:

$$c'(e) + \alpha p_h - [p_h - p_l] \leq \frac{\delta g(e, p_h, p_l)}{1-\delta} \tag{$IC-DE$}$$

Comparing this to (1), the equivalent in the principal-agent case, we see that the requirement for contracts to be self-enforcing has a more complex impact in the manager-agent game. In particular, as the future surplus in the relationship decreases, a reduction in effort is now only one possible effect. The manager and agent may instead choose to keep effort constant by replacing variation in side payments with variation in prices. A further

[10]Note that, if there were no side payments, then $v = \alpha(y - c(e))$ and (DE_M) becomes equivalent to the one faced by the principal since α will cancel out.

option is for the manager to decrease the prices, and hence make them more credible, but keep their variation constant.

We define δ^{FB} as the critical level of δ at which the manager and the agent can implement their first-best contract, i.e. $\frac{\delta^{FB}}{1-\delta^{FB}} = \frac{\alpha y + \alpha \overline{p}}{g(e^{FB}_{MA}, \overline{p}, \overline{p})}$. Then we obtain the following lemma:

Lemma 3. *If $\delta \leq \delta_{FB}$, then $(IC - DE)$ is binding.*

The ability to transfer utility through side payments ensures that there cannot be a second-best optimal contract where one of the dynamic enforcement constraints has slack. Hence, in any optimal contract that does not achieve first best, both (DE_M) and (DE_A) will be binding, and therefore so will $(IC - DE)$.

The following proposition characterizes the optimal contract as a function of δ and shows that the relationship between effort and δ is non-monotonic. We can think of δ as one determinant of the potential future discounted surplus, and indeed the proposition could be written similarly in terms of the agent's outside option $\underline{u}$.

Proposition 1. *Agent's effort may be a non-monotonic function of the future discounted surplus. In particular, there exist values δ^0, δ^L and δ^H, where $\delta^0 \leq \delta^L \leq \delta^H$, such that $e > 0$ if and only if $\delta > \delta^0$, and the optimal manager-agent relational contract can be characterized as follows:*

- ***High surplus:*** *If $\delta \geq \delta^H$, then prices are not used to induce effort, i.e. $p_h = p_l = \overline{p}$, and effort is weakly increasing in δ.*
- ***Intermediate surplus:*** *If $\delta^H > \delta > \delta^L$, then both side payments and prices are used to induce effort, i.e. $p_l < p_h$ and $s_h < s_l$, and prices are at the maximum when quality is high, i.e. $p_h = \overline{p}$. When $p_l > 0$, then effort is decreasing in δ, and otherwise it is increasing in δ.*
- ***Low surplus:*** *If $\delta \leq \delta^L$ then side payments are not used to induce effort, i.e. $s_h = s_l$, and effort is weakly increasing in δ.*

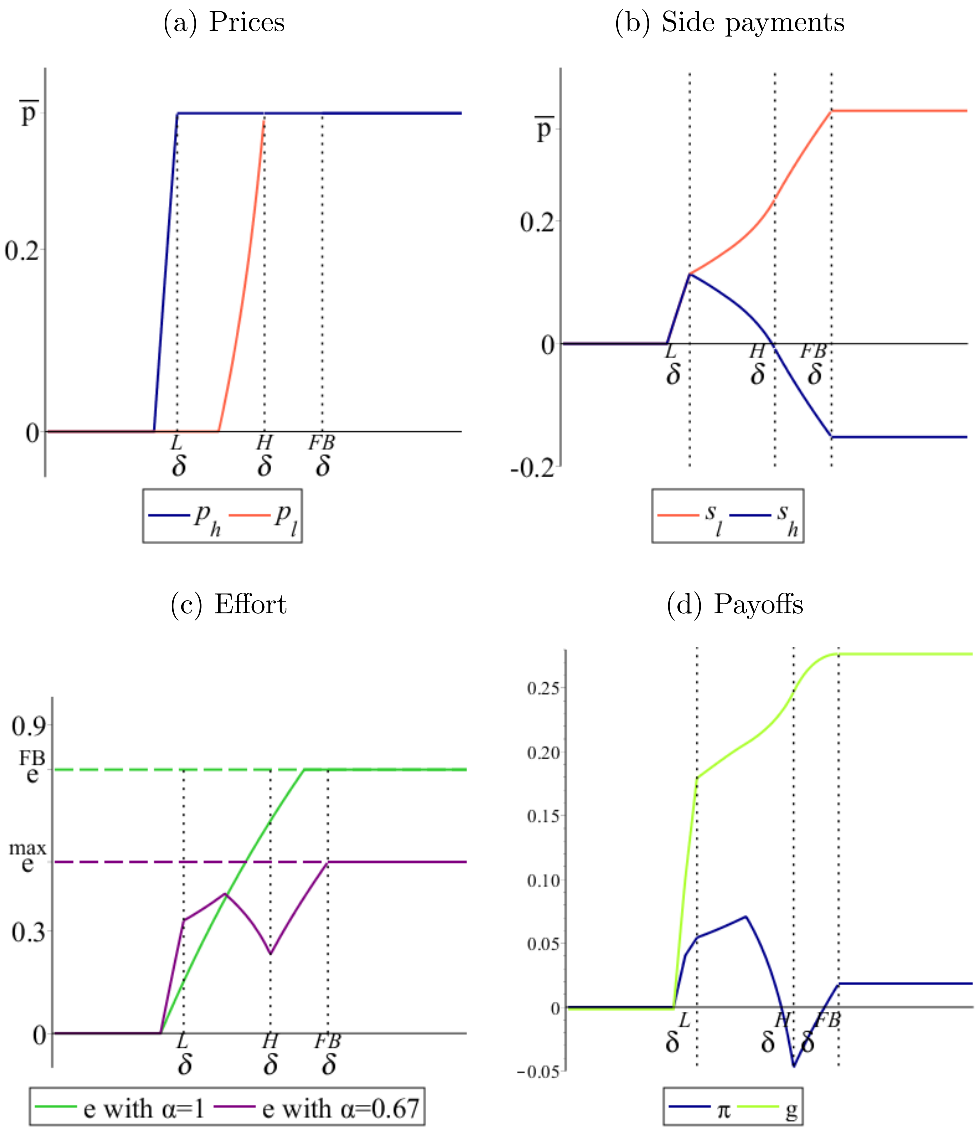
The proof of the proposition starts off by noting that the manager would always like to increase p_h in exchange for side payments (since this both increases g and incentivizes effort), so she must be bound from doing so by either $\overline{p}$ or the relational contracting constraint. Which is binding will depend on whether $\delta \gtrless \delta^L$, and the proof characterizes the optimal contract in both cases by considering how surplus is maximized.

The basic intuition behind the non-monotonicity result can be understood as follows. When future discounted surplus is high, the relationship can sustain both large unvarying prices and a large variation in side payments to induce effort. When future discounted surplus falls below a certain level, the manager replaces some of the variation in side payments with variation in prices, since these are easier to sustain. By doing so, the effort benefits the manager and the agent more, since high quality now not only triggers y but also a higher price. The manager therefore increases variation in prices further to induce more effort. Lower future discounted surplus makes inducing effort more difficult, but this is more than compensated for by the increase in the value of effort to them.

Note that the moral hazard in the model is important in generating effort decreasing with future discounted surplus. If the manager could observe the agent's effort directly, she would make the price and side payment depend directly on effort, and hence neither would vary in equilibrium. An increase in δ would then simply ease the dynamic enforcement constraint, allowing for larger prices or side payments, and hence weakly increase effort.

In order to better understand the nature of the optimal manager-agent contract, we now briefly detail the three cases outlined in Proposition 1. We also depict in Figure 2 the optimal contract for particular parameter values when $c(e) = \frac{1}{2}ce^2$. Figures 2a and 2b plot the prices and side payments as a function of δ.[11] Figure 2c then plots the induced effort levels, and for comparison we also include the effort level that would be exerted with principal-agent contracting. Finally, Figure 2d plots the principal's payoff and the manager-agent surplus g.

[11] The value of α used in this example may seem large compared to real world incentive schemes, but this stems from our assumption that side-payments are costless. In the case where there is a cost to side-payments due, for instance, to a risk of being caught, the relevant values of α will be lower - see Section 4.3 for more details.



$c(e) = 0.54 \times e^2, \overline{p} = 0.39, y = 0.85, \alpha = 0.67,$
$\underline{u} = 0.0015, \delta^L = 0.59, \delta^H = 0.7$ and $\delta^{FB} = 0.75$

Figure 2: Optimal manager-agent contract as a function of δ

2.2.1 High surplus

When future discounted surplus is slightly below the level that allows the manager and agent's first-best contract δ^{FB}, effort will be below first best but prices will remain at the maximum regardless of quality. Since $(IC - DE)$ is binding, effort will be determined by the equation $c'(e) = \frac{\delta g(e,\overline{p},\overline{p})}{1-\delta} - \alpha\overline{p}$. Effort is reduced before prices because, when $e = e_{MA}^{FB}$, a marginal reduction in effort leads to a second-order reduction in g, while the cost of reducing the prices is first-order. There will thus always be a range of future discounted surplus for which the optimal contract involves $p_l = p_h = \overline{p}$ and $e < e_{MA}^{FB}$. Hence $\delta^H < \delta^{FB}$.

When future discounted surplus falls further below δ^H, what happens depends on the relative value that the manager places on quality, α. If α is low, then she will continue to cut effort rather than prices until no effort is sustainable. In this case $\delta^H = \delta^L$ and there is no 'intermediate' range of future discounted surplus. If α is high, then $\delta^H > \delta^L$, and there will be an intermediate future discounted surplus range where prices are used to induce effort.

2.2.2 Intermediate surplus

The manager and agent face a trade-off in deciding upon the price paid when quality is low, p_l. A higher p_l generates greater surplus directly, but it also decreases effort. Maximizing surplus gives us the following expression for p_l when $\overline{p} > p_l > 0$:

$$p_l = \frac{1-\alpha}{\alpha}(1-e)\,c''(e) - y + \frac{1}{\alpha}\frac{\delta g(e, p_h, p_l)}{1-\delta} \tag{2}$$

The first term of this expression stems from the direct gain in surplus that an increase in p_l produces; the more likely low quality is to occur, the higher this gain. The second term is the result of the reduction in expected quality that an increase in p_l produces through the reduction in effort. The third term comes from the relational contracting constraint; higher future discounted surplus means that more effort can be induced through side payments, and hence p_l can be higher.

Since $(IC - DE)$ is binding, the effort level e is given by $c'(e) = \overline{p} -$

$p_l + \frac{\delta g(e,\overline{p},p_l)}{1-\delta} - \alpha\overline{p}$. If we substitute in (2), we can see that effort is weakly decreasing in the future discounted surplus if and only if $p_l > 0$.[12] When $p_l > 0$, a decrease in the future discounted surplus decreases p_l and hence the agent and manager have a greater incentive to increase effort. Instead, when $p_l = 0$, a lower future discounted surplus forces the manager to reduce the variation in side payments.

2.2.3 Low surplus

When the future discounted surplus becomes low, i.e. $\delta = \delta^L$, the manager can only just promise to pay $p_h = \overline{p}$ and will not be able to combine this with any variation in side payments. When $\delta \leq \delta^L$, the price p_h will be the maximum that the manager can credibly promise, i.e. $p_h = \frac{1}{\alpha}\frac{\delta g(e,p_h,p_l)}{1-\delta}$, and p_l will again be either equal to p_h, zero or a solution to (2).

2.3 Discussion

An important implication of Proposition 1 is that there is sometimes, but not always, a trade-off between increasing quality and reducing the surplus captured by intermediaries. The previous literature on relational contracts suggests that non-contractible production can be improved by increasing the discounted surplus within relationships, for instance by increasing tenure or decreasing competitive pressure (Calzolari and Spagnolo, 2009; Board, 2011; Gibbons and Henderson, 2013). Yet those concerned with vertical collusion argue that such policies will facilitate side payments (Martimort, 1999; Lambsdorff and Teksoz, 2005). Our analysis implies that in some cases both effects may indeed occur simultaneously. Examples of such a trade-off can be found in public procurement, where in some instances policies designed to reduce corruption appear to have a negative impact on performance or quality (Coviello, Guglielmo and Spagnolo, 2018; Lichand, Lopes and Medeiros, 2017). We also find, however, that in other cases there is no such trade-off, and decreasing future discounted surplus will reduce collusion without any negative impacts. An example of this can be found in the pubic procurement reforms studied by Lewis-Faupel et al.

[12] To see this, note that $c'(e) + \frac{1-\alpha}{\alpha}(1-e)c''(e)$ is increasing in e by equation (7) in the proof of Proposition 1.

(2016) who find reducing discretion and decreasing interactions appears to reduce corruption with a non-negative effect on quality. In our model, these correspond to situations where the price is positive every period and the manager is sometimes, but not always, paying the maximum possible price.

Furthermore, the example in Figure 2 demonstrates that the ability of the manager and agent to make side payments may facilitate the provision of quality. In particular, we can see from Figure 2(c) that, for a certain range of δ, the agent's effort is higher when he is incentivized by a manager open to collusion (i.e. one with $\alpha < 1$). This is because, when relational contracting is relatively difficult, the ability of the manager to receive kickbacks from the agent helps to make a higher price credible. It is therefore possible that the principal may benefit from the existence of side payments, even if they lead to the manager capturing surplus.[13] This accords with the judgment in the Aéropostale case that the profit that Finazzo made through receiving side-payments did not necessarily come at Aéropostale's expense, since the kickback scheme may have created value greater than the cost of the kickbacks. South Bay was willing, for instance, to go beyond the contract terms by holding and storing inventory, allowing Aéropostale to *"quickly start printing new styles"*, which was very valuable to adapt to its *"fickle"* teenage customer base (Droney, 2017, p.30).

3 Governing managed relational contracts

In the previous section, we ignored the role of the principal and treated the parameters α and $\overline{p}$ as exogenous. This provided insights into situations where the principal's ability to adapt these parameters to the context may be limited if, for instance, she did not observe the strength of relationship the manager has with the particular agent in question. We may also be interested, however, in how the principal should set these parameters were she able to choose them as functions of the underlying parameters, which is likely to be possible in some contexts. In this section, we consider that

[13]Note that this is in the case where $\alpha > 0$. If the principal can choose α, then when side-payments are impossible she can achieve first best by setting $\alpha = 0$. See Section 4.3 for more details.

the principal sets α and $\overline{p}$ at the beginning of the game.

The principal faces the following trade-off when choosing the optimal α. A lower α has two benefits for the principal - it implies giving up less surplus to the manager, and it also makes it easier for the manager to commit to pay the prices. If α is too low, however, then the manager will wish to pay prices even when quality is low. This both costs the principal surplus directly and weakens the agent's effort incentives.

The proposition describes how the principal sets α and $\overline{p}$ to maximize her payoff. In particular, the principal sets α at the lowest possible value that ensures the manager does not misuse prices, and then uses $\overline{p}$ to limit how much surplus is appropriated by the manager and the agent.

Proposition 2. *The principal will set α and $\overline{p}$ such that the optimal manager-agent contract has $p_l = 0$. Moreover, there exist $\delta_h < 1$ such that, if $\delta \geq \delta_h$, then $\overline{p} = 0$ and only side payments are used to induce effort.*

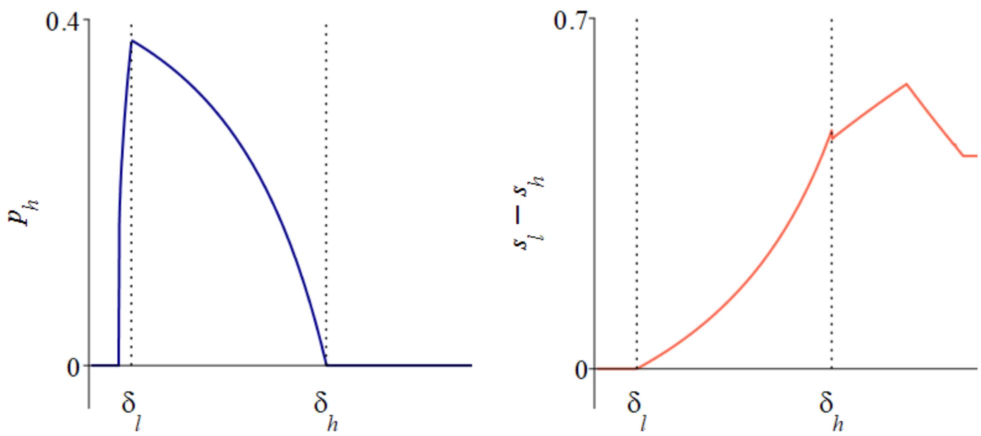
To prove that the principal will always induce $p_l = 0$, we first note that the principal would never induce a contract with $p_l = p_h > 0$, since she could always do better by setting $\overline{p} = 0$. More generally, we show that if the optimal manager-agent contract contained $p_h > p_l > 0$, then the principal could increase her payoff by either lowering $\overline{p}$ or increasing α.

In general, characterizing the principal's optimal solution is complex, but in the case where $c(e) = \frac{ce^2}{2}$ we can derive relatively simple expressions for the principal's optimal behavior. In particular, in this case the principal sets α such that there is zero marginal benefit to the manager of increasing p_l above zero. This is done by setting α such that $p_l = 0$ in equation (2). The maximum price $\overline{p}$ is then set to maximize the principal's profit function.

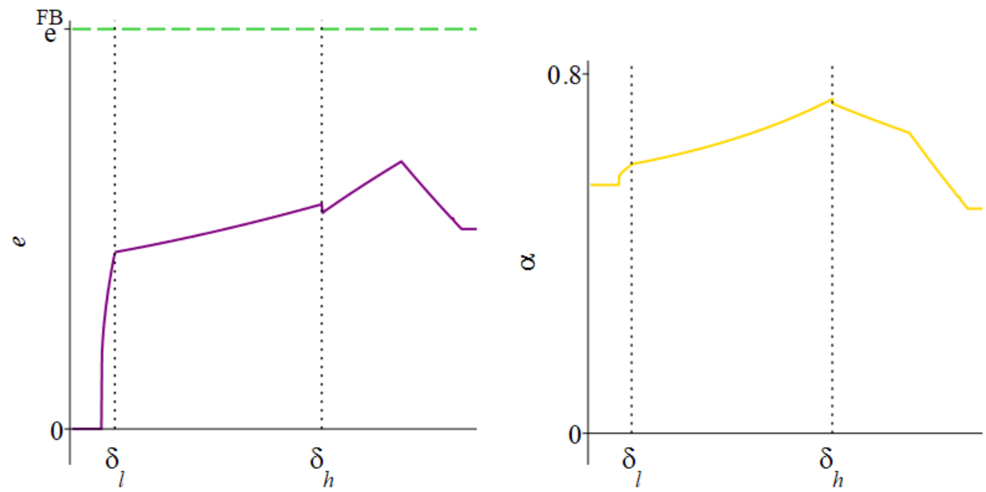
We display in Figure 3 the optimal contract when the principal sets α and $\overline{p}$ and $c(e) = c\frac{e^2}{2}$. For $\delta < \delta_l$, we see that the price paid when quality is high is increasing in δ, and in this range the agent is not constrained by the price cap. As δ increases, the principal increases α to ensure that the manager will be willing to maintain a larger variation in prices while still keeping $p_l = 0$. Between δ_l and δ_h, it is too costly for the principal to ensure $p_l = 0$ exclusively via an increase in α, so she combines it with a decrease in $\overline{p}$. Once $\overline{p} = 0$, at δ_h, the manager can no longer pay a positive p_l. Absent this potential form of collusion, the principal can afford to drop α which may trigger a reduction in effort. From δ_h onward, it is optimal for the principal to reduce α. Finally, when δ is sufficiently high, the relational contracting constraint no longer binds and the manager sets effort to her preferred level, where $ce = \alpha y$.

Figure 3: Optimal contract as a function of δ

(a) High quality price, p_h (b) Variation in side payments, $s_l - s_h$



(c) Effort, e (d) Profit sharing, α



$c(e) = 0.54 \times e^2$, $y = 0.85$, $\underline{u} = 0.0015, \delta_l = 0.58$ and $\delta_h = 0.74$

Note that when δ is low, the principal may not need to use a maximum price - in this case, the price that the manager can pay is sufficiently limited by the relational contracting constraint. For higher δ, however, the principal will use a maximum price to ensure that she does not have to give too large a share of her profit to the manager to achieve $p_l = 0$. By setting a more stringent maximum price, the principal forces the manager to use more variation in side payments as an incentive device. Indeed, if the future discounted surplus is very high, the principal will optimally give no discretion to the manager (i.e. $\overline{p} = 0$) and all the effort incentives will be provided through side payments. That the principal should reduce the manager's discretion in response to potential corruption is a common result in the literature. For instance, in Tirole (1986) and Burguet and Che (2004), an optimal quality is chosen to reduce the rents available to the agent and hence his willingness bribe the manager. Our results suggests that reducing discretion is likely to be most appropriate when the manager and agent have a strong enough relationship to sustain a large amount of vertical collusion.

With the principal's optimal behavior, we can now ask when the principal benefits from delegating to a manager. Sometimes, employing a manager may be obliged; the leader of a government or large firm may simply be unable to manage all relevant relational contracts herself. In other situations, she may have the choice between delegating the relational contract to a manager or managing it herself. In these cases, it is interesting to determine when such delegation may be in the principal's best interest.

The following proposition describes when the principal should delegate to a manager, assuming she sets α and $\overline{p}$ optimally. If she does not delegate, we assume that she undertakes direct relational contracting with the agent, achieving the results outlined in Section 2.1. The proposition states that, in the case of the quadratic cost function, there exists a range of discount factors for which delegating is strictly preferable and a higher range when direct relational contracting is strictly preferable.

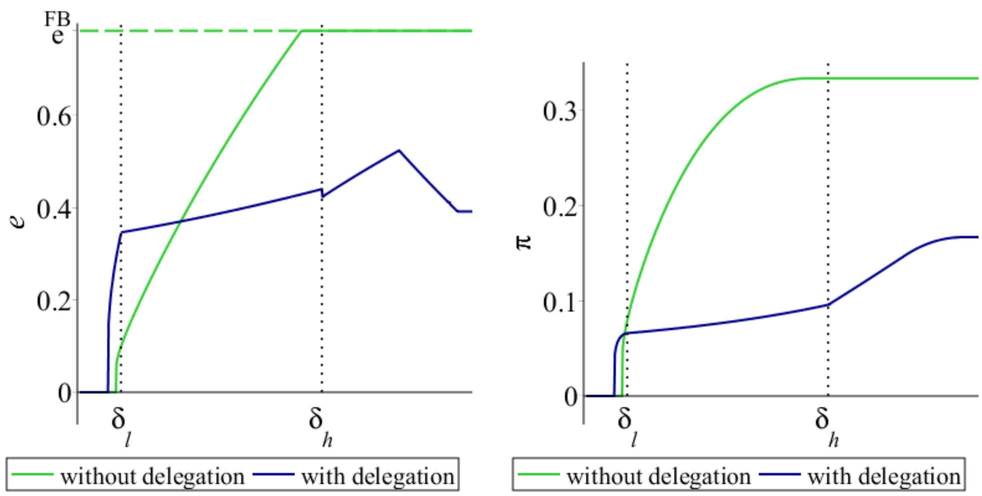
Proposition 3. *If $c(e) = \frac{ce^2}{2}$, then there exist values $\delta_0, \hat{\delta}$ and $\overline{\delta}$ with $0 \leq \delta_0 < \hat{\delta} \leq \overline{\delta} < 1$ such that:*

- *If $\delta > \overline{\delta}$, then the principal's payoff from the optimal managed relational contract is strictly below that from direct relational contracting.*

- *If $\hat{\delta} > \delta > \delta_0$, then the principal's payoff from the optimal managed relational contract is strictly above that from direct relational contracting.*

If the discount factor is high, then relational contracting poses no problem. The principal and agent can implement a large level of effort on their own, and the principal has no reason to share surplus with a manager. On the other hand, if the discount factor is low, then direct relational contracting is difficult and cannot sustain much effort. The principal would therefore prefer to delegate. Since the manager is less likely to renege than the principal, the manager can generate more effort and the extra surplus generated more than compensates for the part given to the manager.

Figure 4: Comparing managed relational contracting to direct relational contracting



$c(e) = .54 \times e^2$, $y = 0.85, \underline{u} = 0.0015, \delta_l = 0.58$ and $\delta_h = 0.74$

Figure 4 demonstrates this result graphically by plotting the best payoffs that the principal can achieve with and without delegation when $c(e) = \frac{ce^2}{2}$. A similar logic applies for other variables affecting the potential future discounted surplus, including the agent's outside option $\underline{u}$. Note that the range of δ for which delegation increases the principal's profit is much smaller than that for which effort increases. Since the principal cannot extract surplus from the manager, she will refrain from delegating on some occasions when doing so would lead to higher quality.

When the principal benefits from delegation, we may be concerned that the principal would like to renegotiate with the manager once the agent has exerted effort (Katz, 1991). This is unlikely to be feasible in our context, however, since the asymmetric information on Y between the principal and the manager would make renegotiation inefficient (Dewatripont, 1988). Moreover, Kockesen and Ok (2004) show using forward induction that strategic delegation is feasible when there is some cost to delegation (like the payment to the manager) and the agent observes who he is playing with. Both of these assumptions are reasonable in the contexts where our model is applicable.

3.1 Discussion

We have seen that the principal benefits from delegating to a manager whose payoffs are partly, but not completely, aligned with her own. The principal needs the manager to care somewhat about profit because otherwise no effort will be induced. This makes manager-agent relational contracting costly, which means that to get more effort the principal has to give up more surplus. As a result, the principal will not wish to induce first-best effort when delegating, and hence there is no need to have a manager whose incentives align completely with her own. Instead, the principal would rather have a manager who cares less than her about profit in order to facilitate relational contracting.

An example of such behavior can perhaps be seen in the way in which businesses in China deal with the practice of Guanxi, a system of informal relationships often formulated through gift exchange. Many firms are well aware of the risks stemming from procurement and sales managers' personal relationships, since these can facilitate side payments and other malpractice (Millington, Eberhardt and Wilkinson, 2005). Yet, when it comes to hiring such personnel, Wiegel and Bamford (2015) find evidence that firms specifically hire people with personal Guanxi, and they cite the ability of Guanxi to smooth inter-firm relationships as an important factor. Indeed, Schramm and Taube (2003) note that including corrupt transactions within Guanxi networks used to facilitate 'regular' transactions can *"lower the average costs of all transactions co-ordinated via this guanxi* *network* ".

4 Alternative specifications and extensions

In this section, we consider the sensitivity of our results to a number of extensions or alternative assumptions. We begin by considering the principal's optimal actions if we were to extend her set of instruments in a number of ways, including allowing transfers between her and the manager. We then consider how the model may be adapted to other contexts where relational contracts are managed on a third party's behalf, beyond the case of inter-firm supplier relationships. We then allow for side payments to be costly actions such as when there is a risk of punishment. Finally, we consider how our model may translate into a setting where the agent is multitasking and one task benefits only the manager.

4.1 Additional instruments for the principal

We previously assumed that the manager simply receives a share α of the profit. In some contexts, however, the principal may have additional tools at her disposal to contract with the manager. In this section, we consider how such additional instruments would affect the main results of the paper.

4.1.1 A transfer between the principal and the manager

In this section, we allow the manager to pay an upfront transfer to the principal. We could interpret this as the principal selling a share of the firm to the manager. This may also approximate situations where it is expected for managers to pay principals large payments in anticipation of future rents, such as paying bribes for jobs (Weaver, 2020). By considering this extension, we show that the key contributions of the paper are not an artifact of an endogenously assumed organizational structure of the firm but they actually hold when the organizational structure is allowed to optimally change. In particular, we show that the principal would not sell the entire venture even if she could in the circumstances where the principal benefits from having the corrupt manager.

Formally, we can define $\Pi = \frac{\pi_0}{1-\delta} + T$ to be the principal's ex-ante discounted payoff and similarly $V = \frac{v_0}{1-\delta} - T$ as the manager's, where T is a transfer set by the principal at the same time as she sets α and $\overline{p}$. Here π_0 and v_0 are the payoffs at $t = 0$, as defined in Section 1.

As in the benchmark model, we consider the case where the manager offers a take it or leave it offer. She sets s_F to give the agent his outside option, i.e. such that $\underline{u} = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\left\{d_\tau[p_\tau - S_\tau - c(e_\tau)] + (1-d_\tau)\,\underline{u}\right\}\right]$. If the agent were to have more bargaining power, the principal would not be able to extract as much surplus from the manager via T. In practice, the bargaining power between the manager and the agent may not be observable to the principal placing limits on realistic values of T.

Since the payment occurs before the relational contract begins, this will not change the manager-agent relational contract. Therefore the results that we discussed in Section 2 are unaffected by this change. With regards to the results set out in Section 3, these will depend on whether there is a bound on the amount the principal can extract from the transfer.

Suppose first that the only constraint on T is the manager's participation constraint. In particular, suppose that we require $V \geq 0$. Then the principal will set T such that this binds and hence we will have $0 = \mathbb{E}\left[\sum_{\tau=0}^{\infty} \delta^\tau d_\tau[\alpha(Y_\tau - p_\tau) + S_\tau]\right] - T$, where S_τ is such that it gives the agent his outside option. Substituting T into the equation for Π gives us the following:

$$\Pi = \mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau \left(Y_\tau - c(e_\tau)\right)\right] - \frac{\underline{u}}{1-\delta}$$

Hence, the principal only cares about total surplus $ey - c(e)$ and now, in her maximization problem in Section 3, will set α and $\overline{p}$ to maximize this subject to the set of potential contracts she can induce set out in Proposition 1. This leads us to the following result, which states that the principal will induce a contract where high prices are only paid as a reward for high quality and that, in general, she will set α such that the manager's incentives do not align exactly with her own.

Lemma 4. *If the principal can receive a transfer T from the manager in the first period, then she will still set α and $\overline{p}$ such that the optimal manager-agent contract has $p_l = 0$. Moreover, in the range of δ where $(IC - DE)$*

binds, the principal will set $\alpha < 1$.

When the principal can extract all the surplus from the manager-agent relationship upfront, she will still set α and $\overline{p}$ such that $p_l = 0$, as before. This is because collusion requires relational capacity in the manager-agent relationship, and the principal would rather ensure this is used to provide greater incentives for effort. Moreover, if providing effort incentives is limited by credibility, then the principal will set $\alpha < 1$. In other words, she does not want the manager to become 'the new principal'. This is because reducing α eases relational contracting and, when α is close to 1, the principal does not need to worry about potential collusion, since the manager's incentives are sufficiently closely aligned.

4.1.2 Sharing profit components to different degrees

There may also be contexts where it is possible for the principal to share the costs of payments to the agents in a different way from the way profit is shared, such that the manager receives $\alpha_y Y - \alpha_p p$ rather than $\alpha(Y - p)$. For a given α_y and α_p, the nature of managed relational contracts studied in Section 2 would not change. In particular, an increase in α_y/α_p has the same impact on $(IC - DE)$ as an increase in y. Manager-agent optimal contracts therefore would not change substantially, and Proposition 1 would remain unchanged. If the principal can set these parameters optimally, however, she would in general not choose $\alpha_Y = \alpha_p$, and the extra degree of freedom will improve the payoff she gets from delegating. Note, however, that delegation will still cost her a share of the surplus and thus Proposition 3 will remain qualitatively unchanged.

4.1.3 Dynamics in α and $\overline{p}$

In the baseline model, the principal observes neither the quality nor the associated payments, therefore α and $\overline{p}$ are time invariant in that the values chosen initially apply to all subsequent periods. This model allows us to focus on the strategic interaction between the manager and the agent (see Che and Yoo (2001) for a paper with a similar setting). We may wonder, however, how the results would change if these parameters are allowed to vary with the history of the relationship. To do so, we need to consider the multi-agent model set out in the online Appendix where the principal observes the aggregate profit. In particular, we can consider whether the principal might improve her payoff by making the manager's payoff at time t depend not only on the aggregate profit at time t but also the profit history. This could be thought of, for instance, as a gradual change of ownership conditional on good performance.

Intuitively, when δ is such that the manager and agent are constrained by the relational contract, the principal may want to help them by 'back-loading' the surplus in the relationship Ray (2002). By doing so, the principal can facilitate the relational contract between the manager and agent in the earlier periods by pushing the surplus the manager receives into the future, hence making it conditional on the manager not reneging. This may be beneficial for the principal if she wants to increase the total amount of surplus received by the manager and agent. Of course, she may alternatively want to do the opposite, if there is so much surplus in the manager-agent relationship that it is facilitating collusion. In this case, the principal will gradually decrease the stakes given to the manager, as in Martimort (1999).

Analyzing exactly how the principal would make α and $\overline{p}$ a function of profit history is complicated, however, by the fact that this would also make the contract between the manager and the agent non-stationary. Moreover, varying α not only changes the amount of surplus shared with the manager, but also determines how costly it is to pay the prices as opposed to side-payments. Lowering α therefore also makes relational contract easier and collusion more tempting, which will limit the principal's willingness to use it to create dynamic incentives. She is therefore more likely to vary $\overline{p}$ as a function of the profit history.

Nonetheless, we do not see any reason why allowing for dynamics would substantially change the results of the paper, since the main intuitions would still apply. For instance, the principal will still set $\overline{p} = 0$ when δ is very high since here the future discounted surplus is enough for the manager to incentivize effort via side payments exclusively. Moreover, even if the principal were to promise to eventually transfer the entire firm to the manager (i.e. set $\alpha = 1$), collusion would still be an important concern in all of the periods leading up to this.

Overall, therefore, we can see that allowing the principal to have extra instruments will improve the payoff she gets when the relational contract is implemented by a manager. In the extreme, allowing the principal to sell the firm to the manager would mean delegation would always be weakly beneficial, though the benefits would still be zero for large δ. In general, the fundamentals of the relational contract between the manager and the agent will not change - the manager not caring about the principal's profit share will still facilitate relational contracting and, since the principal may still find a price cap beneficial, she will need to adapt parameters to changes in δ to avoid the non-monotonic relationship with effort of Proposition 1.

4.2 Adapting the model to other contexts

We have focused our model on supply relationships between firms, but there are many other contexts where relational contracts are important and are managed on a third party's behalf. Adapting our model appropriately may generate useful insights for these situations.

4.2.1 Bureaucracies

In the main model, we have assumed that the manager's incentives are aligned with the principal's through some form of profit sharing or shared ownership, and hence incentivizing the manager comes at a direct cost to the principal. In large government bureaucracies, this is unlikely to be the case, and instead a manager's motivation may come from intrinsic motivations or reputation (Wilson, 1989). In this case, α would represent how much the public manager intrinsically cares about the government's goal and hence the principal's payoff function would simply be $ye_t - c(e_t)$. The results of Section 2 therefore would be unchanged and, if the principal chooses α (through, for instance, hiring decisions), then the model is equivalent to the case with an upfront transfer analyzed in Section 4.1.1. In particular, α would be weakly greater than that found in Section 3, but it would still not be optimal for the principal to set $\alpha = 1$ unless δ was sufficiently large that first-best effort could be achieved.

4.2.2 Employment relationships

Another setting where self-enforcing contracts are important is in relationships between employers and employees (Levin, 2003). As with procurement, these relational contracts are frequently managed by mid-level managers within an organization, and hence there is the possibility of collusion through 'influence activities' (Milgrom, 1988).[14] For instance, Nkamleu and Kamgnia (2014) document that in African governments per-diems are *"mainly given to provide financial incentives to employees in order to increase their motivation"* but managers may *"expect the staff member to share or kickback a portion of the per-diem"*.

The case of Credit Suisse First Boston described by Stewart (1993) suggests that some insights of our model are likely to carry over to the labor relationship setting.[15] The investment banking firm First Boston (FB), which we can consider as the manager, had been very successful at maintaining long-term relationships with its bankers (the agent). At the same time, it financed very precarious transactions that brought bonuses for managers and bankers at FB at the expense of decreasing the firm's long-term value, thereby hurting shareholders like Credit Suisse (CS), the principal.[16] CS introduced measures to change the *"freewheeling atmosphere (...) notable for an absence of the layers of controls... [and] for huge salaries and bonuses."*[17] In particular, it imposed a tighter bonus cap (akin to our maximum price), and as a result, FB top management could no longer pay bonuses that they felt were sufficient to reward their employees. Stewart (1993) then notes that at least one manager *"dipped into his*

[14] Examples of bribing for promotions can be found in a recent study by Weaver (2020), while Bebchuk and Fried (2004, p.93) note that directors give CEOs large retirement gifts partly *"to express gratitude for what the CEO has done for them"*.

[15] We thank Jin Li for suggesting this case study.

[16] Following the 1987 Wall Street crash, CS had to rescue FB *"by sinking more than $300 million"* and removing *"more than $400 million in troubled loans."* Moreover, FB's reputation with its clients was damaged: *"some corporations are asking why they should seek advice from a firm that managed its own finances so disastrously, and helped arrange such ill-fated deals"* (Greenhouse, 1991).

[17] Hierarchical collusion to oversell was a common feature in the Wall Street's 1990s-era: *"So much of communication wasn't captured in e-mails or directly mentioned in meetings. It was implicit—understood without words. If your chairman asked you to take a look at a stock, (...) you didn't need to be told explicitly what to say or write. It was understood, (...) that you were to comply by lavishing the stock or the deal with positive comments"* (Prins, 2006).

own pocket to pay them more", which mirrors what happens in our model when the manager's discretion is reduced.[18]

Overall, we believe the main results of our model are likely to apply in many settings with managed relational contracts when there is an asymmetry between the tools used to incentivize the manager and those used to incentivize the agent. In employment relations, for instance, this may be the case when the manager is the CEO, the managing partner or a controlling shareholder in the firm. Further down the hierarchy, however, a more fundamental change to the model may be required to consider that the manager herself is incentivized relationally.

4.3 Costly side payments

We have assumed for simplicity that side payments between the manager and agent are costless except to the extent that they need to be self-enforced. In reality, however, side payments may be intrinsically costly. For instance, there may be a risk of punishment, and payments may be made in an inefficient way to avoid detection. Alternatively, side payments may not represent cash transactions, but favors that the agent can do for the manager, where the manager's benefit is not necessarily equal to the agent's cost.

In this subsection we consider how our main model would change if we make side payments costly. In particular, we assume that a side payment which costs an agent S only gives a benefit of κS to the manager, where $0 < \kappa \leq 1$. This assumption makes sense when side payments are on net paid from the agent to the manager, which will happen if the maximum price is sufficiently high.[19] We work on the case where $\alpha < \kappa$ since, if $\alpha \geq \kappa$, then side payments are no longer relevant, as the manager does not

[18]Cases of managers paying bonuses from their own money are rarely documented due to their informal nature, but anecdotes suggest that the phenomenon does occur - see, for instance, Green (2013).

[19]For other contexts, we could alternatively assume that the side payment S was always positive, or that a cost was born by the agent for receiving side payments. Either assumption would lead to optimal contracts being potentially non-stationary, since the agent is limited in his ability to extract surplus from the manager and will therefore use a threat of lower production instead. The model would then share similarities with that of Fong and Li (2017*b*), which can be seen as an example of the 'backloading' principal expounded by Ray (2002).

gain from having prices paid to the agent and kicked back to her.

The optimal manager-agent contracts will not change significantly with these new assumptions. In particular, a monotonic transformation of the manager's payoff function tells us that introducing a cost of side payments κ is equivalent to the costless case where she receives a share of profit α/κ.

Proposition 4. *The impact on managed relational contracts of an increase in the cost of side payments, i.e. a decrease in κ, is equivalent to an increase in α. When side payments are more costly, the principal will benefit from delegation more often.*

In other words, Proposition 1 will not change, and in this important sense we can interpret the parameter α/κ as both a measure of how closely aligned incentives are and a measure of the manager's corruptibility.

The principal's optimal behavior will be impacted by κ. When κ is smaller, the risk of collusion is reduced, and hence the principal can set a lower value of α and still avoid $p_l > 0$. In other words, the manager's behavior depends on κ/α, so the principal gets the same commitment benefit of side payments when α and κ are low as when they are high, but the principal prefers to have them both low since then she shares less profits. Delegation is therefore more beneficial for the principal since she can achieve the same level of effort by sharing a smaller share of the profit. This result is consistent with Bloom, Sadun and Van Reenen (2012) and Bloom et al. (2013) who find that firms delegate more when there is either stronger rule of law or management practices which allow better monitoring of managers. The impact of potential collusion on delegation can have important consequences - Akcigit, Alp and Peters (2020), for instance, show that the relative difficulty of delegation in India compared to the US can account for 15 % of the difference in income between the two countries.

If side payments were impossible ($\kappa = 0$), the principal could set $\alpha = 0$ and induce first-best effort. Thus in general, if the principal can set α, she prefers for collusion to be more costly.[20] This is because she can

[20] Our results thus contrast with Strausz (1997), who in an alternative model of intermediation finds outcomes are the same whether or not supervisor-agent collusion is possible (i.e. $\kappa = 1$ or $\kappa = 0$). One reason for this difference is that Strausz (1997) considers a model where the supervisor's monitoring creates verifiable information, and hence the principal can write a contract with the supervisor that depends both on her effort and the agent's performance.

always reduce α accordingly, and thus keep the commitment benefits of side payments while reducing the profit-sharing cost. If she could not reduce α, however, more costly collusion could be damaging for her since it would reduce the effort the manager could induce relationally with the agent.

4.4 An application to multitasking

We have so far modeled manager-agent collusion operating through the transfer of side-payments, as is typical in models of collusion in hierarchies. An alternative approach is in the style of a multitasking model where the agent can undertake two types of activity - one which benefits the principal, and one which benefits only the manager. In this section, we demonstrate with a simple model of multitasking that this form of vertical collusion can also aid relational contracting.

Let us suppose that the agent cannot pay side payments to the manager, but instead can now exert effort on two alternative tasks. He exerts effort e_1, which probabilistically generates a revenue of Y accruing to the principal as before, and e_2 which benefits the manager by an amount $f(e_2)$. The two tasks are substitutes, and hence the cost to agent of exerting effort is $c(e_1 + e_2)$. For simplicity, we assume now that the manager observes e_1 and e_2 directly. The principal's only action is to set α at the beginning of the game, such that the manager's payoff function is $v_t = (1-\delta)\sum_{t=\tau}^{\infty} \delta^{\tau-t} d_\tau[\alpha Y_\tau - \alpha p_\tau + f(e_{2\tau})]$.

If the manager's contract with the agent is enforceable by a third party, she can set e_1 and e_2 and pay the agent a compensation of $p = c(e_1 + e_2)$. She will therefore choose e_1 and e_2 such that $\alpha c'(e_1 + e_2) = \alpha y = f'(e_2)$. From the principal's point of view, therefore, in this setting increasing α decreases the 'collusive' effort e_2 and increases the 'productive' effort e_1.

Now, alternatively, suppose that the manager's contract with the agent is relational. In this case we will still have $p = c(e_1 + e_2)$, but this will now be subject to a dynamic enforcement constraint $p \leq \frac{\delta}{1-\delta}(\alpha y e_1 + f(e_2) - \alpha c(e_1 + e_2))$. Solving the manager's constrained maximization problem gives us that, when this dynamic enforcement constraint is binding, she will set $\alpha y = f'(e_2)$ and $\alpha c(e_1 + e_2) = \delta(\alpha y e_1 + f(e_2))$. This implies that, if $\delta < \min\left\{\frac{c'(e_1+e_2)}{y}, \frac{\alpha y c'(e_1+e_2)+f''(e_2)c(e_1+e_2)}{y(e_1 f''(e_2)+f'(e_2))}\right\}$, then $\frac{de_1}{d\alpha} < 0$. In other words, if relational contracting is sufficiently difficult, reducing the weight which the manager places on the principal's payoff can facilitate effort that is beneficial for the principal. The mechanism is the same one as in our model with side payments - by increasing the value the manager places on collusion as compared to the principal's payoff, the manager has more credibility in promising to reward the agent's effort.

5 Conclusion

This is the first paper that studies the impact of vertical collusion on relational contracts. The main takeaway messages are the following: First, when trust is a scarce resource, managed relational contracts are more credible and can incentivize more quality than direct relational contracts. Second, when relational contracts are overseen by a manager, a stronger relationship between the manager and agent can lead to lower quality.

Before the recent Aéropostale judgment, it was common to use "*the value of the kickbacks*" as "*a reasonable measure of the pecuniary loss suffered*" by the third party (Droney, 2017, p.70). Judge Droney, however, argued that this "*negative correlation*" between kickbacks and loss should not be taken for granted. Indeed, our model shows when this negative correlation may not exist. Hence, our conclusions help explain why politicians and firm owners frequently turn a blind eye to employees accepting side payments (Banfield, 1975). On the other hand, our model also identifies when side payments undermine effort. This occurs when they lead to prices becoming insensitive to the agent's performance, which appears to have been part of the trigger for Aéropostale firing and suing the colluding manager. Overall, the model therefore helps us to understand the complex relationship between kickbacks and productive relational contracts.

The model produces a number of testable implications. We could, for instance, test directly for a non-monotonic relationship between quality and factors that facilitate relational contracting in situations where the principal is constrained in her ability to govern the manager-agent relationship, such as the public sector or when the manager is a controlling shareholder. In some circumstances it may also be possible to observe the extent to which managers use their discretion and test for the type of misuse predicted by the model. For instance, Rasul and Rogger (2015) find Nigerian public projects to be better implemented when the overseeing bureaucrats are ethnically diverse - might this be because collusion is harder in such contexts? In other contexts, we may use the model to analyze the principal's behavior by observing variation in the incentives and discretion given to managers. Variation in the value of manager-agent relationships may be obtained by considering connections between individuals outside work or the importance of contracts in light of the business cycle or changes in competition. A potentially under-explored area may be investigating firm owners' concerns with employee fraud in procurement, particularly in developing countries where courts are weak.

There are also multiple theoretical extensions to the model that would be valuable to pursue. For instance, we have assumed that the manager's preferences are known, but in reality there is uncertainty as to 'how corrupt' any individual is. Removing this assumption, in the spirit of Chassang and Miquel (2019) or through uncertainty over the manager's outside option as in Halac (2012), may reveal insights into how corruption and effort evolve over time. We may also ask whether collusive relational contracts make managers more likely to stick with the same firm over time. In this regard, the papers by Board (2011) and Calzolari and Spagnolo (2009) that consider relational contracts with potential competitors may provide useful approaches.

Appendix

Proof of Lemma 1. We provide a sketch of the proof since it is analogous to that of Theorem 2 in Levin (2003). In a nutshell, the proof shows that any optimal contract can be replicated by a stationary contract by transferring variation in continuation values to side payments. Consider a manager-agent contract that in its first period calls for payments $p(y)$, s^F, $s(y)$ and effort e. If the offer is made and accepted and the discretionary payments made, the continuation contract gives payoffs $u(y)$ and $v(y)$ as a function of the observed outcome y. Let u, v be the expected payoffs under this contract:

$$u \equiv (1-\delta)E\left[p(y) - s^F - s(y) - c(e)|e\right] + \delta E\left[u(y)|e\right]$$
$$v \equiv (1-\delta)E\left[\alpha(y - p(y)) + s^F + s(y)|e\right] + \delta E\left[v(y)|e\right]$$

We follow Levin (2003) in defining this contract as self-enforcing if and only if the following conditions hold:

i. Parties willing to initiate the contract: $u \geq \underline{u}$ and $v \geq 0$

ii. The agent is willing to choose e: $e \in \arg\max_e E\left[p(y) - s(y) + \frac{\delta}{1-\delta}u(y)|e\right] - c(e)$

iii. For all y, both parties willing to pay p:

$$(1-\delta)\left(-\alpha p(y) + s(y)\right) + \delta v(y) \geq 0$$
$$(1-\delta)\left(p(y) - s(y)\right) + \delta u(y) \geq \delta \underline{u}$$

iv. For all y, both parties willing to pay s:

$$\begin{aligned} (1-\delta)s(y) + \delta v(y) &\geq 0 \\ -(1-\delta)s(y) + \delta u(y) &\geq \delta \underline{u} \end{aligned}$$

v. Each continuation contract is self-enforcing: $u(y)$, $v(y)$ correspond to a self-enforcing contract that will be initiated in the next period.

Let g^* be the maximum surplus generated by any self-enforcing contract. Consider an optimal non-stationary contract with continuation payoffs $u(y)$

and $v(y)$ (such that $u(y)+v(y)=g^*$), a side payment $s(y)$ and a price $p(y)$. We must define new side payments, $s^*(y)$ and s^{F*}, to produce the stationary contract that gives u^* to the agent and v^* to the manager, where $v^*=g^*-u^*$

$$\begin{aligned} s^*(y) &= s(y) - \frac{\delta}{1-\delta}u(y) + \frac{\delta}{1-\delta}u^* \\ u^* &= \mathbb{E}_Y\left[p(y) - s^{F*} - s^*(y) - c(e)|e\right]. \end{aligned}$$

□

Proof of Lemma 2. For the first part, consider an optimal contract with $p_h<0$. Then consider an alternative contract with price $p'_h=0$ and side payment $s'_h=s_h-p_h$. It is simple to check that all the self-enforcing constraints are still satisfied. Moreover, this contract has a higher surplus g, and therefore the original contract cannot be optimal. The same logic holds if $p_l<0$.

For the second part, first suppose that $s_h>s_l$. If positive effort is being made, we must have $p_h>p_l$. Then, consider an alternative contract with $s'_l=s_h$, $p'_l=p_l+s_h-s_l$. This alternative contract must also be self-enforcing, yet surplus g is greater. Hence the original contract is not optimal. In the case of prices, if $p_h<p_l$, then we can similarly consider an alternative contract with $p'_h=p_l$ and $s'_h=s_h+p_l-p_h$. □

Proof of Lemma 3. First, consider an optimal contract with (DE_A) not binding. If $e<e^{FB}_{MA}$, then consider an alternative contract with $s'_l=s_l+\epsilon$. This contract induces higher effort and, for some $\epsilon>0$, is self-enforcing. By Lemma 2 and $c'(e^{FB}_{MA})=\alpha y$, a higher effort increases the surplus $\frac{\partial g(e,p_h,p_l)}{\partial e}=\alpha y+(1-\alpha)(p_h-p_l)-c'(e)>0$ - hence the original contract cannot have been optimal. Thus we must have $e\geq e^{FB}_{MA}$. If $p_l<\overline{p}$, then consider a contract with $p'_l=p_l+\epsilon$ and $s'_l=s_l+\epsilon$. This contract implements the same effort but generates higher surplus and, for some $\epsilon>0$, is self-enforcing. Thus we must have $p_l=\overline{p}$. Lemma 2 then implies $p_h=\overline{p}$. Hence any optimal contract with (DE_A) not binding must have $p_h=p_l=\overline{p}$ and $e\geq e^{FB}_{MA}$.

Second, consider an optimal contract with (DE_M) not binding. If $e<$

e^{FB}_{MA}, then consider an alternative contract with $s'_h = s_h - \epsilon$. This contract generates higher effort and, for some $\epsilon > 0$, will be self-enforcing. As before, a higher effort increases the surplus and hence the original contract cannot have been optimal. Thus we must have $e \geq e^{FB}_{MA}$. If $p_h < \overline{p}$, then we must have $s_l = s_h$, since otherwise we can construct an alternative self-enforcing contract that implements the same effort but generates higher surplus with $p'_h = p_h + \epsilon$ and $s'_h = s_h + \epsilon$, for some $\epsilon > 0$. Since $e > 0$, it therefore follows that $p_l < p_h$, but now we can construct a self-enforcing contract with $p'_h = p_h + \epsilon$ and $p'_l = p_l + \epsilon$, for some $\epsilon > 0$. Hence we must have $p_h = \overline{p}$. Finally, if $p_l < \overline{p}$, then we can consider a contract with $p'_l = p_l + \epsilon$ and $s'_h = s_h - \epsilon$ (since $s_l \geq s_h$ from Lemma 2). But this contract is self-enforcing for some $\epsilon > 0$ and has higher surplus. Hence any optimal contract with (DE_M) not binding must have $p_h = p_l = \overline{p}$ and $e \geq e^{FB}_{MA}$.

Therefore, if either (DE_A) or (DE_M) is not binding, we must have $p_h = p_l = \overline{p}$ and $e \geq e^{FB}_{MA}$. Summing (DE_A) and (DE_M) and substituting into (IC) gives $\alpha\overline{p} + c'(e) < \frac{\delta}{1-\delta}(v + u - \underline{u}) \leq \frac{\delta g(e^{FB}_{MA}, \overline{p}, \overline{p})}{1-\delta}$. But, since $e \geq e^{FB}_{MA}$, we must have $c'(e) \geq \alpha y$, which implies $\delta \geq \delta^{FB}$. □

Proof of Proposition 1. If $\delta \geq \delta^{FB}$, then the first-best contract is self-enforcing. This contract is 'high surplus' in the sense of the proposition. For the rest of the proof we consider the case when $\delta < \delta^{FB}$ and hence, from Lemma 3, $(IC - DE)$ is binding. We first consider how the variation in prices and side payments in the optimal contact change as a function of δ, and then how effort e changes as a function of δ for each contract type.

First, note that both the surplus and effort level are increasing in p_h. The manager must therefore be bound from increasing p_h by either the price cap or the $(IC - DE)$. If $p_h < \overline{p}$, then we must have $s_h = s_l$, since otherwise we can consider an alternative contract with $p'_h = p_h + \epsilon$ and $s'_h = s_h + \epsilon$, which will be enforceable for some $\epsilon > 0$ and generate greater surplus. When $s_h = s_l$, the $(IC - DE)$ can be rewritten as $p_h \leq \frac{1}{\alpha}\frac{\delta g(e,p_h,p_l)}{1-\delta}$, so we have that $p_h = \min\{\overline{p}, \frac{1}{\alpha}\frac{\delta g(e,p_h,p_l)}{1-\delta}\}$. Which of these two values p_h takes will depend on the value of $\overline{p}$. Since $g(e, p_h, p_l)$ is weakly increasing in δ (because a larger δ relaxes (DE_A) and (DE_M), we can define δ^L as the unique value which solves the equation $\overline{p} = \frac{1}{\alpha}\frac{\delta^L g(e,\overline{p},p_l)}{1-\delta^L}$. We now consider in turn what the optimal contract looks like in the cases when $\delta \geq \delta^L$ and

$\delta \leq \delta^L$.

If $\delta \geq \delta^L$, then by definition we have $\overline{p} \leq \frac{1}{\alpha}\frac{\delta g(e,p_h,p_l)}{1-\delta}$ and hence $p_h = \overline{p}$. If $p_l \in (0, \overline{p})$, then it is then determined by $(IC - DE)$: $p_l = \frac{\delta g(e,\overline{p},p_l)}{1-\delta} + (1-\alpha)\overline{p} - c'(e)$. Moreover, if $\delta > \delta^L$, then by rearranging this expression and noting that $\frac{\delta g(e,\overline{p},p_l)}{1-\delta} > \alpha\overline{p}$ we can see $c'(e) > \overline{p} - p_l$, implying $s_l > s_h$. When $\delta > \delta^L$, therefore, side payments are used to incentivize effort. To calculate how much side payments are used to incentivize effort, we need to solve the manager's optimization problem. Denote the surplus function $g_1(e,\overline{p}) = g(e,\overline{p},p_l)$ where p_l is given as a function of e by $(IC - DE)$:

$$g_1(e,\overline{p}) = \frac{\alpha e y + (1-\alpha)e\overline{p} + (1-\alpha)(1-e)\left((1-\alpha)\overline{p} - c'(e)\right) - c(e) - \underline{u}}{1 - \frac{\delta(1-e)(1-\alpha)}{1-\delta}} \tag{3}$$

The manager wishes to maximize $g_1(e,\overline{p})$ with respect to e, where p_l is subject to the boundary conditions $0 \leq p_l \leq \overline{p}$. Let $\overline{g}_1$ and $\underline{g}_1$ be the surpluses at the upper and lower boundaries. Effort levels $\overline{e}_1$ and $\underline{e}_1$ are determined by $(IC - DE)$ at these two potential solutions. Let $\tilde{g}_1$ be the surplus at the interior solution that maximizes surplus. This will involve effort level $\tilde{e}_1$ where $g_1'(\tilde{e}_1,\overline{p}) = 0$. Differentiating $g_1(e,\overline{p})$ gives

$$g_1'(e,\overline{p}) = \frac{\alpha y + (1-\alpha)\left(\alpha\overline{p} - (1-e)c''(e)\right) - \alpha c'(e) - (1-\alpha)\frac{\delta g_1(e,\overline{p})}{1-\delta}}{1 - \frac{\delta(1-e)(1-\alpha)}{1-\delta}} \tag{4}$$

This thus completes the determination of the optimal contract when $\delta > \delta^L$. To arrive at the value of p_l written in equation (2), we simply combine the expression $g_1'(e,\overline{p}) = 0$ and the binding $(IC - DE)$.

We now wish to characterize the optimal contract as a function of δ when $\delta > \delta^L$. To do this, we first differentiate each of the possible expressions for the surpluses with respect to δ, giving the following equations:

$$\frac{d\overline{g}_1}{d\delta} = \frac{\overline{g}_1}{1-\delta}\frac{\alpha y - c'(\overline{e}_1)}{(1-\delta)c''(\overline{e}_1) - \delta(\alpha y - c'(\overline{e}_1))}$$
$$\frac{d\tilde{g}_1}{d\delta} = \frac{\tilde{g}_1}{1-\delta}\frac{(1-\alpha)(1-\tilde{e}_1)}{1-\delta-(1-\alpha)(1-\tilde{e}_1)\delta}$$
$$\frac{d\underline{g}_1}{d\delta} = \frac{\underline{g}_1}{1-\delta}\frac{\alpha y + (1-\alpha)\overline{p} - c'(\underline{e}_1)}{(1-\delta)c''(\underline{e}_1) - \delta(\alpha y + (1-\alpha)\overline{p} - c'(\underline{e}_1))}$$

At any δ where $\overline{g}_1 = \tilde{g}_1$, from $(IC-DE)$ we know that $c'(\overline{e}_1) < c'(\tilde{e}_1)$. From (4), at any interior solution we have $c'(\tilde{e}_1) = y - \frac{1-\alpha}{\alpha}(1-\tilde{e}_1)c''(\tilde{e}_1) - \frac{1-\alpha}{\alpha}\left(\frac{\delta\tilde{g}_1}{1-\delta} - \alpha\overline{p}\right)$. Using $(IC-DE)$ for $\bar{e}_1$, we obtain $\alpha c'(\tilde{e}_1) = \alpha y - (1-\alpha)(1-\tilde{e}_1)c''(\tilde{e}_1) - (1-\alpha)c'(\bar{e}_1)$. Rewriting, it follows that

$$0 < \alpha(c'(\tilde{e}_1) - c'(\bar{e}_1)) = \alpha y - (1-\alpha)(1-\tilde{e}_1)c''(\tilde{e}_1) - c'(\bar{e}_1) <$$
$$< \alpha y - (1-\alpha)(1-\tilde{e}_1)c''(\bar{e}_1) - c'(\bar{e}_1)$$

Then, we use $\alpha y - c'(\bar{e}_1) > (1-\alpha)(1-\tilde{e}_1)c''(\bar{e}_1)$ to show that $\frac{d\overline{g}_1}{d\delta} > \frac{d\tilde{g}_1}{d\delta}$ at any δ where $\bar{g}_1 = \tilde{g}_1$. There therefore exists a single value of δ such that for all higher values the upper boundary is preferable to an interior solution, and for all lower values the interior solution is preferable. We can show similarly that, at any δ where $\overline{g}_1 = \underline{g}_1$, we have $\frac{d\overline{g}_1}{d\delta} > \frac{d\underline{g}_1}{d\delta}$. Hence there exists a value δ^H such that the optimal solution has $p_l = \overline{p}$ if and only if $\delta \geq \delta^H$.

If $\delta < \delta^L$, then by definition we have $\overline{p} > \frac{1}{\alpha}\frac{\delta g(e,p_h,p_l)}{1-\delta}$ and hence $p_h < \overline{p}$. From before, we therefore have that $s_h = s_l$, and hence side payments are not used to incentivize effort. We can therefore write the surplus as $g_2(e,\delta)$ where $g_2(e,\delta) = g\left(e, \frac{1}{\alpha}\frac{\delta}{1-\delta}g_2(e), \frac{1}{\alpha}\frac{\delta}{1-\delta}g_2(e) - c'(e)\right)$. Expanding and rearranging gives:

$$g_2(e,\delta) = \frac{\alpha(1-\delta)}{\alpha-\delta}\left(\alpha ye + (1-\alpha)(-(1-e)c'(e)) - c(e) - \underline{u}\right) \qquad (5)$$

In this case, effort will be chosen to maximize the manager's surplus, i.e. by maximizing $g_2(e,\delta)$ with respect to e subject to the constraints $0 \leq p_l \leq p_h$. To arrive at the value of p_l written in equation (2), we simply combine the expression $g_2'(e) = 0$ and the binding $(IC-DE)$: $\frac{1}{\alpha}\frac{\delta}{1-\delta}g_2(e) - c'(e) = p_l$.

To finish characterizing the optimal contract as a function of δ, we need to show that there is a level of δ which determines when effort will be zero. To do this, we define δ^0 to be the maximum δ such that all optimal contracts have $e = 0$. Note that $\delta^0 \leq \delta^L$ since if $\delta > \delta^L$ we have $p_h = \overline{p} < \frac{1}{\alpha}\frac{\delta}{1-\delta}g(e,\overline{p},p_l)$ and hence $e > 0$ from the binding $(IC-DE)$. Now suppose that there exists a value of $\delta < \delta^0$ such that the optimal contract has $e > 0$. Then $g_2(e,\delta) > g_2(0,\delta)$, and it follows that $g_2(e,\delta^0) = \frac{(1-\delta^0)(\alpha-\delta)}{(\alpha-\delta^0)(1-\delta)}g_2(e,\delta) > \frac{(1-\delta^0)(\alpha-\delta)}{(\alpha-\delta^0)(1-\delta)}g_2(0,\delta) = g_2(0,\delta^0)$, which contradicts the definition of δ^0. Hence we must have $e = 0$ in all optimal contracts when $\delta \leq \delta^0$.

Finally, let us consider the relationship between e and δ in the optimal contracts. For high surplus contracts, a binding $(IC - DE)$ implies that the LHS of the equation $\frac{c'(e)+\alpha\overline{p}}{g(e,\overline{p},\overline{p})} = \frac{\delta}{1-\delta}$ is increasing in e. For low surplus contracts with $p_l = 0$ we can similarly transform the binding $(IC - DE)$ to be $\frac{c'(e)}{g(e,c'(e),0)} = \frac{\delta}{1-\delta}\frac{1}{\alpha}$ and for intermediate surplus contracts with $p_l = 0$, we have $\frac{c'(e)+\alpha\overline{p}}{g(e,\overline{p},0)} = \frac{\delta}{1-\delta}$. In each case, the LHS does not depend directly on δ and hence it is straightforward to see that $\frac{de}{d\delta} > 0$. For low surplus contracts with $p_l > 0$, combining equation (2) and the binding $(IC - DE)$ gives

$$c'(e) = y - \frac{1-\alpha}{\alpha}(1-e)c''(e) \tag{6}$$

and hence $\frac{de}{d\delta} = 0$.

For intermediate surplus optimal contracts with $p_l > 0$, we have $e = \tilde{e}_1$ where $g_1'(\tilde{e}_1, \overline{p}) = 0$ and $g_1''(\tilde{e}_1, \overline{p}) < 0$. Differentiating (4) by e and using $g_1'(\tilde{e}_1, \overline{p}) = 0$ gives:

$$g_1''(\tilde{e}_1, \overline{p}) = (1-\delta)\frac{(1-2\alpha)c''(\tilde{e}_1) - (1-\alpha)(1-\tilde{e}_1)c'''(\tilde{e}_1)}{1 - \frac{\delta(1-\tilde{e}_1)(1-\alpha)}{1-\delta}} \tag{7}$$

Note that for this contract to be optimal we must have $1-\delta-(1-\alpha)\delta(1-\tilde{e}_1) > 0$, since otherwise increasing p_l and s_l simultaneously relaxes $(IC - DE)$. $g_1''(\tilde{e}_1, \overline{p}) < 0$ therefore implies

$$(1-2\alpha)c''(\tilde{e}_1) - (1-\alpha)(1-\tilde{e}_1)c'''(\tilde{e}_1) < 0$$

We then differentiate $g_1'(\tilde{e}_1, \overline{p}) = 0$ implicitly by δ to obtain:

$$\frac{d\tilde{e}_1}{d\delta} = \frac{g_1(\tilde{e}_1, \overline{p})}{(1-\delta)^2\left((1-2\alpha)c''(\tilde{e}_1) - (1-\alpha)(1-\tilde{e}_1)c'''(\tilde{e}_1)\right)}$$

This expression is negative in any optimal intermediate contract with $p_l > 0$, and this thus completes the proof. □

Proof of Proposition 2.

To show that the principal will set $\overline{p}$ and α to ensure $p_l = 0$, we first note that, if the principal was to induce a contract with $p_l = p_h$, it must be that which sets $p_l = 0$. To see this, note that if $p_l = p_h$ then $(IC - DE)$ is $c'(e) = \frac{\delta g^*}{1-\delta} - \alpha p_h$ and the principal can achieve a higher effort by decreasing $\overline{p}$ to zero and increasing α to keep g^* constant.

The remaining possibility for a contract with $p_l > 0$ is therefore one with $0 < p_l < p_h$. Since p_l is not at a boundary, it must be that $g_1'(e) = 0$, since otherwise the manager would marginally increase or decrease p_l. The principal is maximizing $\pi = ey - c(e) - g^*$ subject to $(IC - DE)$ and (2). To simplify the algebra, we define $\gamma = \frac{\delta}{1-\delta}$. We now consider first the case where $\alpha \leq \delta$, and then the case where $\alpha \geq \delta$.

If the principal were to set $\alpha \leq \delta$, then substituting out for p_l and p_h gives

$$\begin{aligned} g^* =& \alpha e y + e\left(c'(e) + \frac{1-\alpha}{\alpha}(1-e)\,c''(e) - y + \gamma\frac{1-\alpha}{\alpha}g^*\right) \\ &+ (1-\alpha)(1-e)\left(\frac{1-\alpha}{\alpha}(1-e)\,c''(e) - y + \frac{\gamma}{\alpha}g^*\right) - c(e) - \underline{u} \end{aligned} \tag{8}$$

Rearranging gives

$$\begin{aligned} (\alpha - \gamma(1-\alpha))g^* =& \alpha^2 e y - \alpha c(e) - \alpha\underline{u} - \alpha(1-\alpha)(1-e)c'(e) \\ &+ (1-\alpha+\alpha e)\left(\alpha c'(e) + (1-\alpha)(1-e)\,c''(e) - \alpha y\right) \end{aligned}$$

Note that, since $c'''(e) \geq 0$, the right hand side is strictly increasing in e, and hence, since $(\alpha - \gamma(1-\alpha)) < 0$, g^* is strictly decreasing in e. This implies that π is strictly increasing in e, since $\pi = ey - c(e) - g^* - \underline{u}$, and hence the principal can do better by decreasing g^*, which she does through lowering the maximum price $\overline{p}$. The principal will therefore never induce a contract with $p_l > 0$ and $\alpha \leq \delta$.

If the principal were to set $\alpha > \delta$, then we can go one step further and substitute out for g^* in the principal's profit function, giving

$$\pi = ey - c(e) - \frac{\alpha^2 e y + \alpha e c'(e) + (e + (1-\alpha)(1-e))\left((1-\alpha)(1-e)\,c''(e) - \alpha y\right) - \alpha c(e) - \alpha\underline{u}}{\alpha - \gamma(1-\alpha)}$$

Since the principal is choosing α to maximize π, we must have $\frac{d\pi}{d\alpha} = 0$ (as it is clearly not maximized at either corner solution). Solving out for $\frac{d\pi}{d\alpha}$

gives us that it has the same sign as the following expression:

$$\begin{aligned}&[1+\alpha-\gamma+\gamma\alpha]\left[(1-\alpha)(1-e)c''(e)+c'(e)-\alpha y\right]\\&+e[1-(\alpha-2\gamma+\gamma\alpha)(1-\alpha)](1-e)c''(e)\\&+ec''(e)-c'(e)+\gamma[ec'(e)-c(e)]+[\alpha+\alpha\gamma-\gamma](y-c'(e))\end{aligned}$$

Note that, since $c'''(e) \geq 0$ and $(1-\alpha)(1-e)c''(e)+c'(e)-\alpha y = (1-\alpha)(p_h - p_l) > 0$, together these terms are strictly positive. This implies that $\frac{d\pi}{d\alpha} > 0$, which is not compatible with the principal choosing α to maximize her profits. She will therefore never induce a contract with $\alpha > \delta$ and $p_l > 0$. Together with the previous paragraph, this implies the principal will always ensure $p_l = 0$.

To characterize the optimal contract the principal induces as a function of δ, let us take the definitions of δ_h, δ_l and δ_0 as implied by the proposition. In particular, let δ_h be the highest level of δ such that the principal sets $\overline{p} = 0$ for all $\delta > \delta_h$. Let δ_l be the lowest level of δ such that side payments are not used to induce effort (i.e. $s_h = s_l$) for all $\delta < \delta_l$. Let δ_0 be the lowest level of δ such that $e > 0$ for all $\delta > \delta_0$. Note, therefore, by definition, $\delta_l \geq \delta_0$. To complete the proposition, we need to show that $\delta_h < 1$ and $\delta_l \leq \delta_h$.

To see that $\delta_h < 1$, note that when δ is large, the $(IC-DE)$ will not bind for any reasonable value of $\overline{p}$. The principal can only ensure that $p_l = 0$ therefore by setting $\alpha = 1$ or $\overline{p} = 0$. Since the former gives her a zero payoff and the latter gives her a positive one, she will set $\overline{p} = 0$. □

Proof of Proposition 3. The existence of $\delta_0 \in (0,1)$ is straightforward. Direct relational contracting is only a constraint when (1) does not bind at e^{FB}. When e^{FB} is achievable without delegation, it is better for the principal to contract directly than to involve a manager, since this way she does not have to cede surplus g to the manager.

To show the existence of $\hat{\delta} \in (0, \overline{\delta})$, we first consider the case where $\underline{u} > 0$, and show that there exists a range of δ such that the principal can achieve positive profit with delegation but no effort without. With direct relational contracting, there will be effort if and only if there exists a positive solution to the equation $ce = \frac{\delta}{1-\delta}(ey - ce^2/2 - \underline{u})$, which is equivalent to the condition that $\delta \geq \frac{c}{y+c-\sqrt{2c\underline{u}}}$. With optimal managed relational contracting, the principal always receives a strictly positive payoff when $e > 0$, since $\pi = (1-\alpha)e(y-\overline{p})$. From Proposition 2, the optimal solution for the principal involves setting α such that $p_l = 0$ in (2). For low δ, the principal can do this without using $\overline{p}$, and hence $p_h = ce$ by $(IC-DE)$ implying $2ce = y+c-\frac{1}{\alpha}(1-e)c$. Combining this with $(IC-DE)$ $c = \frac{\delta}{1-\delta}\left((y-ce)+\frac{ce-2\underline{u}}{2\alpha}\right)$ gives us an expression for e which is positive when $\delta > \frac{c^2}{(c+y)(c-\underline{u})}$. Since $\underline{u}$ is smaller than the expected profits with the first best effort $\frac{y^2}{2c}$ and $y < c$, this expression is strictly smaller than $\frac{c}{y+c-\sqrt{2c\underline{u}}}$.

If $\underline{u} = 0$, then in both cases profit is zero at $\delta = \frac{c}{y+c}$ and positive for larger δ. We therefore compare $\frac{d\pi}{d\delta}$ at $\delta = \frac{c}{y+c}$ in both cases. With direct relational contracting, $\frac{d\pi}{d\delta}\Big|_{\delta=\frac{c}{y+c}} = 2\frac{y}{\delta^2}$. With managed relational contracting, $\frac{d\pi}{d\delta}\Big|_{\delta=\frac{c}{y+c}} = \frac{2yc}{\delta^2(c-y)}$. Hence for δ just larger than $\delta = \frac{c}{y+c}$, managed relational contracting is more profitable for the principal than direct relational contracting. □

Proof of Lemma 4. To show that the principal will induce a contract with $p_l = 0$, we consider each of the alternative contracts involving $p_l > 0$ and show that in each case the principal can increase effort (and hence her payoff) by changing either α or $\overline{p}$.

First, suppose that the contract induced involves $s_l > s_h$ and $0 < p_l < p_h = \overline{p}$. Then from Section 2.2.2, we have a positive p_l defined by $p_l = \frac{1-\alpha}{\alpha}(1-e)\,c''(e) - y + \frac{1}{\alpha}\frac{\delta g(e,\overline{p},p_l)}{1-\delta}$. Together with $(IC-DE)$ $c'(e) = \overline{p} - p_l + \frac{\delta g(e,\overline{p},p_l)}{1-\delta} - \alpha\overline{p}$, it gives us:

$$c'(e) + \frac{1-\alpha}{\alpha}(1-e)c''(e) = y - \frac{1-\alpha}{\alpha}\left(\frac{\delta g(e,\overline{p},p_l)}{1-\delta} - \alpha\overline{p}\right) \qquad (9)$$

We know that $c'(e) + \frac{1-\alpha}{\alpha}(1-e)c''(e)$ is increasing in e by equation (6) in the proof of Proposition 1. We can show that the principal can achieve a higher effort by changing $\overline{p}$. After replacing $g(e,\overline{p},p_l)$ by $g_1(e,\overline{p})$ in (3), the effect of changing $\overline{p}$ on the RHS is

$$\frac{\partial\left(y-\frac{1-\alpha}{\alpha}\left(\frac{\delta g_1(e,\overline{p})}{1-\delta}-\alpha\overline{p}\right)\right)}{\partial\overline{p}}=-\frac{1-\alpha}{\alpha}\left(\frac{\frac{\delta}{1-\delta}(1-\alpha)-\alpha}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\right)$$

where we have used $\frac{\partial g_1(e,\overline{p})}{\partial e}=0$ from (4). This expression is non-zero unless $\delta=\alpha$, and hence, when $\delta\neq\alpha$, the principal can increase effort by simply increasing or decreasing $\overline{p}$. If $\delta=\alpha$, then the principal cannot increase effort by marginally changing $\overline{p}$, and so for this case we instead show that the principal can increase effort by increasing α. We take the FOC that defines e in equation (4) and differentiate it fully with respect to α to find $\frac{\partial e}{\partial\alpha}=-\frac{\frac{\partial g_1'(e,\overline{p})}{\partial\alpha}}{g_1''(e,\overline{p})}$. Since we know that $g_1''(e,\overline{p})<0$ by the SOC, we just need to show that $\frac{\partial g_1'(e,\overline{p})}{\partial\alpha}>0$. Expanding out the expression for $g_1'(e,\overline{p})$ and partially differentiating by α gives us:

$$\begin{aligned}\frac{\partial g_1'(e,\overline{p})}{\partial\alpha} &= \frac{y+(1-2\alpha)\overline{p}+(1-e)c''(e)-c'(e)+\frac{\delta g_1(e,\overline{p})}{1-\delta}}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\\ &-\frac{\frac{\delta}{1-\delta}(1-\alpha)\left[\frac{ey+\overline{p}[-(1-\alpha+\alpha e)-(1-\alpha)(1-e)]+(1-e)c'(e)}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\right]}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\\ &+\frac{\frac{\delta}{1-\delta}(1-\alpha)\left[\frac{\delta g_1(e,\overline{p})}{1-\delta}\frac{(1-e)}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\right]}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}\end{aligned}$$

where we have used $\frac{\partial g_1(e,\overline{p})}{\partial e}=0$. After multiplying by $1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)$ and using $\alpha=\delta$, we obtain:

$$\frac{\partial g_1'(e,\overline{p})}{\partial\alpha}\propto\frac{y-\frac{\delta}{1-\delta}(1-\alpha)y-c'(e)+\left(1-\alpha+\frac{\delta-\alpha}{1-\delta}\right)\overline{p}+\frac{\delta g_1(e,\overline{p})}{1-\delta}}{1-\frac{\delta}{1-\delta}(1-e)(1-\alpha)}+(1-e)c''(e)$$

which is positive if $\alpha=\delta$.

Second, suppose that the contract involves $s_l=s_h$ and $0<p_l<p_h<\overline{p}$. Then, by $(IC-DE)$ we know that $p_h=\frac{1}{\alpha}\frac{\delta g(e,\overline{p},p_l)}{1-\delta}$ so equation (9) becomes

$$c'(e)+\frac{1-\alpha}{\alpha}(1-e)c''(e)=y$$

The RHS is constant while the LHS decreases in α, thereby resulting in a larger effort. Hence, the principal will have an incentive to increase α to increase effort.

Third, suppose that the contract involves $p_l = \overline{p}$. Then the principal can achieve just as good a payoff by setting $\alpha = 1$ and setting no limits on prices (i.e. a very large $\overline{p}$) since the manager will be able to incentivize at least the same amount of effort — because using variation in prices is just as easy.

For the second part of the proof, we show that, when the $(IC - DE)$ is binding, the principal can improve over any contract with $\alpha = 1$. In particular, we will show that the principal's payoff improves by setting $\overline{p}$ sufficiently high such that it will not bind and marginally decreasing α. To see this, first note that when $\alpha = 1$ the price cap $\overline{p}$ plays no role, since side payments and prices are equivalent. Setting $\overline{p}$ sufficiently high such that it will not bind will therefore not change the principal's payoff.

From the first part of the proof we know that the principal will ensure $p_l = 0$. Hence, effort in any contract with a non-binding price cap and a binding $(IC - DE)$ will be given by the equation

$$c'(e) = \frac{1}{\alpha}\frac{\delta}{1-\delta}\left(\alpha e y + (1-\alpha)ec'(e) - c(e) - \underline{u}\right)$$

Differentiating this equation by α gives:

$$\frac{de}{d\alpha}\left(\left(\alpha(1-\delta) - \delta(1-\alpha)e\right)c''(e) - \alpha\delta(y - c'(e))\right) = -\frac{\delta}{\alpha}\left(ec'(e) - c(e) - \underline{u}\right)$$

When $\alpha = 1$ this becomes

$$\left.\frac{de}{d\alpha}\right|_{\alpha=1} = -\frac{\delta\left(ec'(e) - c(e) - \underline{u}\right)}{(1-\delta)c''(e) - \delta(y - c'(e))}$$

Since $\overline{p}$ does not bind, then $p_h = c'(e)$. In the numerator, we have then the expected price minus the cost of effort and outside option which is positive.

When $\alpha = 1$ and the $(IC - DE)$ is binding, e is the maximum solution to the equation:

$$c'(e) - \frac{\delta}{1-\delta}(ey - c(e)) = 0$$

Since $(IC - DE)$ is binding at e, the LHS is positive at $e^*(> e)$. Hence at the maximum level of effort which satisfies this equation it must be the case that the LHS is increasing in e, i.e.

$$c''(e) - \frac{\delta}{1-\delta}(y - c'(e)) > 0.$$

□

Proof of Proposition 4. By dividing the manager's payoff by κ, we can see that the manager-agent contract with a cost of corruption κ and profit sharing α will be equivalent to one with costless corruption and profit sharing α/κ. It is therefore straightforward to see that the principal can react to any decrease in κ by a similar decrease in α, and the manager-agent contract will remain unchanged. Since this new contract involves the principal keeping a larger share of the surplus, the principal's payoff is improved, and $\frac{d\pi}{d\kappa} < 0$. □

Online appendix

In this extension, we consider a model with a principal, a manager, and N identical agents. Each agent produces an individual non contractible output, but the aggregate noisy output is contractible. As in the benchmark model of Section 1, we allow the principal to share profits with the manager and limit the manager's discretion with a price cap. Contracts between the manager and agents are bilateral, so we treat the relationship between the manager and each agent as a separate game (Levin, 2002). The timing of the game is unchanged. If agent i accepts, i.e. $d_{it} = 1$, then he chooses an effort $e_{it} \in [0, 1]$ which generates a stochastic quality which is associated with agent's i contribution to the firm $Y_{it} \in \{0, y\}$. Collective revenue Y_t is then the sum of these individual contributions and some noise, i.e. $Y_t = \sum_i Y_{it} + \epsilon_t$, where ϵ_t i.i.d. with $\mathbb{E}[\epsilon] = 0$. We assume that Y_t is contractible but Y_{it} is not. Denote $p_{it} \leq \overline{p}$ the price paid to agent i and s_{it}^F and s_{it} the set of side payments. A share α_M of the total profits is shared with the manager, while a share α_i is given to agent i, with $\alpha_M + \sum_i \alpha_i \leq 1$. If an agent i rejects, i.e. $d_{it} = 0$, then nothing is produced by this agent and he receives a per-period payoff of $\underline{u}$ plus his profit share.

The expected payoff functions are therefore:

$$\Pi_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \left(1 - \alpha_M - \sum_{i=1}^{i=N} \alpha_i\right)\left(Y_\tau - \sum_{i=1}^{i=N} p_{i\tau}\right)\right]$$

$$V_t = (1-\delta)\mathbb{E}\left[\sum_{t=\tau}^{\infty} \delta^{\tau-t} \left[\alpha_M \left(Y_\tau - \sum_{i=1}^{i=N} p_{i\tau}\right) + \sum_{i=1}^{i=N} S_{i\tau}\right]\right]$$

$$u_{it} = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{\alpha_i \left(Y_\tau - \sum_{i=1}^{i=N} p_{i\tau}\right) + d_{i\tau}[p_{i\tau} - S_{i\tau} - c(e_{i\tau})] + (1-d_{i\tau})\,\underline{u}\right\}\right]$$

Effort is the agent's private information, while the individual contributions Y_{it} and the agent's compensation are observed by both the manager and the particular agent. Agents cannot observe the individual contributions of the other agents. The principal cannot observe the individual contributions and compensations, only the aggregate profit $Y_t - \sum_{i=1}^{i=N} p_{it}$.

If, as we have assumed in the main article, the agents do not receive a share of the aggregate profit (i.e. $\alpha_i = 0\ \forall i$), then we achieve identical results to before. Since the relationships between the manager and each agent are technologically independent of one another and contracts are bilateral, we can treat the manager's relationship with each agent as a separate game. As a result, Propositions 1, 2 and 3 go through unaffected.

We now consider the case where it is possible for the principal to share profits with the agents. When there are relatively few agents, the principal may want to incentivize them explicitly through profit sharing. For instance, if there is just one agent, then instead of only sharing profit with the manager, the principal could do better by only sharing the same amount of profit with the agent.[21] If there are multiple agents, however, Rayo (2007) shows that relational contracting based on individual performance may be optimal when it is impossible for each agent to receive the entire profit. This is because profit sharing on aggregate output becomes less effective in providing effort incentives in the presence of the moral hazard in teams problem as in Holmstrom (1982).

As the number of agents becomes larger, profit sharing with the agents becomes less attractive to the principal. The following lemma shows that it is not optimal for the principal to share profit with the agents when their number, N, becomes large.

Lemma 5. *Let $\alpha_i^*(N)$ be the amount of profit shared with agent i that maximizes the principal's payoff when there are N agents. Then, as $N \to \infty$, $\alpha_i^*(N) \to 0$ for all i when $\delta > \delta_0$.*

The intuition behind the lemma is that profit sharing with agents becomes less and less effective as N gets larger. This is because, when N is large, each agent can only receive a very small fraction of the total profit, and hence only reaps a small fraction of the benefits of their actions. When $\delta > \delta_0$, the principal can induce a positive profit effort through sharing profit with the manager, and hence it is not in her interest to share part of this with the agents for only a small additional amount of effort.

Proof of Lemma 5. Since agents are identical, there is no reason to

[21] To see this, consider the example shown in Figure 4. By sharing with only the agent, the principal would achieve the same payoff as she does by sharing with the manager when relational contracting is not a constraint - i.e. when δ is very high. From Figure 4 (b), we can see that, for most δ, this payoff is higher than that she would achieve sharing profit only with the manager.

give them different shares of profit, and we therefore consider that each agent receives $\alpha_i = \frac{\alpha_A}{N}$. Each agent's effort is therefore given by the following incentive compatibility constraint:

$$\frac{\alpha_A}{N}(y - (p_h - p_l)) + p_h - p_l - s_h + s_l = c'(e)$$

It is straightforward to see that, as $N \to \infty$, this equation converges to IC. Let v_i be the surplus the manager obtains with agent i : $v_i = E\left[\alpha_M (Y_i - p_i) + S_i\right]$. The dynamic enforcement constraints for the agent and the manager respectively are:

$$-(1-\delta)s_l + \delta u_i \geq \delta \underline{u}$$

$$(1-\delta)\left[-\alpha_M p_h + s_h\right] + \delta v_i \geq 0$$

Since the profit share is paid regardless of whether the promises are honored, the dynamic enforcement constraints are independent of profit shares. As $N \to \infty$, therefore, the optimal manager-agent relational contract will converge to that characterized in Proposition 1.

Let us now consider the optimal level of α_A for the principal to set. We know that, when $\delta > \delta_0$, she can achieve a positive payoff by setting $\alpha_A = 0$ and setting α_M and $\overline{p}$ to the values she would use in the main benchmark model. Let us call this payoff π^B. By setting $\alpha_A > 0$, she decreases her payoff by $\frac{1-\alpha_A-\alpha_M}{1-\alpha_M}\pi^B$, and then increases it by incentivizing more effort. For sufficiently large N, however, the extra amount of effort generated by having $\alpha_A > 0$ will be smaller than the first effect, since the effort generated goes to zero as $N \to \infty$. Hence, for sufficiently large N, the principal will set $\alpha_A = 0$. □

References

Akcigit, Ufuk, Harun Alp, and Michael Peters. 2020. "Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries." *American Economic Review*, Forthcoming.

Banfield, Edward C. 1975. "Corruption as a Feature of Governmental Organization." *Journal of Law & Economics*, 18(3): 587–605.

Bebchuk, Lucian Arye, and Jesse M Fried. 2004. *Pay without Performance: The Unfulfilled Promise of Executive Compensation.* Harvard University Press.

Bhattacharyya, Sugato, and Francine Lafontaine. 1995. "Double-sided moral hazard and the nature of share contracts." *RAND Journal of Economics*, 761–781.

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts. 2013. "Does Management Matter? Evidence From India." *Quarterly Journal of Economics*, 128(1): 1–51.

Bloom, Nicholas, Raffaella Sadun, and John Van Reenen. 2012. "The Organization of Firms Across Countries." *Quarterly Journal of Economics*, 127(4): 1663–1705.

Board, Simon. 2011. "Relational Contracts and the Value of Loyalty." *American Economic Review*, 101(7): 3349–3367.

Burguet, Roberto, and Yeon-Koo Che. 2004. "Competitive procurement with corruption." *RAND Journal of Economics*, 50–68.

Calzolari, Giacomo, and Giancarlo Spagnolo. 2009. "Relational contracts and competitive screening." CEPR Discussion Paper No. DP7434.

Chassang, Sylvain, and Gerard Padró I Miquel. 2019. "Crime, intimidation, and whistleblowing: A theory of inference from unverifiable reports." *Review of Economic Studies*, 86(6): 2530–2553.

Che, Yeon-Koo, and Seung-Weon Yoo. 2001. "Optimal incentives for teams." *American Economic Review*, 91(3): 525–541.

Chow, Gregory C. 1997. "Challenges of China's economic system for economic theory." *American Economic Review*, 87(2): 321–327.

Cole, Shawn, and Anh Tran. 2011. "Evidence from the Firm: A New Approach to Understanding Corruption." In *International Handbook on the Economics of Corruption Vol. II.* , ed. Susan Rose-Ackerman and Tina Soriede, 408–427. Edward Elgar Publishing.

Compte, O., A. Lambert-Mogiliansky, and T. Verdier. 2005. "Corruption and Competition in Procurement Auctions." *The Rand journal of economics*, 36(1): 1–15.

Coviello, Decio, Andrea Guglielmo, and Giancarlo Spagnolo. 2018. "The effect of discretion on procurement performance." *Management Science*, 64(2): 715–738.

Department of Justice. 2013. "Former Chief Merchandising Officer Of Aeropostale, Inc. Found Guilty Of Receiving More Than $25 Million In Kickbacks."

Desrieux, Claudine, Eshien Chong, and Stéphane Saussier. 2013. "Putting all one's eggs in one basket: Relational contracts and the management of local public services." *Journal of Economic Behavior and Organization*, 89: 167 – 186.

Dewatripont, Mathias. 1988. "Commitment through renegotiation-proof contracts with third parties." *Review of Economic Studies*, 55(3): 377–390.

Droney, J. 2017. "United States v. Finazzo." 14-3213-cr, 14-3330-cr.

Dufwenberg, Martin, and Giancarlo Spagnolo. 2015. "Legalizing bribe giving." *Economic Inquiry*, 53(2): 836–853.

Fong, Yuk-Fai, and Jin Li. 2017*a*. "Information Revelation in Relational Contracts." *The Review of Economic Studies*, 84(1): 277–299.

Fong, Yuk-fai, and Jin Li. 2017*b*. "Relational Contracts, Limited Liability, and Employment Dynamics." *Journal of Economic Theory*, 169: 270 – 293.

Freeman, R Edward, Christopher Hernadi, and Cheryl Tolbert. 2000. "Questionable Payments." *Darden Case No. UVA-E-0178.*

Garicano, Luis, and Andrea Prat. 2013. "Organizational economics with cognitive costs." In *Advances in Economics and Econometrics.* Vol. 1, 342–387. Cambridge University Press New York.

Gibbons, Robert, and Rebecca Henderson. 2013. "What Do Managers Do? Exploring Persistent Performance Differences Among Seemingly Similar Enterprises." In *The Handbook of Organizational Economics.* , ed. Robert Gibbons and John Roberts, 680–731. Princeton University Press.

Green, Alison. 2013. “My boss gave me a bonus — from her own personal money.” http://www.askamanager.org/2013/06/my-boss-gave-me-a-bonus-from-her-own-personal-money-should-i-accept-it.html. Accessed: 2017-10-11.

Greenhouse, Steven. 1991. “Reviving a Humbled First Boston.” *The New York Times*, March 11.

Halac, Marina. 2012. “Relational Contracts and the Value of Relationships.” *American Economic Review*, 102(2): 750–779.

Hermalin, Benjamin E. 2015. “Why Whine about Wining and Dining?” *Journal of Law, Economics, and Organization*, 32(1): 37–60.

Holmstrom, Bengt. 1982. “Moral hazard in teams.” *Bell Journal of Economics*, 324–340.

Katz, Michael L. 1991. “Game-playing Agents: Unobservable Contracts as Precommitments.” *RAND Journal of Economics*, 22(3): 307–328.

Kockesen, Levent, and Efe A Ok. 2004. “Strategic Delegation by Unobservable Incentive Contracts.” *Review of Economic Studies*, 71(2): 397–424.

Kofman, Fred, and Jacques Lawarree. 1993. “Collusion in hierarchical agency.” *Econometrica*, 629–656.

KPM. 2018. “Why hotel owners should keep an eye on their management companies.” https://www.kpmcpa.com/why-hotel-owners-should-keep-an-eye-on-their-management-companies/. Accessed: 2020-07-17.

Kroll. 2018. “Global Fraud & Risk Report.” https://www.kroll.com/en-us/global-fraud-and-risk-report-2018. Accessed: 2018-08-11.

Lafontaine, Francine, Rozenn Perrigot, and Nathan E Wilson. 2017. “The quality of institutions and organizational form decisions: Evidence from within the firm.” *Journal of Economics & Management Strategy*, 26(2): 375–402.

Lambsdorff, Johann Graf, and Sitki Utku Teksoz. 2005. “Corrupt Relational Contracting.” In *The New Institutional Economics of Corruption.* , ed. Johann Graf Lambsdorff, Markus Taube and Matthias Schramm, 154–168. Routledge.

Ledeneva, Alena V. 2013. *Can Russia Modernise? Sistema, Power Networks and Informal Governance.* Cambridge University Press.

Levin, Jonathan. 2002. "Multilateral Contracting and the Employment Relationship." *Quarterly Journal of Economics*, 1075–1103.

Levin, Jonathan. 2003. "Relational Incentive Contracts." *American Economic Review*, 93(3): 835–857.

Lewis-Faupel, Sean, Yusuf Neggers, Benjamin A. Olken, and Rohini Pande. 2016. "Can Electronic Procurement Improve Infrastructure Provision? Evidence from Public Works in India and Indonesia." *American Economic Journal: Economic Policy*, 8(3): 258–83.

Lichand, Guilherme, Marcos Lopes, and Marcelo Medeiros. 2017. "Is Corruption Good For Your Health?" *Mimeo.*

Malcomson, James M. 2013. "Relational Incentive Contracts." In *The Handbook of Organizational Economics.* , ed. Robert Gibbons and John Roberts, 1014–1065. Princeton University Press.

Malcomson, James M. 2016. "Relational Incentive Contracts With Persistent Private Information." *Econometrica*, 84(1): 317–346.

Martimort, David. 1999. "The Life Cycle of Regulatory Agencies: Dynamic Capture and Transaction Costs." *Review of Economic Studies*, 66(4): 929–947.

Martimort, David, and Thierry Verdier. 2004. "The Agency Cost of Internal Collusion and Schumpeterian growth." *Review of Economic Studies*, 71(4): 1119–1141.

Milgrom, Paul R. 1988. "Employment Contracts, Influence Activities, and Efficient Organization Design." *Journal of Political Economy*, 96(1): 42–60.

Millington, Andrew, Markus Eberhardt, and Barry Wilkinson. 2005. "Gift Giving, Guanxi and Illicit Payments in Buyer–Supplier Relations in China: Analysing the Experience of UK Companies." *Journal of Business Ethics*, 57(3): 255–268.

Nkamleu, Guy Blaise, and Bernadette Dia Kamgnia. 2014. "Uses and Abuses of Per-diems in Africa: A Political Economy of Travel Allowances." African Development Bank Group, AfDB Working Paper 196.

Olsen, Trond E, and Gaute Torsvik. 1998. "Collusion and Renegotiation in Hierarchies: A Case of Beneficial Corruption." *International Economic Review*, 413–438.

Paine, Lynn S. 2004. "Becton Dickinson: Ethics and Business Practices (A)." Harvard Business School Case 399-055.

Prins, Nomi. 2006. *Other People's Money: The Corporate Mugging of America.* The New Press.

Rasul, Imran, and Daniel Rogger. 2015. "The Impact of Ethnic Diversity in Bureaucracies: Evidence from the Nigerian Civil Service." *American Economic Review*, 105(5): 457–61.

Rauh, Michael T. 2020. "Sticky Contracts and Involuntary Unemployment." *Working Paper.*

Ray, Debraj. 2002. "The Time Structure of Self-Enforcing Agreements." *Econometrica*, 70(2): 547–582.

Rayo, Luis. 2007. "Relational Incentives and Moral Hazard in Teams." *Review of Economic Studies*, 74(3): 937–963.

Schramm, Matthias, and Markus Taube. 2003. "The Institutional Economics of Legal Institutions, Guanxi, and Corruption in the PR China." In *Fighting Corruption in Asia: Causes, Effects, and Remedies.* 271–96. World Scientific.

Spagnolo, Giancarlo. 2000. "Stock-related compensation and product-market competition." *RAND Journal of Economics*, 22–42.

Spagnolo, Giancarlo. 2005. "Managerial Incentives and Collusive Behavior." *European Economic Review*, 49(6): 1501–1523.

Stewart, James. 1993. "Taking the Dare." *The New Yorker*, 69(23): 34–9.

Strausz, Roland. 1997. "Delegation of Monitoring in a Principal-Agent Relationship." *Review of Economic Studies*, 64(3): 337–357.

Tirole, Jean. 1986. "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations." *Journal of Law, Economics, and Organization*, 2(2): 181–214.

Vanhonacker, Wilfried R. 2004. "When Good Guanxi Turns Bad." *Harvard Business Review*, 82(4): 18.

Vickers, John. 1985. "Delegation and the Theory of the Firm." *Economic Journal*, 95: 138–147.

Weaver, Jeff. 2020. "Jobs for Sale: Corruption and Misallocation in Hiring." Avaialble at: https://ssrn.com/abstract=3590721.

Webb, Jonathan. 2017. "Why Do Supplier Collaborations Go Wrong? What Can Be Done About It?" https://www.forbes.com/sites/jwebb/2017/09/28/why-do-supplier-collaborations-go-wrong-and-how-to-increase-success-rates. Accessed: 2017-10-11.

Wells, Joseph T. 2007. *Fraud casebook: Lessons from the bad side of business.* John Wiley & Sons.

Wiegel, Wladimir, and David Bamford. 2015. "The Role of Guanxi in Buyer–Supplier Relationships in Chinese Small-and Medium-Sized Enterprises–a Resource-Based Perspective." *Production Planning & Control*, 26(4): 308–327.

Wilson, James Q. 1989. *Bureaucracy: What government agencies do and why they do it.* Basic Books New York.