

The Cost of Transportation: Spatial Analysis of US Fuel Prices

Juste Raimbault, Antonin Bergeaud

▶ To cite this version:

Juste Raimbault, Antonin Bergeaud. The Cost of Transportation : Spatial Analysis of US Fuel Prices. EWGT 2017, Sep 2017, Budapest, Hungary. pp.0 - 0. halshs-01582909

HAL Id: halshs-01582909 https://shs.hal.science/halshs-01582909v1

Submitted on 6 Sep 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Available online at www.sciencedirect.com

SciVerse ScienceDirect

Transportation Research Procedia 00 (2017) 000-000



20th EURO Working Group on Transportation Meeting, EWGT2017, 4-6 September 2017, Budapest, Hungary

The Cost of Transportation : Spatial Analysis of US Fuel Prices

Juste Raimbault^{a,b,*}, Antonin Bergeaud^c

^aUMR CNRS 8504 Géographie-cités, 13 rue du Four, Paris, France ^bUMR-T IFSTTAR 9403 LVMT, Cité Descartes, Champs-sur-Marne, France ^cParis School of Economics - EHESS, Paris, France

Abstract

The geography of fuel prices has many various implications, from its significant impact on accessibility to being an indicator of territorial equity and transportation policy. In this paper, we study the spatio-temporal patterns of fuel price in the US at a very high resolution using a newly constructed dataset collecting daily oil prices for two months, on a significant proportion of US gas facilities. These data have been collected using a specifically-designed large scale data crawling technology that we describe. We study the influence of socio-economic variables, by using complementary methods: Geographically Weighted Regression to take into account spatial non-stationarity, and linear econometric modeling to condition at the state and test county level characteristics. The former yields an optimal spatial range roughly corresponding to stationarity scale, and significant influence of variables such as median income or wage per job, with a non-simple spatial behavior that confirms the importance of geographical particularities. On the other hand, multi-level modeling reveals a strong state fixed effect, while county specific characteristics still have significant impact. Through the combination of such methods, we unveil the superposition of a governance process with a local socio-economical spatial process. We discuss one important application that is the elaboration of locally parametrized car-regulation policies.

© 2017 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the Scientific Committee of EWGT2017.

Keywords: Fuel Price, Data Crawling, Spatial Analysis, Geographically Weighted Regression, Multi-level Modeling

1. Introduction

What drives the price of fuel? Using a new database on oil price at a gas station level collected during two months, we explore its variability across time and space. Variation in the cost of fuel can have many causes, from the crude oil price to local tax policy and geographical features, all having heterogeneous effect in space and time. If the evolution of the average fuel price in time is an indicator that is carefully followed and analyzed by many financial institution,

2214-241X © 2017 The Authors. Published by Elsevier B.V. Peer-review under responsibility of the Scientific Committee of EWGT2017.

^{*} Corresponding author. Tel.: +0-000-000-0000; fax: +0-000-000-0000. *E-mail address:* juste.raimbault@polytechnique.edu

its variability across space remain a rather unexplored topic in the literature. Yet, such differences can reflect variation in more indirect socio-economic indicators such as territorial inequalities and geographical singularities or consumer preferences.

There exists to our knowledge no systematic mapping in space and time of retail fuel prices for a large country such as the US. The main reason is probably that the availability of data have been a significant obstacle. It is also likely that the nature of the problem may also have influence, as it lies at the crossroad of several disciplines. While economists study price elasticity and measurement in different markets, transportation geography with methods such as transportation prices in spatial models, puts more emphasis on spatial distribution than on precise market mechanisms. Nevertheless, examples of somehow related works can be found. For example, Rietveld et al. (2001) studies the impact of cross-border differences in fuel price and the implications for gradual spatial taxation in Netherlands. At the country-level, Rietveld and van Woudenberg (2005) provides statistical models to explain fuel price variability across European countries. Macharis et al. (2010) models the impact of spatial fuel price variation on patterns of inter-modality, implying that the spatial heterogeneity of fuel prices has a strong impact on user behavior. With a similar view on the geography of transportation, Gregg et al. (2009) studies spatial distribution of gas emission at the US-state level. The geography of fuel prices also have important implications on effective costs, as shows Combes and Lafourcade (2005) by determining accurate transportation costs across urban areas for France. More closely related to our work, and using very similar daily open data for France, Gautier and Saout (2015) investigate dynamics of transmission from crude oil prices to fuel retail prices. However, they do not introduce an explicit spatial model of prices diffusion and do not study spatio-temporal dynamics. Deltas (2008) studies a long-time panel of prices in the US to show an asymmetry in response to wholesale price variations, but stays at the State level. A review on empirical studies of gasoline retailing done by Eckert (2013) confirms that station-level studies are few and imply small countries only such as Belgium or Austria, or single US States.

In this paper we adopt a broader approach by proceeding to exploratory spatial analysis on fuel prices for the whole US. We show that most of the variation occurs between counties and not across time, although crude oil price was not constant during the period considered. We therefore turn to a spatial analysis of the distribution of fuel prices. Our main findings are twofold: first we show that there are significant spatial pattern in some large US regions, second we show that even if most of the observed variation can be explained by state level policies, and especially the level of tax, some county level characteristics are still significant.

The rest of the paper is organized as follows: in the next section, we describe a generic procedure and the tool used for a systematic data collection. We also present our dataset. In section 3 we conduct statistical analysis in order to study the spatio-temporal variation of fuel price and test the potential correlation with some covariates. Finally, in section 4 we discuss our results and conclude.

2. Dataset

Our dataset contain daily information on fuel price at the gas station level for the whole US mainland territory. These information have been constructed from self-reported fuel price and span almost the entire universe of gas station in the US. We start by describing data collection and then give some statistics about this new dataset.

2.1. Collecting large scale heterogeneous data

The availability of new type of data has induced consequent changes in various disciplines from social science (e.g. online social network analysis (Tan et al. (2013))) to geography (e.g. new insights into urban mobility or perspectives on "smarter" cities (Batty (2013))) including economics where the availability of exhaustive individual or firm level data is seen as a revolution of the field. Most studies involving these new data are at the interface of implied disciplines, what is both an advantage but also a source of difficulties. Collection and use of new data has therefore become a crucial stack in social-science. The construction of such datasets is however far from straightforward because of the incomplete and noisy nature of data. Specific technical tools have to be implemented but have often been designed to overcome one specific problem and are difficult to generalize. We develop such a tool that fills the following constraints that are typical of large scale data collection, namely a reasonable level of flexibility and generality, an optimized performance through parallel collection jobs, and the anonymity of collection jobs to avoid any possible

Table 1. Descriptive statistics on Fuel Price (\$ per gallon)										
	Mean	Std. Dev.	p10	p25	p50	p75	p90			
	2.28	0.27	2.02	2.09	2.21	2.39	2.65			

bias in the behavior of the data source. The architecture, at a high level, has the following structure: (i) an independent pool of tasks runs continuously socket proxies to pipe requests through tor; (ii) a manager monitors current collection tasks, split collection between subtasks and launches new ones when necessary; (iii) Subtasks can be any callable application taken as argument destination urls, they proceed to the crawling, parsing and storage of collected data. The application is open and its modules are reusable: source code is available on the repository of the project.¹ We constructed our dataset by using the tool continuously in time during two months to collect crowdsourced data available from various online sources.

2.2. Dataset

Our dataset comprises around $41 \cdot 10^6$ unique observations of retail fuel prices at the station level, spanning the period starting the 10^{th} of January 2017 and ending the 19^{th} of March 2017 and corresponds to 118,573 unique retail stations. For each of these stations, we associate a precise geographical location (city resolution). On average we have 377 price informations by station. Prices correspond to a unique purchase mode (credit card, other modes such as cash being less than 10% in test datasets, they were discarded in the final dataset) and four possible fuel types: Diesel (18% of observations), Regular (34%), Midgrade (24%) and Premium (24%). The best coverage of stations is for Regular fuel type with on average 4,629 price information by county. We therefore choose to focus the study to this type of fuel, keeping in mind that further developments with the dataset may include comparative analysis on fuel types. Our final dataset thus contains 14,192,352 observations from 117,155 gas stations, followed during 68 days. We further aggregate these data by day, taking the average of the observed price per gallon, to obtain a panel of 5,204,398 gas station - day observations.² Table 1 gives some basic descriptive statistics of on price data showing that the distribution of oil price is highly concentrated with a small skewness (the ratio of the 99th to the 1st percentile is 1.6). Finally, in the spatial analysis, we will also use socio-economic data at the county level, available from the US Census Bureau. We shall use the latest available, what most of the time implies relying to the 2010 Census.

3. Results

3.1. Spatio-temporal Patterns of Prices

Before moving to a more systematic study of the variation of fuel price, we propose a first exploratory introduction to give insight about its spatio-temporal structure. This exercise is a crucial stage to guide further analyses, but also to understand their implications in a geographical context. To explore the data, we built a simple web application which allow to map the data in space and time. This application is available on this page. We also show one example of mapping the data at the county level in Figure 1 where we used average price over the whole period. We clearly see regional patterns with the Southcentral and Southeast regions having the lowest prices and the Pacific cost and Northeast the highest prices. Of course, plotting aggregated data over the whole period does not bring much information about the time variation of the data. As we will show more in detail below most of the variation of fuel price occurs across space. A variance decomposition of fuel price yields only 11% of the total variance is explained by within gas station variations. Similarly, the Spearman's rank correlation coefficient between the gas station price of regular fuel in the first day of dataset and in the last day is 0.867, and the null hypothesis that these two information are independent is strongly rejected.

¹ at https://github.com/JusteRaimbault/EnergyPrice

 $^{^{2}}$ The panel is not balanced as prices are not reported every day in every station. The average gas station has information on price for 44 days (over 68).

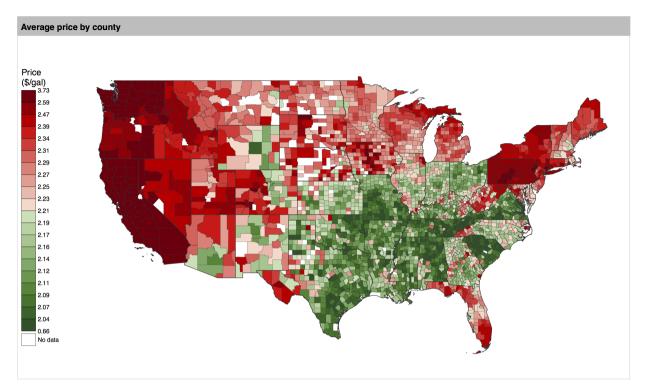


Fig. 1. Map of mean price for counties, regular fuel, averaged over the whole period.

Since most of the variation in oil price is between gas station, we now focus mainly on spatial correlations. We will conduct the analysis at the county level for various reasons. First it appears that a variance decomposition of fuel price between and within county shows that more than 85% of the variance is between-county, second because the localization of gas station is not reliable enough to allow for a smaller granularity and third because we have many socio-economic information at this level. We therefore study the spatial autocorrelation of prices at the county level. Spatial autocorrelation can be seen as an indicator of spatial heterogeneity which we measure using the Moran index (Tsai (2005)), with spatial weights of the form $\exp(-d_{ij}/d_0)$ with d_{ij} being the distance between spatial entities i and j, and d_0 a decay parameter giving the spatial range of interactions accounted for in the computation. We show in Fig. 2 its variations for each day and also as a function of the decay parameter. The fluctuations in time of the daily Moran index for low and medium spatial range, confirms geographical specificities in the sense of locally changing correlation regimes. These are logically smoothed for long ranges, as price correlations drop down with distance. The behavior of spatial autocorrelation with decay distance is particularly interesting: we observe a first regime change around 10km (from constant to piecewise linear regime), and a second important one around 1000km, both consistent across weekly time windows. We postulate that these correspond to typical spatial scales of the involved processes: the low regime would be local specificities and the middle one the state level processes. This behavior confirms that prices are non-stationary in space, and that therefore appropriate statistical techniques must be used to study potential drivers at different level. The two next subsections follow this idea and investigate potential explicative variables of local fuel prices, using two different techniques corresponding to two complementary paradigms: geographically weighted regression that puts the emphasis on neighborhood effects, and multi-level regression taking into account administrative boundaries.

3.2. Geographically Weighted Regression

The issue of spatial non-stationarity of geographical processes has always been a source of biased aggregated analyses or misinterpretations when applying general conclusions to local cases. To take it into account into statistical models, numerous techniques have been developed, among which the simple but very elegant Geographically

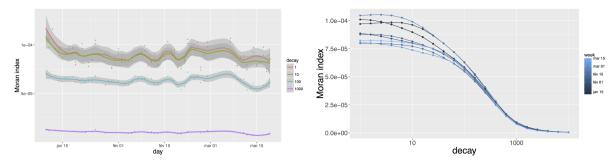


Fig. 2. Behavior of Moran spatial-autocorrelation index. (Left) Evolution in time of Moran index computed on daily time windows, for different decay parameter values. (Right) Moran index as a function of decay parameter, computed on weekly time windows.

Weighted Regression (GWR), that estimates non-stationary regressions by weighting observations in space similarly to kernel estimation methods. This was introduced in a seminal paper by Brunsdon et al. (1996) and has been subsequently used and matured since then. The significant advantage of this technique is that an optimal spatial range in the sense of model performance can be inferred to derive a model that yields the effect of variables varying in space, thus revealing local effects that can occur at different spatial scales or across boundaries. We proceed to multi-modeling to find the best model and associated kernel and spatial range. More specifically, we do the following: (i) we generate all possible linear models from the five potential variables (income, population, wage per job, jobs per capita, jobs); (ii) for each model and each candidate kernel shape (exponential, gaussian, bisquare, step), we determine the optimal bandwidth in the sense of both cross-validation and corrected Akaike Information Criterion (AICc) which quantifies information included in the model; (iii) we fit the models with this bandwidth. We choose the model with the best overall AICc, namely *price* = $\beta \cdot (income, wage, percapjobs)$ for a bandwidth of 22 neighbors and a gaussian kernel,³ with an AICc of 2,900. The median AICc difference with all other models tested is 122. The global R-squared is 0.27, what is relatively good also compared to the best R-squared of 0.29 (obtained for the model with all variables, which clearly overfits with an AICc of 3010; furthermore, effective dimension is less than 5 as 90% of variance is explained by the three first principal components for the normalized variables).

The coefficients and local R-squared for the best model are shown in Fig. 3. The spatial distribution of residuals (not shown here) seems globally randomly distributed, which confirms in a way the consistency of the approach. Indeed, if a distinguishable geographical structure had been found in the residuals, it would have meant that the geographical model or the variable considered had failed to translate spatial structure. Let now turn to an interpretation of the spatial structures we obtain. First of all, the spatial distribution of the model performance reveals that regions where these simple socio-economic factors explain do a good job in explaining prices are mostly located on the west coast, the south border, the north-east region from lakes to the east coast, and a stripe from Chicago to the south of Texas. The corresponding coefficients have different behaviors across the areas, suggesting different regimes.⁴ For example, the influence of income in each region seems to be inverted when the distance to the coast increases (from north to south-east in the west, south to north in Texas, east to west in the east), what may be a fingerprint of different economic specializations. On the contrary, the regime shifts for wage show a clear cut between west (except around Seattle) and middle/east, that does not correspond to state-policies only as Texas splits in two. The same way, jobs per capita show an opposition between east and west, what could be due for example to cultural differences. These results are difficult to interpret directly, and must be understood as a confirmation that geographical particularities matters, as regions differ in regimes of role for each of the simple socio-economic-variables. Further precise knowledge could be obtained through targeted geographical studies including qualitative field studies and quantitative analyses, that are beyond the scope of this exploratory paper and left for further research.

Finally, we extract the spatial scale of the studied processes, that is, by computing the distribution of distance to nearest neighbors with the optimal bandwidth. It yields roughly a log-normal distribution, of median 77km and

³ note that the kernel shape does not have much influence as soon as gradually decaying functions are used

⁴ We comment their behavior in areas where the model has a minimal performance, that we fix arbitrarily as a local R-squared of 0.5

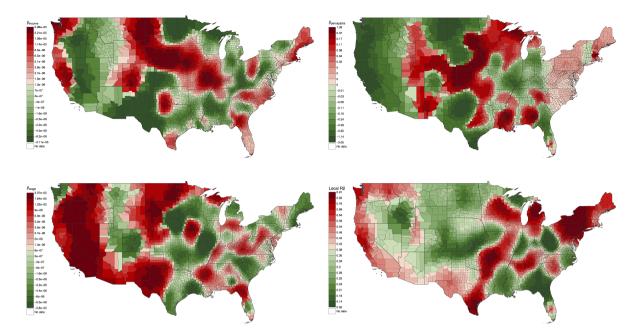


Fig. 3. **Results of GWR analyses.** For the best model in the sense of AICc, we map the spatial distribution of fitted coefficient, in order from left to right and top to bottom, β_{income} , $\beta_{percap jobs}$, β_{wage} , and finally the local r-squared values.

interquartile 30km. We interpret this scale as the spatial stationarity scale of price processes in relation with economic agents, which can also be understood as a range of coherent market competition between gas stations.

3.3. Multi-level Regression

Since our initial database enables to look at the level of variable $x_{i,s,c,t}$, the fuel price in day *t*, in gas station *i*, in state *s* and in county *c*, we start by running high dimensional fixed effect regressions following the model:

$$\begin{aligned} x_{i,s,c,t} &= \beta_s + \varepsilon_{i,s,c,t} \\ x_{i,s,c,t} &= \beta_c + \varepsilon_{i,s,c,t} \end{aligned} \tag{1}$$

$$x_{i,s,c,t} = \beta_i + \varepsilon_{i,s,c,t} \tag{3}$$

where $\varepsilon_{i,s,c,t}$ contains an idiosyncratic error and a day fixed effect. This first analysis confirm that most of the variance can be explained by a state fixed effect and that integrating more accurate levels has only small effect on the fit of our model as measured by the R-squared.

We now turn to a different analysis, aiming at capturing the explanatory variables that account for spatial price variation of fuel. We consider the following linear model:

$$log(x_i) = \beta_0 + X_i \beta_1 + \beta_{s(i)} + \varepsilon_i, \tag{4}$$

where x_i denotes average measured fuel price in county *i* aggregated across all days, X_i is a set of county specific variables and s(i) is the state to which the county belongs so that $\beta_{s(i)}$ capture all state specific variation. Finally ε_i is an error term satisfying $Cov(\varepsilon_i, \varepsilon_j) = 0$ if $s(i) \neq s(j)$. This clustering of standard error at the state level is motivated by finding of the previous section, showing that spatial autocorrelation of fuel price at the state level is still potentially strong. This specification aims at capturing the effect of various socio-economic variable at the county level after a state fixed effect has been removed. We present our results in Table 2. Column (1) shows that regressing the log of price on a state fixed-effect is already enough to explain 74% of the variance. This is mostly due to tax on fuel which

are set at the state level in the US. In fact, when we regress the log of oil price on the level of state tax, we find a R-squared of 0.33%. The remaining explanatory variables show that dense urban counties have higher fuel price, but this price decreases with population. This result seems sensible, desert areas have on average higher oil price. Fuel price increases with total income, decreases with poverty and decrease with the extent to which a county has voted for a Republican candidate. This last finding suggests a circular link: counties that use car the most tend to vote to politician that promote pro car policies. Adding these explanatory variables slightly increase the R-squared, suggesting that even after having removed a state fixed-effect, the price of fuel can be explained by local socio-economic features.

Table 2. Regressions at the county level

(1)				
(1)	(2)	(3)	(4)	(5)
	0.016*** (0.002)	0.016*** (0.001)	0.016*** (0.001)	0.015*** (0.001)
	-0.007***	-0.040***	-0.041***	-0.039***
	(0.001)			(0.010)
		0.031*** (0.010)	0.031*** (0.010)	0.027*** (0.009)
		0.001 (0.001)	0.000 (0.001)	0.000 (0.001)
		-0.028** (0.011)	-0.030*** (0.011)	-0.029** (0.011)
			0.000*** (0.000)	-0.000 (0.000)
				-0.072*** (0.015)
0.743 3,066	0.767 3,011	0.774 3,011	0.776 3,011	0.781 3,011
		(0.002) -0.007*** (0.001) 0.743 0.767	(0.002) (0.001) -0.007*** -0.040*** (0.001) (0.011) 0.031*** (0.010) 0.001 (0.001) -0.028** (0.011) 0.021** (0.011) 0.021*** 0.001 0.001 (0.001) -0.028** (0.011) 0.743 0.767 0.774	$\begin{array}{ccccccc} (0.002) & (0.001) & (0.001) \\ -0.007^{***} & -0.040^{***} & -0.041^{***} \\ (0.001) & (0.011) & (0.011) \\ & 0.031^{***} & (0.010) & (0.010) \\ & 0.001 & (0.001) \\ & 0.001 & (0.001) \\ & -0.028^{**} & -0.030^{***} \\ (0.011) & (0.011) \\ & & 0.000^{***} \\ (0.000) \end{array}$

Notes: This table plots results from an Ordinary Least Square regression of model presented in equation (4). Density is measured as the number of inhabitant by square miles and total income is given in dollars. Poverty is measured as the number of people below the poverty threshold per inhabitant. Percentage black is the percentage of black people living in the county and vote GOP is the share of people having voted for Donald Trump in the 2016 elections. Regression includes a state fixed effect. Robust standard errors clustered at the state level are reported in parenthesis. ***, ** and * respectively indicate 0.01, 0.05 and 0.1 levels of significance.

4. Discussion

On the complementarity of Econometric and Spatial Analysis methods. One important aspect of our contribution is methodological. We show that to explore a new panel dataset, geographers and economists have different approaches, leading to similar generic conclusions but with different paths. Some studies have already combined GWR and multi-level regressions (Chen and Truong (2012)), or compared them in terms of model fit or robustness (Lee et al. (2009)). We take here a multi-disciplinary point of view and combine approaches answering to different questions, GWR aiming at finding precise explicative variables and to measure the extent of spatial correlation, whereas

econometric models explain with more accuracy the effect of factors at different levels (state, county) but take these geographical characteristics as exogenous. We claim that both are necessary to understand all dimensions of the studied phenomenon.

Designing localized car-regulation policies. Another potential application of our analysis is to help better designing car-regulation policies. Environmental and health issues nowadays require a reasoned use of cars, in cities with the problem of air pollution but also overall to reduce carbon emissions. Fullerton and West (2002) showed that a taxation of fuel and cars can be equivalent to a taxation on emissions. Brand et al. (2013) highlight the role of incentives for the transition towards a low carbon transportation. However, such measures can't be uniform across states or even counties for obvious reasons of territorial equity: areas with different socio-economic characteristics or with different amenities shall contribute regarding their capabilities and preferences. Knowing local prices dynamics and their drivers, in which our study is a preliminary step, may be a path to localized policies taking into account the socio-economic configuration and include an equity criterion.

5. Conclusion

We have described a first exploratory study of US fuel prices in space and time, using a new database at the gas station level spanning two months. Our first result is to show the high spatial heterogeneity of price processes, using interactive data exploration and auto-correlation analyses. We proceed with two complementary studies of potential drivers: GWR unveils spatial structures and geographical particularities, and yields a characteristic scale of processes around 75km; multi-level regressions show that even though most of the variation can be explained by state level characteristics, and mostly by the level of the tax on fuel that is set by the state, there are still socio-economic specificities at the county level that can explain spatial variation of fuel price. Our contribution paves the way for further detailed studies using similar micro-level datasets.

References

Batty, M., 2013. Big data, smart cities and city planning. Dialogues in Human Geography 3, 274–279.

- Brand, C., Anable, J., Tran, M., 2013. Accelerating the transformation to a low carbon passenger transport system: The role of car purchase taxes, feebates, road taxes and scrappage incentives in the uk. Transportation Research Part A: Policy and Practice 49, 132–148.
- Brunsdon, C., Fotheringham, A.S., Charlton, M.E., 1996. Geographically weighted regression: a method for exploring spatial nonstationarity. Geographical analysis 28, 281–298.
- Chen, D.R., Truong, K., 2012. Using multilevel modeling and geographically weighted regression to identify spatial variations in the relationship between place-level disadvantages and obesity in taiwan. Applied Geography 32, 737–745.
- Combes, P.P., Lafourcade, M., 2005. Transport costs: measures, determinants, and regional policy implications for france. Journal of Economic Geography 5, 319–349.
- Deltas, G., 2008. Retail gasoline price dynamics and local market power. The Journal of Industrial Economics 56, 613-628.
- Eckert, A., 2013. Empirical studies of gasoline retailing: A guide to the literature. Journal of Economic Surveys 27, 140–166.
- Fullerton, D., West, S.E., 2002. Can taxes on cars and on gasoline mimic an unavailable tax on emissions? Journal of Environmental Economics and Management 43, 135–157.
- Gautier, E., Saout, R.L., 2015. The dynamics of gasoline prices: Evidence from daily french micro data. Journal of Money, Credit and Banking 47, 1063–1089.
- Gregg, J.S., Losey, L.M., Andres, R.J., Blasing, T., Marland, G., 2009. The temporal and spatial distribution of carbon dioxide emissions from fossil-fuel use in north america. Journal of Applied Meteorology and Climatology 48, 2528–2542.
- Lee, S., Kang, D., Kim, M., 2009. Determinants of crime incidence in korea: a mixed gwr approach, in: World conference of the spatial econometrics association, pp. 8–10.
- Macharis, C., Van Hoeck, E., Pekin, E., Van Lier, T., 2010. A decision analysis framework for intermodal transport: Comparing fuel price increases and the internalisation of external costs. Transportation Research Part A: Policy and Practice 44, 550–561.
- Rietveld, P., Bruinsma, F., Van Vuuren, D., 2001. Spatial graduation of fuel taxes; consequences for cross-border and domestic fuelling. Transportation Research Part A: Policy and Practice 35, 433–457.
- Rietveld, P., van Woudenberg, S., 2005. Why fuel prices differ. Energy Economics 27, 79–92.
- Tan, W., Blake, M.B., Saleh, I., Dustdar, S., 2013. Social-network-sourced big data analytics. IEEE Internet Computing 17, 62-69.
- Tsai, Y.H., 2005. Quantifying urban form: compactness versus' sprawl'. Urban studies 42, 141–161.