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Abstract

This article aims to show how a market exposed to a catastrophic event finds the balance between adaptation and mitigation policies through R&D policy. We also study the effect of pollution tax on the long-run growth rate and the implications of catastrophe probability on this effect. Our results suggest that the economy can increase its R&D level even with a higher catastrophe probability. This is possible only if the penalty rate due to an abrupt event is sufficiently high. We show that pollution tax could increase the long-run growth. Additionally, the catastrophe probability increases the amplitude of this positive effect if the penalty rate is high enough. Lastly, we show that the pollution growth could be higher with less polluting inputs, which we call a Jevons type paradox.

Keywords : Abrupt damage, Occurence Hazard, Endogenous Technological Change, Adaptation, Mitigation.

JEL Classification : D81, O3, Q54, Q55

1 Introduction

In this paper, we take a step further to answer the following questions : How catastrophic event probability affects the creative destruction process in the economy ? What is the effect of pollution

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tax on the growth rate and the implications of catastrophe probability regarding this effect ? How market adjusts the equilibrium level of adaptation and mitigation when it faces a higher catastrophe probability ?

Many recent reports (see European Commission- Road map for Climate Services 2015[1]) started to highlight how important is to build a market economy through R&D innovations that handles adaptation and mitigation services to create a low carbon and climate-resilient economy. Climate services market aims at providing the climate knowledge to the society through informational tools.[2] These services involve very detailed analysis of existing environmental knowledge and R&D activity that inform the society about climate impacts. In addition, these services give necessary information to take action against extreme events. To summarize, one can say that the purpose of climate services is to bridge innovation with entrepreneurship that could create new business opportunities and market growth.

"Significantly strengthening the market for climate services towards supporting the building of Europe's resilience to climate change and its capacity to design a low-carbon future will require targeted research and innovation investments. These investments are required to provide the evidence, knowledge and innovations that would identify opportunities, and explore and deliver the means for fuelling the growth of this market." (European Commission - Road map for Climate Services 2015, p.19)

Indeed, it is interesting to see the words "service" and "market" for adaptation and mitigation activities since the existing literature has treated adaptation and mitigation policies in a social optimum and not in a market economy framework (See Zemel (2015), Tsur and Zemel 2015, Bréchet et al. 2012).

In recent years, climate change started to be considered as a business opportunity[3] since companies could develop a new service or product to adapt to catastrophic events. These products and services are expected to ensure competitiveness and advantage for companies on the market which promotes the growth. Regarding this recent evolution about adaptation and mitigation activities, a decentralized market analysis is more than necessary to be able to analyze rigorously the long term implications of adaptation and mitigation.

The world faces undesirable extreme events entailing significant environmental damages. Our aim in this paper is to see how adaptation and reducing the pollution sources (mitigation)[4] can be

[1]The definition for climate services given in this report is the following : "We attribute to the term a broad meaning, which covers the transformation of climate-related data — together with other relevant information — into customized products such as projections, forecasts, information, trends, economic analysis, assessments (including technology assessment), counselling on best practices, development and evaluation of solutions and any other service in relation to climate that may be of use for the society at large. As such, these services include data, information and knowledge that support adaptation, mitigation and disaster." (A European Research and Innovation Roadmap for Climate Services, Box1. pp 10.)

[2]An example can be a smartphone application that informs farmers about weather and how to proceed in extreme weather events.

[3]see European Commission- Road map for Climate Services 2015 and National and Sub-national Policies and Institutions. In: Climate Change 2014: Mitigation of Climate Change.

[4]In this study, R&D aims at decreasing the pollution intensity of machines used for final good production.

possible through the R&D activity handled by the market economy exposed to an abrupt event. To our knowledge, there does not exist any study treating the adaptation and mitigation activities in a decentralized framework with taking into account the uncertainty about catastrophic events. Our contribution relies on building a decentralized growth model that analyzes adaptation and mitigation policies. Moreover, existing studies examine these policies on exogenous growth models and endogenous technological progress is a missing component (See Zemel (2015), Tsur and Zemel 2015, Bréchet et al 2012, Tsur and Withagen 2012, Zemel and Art de Zeeuw 2012). In this sense, our study is the first one that focuses on adaptation and mitigation through an endogenous R&D process.

Firstly, our article builds on the literature on adaptation and mitigation (Bréchet et al. 2012) and also includes the uncertainty about abrupt climate event the effects (see Tsur and Zemel (1996), (1998)) Secondly, our model belongs to Schumpeterian growth literature which started with the seminal paper of Aghion and Howitt (1992).

To inform the reader about adaptation and mitigation analysis, Bréchet et al. (2012), Ayong Le Kama and Pommeret (2014), Kane and Shogren (2014) and Buob and Stephan (2010) are first analytical studies that treat the optimal design of adaptation and mitigation. However, these studies focusing on trade-offs between adaptation and mitigation neglect the uncertainty about abrupt climate events. To fill this gap in the literature, Zemel (2015) and Tsur and Zemel (2016) introduce Poisson uncertainty in Bréchet et al. (2012) framework and shows that a higher catastrophic event probability induces more adaptation capital at long-run.

Now, we return to the Schumpeterian growth literature. Very first study that combines the environment and Schumpeterian growth models is Aghion and Howitt (1998). Authors introduce the pollution in a Schumpeterian growth and make a balanced growth path analysis by taking into account the sustainable development criterion[5]. Grimaud (1999) extends this model to a decentralized economy in which he implements the optimum by R&D subsidies and pollution permits.

One of the first attempts to model environmental aspects in a Schumpeterian growth model is Hart (2004). He studies the effect of a pollution tax and finds that environmental policy can be a win-win policy by increasing the pollution intensity and promoting the growth rate at the long run. In the same line, Ricci (2007) shows in a Schumpeterian growth model that long-run growth of the economy is driven by knowledge accumulation. In his model, environmental regulation pushes the final good producers to use cleaner vintages. The important difference between these Hart (2004) and Ricci (2007) is that Ricci (2007) treats a continuum of different vintages. However, Hart (2004) proposes a model in which there exist only two young vintages on sale. Due to this modeling difference, Ricci (2007) shows that tightening environmental policy does not foster the economic growth since the marginal contribution of R&D to economic growth falls. However, uncertainty about abrupt climate events is totally overlooked in these models. One of the focus of this study is to analyze how the results stated by Hart (2004) and Ricci (2007) change with respect

[5] The sustainable development criterion requires that the utility from consumption follows a constant or an increasing path at the long run. i.e, $\frac{du(c)}{dt} \geq 0$

to a catastrophic event possibility.

In this article, differently from Hart (2004) and Ricci (2007), the benefit of R&D is twofold ; firstly, with the assumption that wealthier countries resist more easily to catastrophic events (see Mendhelson, Dinar and Williams (2006)), we show that making R&D increases the wealth of the economy and make it more resilient to catastrophic events. The knowledge serves as a tool of adaptation only if the abrupt event occurs. In this sense, knowledge plays also a proactive role for adaptation[6]. Secondly, R&D decreases the pollution intensity of intermediate goods (i.e, mitigation) as in Ricci (2007) and increases the total productivity which allows a higher growth rate at the balanced growth path.

In this paper, we show that there are two opposite effects of catastrophe probability on the creative destruction rate. A first channel is straightforward, a higher abrupt event probability increases the impatience level of agents. It follows that the interest rate in the market tends to increase. Consequently, the expected value of an R&D patent decreases as well as the labor allocation in this sector. This one can be called *discount effect*.

The second channel is more interesting : when the abrupt event probability is higher, the marginal benefit of R&D activity increases since the knowledge stock helps to increase the resilience of the economy against the inflicted penalty due to an abrupt event. Consequently, the interest rate in the market decreases and the expected value of R&D patents increases. This one can be called *adaptation effect*.

In other words, more the hazard rate increases, more the opportunity cost of not investing in R&D increases. In a nutshell, a higher hazard rate pushes the economy to invest more in R&D activity. We show that after some threshold of penalty amount, an increase in catastrophe probability boosts the creative destruction rate in the economy. This is due to the fact that adaptation effect dominates the discount effect.

Our results indicate that market's adaptation level relative to mitigation level depends on the ratio between the pollution intensity and total productivity rate. In addition, more the R&D sector offers cleaner intermediate goods, less the economy adapts to abrupt climate damages. This relies on the usual assumption that cleaner intermediate goods are less productive. Then, with cleaner intermediate goods, there is a lower growth rate and knowledge accumulation. Indeed, the trade-off between adaptation and mitigation (see Bréchet et al. 2012) is present in our model. However, this trade-off is less crucial in an economy at the balanced growth path. Since the adaptation and mitigation are shown to grow at the balanced growth path, the economy increases both adaptation and mitigation at each date. Interestingly, there is a new trade-off between adaptation and pollution that can arise in the economy. R&D activity decreases the pollution intensity but at the same time, it seeks to increase the total productivity of the economy. Then, the scale of the economy increases with R&D activity. If the scale effect dominates the emission intensity decrease, then the growth rate increase. However, in this case, the pollution growth is higher even with cleaner intermediate goods since the scale of the economy increases. This is close to the so-called Jevons Paradox which

[6]See Zemel (2015) for a detailed discussion about proactive adaptation policy.

states that technological improvements increases energy efficiency but results in a higher pollution in long term[7].

Before coming to pollution tax analysis, it is worthwhile to note that firms mitigates since they face a pollution tax levied on the use of polluting intermediate goods. Hence, they are making R&D to decrease pollution intensity in order to lower the tax burden. Our model shows a positive effect of pollution tax effect on growth as in Ricci (2007) since the lower demand for intermediate goods implies a shift of labor from final good sector to R&D sector which promotes the economic growth. We show that a higher hazard rate can increase the positive effect of green tax burden on growth rate of the economy at long run, if penalty rate is sufficiently high. This effect is due to a higher marginal benefit of R&D since it helps an economy to better respond to catastrophic events.

The remainder of the paper is organized as follows. Section 2 presents the decentralized model while section 3 focuses on the balanced growth path analysis. In section 4, we examine the adaptation and mitigation handled by the market economy and next, in section 5 we study the welfare implications of green tax burden and abrupt event probability. Section 6 concludes.

2 Model

We make an extension of the Schumpeterian model of endogenous growth (Aghion and Howitt 1998) to consider the effect of uncertain abrupt climate events on the market economy. Our model also adds an environmental aspect to Aghion and Howitt, 1998 since the production emits pollutants (see Hart 2004, Ricci, 2007). The production is realized in three stages. First, labor is used in R&D sector to improve the productivity of intermediate goods. Pollution intensity is also a technological variable since successful innovations decrease the emission intensity of intermediate goods. Second, the machines (intermediate goods) are supplied by a monopolistic intermediate good producer because the technology that allows the production of machines is protected by patents. Their production emits pollutants which is imposed by a tax set by the policymaker. Third, the final good is produced by combining intermediate good and labor allocated by the household who faces an abrupt event probability. The possibility of an abrupt event changes the labor allocation decisions of the household.

2.1 Production of Final Good

An homogeneous final good is produced using labor, L_Y , intermediate good x, according to aggregate production function (see Stokey, 1998 and Ricci, 2007).

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v,t)\, z(v,t)\, x(v,t)^{\alpha}\, dv \tag{2.1}$$

where t is the continuous time index. The parameter α stands for the elasticity of intermediate good in the production function. There exists a continuum of different technologies available on

[7] A similar result is shown empirically for the case of India by Ollivier and Barrows (2016).

the market indexed by $v \in [0,1]$. $\phi(v,t)$ is the technology level that can be referred to as the implicit labor productivity index. The important novelty in the production function with respect to Aghion and Howitt (1998) framework is that the emission intensity $z(v,t)$ of intermediate good is heterogeneous across firms which is defined by

$$z(v,t) = \left(\frac{P(v,t)}{\phi(v,t)\, x(v,t)} \right)^{\alpha\beta} \tag{2.2}$$

where $P(v,t)$ represents the polluting emissions of a given firm. The term $\alpha\beta$ is the share of pollution in the production function (see Appendix Production Function). The emission intensity variable z is defined in a close manner to Stokey (1998) since pollution enters as an input in production function and reducing its use decreases the production.

From equation (2.2), the aggregate pollution stemming from the production of intermediate goods can be written as

$$P(t) = \int_0^1 P(v,t) = \int_0^1 (z(v,t))^{\frac{1}{\alpha\beta}}\, \phi(v,t)\, x(v,t)\, dv$$

Contrary to Stokey (1998) and Aghion and Howitt (1998), R&D activity changes progressively the pollution intensity at the long term which is heterogeneous across firms in the economy (see Ricci (2007)). Unlike Stokey (1998) and Aghion and Howitt (1998), we can remark that the productivity of intermediate goods does not depend only on the labor productivity index ϕ but also on the pollution intensity z.

2.2 Final Good Producer's Program

By using the production function (2.1), the instantaneous profit of competitive firms is

$$\underset{x(v,t),L_Y(t)}{max} \psi(t) = Y(t) - \int_0^1 p(v,t)\, x(v,t)\, dv - w(t)\, L_Y(t) \tag{2.3}$$

where $p(v,t)$ and $w(t)$ are the price of intermediate good and wage respectively. The final good sector is in perfect competition and the price of the final good is normalized to one. From the maximization program, we write the demand of intermediate good and labor of final good producer

$$p(v,t) = \alpha\phi(v,t)\, z(v,t) \left(\frac{L_Y(t)}{x(v,t)} \right)^{1-\alpha} \tag{2.4}$$

$$w(t) = (1-\alpha) \int_0^1 (\phi(v,t)) \left(\frac{x(v,t)}{L_Y(t)} \right)^{\alpha} dv = (1-\alpha)\, \frac{Y(t)}{L_Y(t)} \tag{2.5}$$

When the final good producer maximizes its instantaneous profit, it takes the technology level as given.

2.3 Intermediate Good Producer's Program

The intermediate good producer is a monopolist. It faces a factor demand (2.4) and offers intermediate good to the final good sector. The cost of providing intermediate goods implies foregone production which is subtracted from the consumption (see Nakada, 2010). The intermediate good producer faces a green tax $h(t)$ levied on the use of polluting machines. The maximization program of intermediate good producer is ;

$$\max_{x(v,t)} \pi(t) = p(v,t)\,x(v,t) - \chi x(v,t) - h(t)\,P(v,t) \tag{2.6}$$

where χ stands for the constant marginal cost of producing intermediate goods (Acemoglu, 2009). In the absence of the green tax, market economy will not have incentive to decrease pollution intensity (i.e mitigation) by the means of R&D activity. As pollution enters in the maximization program of the intermediate good producer as a cost, there are incentives to make R&D[8] to reduce this cost.

We write the supply of machines and profits of the intermediate good producer :

$$x(v,t) = \left(\frac{\alpha^2\phi(v,t)\,z(v,t)}{\chi + h(t)\,\phi(v,t)\,z(v,t)^{\eta}}\right)^{\frac{1}{1-\alpha}} L_Y(t) \tag{2.7}$$

By plugging the supply function of intermediate good producer (2.7) in price function (2.4) found in final good producer's program, we can express the profit and the price of intermediate good :

$$p(v,t) = \frac{\chi + h(t)\,\phi(v,t)\,z(v,t)^{\frac{1}{\alpha\beta}}}{\alpha} \tag{2.8}$$

$$\pi(v,t) = (1-\alpha)\,p(v,t)\,x(v,t) \tag{2.9}$$

By plugging equation (2.8) in (2.9) the profit of the intermediate good producer can be written as

$$\pi(v,t) = \frac{(1-\alpha)}{\alpha}\left(\chi + h(t)\,\phi(v,t)\,z(v,t)^{\frac{1}{\alpha\beta}}\right)x(v,t) \tag{2.10}$$

We can notice that profits are decreasing in the marginal cost of firm v : $m(v,t) = \chi + H(v,t)$ where $H(v,t) = h(t)\,\phi(v,t)\,z(v,t)^{\frac{1}{\alpha\beta}}$ represents the green tax burden. The green tax decreases the profits and its effect is heterogeneous across firms since final goods are differentiated in pollution intensity z i.e.

$$z(v,t) \neq z(i,t)\,,\ \ for\ v \neq i,\, h(t) > 0 \implies \pi(v,t) \neq \pi(i,t) \tag{2.11}$$

[8] We will discuss the effect of a pollution tax on R&D in details in further sections.

2.4 R&D Sector

In R&D sector, each laboratory aims at improving the labor productivity and also seeks to decrease the pollution intensity of intermediate goods. R&D innovations are modeled respecting a Poisson process with instantaneous arrival rate λL_R with $\lambda > 0$ which we can be interpreted as the creative destruction rate. Similar to Ricci (2007), to keep things simpler, we adopt only one type of R&D firm, which specializes in both productivity and pollution intensity improvements ϕ and z. However, the reader could consider this feature of modeling R&D unusual. A two-sector R&D model would require that expected profits should be the same to ensure that R&D activity in both sectors is maintained[9].

We can write the dynamics for implicit labor productivity and pollution intensity improvements ;

$$g_\phi = \frac{\dot{\bar{\phi}}_{max}(t)}{\bar{\phi}_{max}(t)} = \gamma_1 \lambda L_R, \quad \gamma_1 > 0 \tag{2.12}$$

$$g_Z = \frac{\dot{\underline{z}}_{min}(t)}{\underline{z}_{min}(t)} = \gamma_2 \lambda L_R, \quad \gamma_2 < 0 \tag{2.13}$$

where L_R is the labor allocated in R&D sector. A successful innovation allows the patent holder to provide the intermediate good with leading-edge technology $\bar{\phi}$ and the lowest pollution intensity $\underline{z}$. The parameter γ_2 shows the direction of the R&D activity. A negative γ_2 means that innovation is environmental friendly and its value shows at which extent innovation allows the production of cleaner intermediate goods. When $\gamma_2 = 0$, all goods have the same pollution intensity as in Nakada (2004). In this case, there is no differentiation of intermediate goods in terms of pollution intensity.

The free-entry condition ensures that arbitrage condition holds ;

$$w(t) = \lambda V(t) \tag{2.14}$$

where $V(t)$ is the present value of expected profit streams. The equation (2.14) states that an agent is indifferent between working in the production sector and R&D sector. This ensures the equilibrium in the model at the balanced growth path. At equilibrium, when there is R&D activity, its marginal cost $w(t)$ is equal to its expected marginal value.

$$V(t) = \int_\tau^\infty e^{-\int_\tau^t (r(s)+\lambda L_R(s))ds} \pi\left(\bar{\phi}(t), \underline{z}(t)\right) dt \tag{2.15}$$

where $\pi\left(\bar{\phi}(t), \underline{z}(t)\right)$ denotes the profit at time t of a monopoly using the leading-edge techno-

[9]In case of asymmetric profits, there will be corner solutions where only one type of R&D will take place. Da Costa (2006) proposes a model with two R&D sectors and finds a balance of labor allocation between two R&D sectors which ensures the same expected value of R&D in both sectors. Recall that in his model, when the allocation of labor increases in one R&D sector, the other one sees its labor allocation increasing as well in order to avoid the corner solutions. This way of modeling two R&D sectors is surely more realistic but does not add different economic insights than the model with one R&D sector.

logy available $\left(\bar{\phi}(t), \underline{z}(t)\right)$. r is the interest rate which is also the opportunity cost of savings and λL_R is the *creative destruction rate* of the economy. The creative destruction rate shows at which extent the incumbent firm is replaced by an entrant. Basically, it is the survival rate of the incumbent firm as an entrant makes the patent of incumbent firm obsolete.

Furthermore, the labor supply is fixed to unity and the market clearing condition is

$$L(t) = L_Y(t) + L_R(t) = 1 \tag{2.16}$$

The labor is allocated between final good production and R&D activity. Then, the cost of R&D activity is measured as a foregone final good production. The cost of producing the intermediate good enters in the resource constraint of the economy which is $Y(t) = c(t) + \chi x(t)$.

2.5 Household

We write the maximization program of household close to Tsur and Zemel (2008). The utility function of the household is

$$maxE_T \left\{ \int_0^T u(c(t)) e^{-\rho t} dt + e^{-\rho T} \Gamma(a(T)) \right\} \tag{2.17}$$

where ρ is the pure time preference of household. $u(c(t))$ is the utility coming from the consumption prior to an abrupt event which occurs at an uncertain date T. $\Gamma(a(t))$ is the value function after the catastrophic event depending on the wealth accumulation $a(t)$. After integrating by parts the equation (2.17), the household's objective function reduces to

$$max \int_0^\infty u\left(c(t) + \bar{\theta}\Gamma(a(t))\right) e^{-(\rho+\bar{\theta})t} dt \tag{2.18}$$

where $\bar{\theta}$ is the constant probability of catastrophic event.

2.5.1 Discussion on the use of a constant hazard rate

Since our focus is on the balanced growth path analysis, the use of a constant hazard rate can be easily justified. To elaborate this, suppose that there is accumulation of the pollution stock and the hazard rate depends on this stock. In this case, it is easy to see that at balanced growth path, hazard would converge to a constant value.

In order to illustrate this, take the following hazard function (see Tsur and Zemel (2007)) $\theta(S) = \bar{\theta} - \left(\bar{\theta} - \underline{\theta}\right) e^{-bS}$ where S is the stock of pollution and $\underline{\theta}$ and $\bar{\theta}$ represent the lower and upper bound of the hazard rate respectively. It is easy to remark that $lim_{S\to\infty} = \theta(S) = \bar{\theta}$ and $lim_{S\to 0} = \theta(S) = \underline{\theta}$.

Indeed, the use of the endogenous hazard rate matters only for transitional path but the endogenous probability converges to a constant hazard rate at the long run. In this paper, we don't focus on transitional dynamics but make a balanced growth path analysis. Then, the scoop of the paper justifies the use of a constant hazard rate.

Another interpretation regarding the use of constant hazard rate could be the following : abrupt events can be also triggered by flow pollution and not only by the stock pollution (see Bretschger and Vinogradova (2016)). Flow pollution highly affects the water, air and soil quality and consequently the agricultural activities. It can also cause consequent damage to the economy. For example, the sulfiric and nitric acid rain damage to man-made capital, buildings etc.

In addition to above mentioned explanations, a recent IPCC Report (2014) claims that the frequency of tropical cyclones would remain unchanged.[10] Moreover, not every climate scientist agree on a variable climate induced changes in catastrophic event frequency. (See IPCC Report 2014).

2.5.2 What happens after the catastrophic event ?

After the occurrence of an abrupt event, the economy is inflicted to a penalty which is proportional to the knowledge accumulation coming from R&D activity. Similar to Bréchet et al. (2012), the penalty function is $\psi(.)$ defined in the following manner :

A1. The penalty function $\psi(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with following properties ; $\psi(a(t)) > 0$, $\psi_a(a(t)) < 0$, $\psi_{aa}(a(t)) > 0, \bar{\psi} > \psi(a(t))$.

$$\psi(a(t)) = \bar{\psi}(\omega - (1-\omega)\,log\,(a(t))) \tag{2.19}$$

where $\bar{\psi}$ is the amount of penalty. It is assumed that $0 < \omega < 1$. We argue that accumulating wealth helps an economy to better respond to negative consequences occurred due to catastrophic event. The empirical evidence suggests as well the higher capability of wealthier countries to adapt to climate change. (See Mendhelson, Dinar and Williams (2006)).

The parameter ω shows at which extent the knowledge accumulation can help the economy to respond against extreme events. The first term $\bar{\psi}\omega$ is the part of the damage that can not be recovered by the knowledge accumulation. The second expression $-\bar{\psi}(1-\omega)\,log\,(a(t))$ stands for the part of the damage that can be reduced by the wealth (knowledge) accumulation which takes place through R&D activity.

In order to ensure that the positive effect of wealth accumulation does not dominate the unrecoverable part of the damage, we make the assumption on the parameter ω (See Appendix Condition on Penalty Function for details about Assumption 2).

A2. $\omega > \frac{(\rho+\bar{\theta})ln(a(0))}{(\rho+\bar{\theta})(1+ln(a(0)))-g_Y}$

The post value function as a function of the wealth can be given as

[10] "In the future, it is likely that the frequency of tropical cyclones globally will either decrease or remain unchanged, but there will be a likely increase in global mean tropical cyclone precipitation rates and maximum wind speed." (IPCC 2014, p.8)

$$\Gamma(a(t)) = u(c_{min}) - \psi(a(t)) \tag{2.20}$$

where $u(c_{min}) = 0$ is the utility function where the consumption is reduced to the subsistence level. Note that the subsistence level consumption does not provide any utility (see Tsur and Zemel (2015)).

The household maximizes the objective function (2.17) subject to the following budget constraint

$$\dot{a}(t) = r(t)a(t) + w(t) - c(t) + T(t) \tag{2.21}$$

where $w(t)$ and $T(t)$ stand for wage and tax collected from the use of polluting intermediate good respectively. We make the assumption that government holds its budget balanced for $\forall t$, i.e., $h(t)P(t) = T(t)$. The solution of the dynamic optimization program should satisfy the no-Ponzi game condition $lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(s) = 0$. Before deriving the Keynes-Ramsey rule from the maximization program of the household, it is crucial to show that the post value function depends on a stock variable[11] to ensure that the maximization problem is well posed.

Lemma 1. *$a(t) = V(t)$. Patents for innovations ($V(t)$ is the expected value of an innovation.) are held by households.*

Proof. See Appendix

From this lemma, we can remark that the wealth a of the household is proportional to the knowledge accumulation $\bar{\phi}_{max}$ and $\underline{z}_{min}$ which is a public good. Recall also that mitigation effort comes at the cost of lower adaptation since a lower pollution intensity z decreases the wealth accumulation. The lemma shows that the wealth is proportional to the expected value of the innovation which can be written by using equation (2.5) and the free-entry condition (2.14)

$$a(t) = V(t) = \frac{w(t)}{\lambda} = \frac{(1-\alpha)}{\lambda}\frac{Y(t)}{L_Y(t)} = \frac{(1-\alpha)}{\lambda}\frac{\gamma_1 \alpha^{\frac{2\alpha}{1-\alpha}}}{1-\gamma_1}\left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}}\Omega_1(H) \tag{2.22}$$

where the aggregate production function[12] is $Y(t) = \frac{\gamma_1}{1-\gamma_1}\alpha^{\frac{2\alpha}{1-\alpha}} L_Y \left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}}\Omega_1(H)$ and $\Omega_1(H)$ is aggregation factor which is a function of the burden of the green tax H (see Appendix for aggregation factor). Indeed, the term $\Omega_1(H)$ stems from the aggregation of different firms indexed by $v \in [0,1]$. The green tax burden H is written

$$H(t) = \int_0^1 H(v,t)\,dv = h(t)\int_0^1 \phi(v,t)\,z(v,t)^{\frac{1}{\alpha\beta}}\,dv \tag{2.23}$$

The green tax burden $H(t)$ should be constant at the long run in order to ensure the existence

[11]In our case, the physical constraint is the knowledge accumulation which stems from R&D activity.

[12]Since there is an infinite number of firms in the economy, one should give an aggregate production function to make a balanced growth path analysis. See the appendix for the derivation of the aggregate production function.

of a balanced growth path. To do this, we should provide a policy rule (see Ricci, 2007 and Nakada, 2010) that makes the green tax burden H constant at the long run. i.e, $\left(\frac{dH(t)}{dt}=0\right)$.

$$g_h = -(\frac{g_Z}{\alpha\beta} + g_\phi) \tag{2.24}$$

g_i represents the growth rate of the variable i. According to the policy rule, the growth rate of the pollution tax $h(t)$ increases when the emission intensity decreases and it decreases while the total productivity increases. The policymaker makes this commitment which is credible since its aim is to keep the budget balanced. When pollution intensity decreases, revenues from tax collection decreases since aggregate pollution decreases. Contrary to this fact, when the total productivity increases, aggregate pollution increases as well as tax revenues. Then, the policy maker is able to decrease the growth rate of the pollution tax since its tax revenues increase.

Once the policy rule is established, it is possible to find the growth rate of the economy by differentiating equation (2.22),

$$g = g_Y = g_a = \frac{1}{1-\alpha}\left(g_\phi + g_Z\right) \tag{2.25}$$

The growth rate is positive if only $\gamma_1 > \gamma_2$.

3 Balanced Growth Path Analysis

In order to proceed to the balanced growth analysis, we start by solving the household's problem which is the maximization of the objective function (2.17) subject to the budget constraint (2.21). We assume a log utility function for household's utility as $u(c(t)) = log(c(t))$[13] for the analytical tractability of the model. By using the lemma 1, the Keynes-Ramsey rule is written[14]

$$g_c = \frac{\dot{c}(t)}{c(t)} = \left(r(t) - \left(\rho + \bar{\theta}\right) + \frac{\bar{\theta}\Gamma_V(V(t))}{\mu(t)}\right) \tag{3.1}$$

where $\mu(t)$ is the marginal utility of consumption per capita[15] (See Appendix Household's Maximization Program). With the resource constraint (7.27), the growth rate of the consumption at the balanced growth rate is $g = g_Y = g_c$.

3.1 The Labor Allocation in Equilibrium

Once we have the Keynes-Ramsey equation, the labor allocation in R&D sector at the balanced growth path is (see Appendix for derivation)

[13] Note that we start our analysis with CRRA utility function $u(c(t)) = \frac{c^{1-\sigma}-1}{1-\sigma}$ where $c_{min} = 1$ and σ is the risk aversion parameter. This is the form of utility function when there is an abrupt event uncertainty. When the extreme climate event occurs, the consumption reduces to a subsistence level c_{min}. With this form of the utility function, it is easy to remark that $\lim\limits_{\sigma\to1}\frac{c^{1-\sigma}-1}{1-\sigma} = log(c(t))$.

[14] In addition, we have $g = g_c = g_Y$ at the balanced growth path (see equation (7.27)).

[15] Since $L(t) = 1$, we have $c(t) = C(t)$

$$L_R = \frac{\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)} - \left(\rho+\bar{\theta}\right)}{\lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\lambda\gamma_1}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)}} \tag{3.2}$$

where $\Omega_2(H)$ is the aggregation term for aggregate intermediate good $x(t)$. One can easily remark that the level of labor allocated in R&D sector depends on both catastrophic event probability, penalty rate and marginal cost of using a polluting intermediate good.

Proposition 1. *The market allocates much more labor to R&D with a higher catastrophe probability if the amount of penalty to due to catastrophic event is higher than a precised threshold*[16].

Proof. See Appendix

This result is counter-intuitive in the sense that catastrophic uncertainty is expected to decrease R&D activity since agents value the future less with a catastrophic event probability. It follows that with the discount effect, the interest rate for innovation patents increases as the impatience level of agents increases. To better understand the discount effect channel, we reformulate the interest rate which is constant at the balanced growth path (see Appendix for derivations).

$$r(t) = \frac{1}{1-\alpha}(g_\phi + g_Z) + \underbrace{\left(\rho+\bar{\theta}\right)}_{Discount\,effect} - \underbrace{\frac{\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Omega_2(H)}{\lambda(1-\alpha)\Omega_1(H)}(1-L_R)}_{Adaptation\,effect} \tag{3.3}$$

Contrary to standard Schumpeterian growth framework, the interest rate implies an additional term we call the adaptation effect. Since the economy becomes more resilient against abrupt events with wealth/knowledge accumulation, a higher abrupt event probability induces a higher marginal benefit from R&D patents. Then, the interest rate decreases through the adaptation effect. Consequently, the expected value of R&D increases with a lower interest rate (see equation (2.15)).

To sum up, it follows that there exist two opposite effects of abrupt event probability $\bar{\theta}$ on the interest rate which guides the investments in R&D activity. One may say that the adaptation effect dominates the discount effect if the penalty rate $\bar{\psi}$ due to the abrupt event exceeds a threshold. This relies on the fact that a higher penalty rate $\bar{\psi}$ implies a higher marginal benefit of R&D.

We illustrate the Proposition 1. graphically to confirm the mechanisms presented above by a numerical exercise.

[16]The threshold is derived in Appendix.

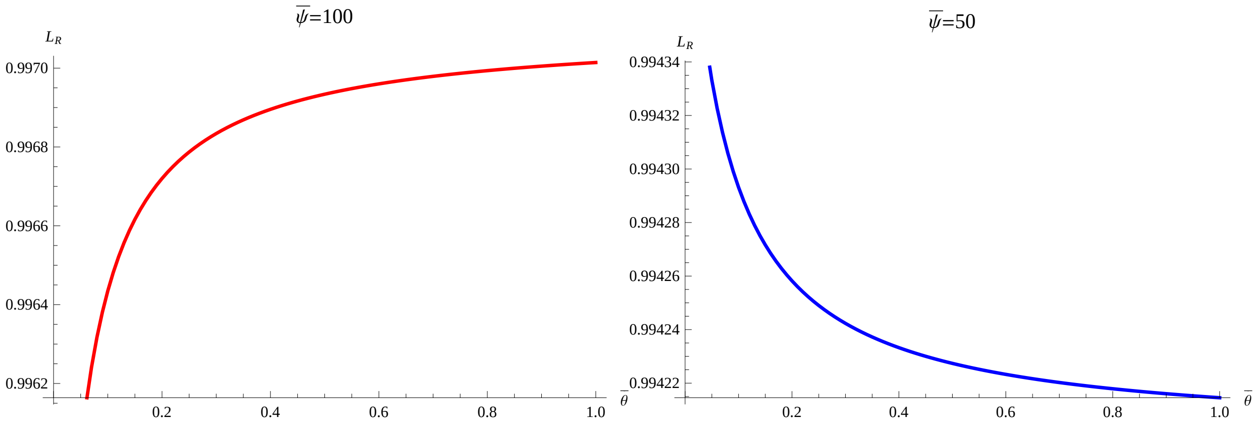
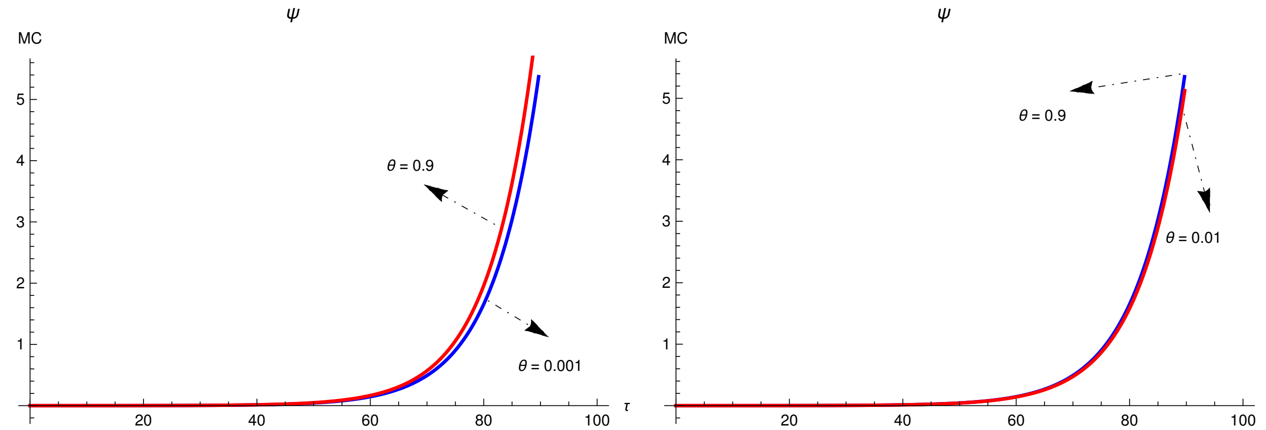


Figure 3.1: *The effect of hazard rate on labor allocation in R&D*

How abrupt events and adaptation can create business opportunities and affect the competitiveness that promotes the long run growth rate in the market economy ? To have an answer for this question, we should focus on the relationship between the labor allocation in R&D and the abrupt event probability. R&D activity changes the distribution of intermediate goods by skewing them towards cleaner ones. Then, the green tax burden becomes more stringent since the policy maker commits to follow an increasing path of pollution tax with cleaner intermediate goods. In order to understand this mechanism, we write the marginal cost of using the intermediate good in the following manner (see Appendix for the derivation)

$$m(\tau) = \chi + e^{g_h \tau} H \tag{3.4}$$

where τ stands for the age of an intermediate good. Recall that older vintages are dirtier than younger vintages. Indeed, the environmental policy rule by the policymaker creates a green crowding-out effect similar to Ricci (2007).



(a) $\bar{\theta}$ increasing L_R (b) $\bar{\theta}$ decreasing L_R

Figure 3.2: The effect of the abrupt event probability on the competitiveness of different vintages

According to the figure (3.2a), the marginal cost of using the intermediate good increases when the abrupt event probability $\bar{\theta}$ increases the labor allocation in R&D. This is because the R&D activity increases and g_h becomes higher[17]. It follows that higher abrupt event probability $\bar{\theta}$ crowds out a higher number of old vintages which are dirtier from the market and replaces them by cleaner intermediate goods. Note that older vintages imply a higher green tax burden which decreases the competitiveness in the economy. Consequently, the abrupt event probability increases the competitiveness of the economy if the market shifts labor to R&D sector.

However, a higher abrupt event probability can also allow a higher number of firms to stay on the market with dirty intermediate goods. This case is possible only if the abrupt event probability decreases the expected value of R&D. In the figure (3.2b), we observe that the marginal cost of using the intermediate good decreases with respect to the abrupt event probability $\bar{\theta}$ since the green tax burden becomes less stringent with a lower level of labor in R&D sector.

Proposition 2 . *(i) The effect of pollution tax is positive on growth if the elasticity of aggregation factor with respect to green tax burden H is high enough. (ii) This effect increases positively with catastrophic event probability if the amount of penalty is sufficiently high.*

Proof. See Appendix Impact of environmental taxation on labor allocation in R&D Sector.

The economic explanation on the positive effect of pollution tax on growth is the following : the pollution tax decreases the demand of intermediate good since it becomes more costly to use polluting intermediate goods in the production as an input. It follows that the labor demand in the final good sector diminishes. As a result, the labor shifts from the final good sector to R&D sector which results in a higher creative destruction rate and hence more economic growth.

Moreover, one can understand this result more rigorously by looking at the elasticity of aggregation factors of production function $\Omega_1(H)$ and intermediate good demand $\Omega_2(H)$ with respect to green tax burden H. As expected, these terms are decreasing with green tax burden H. An

[17] Equivalently, this means that environmental policy becomes more stringent.

important element that explains how pollution tax promotes the growth is the elasticity of these aggregation terms. We show that the elasticity of aggregation factor of production function is higher than the elasticity of aggregation factor of intermediate good factor. (see Appendix) This means that green tax affects negatively the final good sector more than the intermediate good sector. Equivalently, it means that the demand for intermediate good decreases less than the demand of final good. This results in a shift of labor from final good sector to R&D sector which aims to improve the productivity and emission intensity of intermediate goods. We also show that a necessary condition to have the positive effect of the pollution tax on growth is that the marginal cost of producing a machine $m = \chi + H$ is below a threshold.

In order to asses this effect more clearly, one may look at how labor allocation reacts to a change in marginal cost of pollution H. As R&D is known to promote growth in the economy when above mentioned conditions are fulfilled. The graphic shows the effect of the green tax burden H on labor allocation in R&D sector.

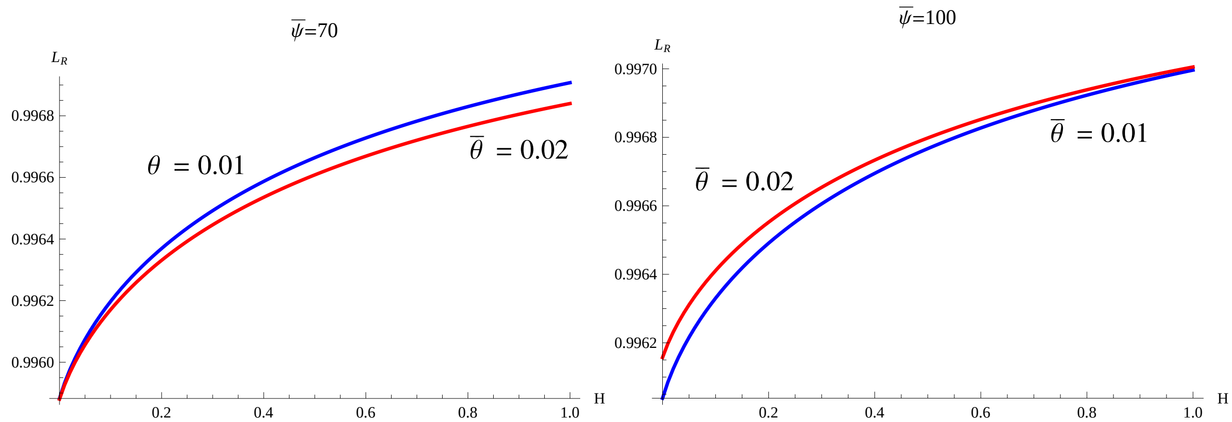


Figure 3.3: *The effect of green tax burden H on labor allocation in R&D*

An important remark is that R&D sector seeks to improve the productivity and emission intensity of intermediate goods. Then, the expected value of R&D is proportional to the profit of the monopolist intermediate good producer (see equation (2.15)). In this sense, we can argue that if the intermediate good demand decreases less than the final good demand, the labor is expected to shift from the final good sector to R&D sector.

It is worth discussing the relation between the abrupt event probability $\bar{\theta}$ and the effect of the pollution tax on growth. The positive effect of pollution tax on growth increases when the penalty rate $\bar{\psi}$ is above a precised threshold (see Appendix). This is due to the fact that the expected value of R&D increases since the interest rate decreases with a higher marginal benefit of R&D. Then, in the case where the adaptation effect dominates the discount effect, the positive effect of pollution tax on growth increases with a higher abrupt event probability $\bar{\theta}$.

4 Adaptation and Mitigation in a Market Economy

It is interesting to look at how the market economy adapts and mitigates when it faces a higher catastrophe event probability $\bar{\theta}$. To assess the implications of the pollution tax on adaptation of the economy, one should observe how the value of R&D $V(t)$ changes with respect to catastrophic event probability. Recall that knowledge accumulation that allows the adaptation stems from R&D activity. An economy that accumulates knowledge becomes wealthier (see lemma 1). On the other hand, the mitigation activity can be captured through variable Z, which stands for the pollution intensity.

Indeed, it is worthwhile to note that the market economy does not target explicitly to do adaptation and mitigation activities. It is clear in our framework that adaptation and mitigation activities are promoted by the means of R&D activity which aims primarily to have R&D patents that provide dividends to shareholders. Then, it is plausible to say that adaptation and mitigation mix are the natural outcome of the R&D in the market. A proxy indicator can be easily constructed to understand how adaptation and mitigation balance is found in the market economy.

The variable $M = \frac{1}{Z}$ can be considered as the mitigation activity. As the pollution intensity decreases, mitigation increases. The economy starts to adapt more when the knowledge stock increases. This means that when wealth accumulation a increases, the resilience against a climatic catastrophe increases. The growth rate of adaptation and mitigation is given by

$$g_A = \frac{1}{1-\alpha}(\gamma_1 + \gamma_2)\lambda L_R$$

$$g_M = -\frac{\gamma_2}{1-\alpha}\lambda L_R$$

$$g_{\frac{A}{M}} = \left(\frac{\gamma_1}{1-\alpha} + \left(1 + \frac{1}{1-\alpha}\right)\gamma_2\right)\lambda L_R$$

Proposition 3. *(i) At the balanced growth path, the growth rate of adaptation is higher than that of mitigation if the cleanliness rate of R&D is not sufficiently high* (γ_2).

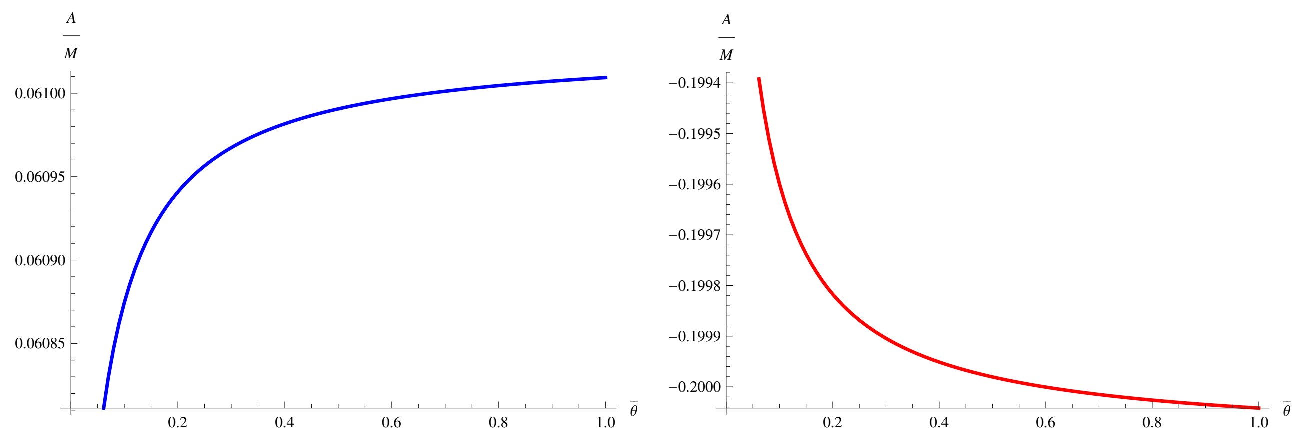
$$Case\,1.\, g_{\frac{A}{M}} > 0\, if\; -\left(\frac{\gamma_1}{\gamma_2}\right) > 2-\alpha$$

$$Case\,2.\, g_{\frac{A}{M}} < 0\, if\; -\left(\frac{\gamma_1}{\gamma_2}\right) < 2-\alpha$$

In case 1, when the cleanliness rate of R&D γ_2 is not high enough relative to the total productivity γ_1, the growth rate for adaptation/mitigation ratio $\frac{A}{M}$ is positive. Then, the economy adapts always much more than it mitigates at the long run. In case 2, the economy offers cleaner innovations compared to the case 1. Therefore, the growth rate of adaptation/mitigation ratio is negative, which means that mitigation is higher than adaptation.

It is interesting to focus on the relation between the catastrophic event probability $\bar{\theta}$ and the equilibrium level of adaptation and mitigation. Taking into consideration proposition 1, when the economy facing a high-level penalty rate allocates more labor to R&D activities, the growth rate of adaptation is higher than the that of mitigation in case 1 and vice versa in case 2.

We illustrate this result numerically :



(a) $g_{\frac{A}{M}} > 0$ with $\left(\frac{\gamma_1}{\gamma_2}\right) > 2 - \alpha$

(b) $g_{\frac{A}{M}} < 0$ with $\left(\frac{\gamma_1}{\gamma_2}\right) < 2 - \alpha$

Figure 4.1: Growth rate of adaptation/mitigation

As one can see, the economy starts to accumulate more wealth with higher catastrophe probability $\bar{\theta}$ in order to adapt to the penalty due to the catastrophic event. In case where the penalty rate is not high, economy would allocate less labor to R&D. Then, ratio adaptation/mitigation would fall, which means that the growth rate of mitigation becomes higher relative to that of adaptation when the economy faces a higher risk of abrupt event.

In the figure (4.1b), the ratio of total productivity and cleanliness of R&D $\left(\frac{\gamma_1}{\gamma_2}\right)$ is low. Therefore, the market mitigates more than it adapts to the catastrophic event. Moreover, we remark an important trade-off between adaptation and mitigation activities. When the cleanliness of R&D is higher, the economic growth decreases since the R&D offers cleaner intermediate goods that are less productive (see Ricci 2007, (see Aghion and Howitt, 1998[18]). This leads to a decrease in final good production Y. Then, it follows that the growth rate of mitigation comes at the cost of the growth rate of adaptation.

A similar trade-off is also present in Tsur and Zemel (2016) and Bréchet et al. (2012) but the difference is that the growth rate of adaptation and mitigation is always positive in the market economy. Consequently, the economy always increases its adaptation and mitigation level at each date. However, in Tsur and Zemel (2016), Bréchet et al. (2012) and Ayong Le Kama and Pommeret (2015), the trade-off relies on the optimal allocation of resources between adaptation and mitigation.

[18]The authors argue that capital intensive intermediate goods are more productive.

It follows that when the economy invests more in adaptation, this comes at the cost of mitigation investments. Nonetheless, when adaptation and mitigation activities come as a natural outcome from the R&D sector and both of them grow at the long run, this trade-off turns out to be less relevant in our framework.

Keeping in mind that the economy grows and adapts to abrupt events at each date, one may ask how the aggregate pollution evolves at the long run. Despite the relaxation of the trade-off between adaptation and mitigation in a decentralized economy, we show that a new trade-off between adaptation and pollution arises in the market economy.

Before presenting the trade-off between adaptation and pollution, we write the aggregate pollution

$$P(t) = \left[\bar{\phi}_{max}(t)\right]\left[\underline{z}_{min}(t)\right]^{\frac{1}{\alpha\beta}} Y(t) \tag{4.1}$$

It is easy to remark that pollution $P(t)$ is proportional to aggregate production $Y(t)$. Differentiating equation (4.1), at the long run, pollution growth can be written

$$g_P = \left(\frac{2-\alpha}{1-\alpha}g_\phi + \frac{1+\frac{(1-\alpha)}{\alpha\beta}}{1-\alpha}g_Z\right) = \frac{1}{1-\alpha}\left((2-\alpha)\gamma_1 + \left(1+\frac{(1-\alpha)}{\alpha\beta}\right)\gamma_2\right)\lambda L_R \tag{4.2}$$

The growth rate of adaptation, mitigation and pollution at the long run is

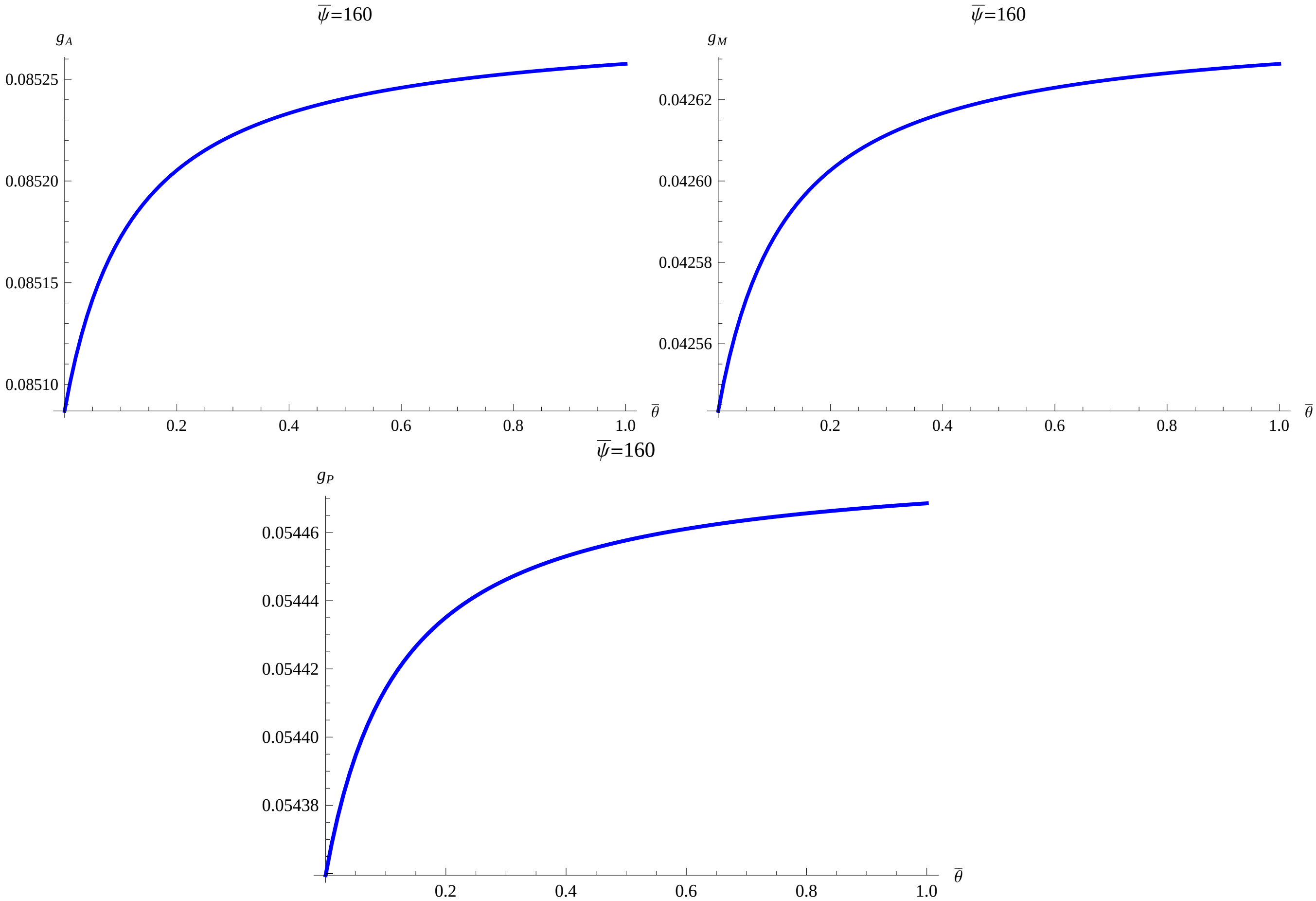


Figure 4.2: Growth rate of adaptation, mitigation and pollution

The numerical exercise confirms that when the economy adapts to abrupt event when it faces a higher abrupt event probability, the pollution growth is higher as well despite the higher growth rate of mitigation at the long run. This outcome is due to scale effect mentioned above. In fact, this result challenges the adaptation and mitigation trade-off but reveals a new trade-off between adaptation and pollution.

Proposition 4. *Pollution growth at the balanced growth path depends on the pollution share $\alpha\beta$, the cleanliness of R&D γ_2 and the total productivity parameter γ_1. In case of higher labor allocation in R&D with abrupt event probability $\bar{\theta}$ and green tax burden H, the growth rate of pollution is positive if the pollution share or cleanliness of R&D are not sufficiently high. In this case, economy faces a Jevons type paradox.*

$$Case\,1.\, g_P > 0\, if\, -\left(\frac{\gamma_1}{\gamma_2}\right) > \frac{\left(1+\frac{(1-\alpha)}{\alpha\beta}\right)}{2-\alpha} \tag{4.3}$$

$$Case\,2.\,g_P < 0\,if\; -\left(\frac{\gamma_1}{\gamma_2}\right) < \frac{\left(1+\frac{(1-\alpha)}{\alpha\beta}\right)}{2-\alpha} \tag{4.4}$$

In the market economy, pollution can grow, albeit the presence of cleaner intermediate goods, when the economy allocates much more labor to R&D. Indeed, total productivity improvements with R&D activity increases the scale of the economy. Due to the scale effect, pollution growth at the long run turns out to be higher if R&D does not offer sufficiently cleaner intermediate goods. This result can be referred to as Jevons Paradox which claims that technological improvements increases the efficiency of energy used in the production but also increases the demand of energy. In the Schumpeterian economy, the intermediate godo demand increases with the scale effect. Consequently, the pollution growth can be higher even with cleaner intermediate goods.

An illustrative example about this topic could be India's increased aggregate pollution despite the reduction of the pollution intensity. Barrows and Ollivier (2016) shows that pollution intensity decreases in India between 1990-2010. However, the emissions have increased in India between this period[19].

5 Welfare Analysis

Once, we show that in a Schumpeterian economy, a new trade-off arises between adaptation and pollution, it is desirable to study the welfare implications regarding this new trade-off with respect to the abrupt event probability $\bar{\theta}$ and green tax burden H at the balanced growth path. We know that with adaptation, pollution growth can be higher if the condition (4.3) is ensured. Then, how the welfare of the household is affected when adaptation (wealth accumulation) increases ? Using equation (2.20), the total welfare can be described as

$$W^* = \int_0^\infty \left[log\left(c\left(t\right)\right) + \bar{\theta}\left(u\left(c_{min}\right) - \bar{\psi}\left(\omega - (1-\omega)\,log\left(a\left(t\right)\right)\right)\right)\right] e^{-\left(\rho+\bar{\theta}\right)t}dt \tag{5.1}$$

By integrating the welfare function (5.1) and using the lemma 1, we have

$$W^* = \frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y\left(0\right)\right) - \bar{\psi}\bar{\theta}\left(\omega + (1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\rho+\bar{\theta}-g} \tag{5.2}$$

A higher abrupt event probability and green tax burden have two opposite effects on the welfare of the household. The first effect is the output effect. When the abrupt event probability increases the labor allocation in R&D sector, the final good production decreases as well as the consumption. Consequently, the welfare decreases with the output effect. On the other hand, since the labor allocation in R&D sector increases, the growth rate of the economy at the long run increases. This can be called the growth effect. If the growth effect is higher than the output effect, the welfare increases (see Appendix).

[19] See World Bank Database : http://data.worldbank.org/indicator/EN.ATM.CO2E.PC?end=2010&locations=IN&start=1990

Proposition 5 . *The welfare is affected negatively with respect to green tax burden and abrupt event probability if they don't enhance the growth rate of the economy. This result depends on the total productivity γ_1 and cleanliness of R&D γ_2.*

Proof. See Appendix Impact of environmental taxation on Welfare

In the numerical exercise, the welfare is shown to be decreasing with respect to green tax burden H and abrupt event probability $\bar{\theta}$ when the cleanliness rate of R&D is close to the total productivity of R&D activity[20]. To sum up, the abrupt event probability and green tax burden do not increase the growth rate of the economy and the welfare is always negatively affected.

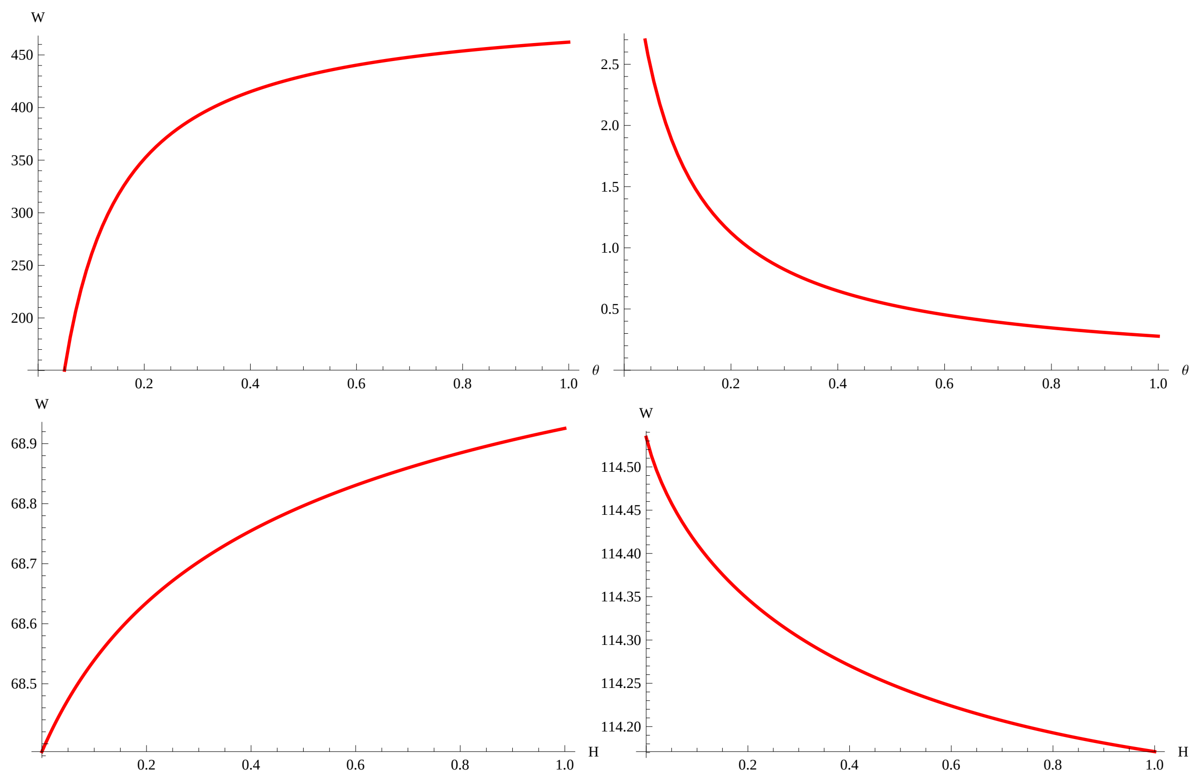


Figure 5.1: The effect of abrupt event probability $\bar{\theta}$ and green tax burden H on welfare

This result is plausible because when the cleanliness rate of R&D γ_2 is high, the intermediate goods are replaced by cleaner ones. In this case, two important thing happens ; first, the output decreases since cleaner intermediate goods are less productive. Second, the growth rate of the economy decreases with cleaner intermediate goods since the burden of green tax increases with cleaner intermediate goods.

Whereas, the welfare increases with abrupt event probability and green tax burden if both of them increases the growth rate of the economy. In this case, there is a reduction of output since the

[20] $\rho = 0.05$, $\omega = 0.1$, $\bar{\psi} = 160$, $\alpha = 0.42$, $\beta = 0.06$, $\lambda = 0.001$, $\chi = 1$. In addition to these parameter values, the welfare with respect to $\bar{\theta}$ and H increases when $\gamma_1 = 0.75$ and $\gamma_2 = -0.25$ and decreases when $\gamma_1 = 0.95$ and $\gamma_2 = -0.9$

labor shifts from final good sector to R&D sector. However, if the total productivity is sufficiently high, the growth effect can compensate the output effect and the welfare can increase.

6 Conclusion

In this paper, our contribution builds on the analysis of adaptation and mitigation through an endogenous R&D process in a decentralized economy. The existing literature treated the adaptation and mitigation policy mix in the social optimum framework without taking into account the presence of an endogenous R&D decision making.

We examine the effect of catastrophe probability on R&D decisions of the market economy. R&D activity aims to improve the total productivity of labor and the emission intensity of intermediate goods. Additionally, R&D serves to adapt to damage from abrupt events as well. We show that higher abrupt event probability increases the R&D if the penalty rate is above a threshold. This result relies on the fact that marginal benefit of R&D increases since innovation patents helps to decrease the vulnerability against abrupt event damage.

Similar to Hart (2004) and Ricci (2007), we show that pollution tax can promote the growth rate of the economy. Differently from these studies, the effect of pollution tax with respect to abrupt event probability is shown to be higher or lower depending on the penalty rate.

The market economy starts to accumulate more knowledge and to adapt more if the total productivity of R&D is higher than the cleanliness of innovations. This fact relies on the assumption that cleaner intermediate goods are less productive. Then, the growth rate turns out to be lower at the long run. This means that mitigation comes at the cost of wealth accumulation at the long run. However, in a growing economy at the long run, the trade-off between adaptation and mitigation is not relevant as much as claimed in many studies (see Tsur and Zemel, 2015, Zemel, 2015) since adaptation and mitigation are both shown to grow at the long run. We show that a new trade-off between adaptation and pollution can arise. Since wealth accumulation (adaptation) increases the growth rate of the economy at the long run, the pollution growth can be higher due to the increased scale of the economy. This result shows the possibility of a Jevons paradox since the economy emits more pollution with cleaner intermediate goods.

Lastly, we analyze the implications of the abrupt event probability and green tax burden on welfare. We show that there exists two opposite effects which are the output and growth effect. When the green tax boosts the economy, there is a shift of labor from final good sector to R&D sector which decreases the output. However, we show that this negative effect on output and hence on welfare can be compensated by the growth effect and the welfare can be higher with a higher abrupt event probability and green tax burden.

7 Appendix

7.1 Production Function

As in Ricci (2007), we define the function as

$$Y(t) = \int_0^1 \left(\phi(v,t) L_Y(t)\right)^{1-\alpha} \left(P(v,t)^{\beta} x(v,t)^{1-\beta}\right)^{\alpha} dv$$

where $P(v,t)$ is the polluting input. From production function, we can define a emissions-intermediate good ratio in order to have simpler form for production function ;

$$z(v,t) = \left(\frac{P(v,t)}{\phi(v,t) x(v,t)}\right)^{\alpha\beta}$$

The production function takes a simpler form

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^1 \phi(v,t) z(v,t) x(v,t)^{\alpha} dv$$

7.2 Household's Maximization Program

The Hamiltonian for the maximization program reads

$$\mathcal{H} = u(c(t)) + \bar{\theta}\Gamma(a(t)) + \mu(r(t) a(t) + w(t) - c(t) + T(t)) \tag{7.1}$$

The first-order conditions can be written

$$u_c(c) = \mu \tag{7.2}$$

$$\frac{\dot{\mu}}{\mu} = \left(\rho + \bar{\theta}\right) - r - \frac{\bar{\theta}\Gamma(a(t))}{\mu} \tag{7.3}$$

With $u(c) = log(c)$. The Keynes-Ramsey equation yields

$$\frac{\dot{c}}{c} = \left(r - \left(\rho + \bar{\theta}\right)\right) + \frac{\bar{\theta}\Gamma_a(a)}{u_c(c)} \tag{7.4}$$

By making trivial algebra, we can reformulate equation (7.4) as (7.37).

7.3 Proof of Lemma 1

We can reformulate the budget constraint in the form

$$\dot{a}(t) = r(t) a(t) + w(t) - c(t) + T(t) \tag{7.5}$$

With the perfect competition assumption in final good sector, the profits are equal to zero.

$$c(t) + \chi x(t) = Y(t) = w(t) L_Y(t) + \int_0^1 p(v,t) x(v,t) \tag{7.6}$$

By replacing zero profit condition (7.6) in budget constraint of the household (7.5), the budget constraint becomes

$$\dot{a}(t) = r(t) a(t) + w(t) L_R(t) - \left[\int_0^1 p(v,t) x(v,t) - h(t) P(t) - \chi x(t) \right]$$

From free-entry condition in R&D sector, we know $\lambda L_R(t) V(t) - w(t) L_R(t) = 0$. Recall that the term in brackets is the total profit $\pi(t) = \int_0^1 \pi(v,t)$ in intermediate good sector. Then, the budget constraint becomes

$$\dot{a}(t) = r(t) a(t) + \lambda L_R(t) V(t) - \pi(t)$$

Consequently, the Hamilton-Jacobi-Bellman equation for expressing the expected value of an innovation in R&D sector allows us to conclude that

$$a(t) = V(t)$$

This completes the proof of Lemma 1.

7.4 Cross-Sectoral Distribution

7.4.1 Productivity Distribution

We follow a method similar to Aghion and Howitt (1998) in order to characterize long-run distribution of relative productivity terms, both for technology improvements $\phi(v,t)$ and emission intensity $z(v,t)$. Let $F(.,t)$ be the cumulative distribution of technology index ϕ across different sectors at a given date t and write $\Phi(t) \equiv F(\phi, t)$. Then

$$\Phi(0) = 1 \tag{7.7}$$

$$\frac{\dot{\Phi}(t)}{\Phi(t)} = -\lambda L_R(t) \tag{7.8}$$

Integrating this equation yields

$$\Phi(t) = \Phi(0) e^{-\lambda \gamma_1 \int_0^t L_R(s) ds} \tag{7.9}$$

The equation (7.7) holds because it is not possible that a firm has a productivity parameter ϕ larger than the leading firm in the sector. The equation (7.8) means that at each date a mass of λn firm lacks behind, due to innovations that take place with Poisson distribution. From equation (2.12), we write

$$\frac{\dot{\bar{\phi}}_{max}(t)}{\bar{\phi}_{max}(t)} = \gamma_1 \lambda L_R \tag{7.10}$$

Integrating equation (7.10), we have ;

$$\bar{\phi}_{max}(t) = \bar{\phi}_{max}(0)\, e^{\lambda\gamma_1 \int_0^t L_R(s)ds} \tag{7.11}$$

where $\bar{\phi}_{max}(0) \equiv \bar{\phi}$. By using equations (7.9) and (7.11), we write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}}\right)^{\frac{1}{\gamma_1}} = e^{-\lambda \int_0^t L_R(s)ds} = \Phi(t) \tag{7.12}$$

We define a to be the relative productivity $\frac{\bar{\phi}}{\bar{\phi}_{max}}$. Basically, $\Phi(t)$ is the probability density distribution.

7.4.2 Emission Intensity Distribution

By proceeding exactly in same manner, we have

$$\frac{\dot{\underline{z}}_{min}(t)}{\underline{z}_{min}(t)} = \gamma_2 \lambda L_R \tag{7.13}$$

By integrating equation (7.13), we have

$$\underline{z}_{min}(t) = \underline{z}_{min}(0)\, e^{\lambda\gamma_2 \int_0^t L_R(s)ds} \tag{7.14}$$

We rewrite the equation as

$$\left(\frac{\underline{z}}{\underline{z}_{min}}\right)^{\frac{1}{\gamma_2}} = e^{-\lambda \int_0^t L_R(s)ds} \tag{7.15}$$

We can easily remark that this last equation is the same that we have found in equation (7.12). We write

$$\left(\frac{\bar{\phi}}{\bar{\phi}_{max}}\right)^{\frac{1}{\gamma_1}} = \left(\frac{\underline{z}}{\underline{z}_{min}}\right)^{\frac{1}{\gamma_2}} \tag{7.16}$$

From equation (7.16), We can find the relative distribution for emission intensity across firms

$$\frac{\underline{z}}{\underline{z}_{min}} = \left(\frac{1}{a}\right)^{-\frac{\gamma_2}{\gamma_1}}$$

7.5 Marginal cost of using the intermediate good

We know that the marginal cost of using a given machine v is the following

$$m\left(v,t\right)=\chi+H\left(v,t\right) \tag{7.17}$$

where $H\left(v,t\right)=h\left(t\right)\phi\left(v,t\right)z\left(v,t\right)^{\frac{1}{\alpha\beta}}$. It is possible to represent equations (7.15) and (7.16) in terms of their vintage v,

$$\left(\frac{\bar{\phi}_{max}\left(t-v\right)}{\bar{\phi}_{max}\left(v\right)}\right)^{\frac{1}{\gamma_1}}=e^{-\lambda\int_0^v L_R(s)ds} \tag{7.18}$$

$$\left(\frac{\underline{z}_{min}\left(t-v\right)}{\underline{z}_{min}\left(v\right)}\right)^{\frac{1}{\gamma_2}}=e^{-\lambda\int_0^v L_R(s)ds} \tag{7.19}$$

Using equations (7.18) and (7.19), we find the equation

$$m\left(v\right)=\chi+e^{\left(\frac{g_Z}{\alpha\beta}+g_\phi\right)v}H \tag{7.20}$$

7.6 Aggregate Economy

We replace equation of supply of machines (2.7) in equation (2.1) and write

$$Y\left(t\right)=L_Y\left(t\right)\int_0^1\phi\left(v,t\right)z\left(v,t\right)\left(\frac{\alpha^2\phi\left(v,t\right)z\left(v,t\right)}{\chi+h\left(t\right)\phi\left(v,t\right)z\left(v,t\right)^{\frac{1}{\alpha\beta}}}\right)^{\frac{\alpha}{1-\alpha}}dv \tag{7.21}$$

We proceed to reformulate the production in a way that it is possible to write productivity and emission intensity gaps. Note that according to Aghion and Howitt (1992), they are constant along time. By dividing and multiplying nominator and denominator by $\bar{\phi}_{max}\,\underline{z}_{min}$;

$$Y\left(t\right)=\alpha^{\frac{2\alpha}{1-\alpha}}L_Y\left(\bar{\phi}_{max}\,\underline{z}_{min}\right)^{\frac{1}{1-\alpha}}\int_0^1\left(\frac{\phi\left(v,t\right)}{\bar{\phi}_{max}}\frac{z\left(v,t\right)}{\underline{z}_{min}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{\left(\chi+h\left(t\right)\bar{\phi}_{max}\,\underline{z}_{min}^{\eta}\left(\frac{z(v,t)}{\underline{z}_{min}}\right)^{\frac{1}{\alpha\beta}}\frac{\phi(v,t)}{\bar{\phi}_{max}}\right)}\right)^{\frac{\alpha}{1-\alpha}}dv \tag{7.22}$$

By using the productivity and emission intensity distributions in Appendix 2.1, We find the aggregate production function as follows ;

$$Y\left(t\right)=\frac{\gamma_1}{1-\gamma_1}\alpha^{\frac{2\alpha}{1-\alpha}}L_Y\left(\bar{\phi}_{max}\left(t\right)\underline{z}_{min}\left(t\right)\right)^{\frac{1}{1-\alpha}}\Omega_1\left(H\right) \tag{7.23}$$

where the aggregation function for production $\Omega_1\left(H\right)$;

$$\Omega_1\left(H\right)=\int_0^1\frac{a^{\frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right)}}{\left(1+\frac{H}{\chi}a^{1+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}}\right)^{\frac{\alpha}{1-\alpha}}}\nu'\left(a\right)da \tag{7.24}$$

where $H = h(t)\,\bar{\phi}_{max} \underline{z}_{min}^{\eta}$ which is a constant term along time t by the policy rule and $\nu'(a)$ is the density function for the function $\nu(a) = F(.,t) = a^{\frac{1}{\gamma_1}}$.

The aggregation of intermediate factor $x(t)$ is obtained in same manner.

$$x(t) = \int_0^1 x(v,t)\,dv = \frac{\gamma_1}{1-\gamma_1}\alpha^{\frac{2}{1-\alpha}} L_Y \left(\bar{\phi}_{max}(t)\,\underline{z}_{min}(t)\right)^{\frac{1}{1-\alpha}} \Omega_2(H) \tag{7.25}$$

where the aggregation factor $\Omega_2(H)$ for intermediate good $x(t)$ is

$$\Omega_2(H) = \int_0^1 \frac{a^{\frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right)}}{\left(1+\frac{H}{\chi}a^{1+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}}\right)^{\frac{1}{1-\alpha}}}\nu'(a)\,da \tag{7.26}$$

The final good market equilibrium yields $Y(t) = c(t) + \chi x(t)$, since some part of the final good is used for the production of intermediate good. From equation (2.6), we know that aggregate cost of the production good $x(t)$is given by $\chi x(t)$.

$$c(t) = Y(t) - \chi x(t) = \alpha^2 \frac{\Omega_2(H)}{\Omega_1(H)} Y(t) \tag{7.27}$$

which gives the consumption $c(t)$ as a function of production function $Y(t)$.

7.7 Aggregation Factor

From production function, in order to solve the integral (7.24),

$$\Omega_1(H) = \int_0^1 \frac{a^{\bar{\gamma}}}{\left(1+\frac{H}{\chi}a^{1+\frac{\gamma_2}{\gamma_1}}\right)^{\frac{\alpha}{1-\alpha}}}da \tag{7.28}$$

where $\bar{\gamma} = \frac{1}{1-\alpha}\left(1+\frac{\gamma_2}{\gamma_1}\right) + \frac{1}{\gamma_1} - 1$. We use the substitution method. We define

$$y = -\frac{H}{\chi}a^{1+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}} \quad and \quad dy = -\left(1+\frac{\gamma_2}{\gamma_1}\right)\frac{H}{\chi}a^{1+\frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}-1}da \tag{7.29}$$

We rewrite the aggregation factor,

$$\Omega_1(H) = \int_0^{-\frac{H}{\chi}} y^{\frac{\bar{\gamma}+1-b}{b}}(1-y)^{-\frac{\alpha}{1-\alpha}}\,dy \tag{7.30}$$

where $b = 1 + \frac{\gamma_2}{\gamma_1}\frac{1}{\alpha\beta}$. It is easy to remark that expression in the integral is the incomplete beta function. Then, we can express this integral by using Gaussian hypergeometric function as follows

$$\Omega_1(H) = \left(\frac{1}{1+\bar{\gamma}}\right) {}_2F_1\left(\frac{\bar{\gamma}+1}{b}, \frac{\alpha}{1-\alpha}; \frac{\bar{\gamma}+b+1}{b}; -\frac{H}{\chi}\right) \tag{7.31}$$

In order to see the marginal change of aggregation factor with respect to marginal cost of pollution H ;

$$\frac{\partial \Omega_1(H)}{\partial H} = -\frac{1}{\chi}\left(\frac{\alpha(\bar{\gamma}+1)}{(1-\alpha)(\bar{\gamma}+1+b)}\right) {}_2F_1\left(\frac{\bar{\gamma}+1}{b}+1, \frac{\alpha}{1-\alpha}+1; \frac{\bar{\gamma}+b+1}{b}+1; -\frac{H}{\chi}\right) < 0$$

7.8 Condition on Penalty Function

From the household problem, we define the post-value function as

$$\Gamma(a(t)) = u(c_{min}) - \psi(a(t)) \tag{7.32}$$

$$\psi(a(t)) = \bar{\psi}(\omega - (1-\omega)\, log(a(t))) \tag{7.33}$$

At Balanced Growth Path, the post value function can be written in the following manner ;

$$\Gamma^* = -\int_0^\infty \psi(a(t))\, e^{-(\rho+\bar{\theta})t} dt = -\bar{\psi}\left(\frac{\omega}{\rho+\bar{\theta}} - \frac{(1-\omega)\, log(a(0))}{\rho+\bar{\theta}-g_Y}\right) \tag{7.34}$$

$$\omega > \frac{\left(\rho+\bar{\theta}\right) ln(a(0))}{\left(\rho+\bar{\theta}\right)(1+ln(a(0))) - g_Y} \tag{7.35}$$

where $a(0)$ is the level of wealth at initial date.

7.9 Labor allocation in equilibrium

To find the labor allocation in R&D sector, we differentiate equation (2.15) that yields the Hamilton-Jacobi-Bellman equation at the balanced growth path

$$(r+\lambda L_R) - \frac{\dot{V}(t)}{V(t)} = \frac{\pi\left(\bar{\phi}_{max}, \underline{z}_{min}\right)}{V(t)} \tag{7.36}$$

The lemma 1 shows that household owns the firms in market. Household receives dividend from innovation assets on the market.

With the functional forms defined in the text and using the resource constraint $Y(t) = c(t) + x(t)$. The growth rate of economy can be written

$$g_c = \frac{\dot{c}(t)}{c(t)} = r(t) - \left(\rho+\bar{\theta}\right) + \frac{\lambda\bar{\theta}\bar{\psi}(1-\omega)}{(1-\alpha)}\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)} L_Y \tag{7.37}$$

Note that by the free-entry condition, we have $g_V = g_w = g_Y$. Using equations (7.37), (**??**) and (2.9), we reformulate the expected value of an innovation

$$\underbrace{\frac{1}{1-\alpha}\left(g_\phi+g_Z\right)+\left(\rho+\bar{\theta}\right)-\frac{\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Omega_2\left(H\right)}{\lambda\left(1-\alpha\right)\Omega_1\left(H\right)}\left(1-L_R\right)+\lambda L_R}_{=\mathrm{r}+\lambda L_R}-\underbrace{\frac{\lambda L_R}{1-\alpha}\left(\gamma_1+\gamma_2\right)}_{=\frac{\dot{V}(t)}{V(t)}}=\underbrace{\frac{\alpha\gamma_1}{\lambda\left(1-\gamma_1\right)}\frac{\left(\chi+H\right)^{-\frac{\alpha}{1-\alpha}}}{\Omega\left(H\right)}\left(1-L_R\right)}_{=\frac{\pi\left(\bar{\phi}_{max},\underline{z}_{min}\right)}{V(t)}} \tag{7.38}$$

From (7.38), we find the equilibrium level of labor in R&D sector (see equation (3.2)).

7.10 Impact of hazard rate on labor allocation in R&D Sector

To assess the impact catastrophe probability on labor in R&D, we take derivative of L_R (equation (3.2)) with respect to hazard rate $\bar{\theta}$;

$$\frac{\partial L_R}{\partial\bar{\theta}}=\frac{\left(\frac{\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)}-1\right)}{\left(\lambda+\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)} \tag{7.39}$$

$$-\frac{\left(\frac{\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)}\right)\left[\frac{\bar{\theta}\bar{\psi}\lambda(1-\omega)\alpha^2}{(1-\alpha)}\frac{\Omega_2(H)}{\Omega_1(H)}+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)}-\left(\rho+\bar{\theta}\right)\right]}{\left(\lambda+\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)^2} \tag{7.40}$$

The impact depends whether the penalty rate $\bar{\psi}$ is sufficiently high or not.

$$sign\left(\frac{\partial L_R}{\partial\bar{\theta}}\right)>0\ if\ \bar{\psi}>\frac{\left(\lambda+\rho\right)\left(1-\omega\right)+\sqrt{\left(\left(\omega-1\right)\left(\lambda+\rho\right)\right)^2+4\bar{\theta}\omega\left(\omega-1\right)\left(\lambda+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)}}{2\bar{\theta}\omega} \tag{7.41}$$

$$sign\left(\frac{\partial L_R}{\partial\bar{\theta}}\right)<0\ if\ \bar{\psi}<\frac{\left(\lambda+\rho\right)\left(1-\omega\right)+\sqrt{\left(\left(\omega-1\right)\left(\lambda+\rho\right)\right)^2+4\bar{\theta}\omega\left(\omega-1\right)\left(\lambda+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}\right)}}{2\bar{\theta}\omega} \tag{7.42}$$

7.11 Impact of environmental taxation on labor allocation in R&D Sector

Taking the derivative of L_R (equation (3.2)) with respect to marginal cost of pollution H ;

$$\frac{\partial L_R}{\partial H}=\frac{\left[\underbrace{\frac{\lambda(1-\omega)\alpha^2\bar{\theta}\bar{\psi}}{(1-\alpha)}\left[\frac{\partial\Omega_2\left(H\right)}{\partial H}\frac{1}{\Omega_2\left(H\right)}-\frac{\Omega_2\left(H\right)}{\left(\Omega_1\left(H\right)\right)^2}\frac{\partial\Omega_1\left(H\right)}{\partial H}\right]}_{Z_1}+\underbrace{\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\left[-\frac{\partial\Omega_1\left(H\right)}{\partial H\left(\Omega_1\left(H\right)\right)^2}-\frac{\alpha}{1-\alpha}\frac{\left(\chi+H\right)^{-\frac{\alpha}{1-\alpha}-1}}{\Omega_1\left(H\right)}\right]}_{Z_2}\right]\left[\lambda+\rho+\bar{\theta}\right]}{\left(\lambda+\frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}+\frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega_1(H)}\right)^2}>0 \tag{7.43}$$

The impact of pollution tax depends on the relationship between elasticity of aggregation factor of production $\Omega_1(H)$ and that of intermediate good demand. The increase of marginal cost of pollution increases labor allocation in R&D if

Condition 1.

$$-\frac{\frac{\partial\Omega_1(H)}{\Omega_1(H)}}{\frac{\partial H}{H}} > -\frac{\frac{\partial\Omega_2(H)}{\Omega_2(H)}}{\frac{\partial H}{H}} \tag{7.44}$$

A necessary condition to have a positive impact of pollution tax on growth is that the elasticity of aggregation factor of production function is higher than the elasticity of aggregation factor of intermediate good factor. We know that a higher marginal pollution tax implies a lower production of final good which follows a lower intermediate good demand. Then, the term Z_1 is positive.

Condition 2.

$$H + \chi < 2 \tag{7.45}$$

In order to ensure that Z_2 is positive, we impose some conditions on some key parameters of the model. We suppose that $\frac{\bar{\gamma}+1}{b} = 0$ and $\alpha = \frac{1}{3}$. Our purpose in doing this is to gain insight about the mechanism that explains why a higher marginal cost of pollution can boost the economic growth at the long run. If the producing cost of machines is sufficiently low and the Condition 1. is ensured, the nominator is positive. Consequently, the effect of pollution tax is positive on growth.

To assess the impact of hazard rate on the effect of environmental taxation, we compute

$$\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right) = \left(\frac{\left[k^2 - \left(\lambda+\rho+\bar{\theta}\right)\frac{\lambda\alpha^2(1-\omega)\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)}\right]\left[\frac{\lambda(1-\omega)\alpha^2\Upsilon_1}{(1-\alpha)} + \frac{\alpha\gamma_1\lambda\Upsilon_2}{(1-\gamma_1)}\right] - k^2\left(\lambda+\rho+\bar{\theta}\right)\frac{\lambda\bar{\theta}\bar{\psi}(1-\omega)\alpha^2\Upsilon_2}{(1-\alpha)}}{k^4}\right) \tag{7.46}$$

where $k = \lambda + \frac{\lambda\alpha^2(1-\omega)\bar{\theta}\bar{\psi}}{1-\alpha}\frac{\Omega_2(H)}{\Omega_1(H)} + \frac{\alpha\gamma_1\lambda}{(1-\gamma_1)}\frac{(\chi+H)^{-\frac{\alpha}{1-\alpha}}}{\Omega(H)}$ and $\Upsilon_1 = \frac{\partial\Omega_2(H)}{\partial H}\frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2}\frac{\partial\Omega_1(H)}{\partial H} < 0$, $\Upsilon_2 = -\frac{\alpha(\chi+H)^{-\frac{\alpha}{1-\alpha}-1}}{(1-\alpha)(\chi+H)} - \frac{\partial\Omega_1(H)}{\partial H}\frac{1}{(\Omega_1(H))^2} \lesseqgtr 0$.

The derivative of (7.43) yields a complicated term. However, one can remark that it is possible to write a third degree equation $f\left(\bar{\psi}\right)$ in order to find the roots for constant penalty rate $\bar{\psi}$. Since, we will have three different roots, we can analyze the implications of hazard rate on the effect of pollution tax H ;

$$sign\left(\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right)\right) > 0 \; if \; \bar{\psi} > g(.) \tag{7.47}$$

$$sign\left(\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial L_R}{\partial H}\right)\right) < 0 \; if \; \bar{\psi} < g(.) \tag{7.48}$$

where $g(.)$ is the positive root of the third degree equation $f\left(\bar{\psi}\right)$ which is a function of constant parameters of the model. We also verify this condition by a numerical analysis in the text.

7.12 Impact of environmental taxation on Welfare

The differentiation of (5.2) yields ;

$$\frac{dW^*}{dH} = \underbrace{\frac{\frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\left[\frac{\partial\Omega_2(H)}{\partial H}\frac{1}{\Omega_2(H)} - \frac{\Omega_2(H)}{(\Omega_1(H))^2}\frac{\partial\Omega_1(H)}{\partial H} + \frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\frac{dY(0)}{dH}\right]}{\rho+\bar{\theta}-g}}_{\text{Output effect}}$$

$$+\underbrace{\frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y(0)\right)-\bar{\psi}\bar{\theta}\left(\omega+(1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\left(\rho+\bar{\theta}-g\right)^2}\frac{dg}{dH}}_{\text{Growth effect}}$$

where the sign of $\frac{dY(0)}{dH}$ is negative since the green tax decreases the final good production. On the other hand, since the labor in production shifts to the R&D sector, the growth rate of the economy increases. At the end, if the growth effect dominates the output effect, the green tax increases the welfare. It is easy to remark that when the necessary conditions for the positive effect of pollution tax on growth are not satisfied, both output and growth effect are negative. Consequently, the welfare becomes negative.

The effect of catastrophe probability on welfare is

$$\frac{dW^*}{d\bar{\theta}} = \underbrace{\frac{\frac{\Omega_1(H)}{\alpha^2\Omega_2(H)Y(0)}\frac{dY(0)}{d\bar{\theta}} - \bar{\psi}\left(\omega+(1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\rho+\bar{\theta}-g}}_{\text{Output effect}}$$

$$-\underbrace{\frac{log\left(\frac{\alpha^2\Omega_2(H)}{\Omega_1(H)}Y(0)\right)-\bar{\psi}\bar{\theta}\left(\omega+(1-\omega)\,log\left(\frac{\gamma_1(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\phi_{max}(0)\underline{z}_{min}(0)}{1-\gamma_1}\right)\right)}{\left(\rho+\bar{\theta}-g\right)^2}\left(1-\frac{dg}{d\bar{\theta}}\right)}_{\text{Growth effect}}$$

Similar to the effect of green tax burden on welfare, the total effect of catastrophe probability on welfare depends on the growth and output effect. If the abrupt event probability pushes the market economy to invest more in R&D and the increase of the growth rate compensates the output effect, the welfare of the economy increases.

References

[1] IPCC 2014. Ipcc, 2014: Climate change 2014: Synthesis report. contribution of working groups i, ii and iii to the fifth assessment report of the intergovernmental panel on climate change.

2014.

[2] Philippe Aghion and Peter Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323, mar 1992.

[3] Philippe Aghion and Peter W. Howitt. *Endogenous Growth Theory*. MIT PR, 1997.

[4] Thierry Bréchet, Natali Hritonenko, and Yuri Yatsenko. Adaptation and mitigation in long-term climate policy. *Environmental and Resource Economics*, 55(2):217–243, dec 2012.

[5] Lucas Bretschger and Alexandra Vinogradova. *Growth and Mitigation Policies with Uncertain Climate Damage*. August 2014.

[6] Seraina Buob and Gunter Stephan. To mitigate or to adapt: How to confront global climate change. *European Journal of Political Economy*, 27(1):1–16, mar 2011.

[7] Pascal Da Costa. Un progres technique pour le developpement durable ? recherche sur l'intensite et la direction du changement technique propre. phd thesis. 2006.

[8] André Grimaud. Pollution permits and sustainable growth in a schumpeterian model. *Journal of Environmental Economics and Management*, 38(3):249–266, nov 1999.

[9] Rob Hart. Growth, environment and innovation—a model with production vintages and environmentally oriented research. *Journal of Environmental Economics and Management*, 48(3):1078–1098, nov 2004.

[10] Alain Ayong Le Kama and Aude Pommeret. Supplementing domestic mitigation and adaptation with emissions reduction abroad to face climate change. *Environmental and Resource Economics*, jul 2016.

[11] Sally Kane and Jason F. Shogren. Linking adaptation and mitigation in climate change policy. *Climatic Change*, 45(1):75–102, 2000.

[12] Chol-Won Li. Endogenous vs. semi-endogenous growth in a two-r&d-sector model. *The Economic Journal*, 110(462):C109–C122, 2000.

[13] ROBERT MENDELSOHN, ARIEL DINAR, and LARRY WILLIAMS. The distributional impact of climate change on rich and poor countries. *Environment and Development Economics*, 11(02):159, mar 2006.

[14] Minoru Nakada. Does environmental policy necessarily discourage growth? *Journal of Economics*, 81(3):249–275, mar 2004.

[15] Minoru Nakada. Environmental tax reform and growth: Income tax cuts or profits tax reduction. *Environmental and Resource Economics*, 47(4):549–565, aug 2010.

[16] Ollivier and Barrows. Emission intensity and firm dynamics: Reallocation, product mix, and technology in india. *Working Paper*, 2016.

[17] Francesco Ricci. Environmental policy and growth when inputs are differentiated in pollution intensity. *Environmental and Resource Economics*, 38(3):285–310, apr 2007.

[18] Yacov Tsur and Cees Withagen. Preparing for catastrophic climate change. *Journal of Economics*, 110(3):225–239, dec 2012.

[19] Yacov Tsur and Amos Zemel. Accounting for global warming risks: Resource management under event uncertainty. *Journal of Economic Dynamics and Control*, 20(6-7):1289–1305, jun 1996.

[20] Yacov Tsur and Amos Zemel. Pollution control in an uncertain environment. *Journal of Economic Dynamics and Control*, 22(6):967–975, jun 1998.

[21] Yacov Tsur and Amos Zemel. Regulating environmental threats. *Environmental and Resource Economics*, 39(3):297–310, jun 2007.

[22] Yacov Tsur and Amos Zemel. Policy tradeoffs under risk of abrupt climate change. *Journal of Economic Behavior & Organization*, 132:46–55, dec 2016.

[23] Thierry Verdier. Environmental pollution and endogenous growth. *Control and Game Theoretic Models of the Environment, Birkahuser, Boston*, 2006.

[24] Amos Zemel. Adaptation, mitigation and risk: An analytic approach. *Journal of Economic Dynamics and Control*, 51:133–147, feb 2015.

[25] Joshua Graff Zivin and Matthew Neidell. The impact of pollution on worker productivity. *American Economic Review*, 102(7):3652–3673, dec 2012.