Conditional and spontaneous asymmetry of harmonic progressions in madrigal cycles from Verdelot to Monteverdi

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[1] The theory of harmonic vectors (THV) postulates that of the six possible root progressions in a given tonality those up a fourth, down a third and up a second (+4, -3, +2) are present in significantly greater numbers than the complementary root motions down a fourth, up a third and down a second (-4, +3, -2) (Meeùs 1988, 1989, 2000). This imbalance between both root progression categories is considered to shed light on a specific aspect of the tonal system (Meeùs 2001, 63).

[2] Previous studies have confirmed the tendency for the primary group to dominate in genres ranging from Renaissance polyphony to 20th century popular music (Desbordes 2001, Meyer 2009, O'Donnell 2011, Cathé 2012, Guillotel-Nothmann 2013). But while root progressions of the first type do become more prevalent in tonality, the increase is not as dramatic as might have been expected (Tymoczko 2003, 43; Hedges & Rohrmeier 2011). This could suggest that the prevalence is less crucial for harmonic tonality than initially thought. However, the *nature* of these progressions also needs to be considered.

[3] I will argue that it is less the change in *frequency* of these preferred progressions that is critical for harmonic tonality, than the change in their *quality*. The progressions +4, -3, +2 arise almost accidentally in pre-tonal polyphony through the constraints of contrapuntal rules. On the contrary, in later repertoires, they become a decisive syntactical feature which actively constrains tonality.

[4] To test this hypothesis, a model that combines voice-leading and harmonic progressions will be considered against a body of madrigal cycles by Verdelot, Arcadelt, Lassus, Rore, Wert and Monteverdi. These cycles, published between c. 1530 and 1638, contain about 50 000 chord progressions. The empirical results in conjunction with the model will allow for a close examination of how and why one particular group of root progressions dominates. They will show the phenomena that reflect the changing status of the prevalent root progressions and the technical aspects which may have fostered it. Finally, they will help to identify the compositional possibilities which result from the evolution outlined.

1. Vectors, Voice-leading and Asymmetry of root progressions

1.1. The Theory of Harmonic Vectors

[5] The THV is based on a systematic classification of root progressions and provides rules of syntax that constitute the embryo of a tonal grammar (Meeùs 2003, 8). The Theory categorizes harmonic progressions into two distinct groups of dominant and subdominant functions. Each group includes one main progression that moves by fourth and two substitute progressions which move by a third or a second (example 1).

	Dominant vectors	Subdominant vectors
Main progression	+4	-4
Substitutions	-3	+3
	+2	-2

Example 1. Classification of harmonic progressions in the THV.

[6] The progression up a fourth (+4) and the substitutions down a third (-3) and up a second (+2) are classified as dominant vectors. The complementary chord progressions (-4, +3, -2) belong to a category of subdominant vectors. While the main progressions are named with reference to the dominant and subdominant progressions in the perfect (V-I) and plagal cadence (IV-I), the substitutions are inferred – in the theory's initial formulation – from Rameau's *double emploi* (Rameau 1737) and Riemannian functional equivalences (Riemann 1909[7]).

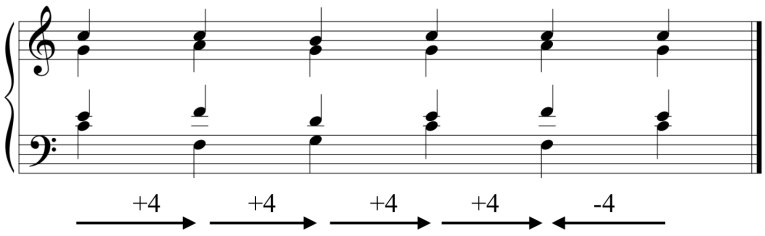
[7] Studies of corpora of sixteenth and seventeenth century music have shown that dominant vectors are always in the majority. This imbalance increases, and gradually stabilizes however from the 17th century onwards (Cathé 2012).

[8] The hegemony of dominant vectors can be deduced most effectively from the tonal cadence I-IV-V-I-IV-I (example 2), where all progressions, with the exception of the plagal closure IV-I, correspond to dominant vectors. Following Tymoczko (2003, 38), I term this imbalance as the 'asymmetry of root progressions'. This asymmetry corresponds to the difference in frequency between dominant vectors (DV) and subdominant vectors (SV):

$$\text{Asymmetry} = DV - SV$$

[9] In the THV the vector categories are assigned opposing directions. These are visualised with arrows to the right, for dominant progressions, and arrows to the left for subdominant

progressions (see example 2). The predominance of the rightward arrows makes explicit the cadential teleology which plays a key role in tonality. This cadential direction will be referred to as the 'privileged direction' of chord progressions. The THV does not claim that tonal harmony can be exclusively reduced to dominant progressions[1]. It does highlight however the crucial role of these progressions in tonal syntax.



Example 2. Paradigmatic cadence with harmonic vectors.

1.2. Model

[10] As shall be demonstrated, the asymmetry of root progressions has its roots in cadence patterns which go back to the late Middle Ages and the early Renaissance.

[11] The earliest madrigals considered in this study date from the early sixteenth century, a time when the cadence becomes a *locus* of theoretical thought. Through the concept of *clausula formalis,* and taking as point of departure the idiomatic melodic and harmonic formulae associated with the cantus and the tenor, theorists of that time describe the additional bassus and altus lines, the voice's permutation, the intermingling of dissonances and the exception of the mi-cadence.

[12] The intervallic progression sixth to octave between the penultimate and the finalis of example 3 has remarkable qualities. It combines a gradual transition from the relative imperfection (imperfect consonance) to the relative perfection (perfect consonance) with stepwise upward (cantus) and downward (tenor) motion. This characteristic progression (and its complementary progression third to unison) becomes established in the 14th century as the canonical cadential formula of the cantus-tenor framework (Eberlein 1992, 34)[2]. From the 15th century onwards, because triadic harmony weakens the distinction between imperfect and perfect consonances, this formula is regularly preceded by a dissonant suspension. This dissonance on the antepenultimate (D4-C5 in example 3) expands and reinforces the cadence by launching the teleological drive earlier (Dahlhaus 1990a).

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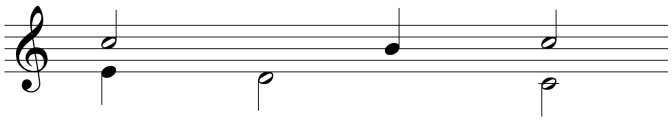
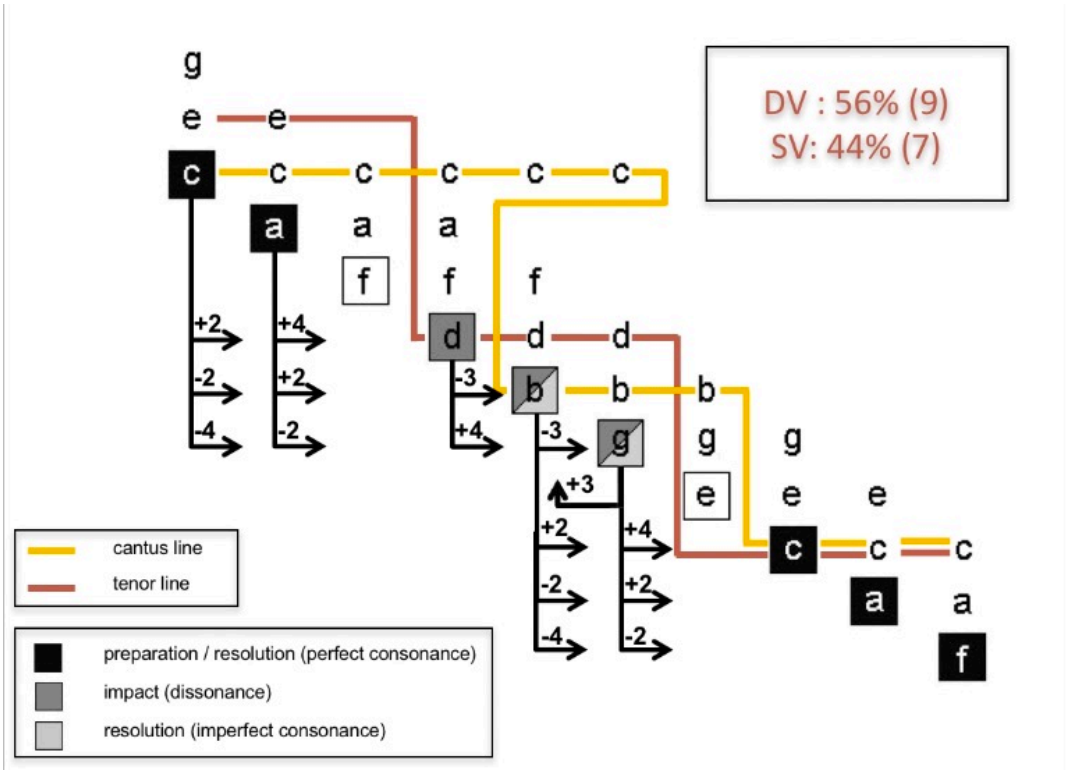
1 Tymoczko (2003, 46-47) draws attention to the particular status of subdominant progressions. Occurring on specific scale degrees, they can play a critical role in establishing tonality, as for example the subdominant progressions included in the a-b-a patterns I-V-I and I-IV-I.

2 Theorists of the 14th century award a special status to two particular progressions: 1. major sixth – octave 2. minor third – unison. They thus systematically apply the principle of voice-leading proximity to intervallic classes.



Example 3. Cantus-tenor framework.

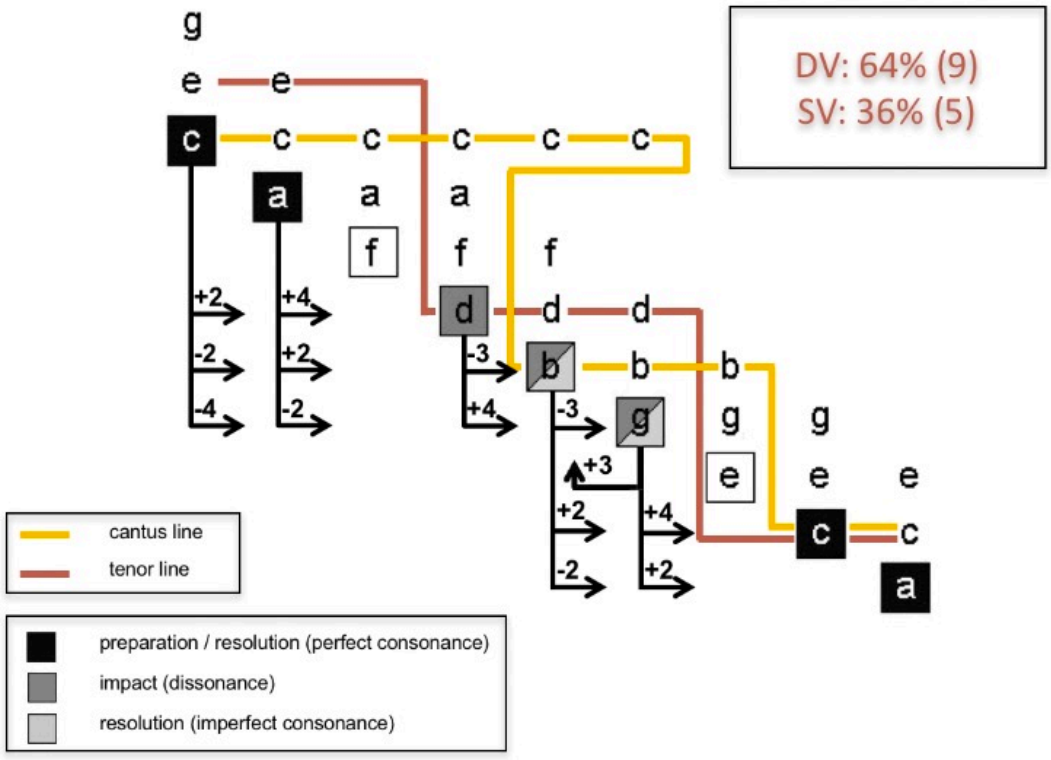
[13] The model of example 4 presents the triadic contexts that may follow from the cantus-tenor framework in 3a. It shows the possible harmonizations and compatible roots through which the cantus and tenor lines may pass. The initial consonance E4-C5 which prepares the suspension may be harmonized by a triad on either C or A (in black). The dissonance D4-C5 that follows can be harmonized by triads on D, B or G (in dark grey). The resolution onto the D4-B4 imperfect consonance that follows is harmonized by triads on B or G (in light grey). Finally, the resolution onto the perfect consonance C4-C5 is tied to the roots C, A and F (in black). These harmonizations lead to several observations.



Example 4a. Model: Harmonic progressions implied by the cantus-tenor framework.

[14] The cadence-pattern as a whole implies a fall through a cycle of thirds between the different intervals of the cadential chain. The arrows in example 4a represent the root progressions that may occur between the preparation, the impact and the resolution of the dissonance. They indicate that in the harmonic progressions generated, dominant vectors (56%) occur more frequently than subdominant vectors (44%). This confirms that under specific conditions the cantus-tenor framework is a potential source of asymmetry. The imbalance between both vector categories is yet notably low.

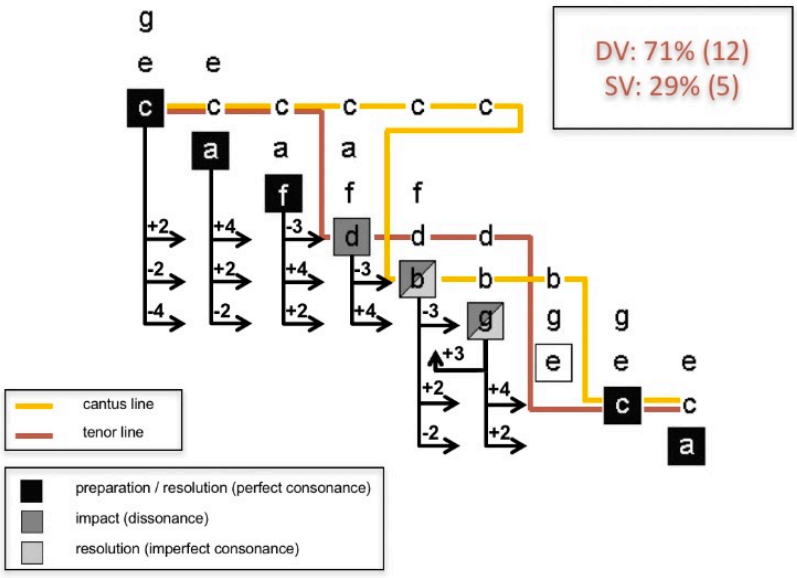
[15] Some roots are however excluded from the cadence pattern. The harmonization of the final consonance C4-C5 using a triad with root F, although theoretically possible, was not included here. With the exception of the Phrygian cadence (the harmonization of a final E with a triad on A), composers tend to avoid the harmonization of the final by the lower fifth. If one exclude the chord on root F as final chord, the asymmetry between dominant (64%) and subdominant (35%) vectors significantly increases, as shown in the animation of example 4b.



Example 4b. Model: Harmonic progressions implied by the cantus-tenor framework with restricted final chord.

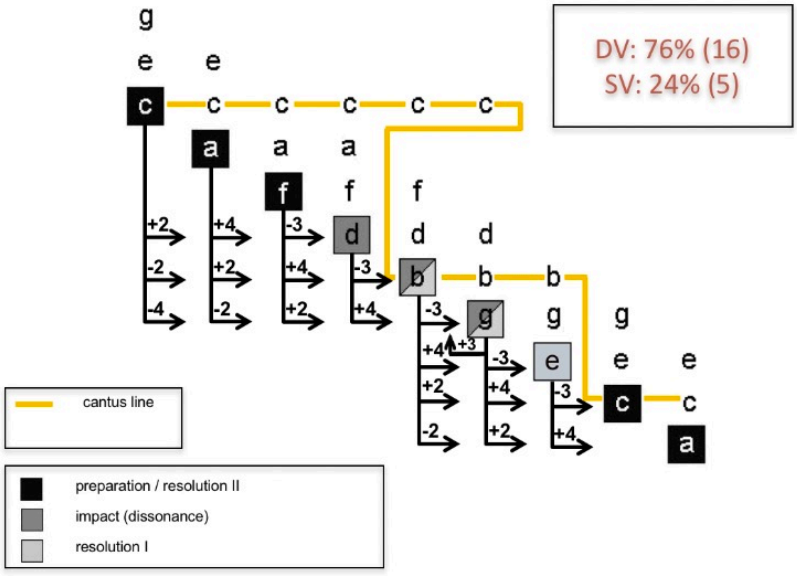
[16] Furthermore, the third triad in the cycle (also on F) cannot be involved in the cadence pattern as the tenor line excludes its use. The same applies to the triad on E, the seventh triad in the cycle.

[17] The tenor line is however not entirely stable in theory and in practice. It can be replaced by the movement ^1-^2-^1 or be absent (see Eberlein 1992, 56-62 and 2.1). In the first case (^1-^2-^1), harmonization with a triad on F becomes possible at the beginning and leads to three additional dominant progressions (F-D, F-B and F-G) thus reinforcing the asymmetry between dominant (71%) and subdominant vectors (29%), as shown by the animation in example 4c.



Example 4c. Model: Harmonic progressions implied by the modified tenor line ^1-^2-^1 in the cantus-tenor framework.

[18] In the second case (absence of the tenor line), the possibilities even increase: a triad with root E becomes possible at the resolution on B. This leads again to intensifying the asymmetry between dominant vectors (76%) and subdoinant vectors (36%) by allowing five additional dominant progressions, as in example 5 (D-E, B-E, G-E, E-C, E-A) [3], as shown by the animation of example 4d.



Example 4d. Model: Harmonic progressions implied by the cantus line.

3 The relationship between the evolution of cadential lines and harmonic progressions has been studied in detail in Guillotel-Nothmann & Meyer 2013 and Guillotel-Nothmann 2015, 459-469.

[19] Interestingly, this model has affinities with Tymoczko's third-based-grammar of elementary tonal harmony (Tymoczko, 2011, 226-30). Both favor the downward direction in the cycle of thirds with the upward motion limited to a restricted number of progressions. They also both restrict the use of the chord on scale degree iii (E in example 3) but allow subdominant progressions, especially in I-V-I, or I-VII-I (C-G-C, C-B-C in example 2).
[20] The model does not claim, however, to be an accurate representation of tonal harmony. Its contrapuntal constraints do not equate to constraints that govern tonal organization. It nevertheless allows several important observations about asymmetry in Western polyphony and its links with other characteristics of harmonic tonality.

[21] 1. The model supports the hypothesis that dominant progressions are not the result of tonality but, on the contrary, help to create some of its characteristics, both ontologically and historically. It suggests that rules of root-motion that constrain tonality might have evolved at an earlier stage through contrapuntal constraints at the cadence. This indirectly corroborates an intuition Lowinsky (1962, 4) had when he qualified the cadence as the "cradle of tonality".

[22] 2. The model also sheds another light on the concept of substitution. Theories of chord progression from Rameau (1721) to De Jong & Noll (2008) and theories of harmonic function such as Riemann's *Funktionstheorie* assume (at least implicitly) a hierarchy between main representatives and substitutions[4]. On the other hand, the contrapuntal perspective presented in this model conceives of the alternative harmonizations as equivalent. This equivalence results from the harmonic affinities between thirds in a distinctly diatonic context, and the position of the harmonizations in the cadence. The concept of substitution then, both in its transformational and in its functional interpretation, is closely linked to the asymmetry of dominant versus subdominant root progressions. It also appears as a hierarchical reinterpretation of a more general principle in a specific tonal context.

[23] 3. The model shows that asymmetry, substitution and tonicisation are inter-related. The confirmation of the tonic through the cadential teleology both elicits dominant chord progressions and concludes them. Without this gravitational force, the harmony would move forward perpetually in an unlimited harmonic space (Meeùs 2003). Directional tendency thus cannot be the only criterion for an advanced theory of harmonic tonality because it is not restrictive enough (Noll 2008, 87). It is nonetheless this criterion, directional tendency, that facilitates the utterance of the tonic and constrains tonality, and not the converse.

[24] 4. Finally, it should be noted that the model does not allow us to infer a causal relationship between contrapuntal constraints and asymmetry. On the one hand, contrapuntal rules actively affect root progressions and encourage asymmetry. On the other hand, dominant vectors always allow the preparation and downward resolution of the dissonance and the upward motion of the leading tone. Therefore, I argue that contrapuntal constraints and the

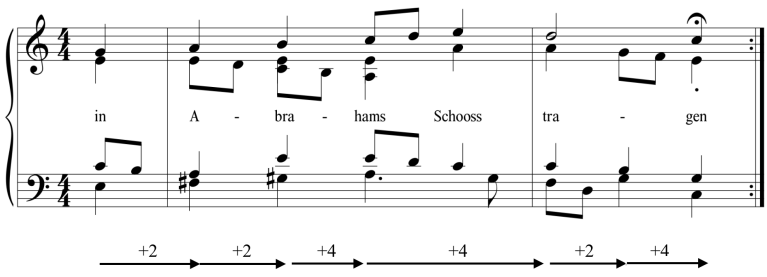
4 These questions of the hierarchy between different representatives of the same tonal function and of a possible distinction between a main representative and substitutes have been discussed in detail in Dahlhaus 1966, 1975.

hegemony of dominant vectors influence each other mutually. However, this link evolves and we see their causal relationship change through centuries of polyphony. This evolution is key to my argument and to the concepts of conditional and spontaneous asymmetry presented below.

1.3. Changing asymmetries

1.3.1. Spontaneous asymmetry

[25] Nicolas Meeùs (1992a) argues that in tonality the chord's mode and the characteristic dissonances – i.e. the subdominant 6/5 or the dominant 7^{th} – are superficial elements. These features are compared to what the linguist Sechehaye (1926, 86) calls "rection", i.e. a characterizing element that confirms a grammatical construction without being decisive for it. This specific situation is illustrated in the harmonization of *Herzlich lieb hab ich Dich, o Herr* by J.S. Bach (example 5). The cadential 6/5 and the passing dominant seventh in the example's last bar reinforce the cadential teleology. These dissonances are however not decisive for the cadence itself. As corroborated by the model cadence (example 2) which uses no dissonances on IV and V, it is *how* the harmonic units are arrived at and are left – i.e. in both cases by dominant vectors between I-IV and IV-V – that is crucial for the grammatical construction. The dissonances reinforce the cadential teleology and contribute to characterize the chord's function as predominant and dominant, but are neither decisive for these functions nor for the prevalent dominant direction. In both cases, the dominant direction, critical for the syntactical organization of the cadence pattern, is established for its own will, independently of contrapuntal constraints such as the preparation or resolution of dissonances. This is what I call *spontaneous asymmetry*.

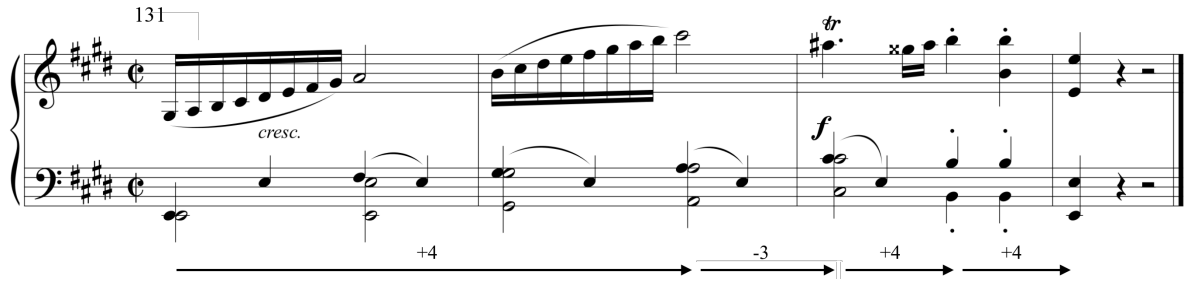


Example 5. J.S. Bach, Herzlich lieb hab ich Dich, o Herr, from Cantata 149 Man singet mit freuden vom Sieg, BWV 149.

[26] The concept of spontaneous asymmetry supposes the assimilation by the listener of the prevalent dominant direction, which thus becomes a key syntactical element. Dahlhaus (1990b, 133) alludes to this when he argues that chordal dissonances are the result of a "reciprocal relationship between root progression and the resolution of dissonance". This means that the chordal dissonance relies on a dynamic interpretation of chord progressions, one based on the expectation of specific chord progressions (i.e. dominant vectors +4 or +2) that coincide with the dissonance's resolution.

[27] Spontaneous asymmetry thus interacts with voice-leading: the upwards direction of the leading tone or the downward resolution of the dissonances in example 5 are the consequence of a specific kind of listening which hears root motion as an essential relationship between the chords. Or put in other words: “The ‘dynamic’ conceptions of root progressions and the resolution of dissonance are two sides of the same coin” (Dahlhaus 2014, 134).

[28] This tonally oriented understanding of musical syntax also means that the grammatical consistency of the polyphony can be preserved despite irregularities in the foreground, as in example 6. Here, the cadential effect is maintained despite the extensive elaboration of the tonic (bars 131-132), and the incomplete dominant and tonic chords, which are deprived of the upward motion of the leading-tone (bars. 133-134) and reduced to the characteristic bass movement down a fifth. Accordingly, spontaneous asymmetry, based as it is on a chordal background that implies directional tendencies, also carries new compositional possibilities such as register transfer, diminution, elision, elaboration or irregular voice-leading, which can be exploited in free composition.

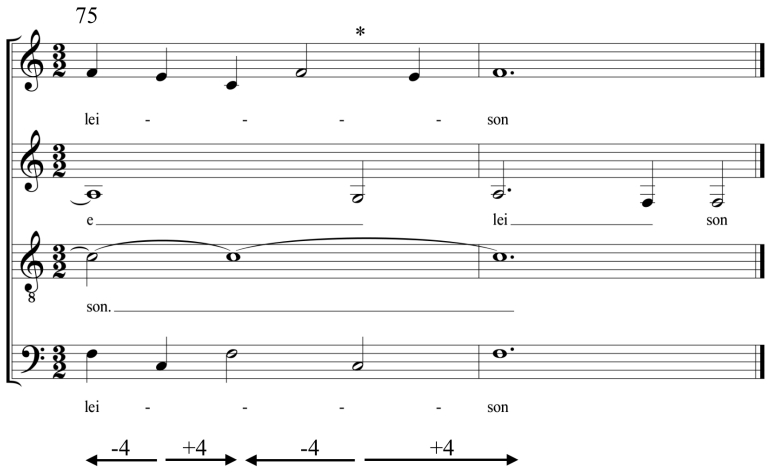


Example 6. Beethoven, Sonata op. 14.1, Rondo, 131-134.

[29] This organization around spontaneous asymmetry is the result of a specific tonally oriented type of listening. But harmonic syntax and the status of asymmetry have changed during the history of Western polyphony. This is why it is necessary to take into account a possible shift between, on the one hand, constitutive elements that are critical for syntactic meaning and, on the other, characterizing elements, which reinforce this meaning without being decisive.

1.3.2 Conditional asymmetry

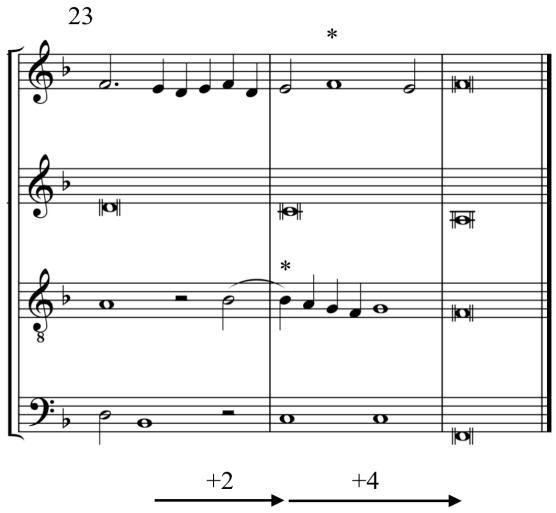
[30] In Dufay's *Missa Se la face ay pale* the progression from the penultimate to the final harmony at the end of the Kyrie (example 7) cannot be distinguished from a tonal dominant-tonic progression, as in the model cadence in example 2. However, in the middle of the 15th century, the +4 progression between penultimate and *finalis* is not the result of an emerging awareness of dominant-tonic relationships as Besseler (1950) suggests. It is instead the result of a strict observation of compositional rules in four-voice modal counterpoint (Eberlein 1992, 39-41).



Example 7. Dufay, Missa Se la face ay pale, Kyrie, bars 75-76.

[31] The impact of writing constraints is also evident earlier in this cadence. In the usual configuration of the *clausula formalis*, the harmonic progression between antepenultimate and penultimate usually consists of a subdominant vector -4 (example 7). However, the introduction of a dissonant seventh in the penultimate harmony, as seen in example 8 with the dissonance C3-Bb3 in bar 24, means the last two progressions now both move in a dominant direction, with a dominant vector +2 between antepenultimate and penultimate harmony. It is the preparation of the dissonance 7th in example 8 that necessitates a harmonisation on the antepenultimate which generates the dominant vector that was previously absent.

[32] In these cases, the dominant direction is in fact conditioned by the contrapuntal rules of preparation and resolution. This specific type of asymmetry, where the dominant progression is induced by contrapuntal constraints, I describe as *conditional* asymmetry.



Example 8. Frottole libro Primo (1504), *Frottola* XXIX, Tromboncino, Ah partiale e cruda morte.

[33] In this type of writing, the syntactic consistency, and more precisely the cadential meaning, results from melodic fluidity – i.e. parsimonious voice-leading – and the change of consonant quality, particularly the alternation between dissonance, imperfect and perfect consonance which plays a critical role for cadential teleology. The preference for the dominant direction of chord progressions arises almost accidentally from contrapuntal constraints and plays only a secondary role as a factor of syntactical coherence.

[34] Correspondingly, compositional techniques are not affected by this type of asymmetry. Although polyphony becomes inherently triadic from the beginning of the 16th century onwards (Lowinsky 1962, 3), the compositional possibilities that result from the triadic background – i.e. register transfers, harmonic elaborations, irregular voice-leading (see 1.3.1 and 4) – are not fully developed. The preferred direction is not an element acting on voice leading but is a phenomenon which results from contrapuntal constraints.

[35] These different features which characterize conditional and spontaneous asymmetry are summarized in the table of example 9:

Conditional asymmetry	Spontaneous asymmetry
• Consequence of contrapuntal constraints.	• Interacts with voice-leading.
• Result of mediation between melodic fluidity and change of consonant quality.	• Consequence of a dynamic interpretation of root progressions.
• Secondary criterion for syntactic coherence.	• Essential criterion of syntactic coherence.
• Does not affect compositional techniques.	• Affects compositional techniques.

Example 9. Conditional asymmetry vs. spontaneous asymmetry.

1.3.2. Empirical verification

[36] How does the relationship between conditional and spontaneous asymmetry evolve in the corpus? To answer this question, the line AsyT in example 10 illustrates the variation of the total asymmetry encountered in the madrigal cycles[5]. Contrary to expectations, the privileged direction does not increase at the same rate from the beginning to the end of the corpus[6]. The asymmetry is significantly high in Arcadelt's and Verdelot's books. It is low in the cycles by

5 The *total asymmetry* (AsyT) corresponds to the difference between all dominant (DV) and subdominant (SV) vectors encountered in the madrigal cycles (DV-SV=AsyT). For example, the total asymmetry in Arcadelt I equates to 62,72% – 32,28% = 35,44%. The line AsyA in example 11 represents the difference between all dominant (DVA) and subdominant (SVA) vectors associated with the preparation and the resolution of the dissonance. See below and note 2.

6 Compare these results with those obtained by Cathe 2012, 25, 129, 159.

Lassus and Rore and then increases with a remarkable regularity over fifty years until Monteverdi's book V, peaking in his books VII and VIII.

[37] The constant imbalance in favor of the dominant vectors confirms that asymmetry is not specific to harmonic tonality. It is a general feature of Western Polyphony, although this feature tends to intensify and – as this article argues – change in quality. Furthermore, the way the asymmetry varies within the corpus implies that the phenomenon is superficial and does not belong to the deep structure of the harmonic language. It must depend instead on compositional techniques (homorhythmic textures, imitative counterpoint, thorough bass etc.) and stylistic criteria (strict counterpoint, free counterpoint, *stile recitativo* etc.).

[38] From this perspective, it is worth noting that the level of asymmetry found in Verdelot's and Arcadelt's cycles – which still share important characteristics with the earlier *frottola* – are almost identical with the level of asymmetry found more than a century later in Monteverdi's books VII and VIII, in which tonality partly crystallizes. Einstein (1949, 865) had already observed "how closely the extremes approach one another – the beginning [of the madrigal], about 1500, and the end, about 1620". These "strange bonds", were also indirectly grasped from a more abstract perspective by Lowinsky (1962, 14), who claimed that some aspects of tonality – especially of the major mode – are anticipated in the lighter polyphonic genres of the early 16th century.

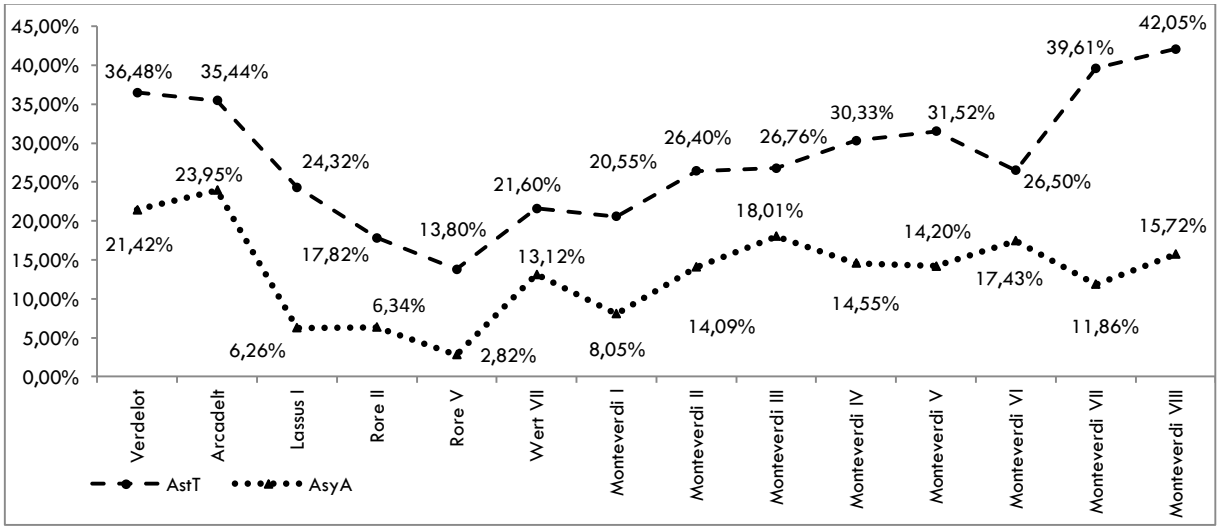
[39] The statistical results here clearly confirm an affinity between the earlier and later repertoires of the corpus, from the point of view of the asymmetry of root progressions. They also show that the madrigal cycles more representative of modal polyphony – such as Lassus' book I of 1555 – stand out through their syntactic properties.

[40] At the same time, a closer examination of the *nature* of the asymmetry suggests that its conceptual background changed dramatically between the beginning and the end of this corpus. Example 10 shows how the variation in overall asymmetry in the composers' work (line AsyT) relates to contrapuntal constraints. To this end, the line AsyA shows the asymmetry that relates specifically to chord progressions which involve dissonances. It takes into account those root progressions that are associated with suspensions and note against note dissonances [7]. A comparison of both lines confirms that the total asymmetry and asymmetry associated with dissonance are strongly correlated from Verdelot to Monteverdi's book III. However, this correlation weakens in Monteverdi's books IV to VI and the significant increase in the asymmetry between books VI and VII occurs independently of the dissonance.

[41] It is not so much the accentuation of the asymmetry which seems critical here, but the fact that this increase is not conditioned by asymmetry associated with the dissonance, which

7 In the case of the suspension dissonance, the chord progressions associated with the preparation, and the successive resolution of the dissonance onto the imperfect and perfect consonance have been taken into account (see the model in example 2). In the case of the dissonance note against note, only the chord progressions associated with the dissonance's impact and immediate resolution have been selected.

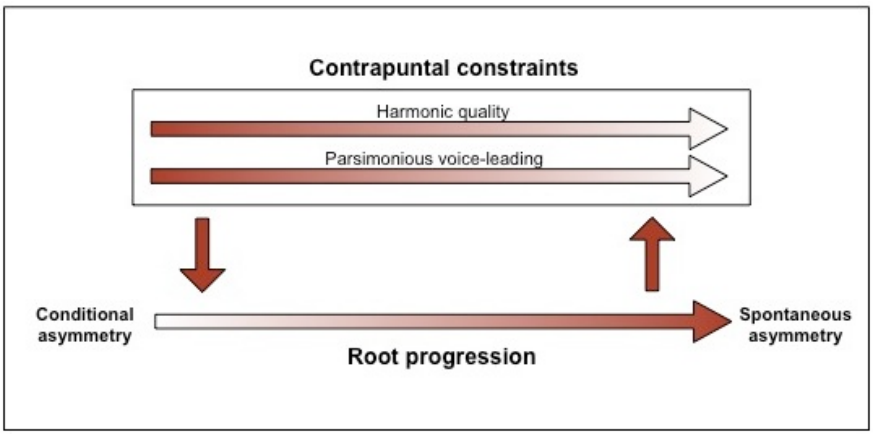
decreases between VI and VII. Where previously the asymmetry was induced by contrapuntal constraints, it now occurs largely independently and spontaneously. Considering the theoretical implications of asymmetry (1.2) and its possible repercussions on compositional techniques (4), I argue that this emancipation is an important hint of the progressive crystallization of tonality.



Example 10. Fluctuation of total asymmetry and asymmetry associated with the dissonance.

[42] This move from conditional to spontaneous asymmetry is modeled in example 11. Contrapuntal constraints, represented by the arrows at the top, give rise to intervallic progressions that mediate between the change of harmonic quality (dissonance, imperfect consonance and perfect consonance) and parsimonious voice-leading. Root progression, indicated by the arrow below, constitutes, on the other hand, an abstract representation of these intervallic patterns.

[43] The contrapuntal and harmonic perspectives are never opposed: harmonic quality and parsimonious voice-leading on the one hand and root progressions on the other are mutually dependent. However, the causal relationship between these criteria evolves. This is suggested by the vertical arrows and by the gradient from red to white (and conversely). Initially, the contrapuntal constraints affect root progression and give rise to *conditional* asymmetry. At the end of this development it is the root progression and its constraints on tonality which becomes the main vector of harmonic meaning and actively determines the polyphonic stream.



Example 11. Evolution from conditional asymmetry to spontaneous asymmetry.

[44] Tymoczko (2011, 232) has pointed out that the circle of thirds, as modeled in example 3, might indirectly have "influenced the developing conventions of functional harmony". The scenario of a shift from conditional to spontaneous asymmetry puts this hypothesis into a tangible historical context and helps, to confirm it through empirical evidence.

[45] This move from conditional to spontaneous asymmetry continues beyond the end of the corpus. It must also be borne in mind that both extremes – a completely conditioned asymmetry and a fully spontaneous one – never appear in music; neither in pre-tonal polyphony nor in common practice harmony.

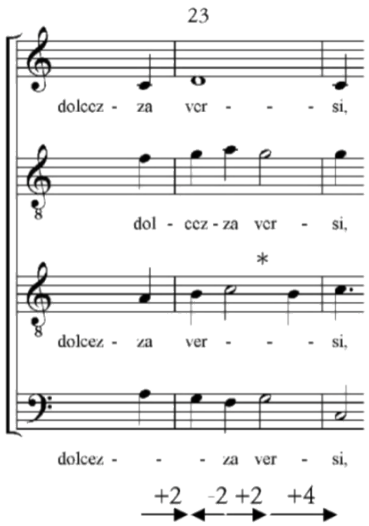
[46] This is corroborated by the empirical results. On the one hand, in all the madrigal cycles considered, there is a residual asymmetry, seen in the gap between both lines of the histogram in example 10. This residual asymmetry, which by definition cannot be explained by contrapuntal constraints, shows that triadic progressions are never completely free as regards their direction, and they are increasingly involved in dominant movement. On the other hand, even in the last two madrigal cycles, characterized by the highest residual asymmetry, the overall asymmetry remains partly correlated to the asymmetry that is tied to the dissonance: between book VII and VIII the increase of the total asymmetry (AsyT) goes hand in hand with the increase of the asymmetry associated with the dissonance (AsyA). Thus, the results and the model do not imply a radical reversal but rather a gradual exchange in the hierarchy between both kinds of asymmetries.

2. Signs of the evolution from conditional to spontaneous asymmetry

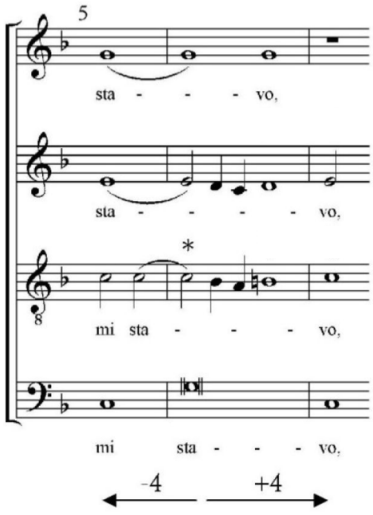
[47] The shift from conditional to spontaneous asymmetry is conceptual in nature. It is not inherent in the harmonic syntax itself but in how the syntax is interpreted. This shift can nevertheless be inferred from the evolution of compositional techniques. From the many criteria which might reflect this evolution, three will be examined in detail here: 1. the realization of the cantus-tenor-framework, 2. the use major and minor chords and 3. the morphology of dissonant chords.

2.1. Cantus-tenor framework

[48] The cantus-tenor progression of the *clausula formalis* has remarkable properties. As the tenor and cantus show in example 12[8], it combines parsimonious voice-leading with a gradual change of harmonic quality. This can be seen in the move from the second quarter note of m. 23 to the first of m. 24 in the tenor and cantus *parts*: the change from the dissonance (C4-D4) to the imperfect consonance (B3-D4) and the perfect consonance (C4-C4). However, this framework is not always complete. The downward stepwise motion of the tenor line (D4-C4 at the cantus in example 12) is frequently replaced by an upward movement to the third of the final chord. This is the case in example 13 where this tenor line is sung by the altus (D4-E4), and where the cantus line is sung by the tenor (B3-C4).



Example 12. Arcadelt (1539), *Benedetti i martiri*, 25-26.



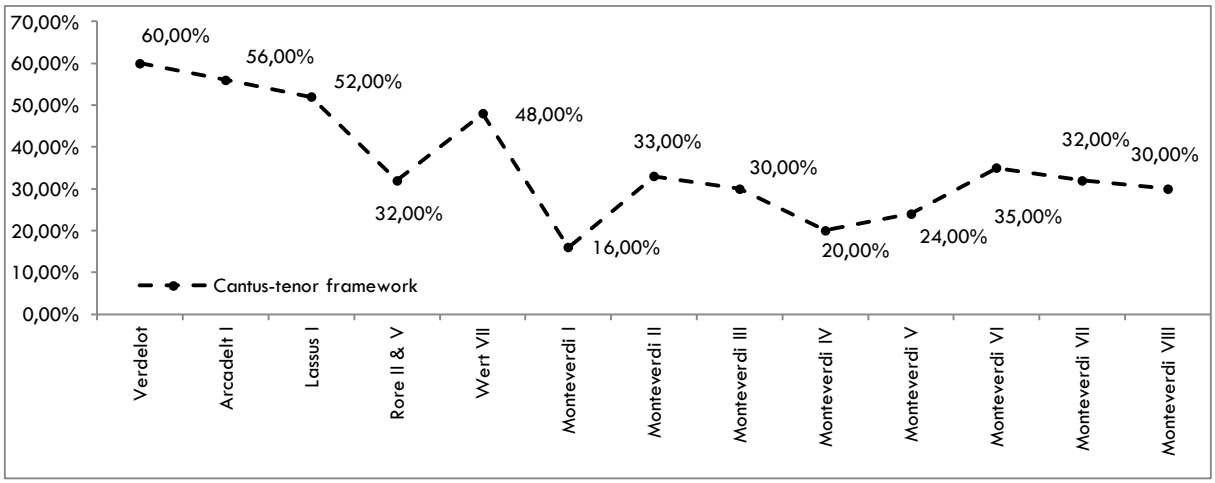
Example 13. Verdelot (1530), *Amor quanto più lieto*, 6-7.

[49] As shown in example 14, the realization of the cantus-tenor framework shows high levels of variation, while exhibiting a gradual decrease. This evolution is linked to questions of voice-leading and to the changing status of triadic harmony. In four-part writing, the regular cantus-tenor framework prevents a conclusion on a full triad and only permits an ending on a third or on an empty fifth, as in example 12. The upward movement of the modified tenor line, however, allows a full triad as is the case in example 13 (if the G in the cantus is prolonged).

[50] What makes a cadence in the middle of the 16th century is no longer the intervallic progression from the imperfect to the perfect consonance since this change of harmonic quality is completely dissolved in the general imperfect sonority of the triads. It is at the same time the passage from the dissonance to the consonance and the characteristic melodic

8 In this and the following examples, arrows pointing to the right and to the left respectively indicate dominant and subdominant vectors. The root progression is identified by the positive (upward progression) and negative (downward progression) numbers over the arrows.

movements of the cantus-bassus framework (Eberlein & Fricke 1992, 57). This leads, on the one hand, to an increased use of dissonant structures which, as has been shown, condition the asymmetry of chord progressions. On the other hand, this change in what constitutes a cadence also brings increased attention to the outer voices. From this point of view, the idiomatic bass movements become, in the long term, a prominent factor of syntactic coherence. The empirical results thus corroborate the hypothesis that the logic behind intervallic cadence patterns coexists with one based on an increasing role for the lowest voice. The latter tends however to increase during the period under consideration.



Example 14. Realization of the cantus-tenor framework in the madrigal cycles examined.

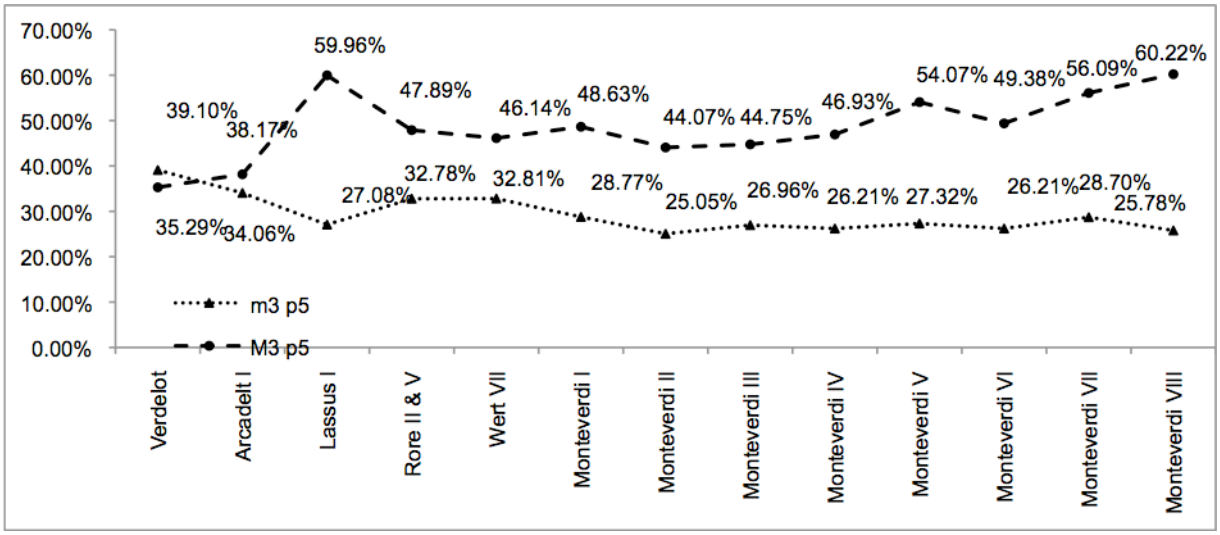
2.2. Major versus minor triads

[51] As shown in the diagram in example 15, the relationship between major triads (M3 p5) and minor triads (m3 p5) evolves in an interesting manner. While Verdelot and Arcadelt use both chord types in equal measure, the use of the major triad increases significantly in frequency with Lassus and then again at the end of the corpus from Monteverdi's Book III onwards.

[52] The French musicologist Serge Gut had already noticed a similar evolution when, in his study of the harmonic third, he examined the final chord in cadences from large polyphonic corpora of the Middle Ages and the Renaissance. Gut (1969, 191) suggests two explanations for this phenomenon. The first proposes a possible link between a) the predominance of the major mode, b) the systematic use of the leading tone in the minor mode and c) the hegemony of the major final chord. The second argues that an awareness of harmonic overtones, together with an evolution from an *intervallic understanding* to a *harmonic understanding*, contributed to the increased use of major chords. Both explanations however are questionable, partly because the hypothesis of a gradual integration of overtones remains debatable and partly because the two arguments fail to explain why the increase in the use of the major triad happens at this particular period in the history of Western music.

[53] I would like to suggest instead that the increased use of major triads may partly be linked to an increasingly tonally oriented type of listening. Once the triad is understood as a whole, once the bass is perceived as the triad's root and once root progression become salient from a cognitive point of view, then the use of the major triad reinforces the link between the syntactical units. This is both because of the harmonic quality of the triad and due to the upward melodic movement, by semitone, of the major third.

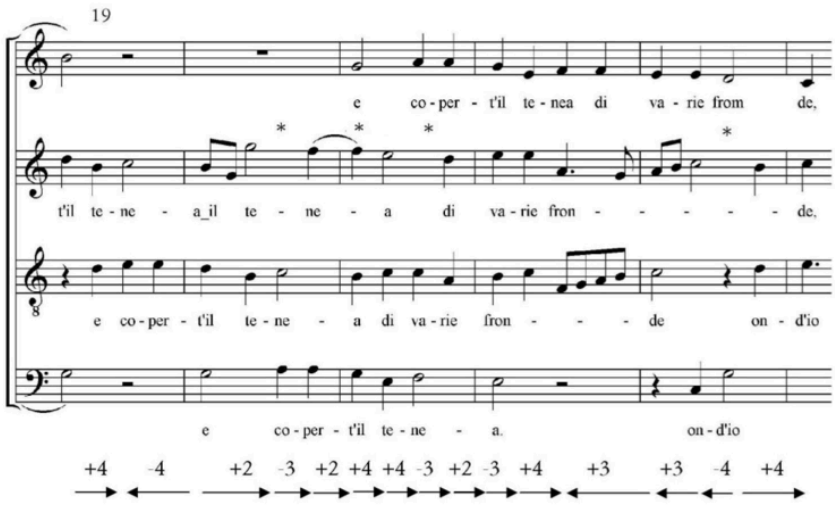
[54] In this explanation, which characterises the chord's mode as "rection" (see 1.2.), the increased use of the major triad is also linked to an increasingly dynamic interpretation of chord progressions. Thus, this explanation also maintains that the greater use of the major third indirectly reflects the increase of spontaneous asymmetry.



Example 15. Major and minor triads in the madrigal cycles examined.

2.3. Morphology of seventh chords

[55] The possible shift from conditional to spontaneous asymmetry can ultimately be deduced by considering the make-up of dissonant chords, especially seventh chords. The chord appears, both with and without the fifth from the beginning of the corpus, as in the madrigal *Fra più bei fiori* by Arcadelt (example 16), where the descending chain of suspensions in bars 20-21 gives rise to three dissonant chords: two sevenths harmonized with only the third (A-C-G and G-B-F), and one seventh to which the third and the fifth have been added (F-A-C-E).



Example 16. Arcadelt (1539), *Fra più bei fiori*, 19-23.

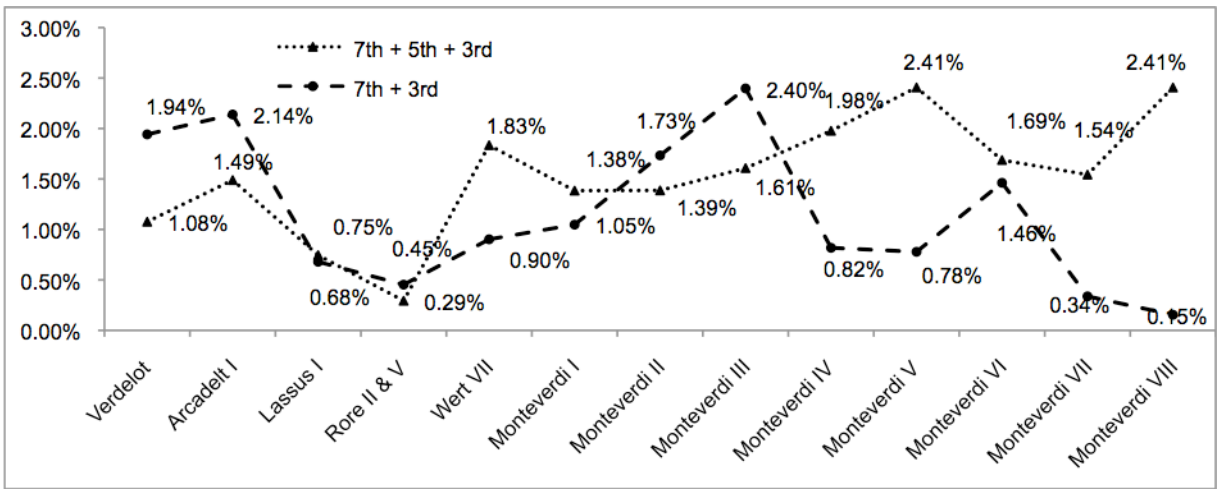
[56] The histogram in example 17 shows that the use of both chord types is correlated in the early madrigal cycles and that both increase in use between Rore and Monteverdi's book III. However, from Monteverdi's Book III on, the use of the two kinds of seventh chords diverges: the full seventh chord increases and peaks in Monteverdi's books V and VIII, while the seventh chord without the fifth is used much less frequently.

[57] This tendency is interesting since, in the 19th century, the Belgian theorist Fétis uses the presence of the fifth, along with the major third and minor seventh, to distinguish a seventh as non-harmonic tone from a *true* dominant seventh chord. For Fétis, the latter appears from Monteverdi's book III onwards, and marks the beginning of the "tonalité moderne" (Fétis 1840, 36).

[58] The evidence from the corpora suggests a real change in the use of the chord both with and without its fifth at the beginning of the 17th century. This evolution however can by now way *explain* the dramatic changes polyphonic syntax undergoes at that time.

[59] As Dahlhaus has shown, the very concept of chordal dissonance depends on a dynamic interpretation of chord progression (see 1.3.1 and Dahlhaus 1990b, 133-135). His deduction reflects the more general conviction that the distinction between essential and non-essential chords made by a listener is not a natural phenomenon but depends on harmonic schemas assimilated in a tonally oriented type of listening. The traditional distinction between suspensions as non-harmonic events and as "real" chordal dissonances, as implied by Fétis and still largely used in rudimentary music theory, is thus irrelevant in syntax which essentially relies on conditional asymmetry i.e. syntax prior to harmonic tonality. On the other hand, the gradual rise of the complete seventh chord may be indicative of a transformation of the way harmonic syntax is implicitly understood. But it can however neither justify the increase of

spontaneous asymmetry nor be considered as a decisive feature that actively contributes to the origin of “tonalité moderne”.

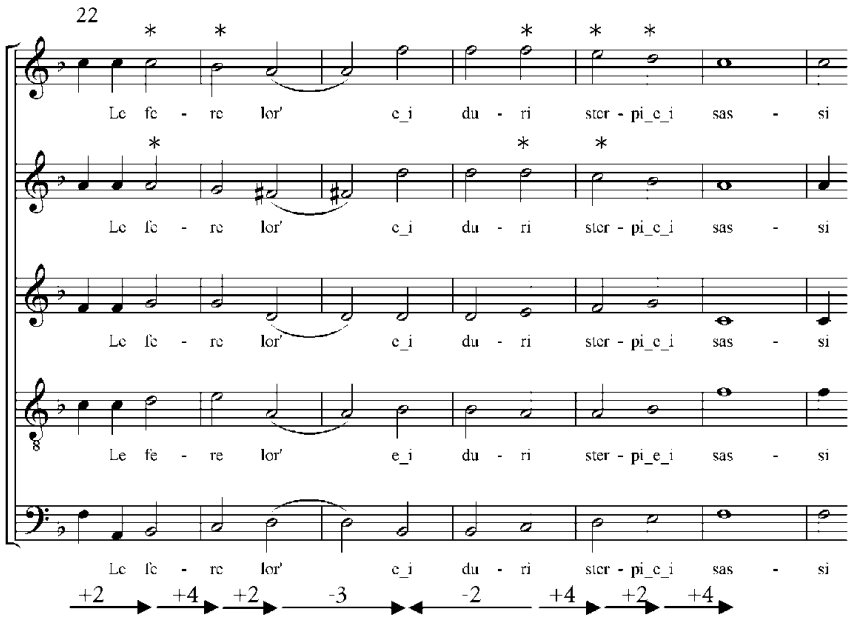


Example 17. Morphology of 7th chords in the madrigal cycles examined.

3. Factors contributing to an intensification of spontaneous asymmetry

[60] Several factors may have fostered the evolution from conditional to spontaneous asymmetry. I already mentioned the hegemony of triadic harmony. The practice of *basso continuo* may also have contributed to this evolution. There is however another factor that appears crucial: the irregular dissonances at the start of the 17th century. The systematic use of contrapuntal licence undermines, in the long term, the relationship between the vertical and horizontal dimensions of polyphony and partly works against syntactical coherence. Since this coherence is no longer derived exclusively from parsimony and the smooth change of harmonic quality, characteristic of intervallic writing, movements of the real bass and thereafter abstract root progression become a prominent factor of harmonic meaning.

[61] Signs of this shift can be seen in bars 22-26 of Monteverdi’s madrigal *Ch’io t’ami* (example 18) where the eight note-against-note dissonances infringe radically on contrapuntal rules. Unquestionably the parsimonious voice-leading and the contrary motion of the outer voices help to soften the harshness of these irregular structures. However, it seems at least as significant from the point of view of syntactic consistency that, despite the contrapuntal irregularity and with only one exception, all harmonic units are arrived at and left by dominant vectors.



Example 18. Monteverdi (1605), *Ch'io t'ami, e t'ami piú della mia vita*, bars 22-28.

[62] These observations lead to the following hypothesis: that opposition to regular counterpoint at the beginning of the 17th century provoked a search for new criteria of syntactic coherence. As a result, root motion becomes the focal point and thereby the main vector of cadential meaning. Contrapuntal licences that are compatible with dominant vectors are maintained, whereas those that are incompatible are gradually excluded. This leads to the strengthening of spontaneous asymmetry.

[63] The table in example 19, showing the rates of dominant and subdominant vectors associated with irregular dissonances in books I-VI and VII-VIII by Monteverdi, supports this hypothesis. Irregular dissonances mainly occur with dominant vectors in both sub-corpora (books I-VI and VII-VIII). On average however, the percentage of irregular structures linked to dominant vectors increases by 6% in the last cycles, regardless of any contrapuntal constraint. These results suggest that irregular structures were gradually integrated in line with the dominant direction of harmonic vectors occurring spontaneously.

[64] Because these irregular dissonances are compatible with the dominant direction of chord progression (which always allows the preparation and the downward stepwise resolution of dissonances, see 1.2.), they can be reduced at a deeper level (as per Schenker's 'middle ground') to strict counterpoint. However, this is not the result of a slavish obedience to the strict counterpoint of the past. It is instead the indirect consequence of the privileged direction of root progressions occurring spontaneously.

Contrapuntal licence	Monteverdi I-VI		Monteverdi VII-VIII	
	DV	SV	DV	SV
Syncopa tutta cattiva	84.44%	15.56%	88.00%	12.00%
Dissonant preparation	69.86%	30.14%	87.10%	12.90%
Note against note	64.44%	35.56%	68.75%	31.25%
Absence of resolution	80.33%	19.67%	78.57%	21.43%
Resolution on a silence	66.67%	33.33%	75.00%	25.00%
Resolution on a dissonance	58.43%	41.57%	65.00%	35.00%
Average	70.69%	29.31%	77.07%	22.93%

Example 19. Rates of dominant and subdominant vectors associated with irregular dissonances in Monteverdi's books I-VI and VII-VIII.

4. Spontaneous asymmetry and compositional techniques

[65] As can be inferred from these bonds between asymmetry and the treatment of irregular dissonance, the emancipation of the privileged direction from contrapuntal constraints grants new compositional possibilities. This large field, worthy of a separate study, will be discussed here exclusively from the point of view of possible links between spontaneous asymmetry and elaboration techniques.

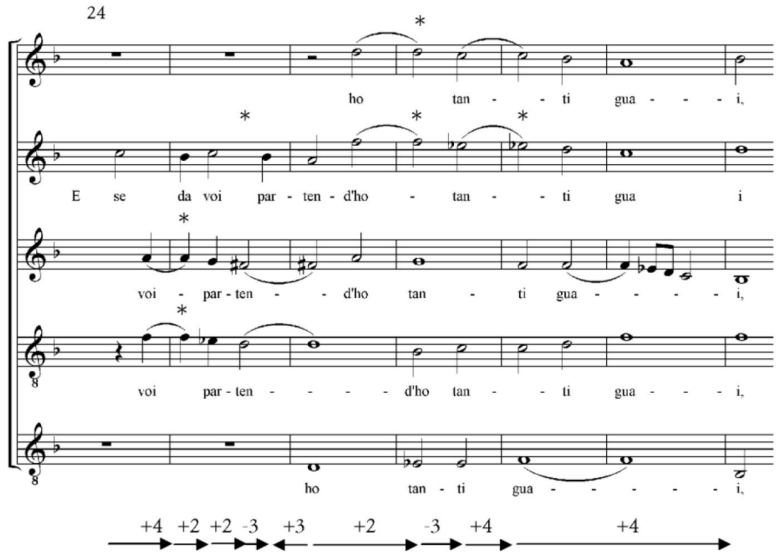
[66] In her study on the transition from modal to tonal organization in the works of Monteverdi, Susan McClary argues that the hierarchical shift between the structural line and the melodic foreground is decisive for the emergence of tonality (1976, 179). According to this view, in pre-tonal counterpoint the harmony-generating structural line corresponds in a one-to-one relationship to the foreground melody. Both the structural line and the foreground melody belong to the same hierarchical level. In tonality on the other hand, each pitch of the structural line gives rise to further elaborations: the structural line corresponds to a higher hierarchical level, distinct from the subordinated melodic foreground. This shift also affects harmony. On the one hand, each structural pitch is able to generate what McClary calls 'harmonic collections'. On the other, harmony actively projects the structural line, articulates it and becomes decisive both for freer voice-leading and for increased elaboration in the foreground.

[67] The increased elaboration and ornamentation of the foreground, which goes hand in hand with a decrease in the harmonic rhythm, supposes that the chords have to be understood as immediate harmonic units and that their progression has to be interpreted from the dynamic point of view, as outlined above. These conditions can also be inferred from Schenker's (1954, p. 155) explanation of free composition:

[68] "In reality however, the tactics of voice-leading become ever freer to the extent to which, in free composition, there erupts suddenly the force of the scale-step, under whose cover the individual parts may manoeuvre in a less inhibited way even than in strict composition. The scale-steps then resemble powerful projector lights: in their illuminated sphere the parts go through their evolution in a higher and freer contrapuntal sense, uniting in harmonic chords, which, however, never become end in themselves but always result from the free movement. […]"

[69] The projector-lights-metaphor reflects the importance of the harmonic degree as a conceptual unit that enhances freer voice-leading. If we accept that the scale degree's meaning depends on how it is arrived at and left, it can be argued that at least some elaboration techniques are closely linked to the in increase in spontaneous asymmetry. Two particular cases of elaboration, closely linked to the dynamic interpretation of chord progressions, will be discussed here: the elaboration of the dominant itself and of the pre-dominant.

4.1. Elaboration of the dominant

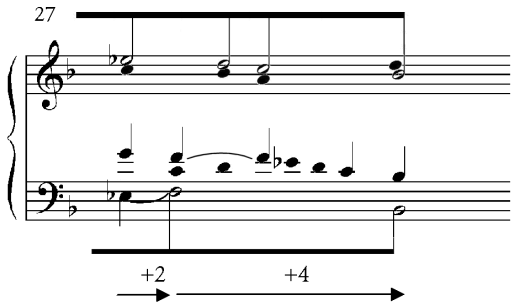


Example 20. Monteverdi (1585), *Se nel partir da voi*, 24-30.

[70] At bar 28 of the madrigal *Se nel partir da voi* from Monteverdi first book (example 20), the syncopated seventh F-Eb between bassus and quintus, resolves correctly onto the major sixth F-D. However, the harmonic unit as a whole contains a dissonant fourth F-Bb between bassus and quintus, which shows that the dominant chord on root F is conceived and elaborated across the two bars that precede the final chord.

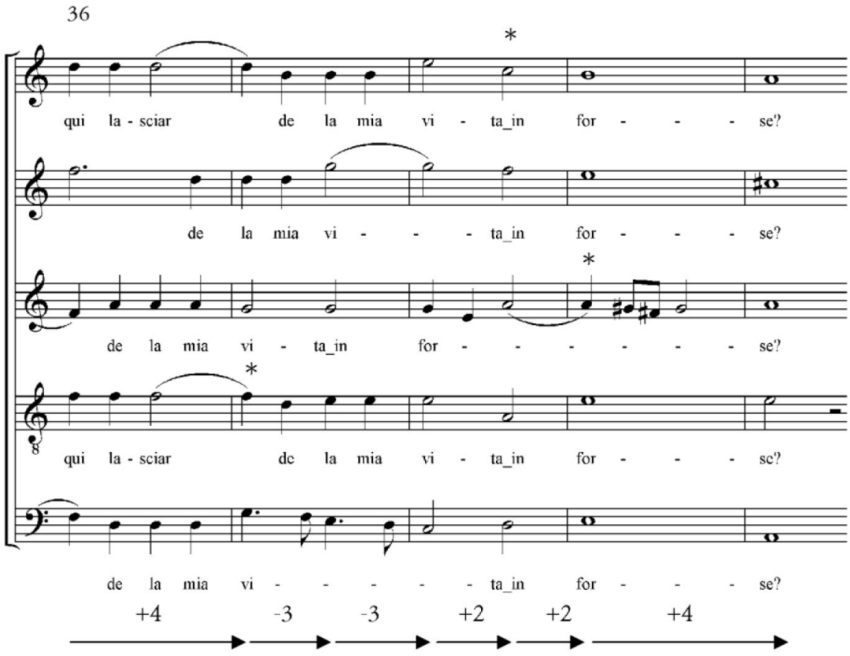
[71] This stagnation has two consequences. On the one hand, it suspends the distinction between the antepenultimate and the penultimate in the cadence. More precisely, this distinction is no longer established from a contrapuntal point of view, but through harmonic progression. As is shown by the reduction in example 21, harmonically the antepenultimate is pulled backwards, and now corresponds to the chord on root C in bar 27.

[72] On the other hand, this stagnation prolongs the penultimate on F by stretching it in time. In particular, the elaboration of the seventh Eb, first treated as a suspension and then as a passing note, reinforces the cadential effect of the following bass leap down a fifth. The reinterpretation of the cadence and the elaboration of the dominant are compositional possibilities that are ultimately granted by a dynamic interpretation of chord progressions.



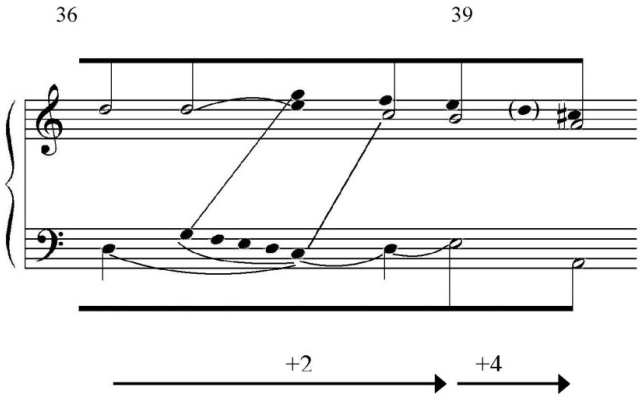
Example 21. Monteverdi (1585), *Se nel partir da voi*, 27-30, reduction.

4.2. Elaboration of the pre-dominant



Example 22. Monteverdi (1592), *Poi ch'ella in sé tornò*, 36-40.

[73] At bar 38 of the madrigal *Poi ch'ella in sé torno* from Monteverdi's book III, the pre-dominant is reached by a note-against-note dissonance D-C (example 22). However, at a local level, the dissonant pitch is implicitly prepared by the root C of the previous chord and could be explained by a register transfer. This is shown by the reduction (example 23) and confirms that free composition can always be reduced to strict parsimony and regular counterpoint if a triadic background is presumed and if the triads move in a dominant direction. What is more, the pre-dominant D is involved in relatively important elaborations from bar 36 onwards. Thus, from a broader perspective, C5 at bar 38 could be interpreted as a mere passing note which resolves onto B4, the fifth of the dominant. The fact that the subdominant involves a note-against-note dissonance and that it is elaborated intensively over three bars grants it relative autonomy. This suggests that it derives its legitimacy and meaning from how it is arrived at and left, harmonically.



Example 23. Monteverdi (1592), *Poi ch'ella in sé tornò*, 36-40, reduction.

[74] The elaboration techniques used in both examples do not lead to a large distance between foreground and background. However, they show a real link between asymmetry and elaboration, which allows us to make two observations. Firstly, the elaborations happen in a tonal context. They focus specifically on the dominant and pre-dominant to enhance tonal coherence. Secondly, spontaneous asymmetry is crucial to the integration of these superficial phenomena. It is the way the harmonic units are reached and moved from that allows for the integration of the contrapuntal licenses and that grants them a particular syntactical meaning. From this perspective, the above elaborations result indirectly from the integration of spontaneous asymmetry.

[75] In his attempt to formalize a generative syntax of tonal harmony, Martin Rohrmeier (2011) tries to reconcile Riemannian tradition with recursive and prolongational approaches. His results tend to confirm that a comparatively simple set of rules suffices for the explanation of a large range of examples because "tonal harmony is fundamentally grounded in elaborations of cadential harmony" (2011, 48).

[76] The empirical results and theoretical reflection outlined here partly corroborate this hypothesis. At the same time, they suggest that both the crystallization of static tonal functions

and the hierarchical articulation of tonal syntax may be historically linked to the changing status of the asymmetry of root progression. A systematic consideration of the asymmetry of root progressions could thus provide a better understanding of the hierarchical and functional characteristics of tonality, historically[9]. What is more, it could also help to integrate contrapuntal structures, that lie outside the tonal framework, into this same understanding of tonal harmony.

5. Conclusion

[77] To conclude, the Theory of Harmonic Vectors argues that the dominant direction of chord progressions is a characteristic feature of tonality. This paper in turn suggests that it is not so much the mere prevalent direction that is crucial but the fact that this directional tendency emancipates itself from contrapuntal constraints and acts on them. This evolution from a conditional to a spontaneous asymmetry, though immaterial, can be deduced from several phenomena: the dissolution of the cantus-tenor framework, the increased use of major triads, the morphology of the seventh chord or the fact that irregular dissonances are increasingly compatible with the preferred dominant direction.

[78] The recurrent use of cadential progressions helped assimilate this preferred direction over decades. Furthermore, irregular dissonances, temporarily suspending contrapuntal logic, may have played a decisive role in the increase of spontaneous asymmetry. Finally, this study has shown that the assimilation of spontaneous asymmetry provides new compositional possibilities and contributes to a greater distance between foreground and background.

[79] This evolution does not come to an end at the turn of the 17th century but continues through the following centuries. However, the corpus examined here evolves drastically and suggests that the shift from conditional to spontaneous asymmetry, with all its theoretical implications, has already taken place, at least in part, in the last madrigal cycles analyzed above.

[80] This paper does not claim that all aspects of tonality can be reduced to asymmetry. It does not even suggest that asymmetry is a characteristic feature of tonality. However, because asymmetry is tightly related to other important tonal features and because it interacts with these features, this analytical and theoretical criterion has a significant heuristic value for understanding tonality.

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9 Tymoczko 2003, 42 has argued that if the historian could confirm that functional tonality appeared when composers gradually began to favor dominant progressions over subdominant progressions, it would constitute a decisive step towards an explanation of tonal harmony. The results obtained here suggest that it is not so much the accentuation of asymmetry that seems decisive from a historical point of view for the crystallization of functional harmony but the changing status of asymmetry.

6. Acknowledgments

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