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Description-dependent Choices

Dino Borie* Dorian Jullien†

Abstract

We propose a theory of choice that accounts for the class of framing effects where a preference reversal is due to different descriptions of the same alternatives. These framing effects violate the principle of “description invariance” which is implicit in the standard theory of choice. We make this principle explicit by taking descriptions of alternatives (instead of alternatives *per se*) as the primary objects of choice. We then explore several ways to relax description invariance to see the different preferences that are generated and we pay specific attention to the role played by the Weak Axiom of Revealed Preference in description-invariant and description-dependent choices. Finally, we extend our theory under risk to show how it can generate risk attitudes that are dependent on descriptions.

JEL Classification: D89, D90, D91.

Keywords: Choice correspondence, framing effects, rational choice, description invariance, description dependence.

1 Introduction

Imagine that the U.S. is preparing for the outbreak of an unusual Asian Disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is $1/3$ probability that 600 people will be saved, and $2/3$ probability that no people will be saved.

If Program C is adopted, 400 people will die.

If Program D is adopted, there is $1/3$ probability that nobody will die, and $2/3$ probability that 600 people will die.

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This famous decision problem was introduced by [Tversky and Kahneman \(1981\)](#). In their experiment, the decision maker (DM) is presented with either the choice set $\{A, B\}$ or $\{C, D\}$. In the first choice set, A is chosen by a majority of subjects, while D is chosen by a majority of subjects in the second one. This result shows that choices are *description-dependent* because both choice sets have identical consequences, they only differ in the descriptions of these consequences¹.

This type of “framing effect” is not representable in the standard theory of choice, in which the objects of choice are *alternatives* abstracted from their empirical descriptions. We propose to build a theory of choice in which the objects of choice are descriptions of alternatives. This allows us to formalize the principle of “Description Invariance” (DI) that is implicit in the standard theory and according to which choice behavior is not sensitive to the specific descriptions of alternatives.

We propose to formalize this principle as the conjunction of two behavioral properties. On the one hand, if we observe the choice of one description of an alternative in a given decision problem, then we should observe the choice of an equivalent description of that alternative in another equivalent decision problem. This first property is easily illustrated by most framing experiments, e.g., in the Asian Disease problem, if A is chosen out of $\{A, B\}$, then C is chosen out of $\{C, D\}$ (and conversely). That is, description invariance requires the absence of sensitivity between equivalent pairs of descriptions of different alternatives. We call this property the Weak Description Invariance (WDI).

On the other hand, if we observe the choice of one description of an alternative in a given decision problem, then we should also observe the choice of all the equivalent descriptions of that alternative whenever they are available. For instance, using the descriptions in the Asian Disease problem, the choice of A out of $\{A, B, C, D\}$ also implies the choice of B from that opportunity set. That is, description invariance requires the absence of sensitivity between equivalent descriptions of a given alternative. We call this property Revealed Identity (RI): the choice of one description of an alternative implies the choice of all the other available descriptions of the same alternative, i.e., the same in the sense of having identical consequences.

Distinguishing these two behavioral properties of description invariance is central to understand framing effects. When formalized as consistency conditions on choice over descriptions of alternatives, these two conditions (separately or in conjunction) tell us nothing about the other determinants of choice behavior (maximization of a preference relation, limited attention and so on). Conversely, these other determinants, by themselves, tell us nothing about description invariance. However, if choice

¹There is a wide literature in psychology that has investigated this type of framing effects after Tversky and Kahneman (1981) and that is overlooked in discussions of the phenomenon within economics. As emphasized in the most cited review and meta-analysis on framing effects in psychology, “the likelihood of obtaining choice reversals [is] directly related to the similarity between features of a given study and features of Tversky and Kahneman’s (1981) original “Asian Disease problem.”” ([Levin et al., 1998](#), p.157). The effect is robust to changes in scenario and type of consequences, e.g., money instead of lives (see [Kühberger \(1997; 1998\)](#), [Kühberger et al. \(1999\)](#), [Piñon and Gambara \(2005\)](#), [Maule and Villejoubert \(2007\)](#), [Keren \(2011\)](#), [Mandel and Vartanian \(2012\)](#), [Takemura \(2014, Part. V\)](#)). The results of [Kühberger et al. \(2002\)](#) is of particular interest for economics, as they show in a monetary version of the Asian Disease that the framing effect disappears with small stakes but is amplified with large stakes.

behavior results from the maximization of a complete preference relation over the set of descriptions of alternatives, then adding RIP alone characterizes description-invariant preferences as in the standard theory. By contrast, adding WDI alone shows the existence of a specific type of preferences that are mildly sensible to descriptions in the sense that the descriptions of a given alternative can be strictly ordered without implying choice reversals between alternatives. The theoretical interest of this type of preferences appears with the introduction of a structure on the set of objects of choice (such as risk, uncertainty, time, other people and so on). For instance, in the case of the Asian Disease problem, this type of preferences over descriptions of sure alternatives can explain the choice reversal over lotteries without introducing *ad hoc* notions such as the positive or negative valence of the objects of choice.

These results are however not totally satisfactory when we consider (as we have done so far) that a choice problem can contain more than one description per alternative (e.g., $\{A, B, C, D\}$). Indeed, this is rarely done in experiments on framing effects (for one exception see, e.g., [Kühberger and Gradle, 2013](#), experiments 3 and 4). We therefore also propose to work on a restricted domain of choice that is more empirically plausible, i.e., decision problems that contain only one description per alternative. This situation is representative of the overwhelming majority of experiments on framing effects. For instance, decision makers are confronted with decision problems that contain either “200 people will be saved” or “400 people will die” but not both at the same time. Arguably, this corresponds to most real world decisions, as we often have to choose between different alternatives that are each described in one way (or in *one* combination of ways) but we rarely have to choose between different descriptions of the same alternative. Such a restriction also prevents potential contextual effects that several descriptions of the same alternative can generate (a phenomenon which is understudied, if at all).

Related literature

Discussions of framing effects in economics usually highlight that description invariance is (1) *one* implicit axiom of the standard model –interpreted as a behavioral translation of the logical principle of extensionality and as a requirement of consequentialism –, the violations of which are (2) descriptively pervasive, (3) mathematically intractable, (4) normatively unjustifiable and (5) explained by prospect theory². Our theoretical contribution consists in the formal definition of description invariance which allows the derivation of the properties that must be satisfied by

²See Tversky and Kahneman (1981; [1986](#), p.253; [1992](#), p.298), [Arrow \(1982](#), pp.6-8), [Smith \(1985\)](#), [Machina \(1987](#), pp.144-146), [Kreps \(1988](#), p.197), [Sugden \(1991](#), p.759), [Quiggin \(1993](#), §14.5; [2014](#), p.713), [Camerer \(1995](#), p.652; [1999](#), p.10577; [2004](#), p.386), [Rabin \(1996](#), pp.46-47; [1998](#), pp.37-38; [2002](#), p.662), [Starmer \(2000](#), pp.338-9, p.352), [Luce \(2000](#), p.8, pp.34-35, pp.216-217), [Camerer and Loewenstein \(2004](#), p.12, p.14), [Samuelson \(2005](#), pp.93-95), [Varian \(2006](#), pp.549-550), [Fudenberg \(2006](#), p.699, p.700, p.708), [Fudenberg and Levine \(2006](#), p.1472), [Bernheim and Rangel \(2007](#), p.66), [Thaler and Sunstein \(2008](#), p.37), [Köszegi and Rabin \(2008](#), p.1824), [Loewenstein and Haisley \(2008](#), pp.219), [Wakker \(2010](#), p.234, pp.241-2, p.250, p.265, p.350, pp.377-378), [Bardsley et al. \(2010](#), pp.130-1), [Kahneman \(2011](#), chap.34), [Angner and Loewenstein \(2012](#), p.663, p.668), [Moscati \(2012\)](#).

a choice correspondence so that its violation can be tractable. Our conclusion discusses the insights that our contribution can offer regarding the normative aspects of violations of description invariance.

The original explanation of the Asian Disease problem through prospect theory relies on the argument that the semantics of “saved” and “die” induce changes in the reference point. Tversky and Kahneman (1981) argued that decision makers are risk averse in the “save frame” because it creates the expectation that everybody will die (and saving people is a positive outcome) and risk seeking in the “die frame” because it creates the expectation that everybody can be saved (and letting people die is a negative outcome). However, this explanation does not work straightforwardly for the choices that are observed in the reformulations of the original Asian Disease problem. The most robust of these effects that show the explanatory limits of prospect theory for framing effects are obtained with the use of negations to reformulate the original versions of the sure program:

“400 people will not be saved” \prec “1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved”

“200 people will not die” \succ “1/3 probability that nobody will die, and 2/3 probability that 600 people will die”

“200 people will be saved and 400 people will not be saved” \prec “1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved”

“400 people will die and 200 people will not die” \prec “1/3 probability that nobody will die, and 2/3 probability that 600 people will die”³.

Other choice-theoretic contributions on framing effects (Masatlioglu and Ok, 2005; Salant and Rubinstein, 2008; Bernheim and Rangel, 2009) do not deal with violation of description invariance. Instead, they focus on the effects created, for instance, by different alternatives of a given decision problem set as the “default” (i.e., automatically chosen if the agent makes no active choice) or different order of presentation of the alternatives. One exception is Dietrich and List (2016). For them, if the decision maker perceives two descriptions as being about the same object, then he or she is necessarily indifferent between the two descriptions. Our framework is more general because description invariance is defined independently of (the existence of) a preference relation. Furthermore, their approach involves more abstract, less observable, primitives (perceptions of objects of choice) than ours (descriptions of objects of choice)⁴.

³These four examples were originally demonstrated by Kühberger (1995). For replications and further examples of reformulations, see Mandel (2001); Kühberger and Tanner (2010); Schulte-Mecklenbeck and Kühberger (2014); Tombu and Mandel (2015). For more detailed discussions of the theoretical problems posed by framing effects to prospect theory, see Kühberger (1997) and Takemura (2014, chap.10)

⁴Comparison with other frameworks proposing choice functions and correspondences for different types of choice reversals (i.e., other than framing effects in general and framing effects triggered by different descriptions in particular) would not be directly enlightening for the basic phenomena we are interested in (for recent references, see Ok et al. 2015 and Furtado et al. 2017). There is nev-

There is also a number of contributions that address violations of description invariance but directly at the level of preference relations (Bacharach, 2003; Gold and List, 2004; Ryan, 2005; Lanzi, 2011; Blume et al., 2013; Lerner, 2014). These contributions are different from ours either because they just state but do not axiomatize description invariance or because they do so from a rather unconventional formalism from the point of view of decision theory (e.g., predicate logic, a programming language). The closest contributions in spirit from ours are the ones of Giraud (2004a, 2004b, 2005) and Bourgeois-Gironde and Giraud (2009). We are indeed inspired from their approach, which consists in broadening the choice set to include descriptions of alternatives. The main difference is that they stick to a prospect-theoretic explanation of framing effects.

We first present our general setup, then our results in the full and restricted domains of choice, before extending our approach under risk.

2 Described choice problem

Following Giraud’s (2005) formalization of framing effects, let \mathcal{X} be a non-empty finite set of objects of choice that we interpret as *described alternatives* and \approx an equivalence relation on \mathcal{X} . For every described alternatives $x, y \in \mathcal{X}$, $x \approx y$ holds if (and only if) they are two descriptions of the same alternative. If $x \approx y$, we say that x and y are *consequentially equivalent*. This framework can be reduced to the standard one as follows. The set of equivalence classes of \mathcal{X} under \approx , denoted by X , is the choice set of the standard framework. We therefore interpret an element of X as an alternative, which we denote by $[x], [y], \dots$ where $x, y, \dots \in \mathcal{X}$. A non-empty subset of X , denoted by S, T, \dots , is a choice problem in the standard framework. By contrast, a *described choice problem* is a non-empty subset of \mathcal{X} , denoted by f, g, \dots and the consequentialist equivalence relation \approx on \mathcal{X} is easily extended⁵ to $2^{\mathcal{X}} \setminus \{\emptyset\}$.

In this paper, we consider two choice spaces, one with full domain and the other with a restricted domain. Let $\Omega_{\mathcal{X}}^f$ be the set of all nonempty finite subsets of \mathcal{X} which is the choice space with full domain. Let $\Omega_{\mathcal{X}}$ be the set of all nonempty finite subsets of \mathcal{X} that contain one and only one description of an alternative which is the choice space with restricted domain⁶.

ertheless some similarities in terms of approach with some of them. Generally, these contributions insist on making the standard model a specific case of a more general framework. More specifically, the approach of expliciting an implicit assumption in order to weaken it so that a change of preference can be rationalized without introducing two preference relations is close to Masatlioglu et al. (2012) – who make explicit the implicit assumption of full attention in the standard model and weaken it to represent limited attention.

⁵Define this extended consequentialist equivalence relation, still denoted \approx by a slight abuse of notation, on $2^{\mathcal{X}} \setminus \{\emptyset\}$ as follows: for all $f, g \in 2^{\mathcal{X}} \setminus \{\emptyset\}$,

$$f \approx g \text{ iff } (\forall x \in f, \exists y \in g, x \approx y) \text{ and } (\forall y \in g, \exists x \in f, y \approx x).$$

It is readily seen that the relation \approx on $2^{\mathcal{X}} \setminus \{\emptyset\}$ is an equivalence relation.

⁶Formally, $\Omega_{\mathcal{X}}$ satisfies the two following properties, (1) for all $x \in \mathcal{X}$, $\{x\} \in \Omega_{\mathcal{X}}$, and (2) if $f, g \in \Omega_{\mathcal{X}}$ are such that for all $x \in f$, $g \cap [x] \subseteq \{x\}$ and for all $y \in g$, $f \cap [y] \subseteq \{y\}$, then $f \cup g \in \Omega_{\mathcal{X}}$.

A choice correspondence on a choice space Ω is defined as any correspondence c which satisfies $\emptyset \neq c(f) \subseteq f$ for every $f \in \Omega$. This definition leaves open the possibility that, given a described choice problem f , the set $c(f)$ of choices of the individual may contain more than one alternative. If the choice correspondence is defined on a choice space of alternatives, we denote it by C . As usual, a reflexive and transitive (but not necessarily complete) binary relation \succsim on a given set is called a preference relation, denoted by \succsim , and its symmetric and asymmetric parts denoted by \sim and \succ , respectively. The incomparable part of such a relation is denoted by \bowtie . That is, \bowtie is the binary relation defined by $x \bowtie y$ iff neither $x \succsim y$ nor $y \succsim x$.

3 Revealed preference theory with described alternatives: full domain

3.1 Revealed description invariance

Consider an individual with a choice correspondence c on $\Omega_{\mathcal{X}}^f$. An informal example to illustrate how this individual should behave to satisfy description invariance run as follows. With the notations of the programs in the original Asian Disease problem, if we observe $c(\{C, D\}) = D$, then we must observe $c(\{B, C, D\}) = B, D$ since B is consequentially equivalent to D : D chosen out of $\{C, D\}$ must imply that B is chosen out of $\{B, C, D\}$ and that what is chosen in $\{C, D\}$ must remain chosen in $\{B, C, D\}$.

To make this point a bit more precise, let us agree to write f_y for the set $f \cup \{y\}$, for any $f \in \Omega_{\mathcal{X}}^f$ with $x \in f$, $y \notin f$ and $x \approx y$. In words, f_y is the described choice problem that is obtained from f , with $x \in f$, by adding the consequentially equivalent described alternative y .

If the decision maker's behavior is description-invariant, then, for x, y such that $x \approx y$, for any $f \in \Omega_{\mathcal{X}}^f$ with $x \in f$, $y \notin f$, it would be sensible to expect that $c(f)$ and $c(f_y)$ are related: $c(f_y)$ is equal to $c(f) \cup \{y\}$ if $x \in c(f)$ and equal to $c(f)$ otherwise. Equivalently, x is chosen in f if and only if y is chosen in f_y , and whatever is chosen in f other than x would remain chosen in f_y , and vice versa. If this is the case, we say that (x, y) is revealed to be a *non-sensitive pair*. If this is not the case for some described choice problem f (with $x \in f$, $y \notin f$ and $x \approx y$), then there is a framing effect. In other words, the decision maker's behavior is description-dependent, at least with respect to the pair (x, y) and we say that (x, y) is revealed to be a *sensitive pair*. This concept captures the idea that description-dependent behavior does not occur for all possible described decision problems, but only in the ones containing descriptions that have specific psychological effects. Description invariance can therefore be defined as the absence of sensitive pair or, which is equivalent, as the requirement that all possible pair are non-sensitive ones.

Notice that description invariance involves two properties. Firstly, it requires that decision makers recognize the identity of the consequence of two descriptions of the same alternative. In a given described choice problem, a decision maker who chooses a described alternative should also choose all the other available equivalent described alternatives (in the above example, $c(\{B, C, D\}) = B, D$ since $B \approx D$).

Secondly, it requires that decision makers chose alternatives independently of the descriptions under which these alternatives are presented (in the above example, $c(\{B, C\}) = B$ since $B \approx D$ and $c(\{C, D\}) = D$). In a given choice problem, a decision maker should always choose the same alternative whatever its description.

Formally, these two properties can be stated as follows. If a decision maker is able to recognize the identity of described alternatives, then for any $f \in \Omega_{\mathcal{X}}^f$ with $x \in f$, $y \notin f$ and $x \approx y$, we should observe that x is chosen in f_y if and only if y is chosen in f_y ; and when both conditions hold, x is revealed identical to y . This property captures the idea that the decision maker is able to abstract from the descriptions of alternatives.

If the descriptions play no role in the alternative chosen, then $\forall x, y \in [x]$ such that $x \neq y$ and $\forall f \in \Omega_{\mathcal{X}}^f$ such that $x \in f$, $y \notin f$, we should observe that $c(f) \approx c(f_y)$. More specifically, a described alternative consequentially equivalent to x is chosen in f if and only if a described alternative consequentially equivalent to y is chosen in f_y , and whatever is chosen in f other than x , there should be a consequentially equivalent described alternative chosen in f_y , and vice versa. In this case, the alternative $[x]$ is revealed to be a *non-sensitive alternative*. This discussion prompts the following definition.

Definition 1. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}^f$. Let $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. We say that,

- (x, y) is revealed to be a non-sensitive pair if $\forall f \in \Omega_{\mathcal{X}}^f$ s.t. $x \in f$, $y \notin f$, $x \in c(f)$ iff $y \in c(f_y)$ and $c(f) = c(f_y) \setminus \{y\}$.
- x is revealed to be identical to y if $\forall f \in \Omega_{\mathcal{X}}^f$ s.t. $x \in f$, $y \notin f$, $x \in c(f_y)$ iff $y \in c(f_y)$.

Let $[x] \in X$, we say that

- $[x]$ is revealed to be a non-sensitive alternative if $\forall x, y \in [x]$ s.t. $x \neq y$ and $\forall f \in \Omega_{\mathcal{X}}^f$ s.t. $x \in f$, $y \notin f$, $c(f) \approx c(f_y)$.

These definitions, in turn, allows to properly define Description Invariance and its two weakenings as follows.

Definition 2. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}^f$. A choice correspondence c satisfies:

- Description Invariance on the full domain (DI_f) if for all $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$, (x, y) is revealed to be a non-sensitive pair,
- Revealed Identity (RI) if for all $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$, x is revealed to be identical to y ,
- Weak Description Invariance on the full domain (WDI_f) if for all $[x] \in X$, $[x]$ is revealed to be a non-sensitive alternative.

These definitions express different forms of description-invariant behavior in terms of observable choices over specific pairs of described alternatives. The following proposition shows that (1) these properties can be expressed more generally (as usually conceived in discussions of description invariance within economics), and (2) it turns out that WDI_f and RI implies DI_f

Proposition 1. *Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}^f$. Then,*

- (i) *c satisfies DI_f if, and only if, for all $f, g \in \Omega_{\mathcal{X}}^f$, if $f \approx g$, then $c(f) \approx c(g)$ and for all $f \in \Omega_{\mathcal{X}}^f$, for all $x \in f$, if $x \in c(f)$, then $[x] \cap f \subseteq c(f)$.*
- (ii) *c satisfies RI if, and only if, for all $f \in \Omega_{\mathcal{X}}^f$, for all $x \in f$, if $x \in c(f)$, then $[x] \cap f \subseteq c(f)$.*
- (iii) *c satisfies WDI_f if, and only if, for all $f, g \in \Omega_{\mathcal{X}}^f$, if $f \approx g$, then $c(f) \approx c(g)$.*
- (iv) *c satisfies DI_f if, and only if, c satisfies RI and WDI_f .*

We emphasize that our definitions and results characterize an arbitrary choice correspondence c in general, independently of whether c comes out as a result of the maximization of a preference relation.

3.2 Choice and preference relations

The classical result of revealed preference theory is that the necessary and sufficient condition for observed behavior to be consistent with the preference maximization is the Weak Axiom of Revealed Preference. This axiom requires that, if $[x] \in c(T)$ and $[y] \in T$, then $[x]$ is at least as good as $[y]$ for the decision maker, and thus whenever $[y]$ is chosen from a set S that contains $[x]$, $[x]$ must be chosen from S . Formally,

Weak Axiom of Revealed Preference (WARP) For any $S \in \Omega_X$ and $[x] \in S$, if there exists an $[y] \in C(S)$ such that $[x] \in C(T)$ for some $T \in \Omega_X$ with $[y] \in T$, then $[x] \in C(S)$.

It seems natural to restate WARP in order to make the connection between choices and preferences over described alternatives. For the choice space with full domain, the Weak Axiom of Revealed Preference is restated as follows

Weak Axiom of Revealed Preference for Description on the Full domain (WARPD_f) For any $f \in \Omega_{\mathcal{X}}^f$ and $x \in f$, if there exists an $y \in c(f)$ such that $x \in c(g)$ for some $g \in \Omega_{\mathcal{X}}^f$ with $y \in g$, then $x \in c(f)$.

The fundamental theorem of revealed preference theory together with Proposition 1 allows to characterize description invariance in terms of preference.

Theorem 1. *Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}^f$. Then, c satisfies WARPD_f if, and only if, there exists a unique complete preference relation \succsim on \mathcal{X} such that for all $f \in \Omega_{\mathcal{X}}^f$*

$$c(f) = \{x \in f \mid x \succsim y, \forall y \in f\}.$$

Moreover,

(i) *c satisfies DI_f if, and only if, c satisfies RI ; and when both conditions hold*

$$\forall x, y \in \mathcal{X}, x \approx y \Rightarrow x \sim y.$$

(ii) *c satisfies WDI_f if, and only if, $\forall x, y, x', y' \in \mathcal{X}$ s.t. $x \approx x', y \approx y'$, and $x \not\approx y$,*

$$x \succsim y \Leftrightarrow x' \succsim y'.$$

Remark that if WARPD_f is satisfied, then DI and RI are equivalent. This means that, under WARPD_f , the full force of DI is not required anymore, RI is sufficient.

The existence of a preference relation on the set of described alternatives does not allow to characterize different types of sensitivity of the decision maker to framing effects or the lack of such sensitivity. From Theorem 1 we can distinguish three interesting cases.

Firstly, when the decision maker is indifferent between all the descriptions of an alternative, this is the case of description-invariant behavior:

$$\forall x, y \in \mathcal{X}, x \approx y \Rightarrow x \sim y.$$

In this situation, the standard model is applicable.

Secondly, when the decision maker can exhibit strict preferences between descriptions of a given alternative but a well-defined preference relation on the alternatives, this is the case of tidy description-dependent behavior: for all $x, y, x', y' \in \mathcal{X}$ such that $x \approx x', y \approx y'$, and $x \not\approx y$,

$$x \succsim y \Leftrightarrow x' \succsim y'.$$

Tidy description-dependent behavior is theoretically relevant in two ways. Firstly, because the preference relation on the alternatives is well-defined, we know from Theorem 1 that the induced choice correspondence will satisfy WARP . In other words, WDI_f allows the equivalence between WARP in the standard domain of alternatives and WARPD_f in the full domain of described alternatives. (This remark trivially applies also to the above case of description-invariant behavior). Secondly, this type of preferences allows to represent framing effects such as the one in the Asian Disease problem when risk is introduced (see Section 5).

Thirdly, when the decision maker exhibits neither description-invariant nor tidy description-dependent behavior (i.e., WDI and RI are not satisfied), this is the case of untidy-description dependent behavior; the preference relation over the alternatives does not exist.

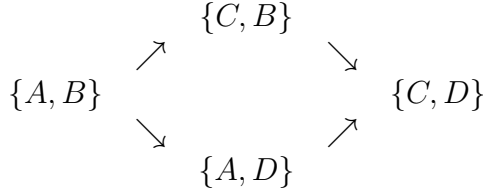


Figure 1: Swapps from the save frame to the die frame.

4 Revealed preference theory with described alternatives: restricted domain

4.1 Revealed description invariance

Consider now an individual with a choice correspondence c on $\Omega_{\mathcal{X}}$. We recall that $\Omega_{\mathcal{X}}$ is the set of all nonempty finite subsets of \mathcal{X} that contain one and only one description of an alternative. In this space, it is no longer possible to add equivalent described alternatives to a given described decision problem to investigate the existence of framing effect. Instead, we consider swapping equivalent described alternatives in described decision problem. For instance, in the original Asian Disease problem, the die frame $\{C, D\}$ is obtained from the save frame $\{A, B\}$ by swapping A with C and then swapping B with D , or in the reverse order.

If we observe $c(\{A, B\}) = A$, then we must first observe $c(\{C, B\}) = C$ or $c(\{A, D\}) = A$, and finally $c(\{C, D\}) = C$, since A is consequentially equivalent to C , as is B to D . A is chosen out of $\{A, B\}$ must imply that C is chosen out of $\{C, B\}$ and that what is chosen in $\{A, B\}$ must remain chosen in $\{C, B\}$, and so on.

Formally, we write $f_{y,-x}$ for the set $(f \cup \{y\}) \setminus \{x\}$, for any $f \in \Omega_{\mathcal{X}}$ with $x \in f$, $y \notin f$ and $x \approx y$. In words, $f_{y,-x}$ is the described choice problem that is obtained from f by swapping x with the consequentially equivalent described alternative y .

In parallel to what we did on the full domain, if the decision maker in question is not influenced by the description of a choice problem, then, for x, y such that $x \approx y$, for any $f \in \Omega_{\mathcal{X}}^f$ with $x \in f$, $y \notin f$, it must be that $c(f)$ and $c(f_{y,-x})$ are consequentially equivalent: the role of x in $c(f)$ would be played by y in $c(f_{y,-x})$. In particular, x is chosen in f if and only if y is chosen in $f_{y,-x}$, and whatever is chosen in f other than x would remain chosen in $f_{y,-x}$, and vice versa. If this is the case, we say that (x, y) is revealed to be a *non-sensitive pair*. If this is not the case for some described choice problem f (with $x \in f$, $y \notin f$ and $x \approx y$), then there is a framing effect.

Definition 3. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Let $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. We say that, (x, y) is revealed to be a non-sensitive pair if $\forall f \in \Omega_{\mathcal{X}}$ s.t. $x \in f$, $y \notin f$, $x \in c(f)$ iff $y \in c(f_{y,-x})$ and $c(f) \setminus \{x\} = c(f_{y,-x}) \setminus \{y\}$.

In the restricted domain, description invariance is fully characterized by this property alone. Intuitively, this is because we constructed the restricted domain to reflect most experimental design of framing effects, in which description invariance can be tested more straightforwardly than in the full domain.

Definition 4. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. A choice correspondence c satisfies the Weak Description Invariance on the restricted domain (WDI_r) if for all $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$, (x, y) is revealed to be a non-sensitive pair.

Proposition 2. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Then, c satisfies WDI_r if and only if for all $f, g \in \Omega_{\mathcal{X}}$, if $f \approx g$, then $c(f) \approx c(g)$.

WDI_r and WDI_f characterize the same property of the choice correspondence, but are however not equivalent. This is because more than one description of an alternative is not allowed in a described choice problem on the restricted domain. Moreover, this makes it impossible to observe directly if a decision maker recognizes the identity of the consequence of two descriptions of the same alternative. Consequently, RI cannot be expressed in the restricted domain. Again, we emphasize that the definition and the result here characterize an arbitrary choice correspondence c in general, independently of whether c comes out as a result of the maximization of a preference relation.

4.2 Choice and preference relations

In the restricted domain, the Weak Axiom of Revealed Preference is restated as follows. The noticeable difference with the full domain is that here x and y must not be equivalent. In other words, it requires that, if $x \in c(g)$, $y \in g$, and x, y are not consequentially equivalent, then x is at least as good as y for the decision maker, and thus whenever y is chosen from a set g that contains x , x must be chosen from S . Formally,

Weak Axiom of Revealed Preference for Description in the Restricted domain ($WARPD_r$) For any $f \in \Omega_{\mathcal{X}}$ and $x \in f$, if there exists an $y \in c(f)$ such that $y \not\approx x$, $x \in c(g)$ for some $g \in \Omega_{\mathcal{X}}$ with $y \in g$, then $x \in c(f)$.

One may wonder whether there is an equivalence between WARP in the standard domain of alternatives and $WARPD_r$ in the restricted domain of described alternatives. The next theorem states that the answer is yes: $WARPD_r$ yields the following counterpart of Theorem 1.

Theorem 2. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Then c satisfies $WARPD_r$ and WDI_r if, and only if, there exists a unique and complete preference relation \succsim on X such that for all $f \in \Omega_{\mathcal{X}}$

$$c(f) = \{x \in f \mid [x] \succsim [y], \forall y \in f\}.$$

This theorem means that, in the restricted domain, we cannot characterize full description invariance (because RI is undefined). The closest to the standard model we can get is tidy-description dependent behavior. Contrary to the full domain, $WARPD_r$ alone is not sufficient to imply the existence of a preference relation over the described alternatives, for which we need the following axiom.

Weak Axiom of Revealed Preference Across Frames (WARPAF) For any $f \in \Omega_{\mathcal{X}}$ and $x \in f$, if there exist an $y \in c(f_{y,-x})$, t, z such that $z \in f$, $t \approx z$, $t \in c((f_{y,-x})_{t,-z})$, and $x \in c(f_{t,-z})$, then $x \in c(f)$.

Consider x, y, z, t such that $x \approx y$ and $z \approx t$. This axiom requires that, if y is chosen from a set where z is available, t is chosen from a set where y is available and x is chosen from a set where t is available, then y is at least as good as z , t is at least as good as y , and x is at least as good as t for the decision maker. And thus, this is the original spirit of WARP, x must be chosen from a set where z is available.

Before we state the proposition, we illustrate how to infer the decision maker's preference in the restricted domain. In the standard framework, x is revealed preferred to y when x is chosen while y is available. Here, the direct binary choice between *only* two descriptions of the same alternative is not allowed. We can nevertheless infer the underlying preference in two steps. The first one consists in observing the decision maker's preference over non consequentially equivalent pairs of described alternatives. The second step consists in inferring the decision maker's preference over consequentially equivalent pairs of described alternatives by transitivity (this is possible if WARPAF is satisfied). Therefore, if two descriptions of the same alternative can be separated by a description of another alternative, then we can infer the preference between the two descriptions of the same alternative indirectly. If it is not possible, presumably the two descriptions of the same alternative are incomparable.

Replacing WDI_r with WARPAF gives the following result.

Theorem 3. *Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Then, c satisfies WARPD_r and WARPAF if, and only if, there exists a unique (not necessarily complete) preference relation \succsim on \mathcal{X} such that for all $f \in \Omega_{\mathcal{X}}$, \succsim restricted to f is complete, and*

$$c(f) = \{x \in f \mid x \succsim y, \forall y \in f\}.$$

Moreover, for all $x, y \in \mathcal{X}$,

$$x \bowtie y \Leftrightarrow (x \approx y \text{ and } x \neq y) \text{ and } (\forall z \not\approx x, z \succ x, y \text{ or } x, y \succ z).$$

Despite its elementary nature, Theorem 3 provides a choice-theoretic foundation for preferences on described alternatives in the restricted domain similarly to how Theorem 1 generates complete preferences from choice behavior on described alternatives in the full domain. In particular, Theorem 3 shows that relaxing the full domain hypothesis has the possible effect of relaxing the completeness property of the rationalizing preference relations. Therefore, choice correspondences that satisfy WARPD_r and WARPAF are consistent with maximizing behavior by means of possibly incomplete preference relations.

5 Description-dependent choice and expected utility representation

As most framing effects (especially the Asian Disease) involve risk, we need to show how the behavior implied by our axioms can be extended under risk. Taking objects

of choice to be lotteries requires to modify our framework, notably to add extra (standard) axioms. Let \mathcal{X} be a non-empty finite set of objects of choice, and denote by $\Delta(\mathcal{X})$ the set of all probability measures (or lotteries) on \mathcal{X} , endowed with the euclidean metric which induces the topology of weak convergence. $\Delta(\mathcal{X})$ is then a compact metric space. In the sequel we shall identify \mathcal{X} with the set of degenerate lotteries in $\Delta(\mathcal{X})$. The consequentialist equivalence relation \approx defined on \mathcal{X} is extended, and still denoted by \approx , on $\Delta(\mathcal{X})$ as follows:

$$l \approx m \Leftrightarrow \forall x \in \mathcal{X}, \sum_{y \in [x]_{\approx}} l(y) = \sum_{y \in [x]_{\approx}} m(y).$$

We let $\Omega_{\Delta(\mathcal{X})}$ denote the set of all nonempty closed subsets of $\Delta(\mathcal{X})$. As is common, we view $\Omega_{\Delta(\mathcal{X})}$ as a metric space on its own right by using the Hausdorff metric. Define for all $(l, m) \in \Delta(\mathcal{X})$ and $\alpha \in [0, 1]$, $\alpha l + (1 - \alpha)m \in \Delta(\mathcal{X})$ by $(\alpha l + (1 - \alpha)m)(x) = \alpha l(x) + (1 - \alpha)m(x)$ for all $x \in \mathcal{X}$. If $f \in \Omega_{\Delta(\mathcal{X})}$, $m \in \Delta(\mathcal{X})$, and $\alpha \in [0, 1]$, then $\alpha f + (1 - \alpha)m$ is defined to be the Minkowski mixture of f and m , and it is clear that $\alpha f + (1 - \alpha)m \in \Omega_{\Delta(\mathcal{X})}$. We write $\mathbb{E}(l, u)$ for the expectation of $u \in \mathbb{R}^{\mathcal{X}}$ with respect to l .

Continuity (C) For any convergent sequences (l_n) and (f_n) with limits l and f , if for each n , $l_n \in c(f_n)$, then $l \in c(f)$.

The following axiom is a reformulation of the standard independence axiom of expected utility theory in our framework.

Independence (I) For all $f \in \Omega_{\Delta(\mathcal{X})}$, for all $l, m \in \Delta(\mathcal{X})$, and for all $\alpha \in (0, 1)$, $l \in c(f)$ if, and only if, $\alpha l + (1 - \alpha)m \in c(\alpha f + (1 - \alpha)m)$.

This axiom states that if the equivalence class of a lottery l is chosen in a described choice problem f , then the equivalence class of any given mixing of l with another fixed lottery n should be chosen out of the same mixing of f with n . In other words, the mixing operation is assumed to be independent of the frames (e.g., descriptions) of the lotteries. Notice that, by contrast with our other axioms, this one is implicit in the standard framework. We just make it explicit here but we do not weaken it because it is enough to explain the framing effects we are concerned with. However, we strongly believe that weakening it may lead to more accurate theories of how framing influence the decision maker's weighting of probability.

Theorem 4. *Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\Delta(\mathcal{X})}$. Then, c satisfies $WARPD_f$, and the (C) and (I)-axioms if, and only if, there exists a function $u : \mathcal{X} \rightarrow \mathbb{R}$ (unique up to a positive affine transformation) such that for all $f \in \Omega_{\Delta(\mathcal{X})}$,*

$$c(f) = \{l \in f \mid \mathbb{E}(l, u) \geq \mathbb{E}(m, u), \forall m \in f\}.$$

In other words, our framework under risk rationalizes description-dependent choices through expected utility theory. This proposition implies that, in our framework, risk attitudes can depend on the frames under which alternatives are described

and not on the alternatives *per se*. More formally, a consequence of Theorem 4 is that it is possible to have for $x, y \in \mathcal{X}$, $x \approx y$ but $u(x) \neq u(y)$. This is indeed the point of the Asian Disease example, which we can now illustrate formally. By adopting a notation compatible with lotteries, the certain program described either as “200 people will be saved” or as “400 people will die” is denoted s_{200} or d_{400} , respectively⁷. The risky program described either as “1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved” or as “1/3 probability that nobody will die, and 2/3 probability that 600 people will die” is denoted $(s_{600}, \frac{1}{3}; s_0, \frac{2}{3})$ or $(d_0, \frac{1}{3}; d_{600}, \frac{2}{3})$, respectively. Observe that choosing between the certain and risky programs is a standard choice problem that can give rise to the two different described choice problems of the Asian Disease: $f = \{s_{200}, (s_{600}, \frac{1}{3}; s_0, \frac{2}{3})\}$ and $g = \{d_{400}, (d_0, \frac{1}{3}; d_{600}, \frac{2}{3})\}$. The choice in the Asian Disease example can be rationalized straightforwardly in expected utility theory as:

$$u(s_{200}) > \frac{1}{3}u(s_{600}) + \frac{2}{3}u(s_0)$$

$$\frac{1}{3}u(d_0) + \frac{2}{3}u(d_{600}) > u(d_{400})$$

It is straightforward to see that these two inequalities hold only because risk attitudes are attached to described alternatives instead of alternatives (i.e., the arguments of u are s_{200} , d_{400} , and so on).

These two inequalities are compatible with the following tidy description-dependent preference relation over the sure described alternatives:

$$s_{600} \succsim d_0 \succsim s_{200} \succsim d_{400} \succsim s_0 \succsim d_{600}.$$

This pattern can be explained by a very small variations of utility that produced no preference reversal under certainty but do produce non-trivial preference reversal under risk, such as in the Asian Disease problem.

The same reasoning carries identically to other framing effects involving description-dependent choices besides the Asian Disease. One further illustration from [De Martino et al. \(2006\)](#) is worth providing. Decision makers are told “You Receive £50”, and then a graphical twist in the two frames shows how the underlying type of preference reversals is driven by the framing of the sure consequence:

In other words, the two different frames have identical descriptions of the risky alternative, they differ only by different descriptions of the sure alternative. Formally, the sure option can be described as k_{20} or l_{30} (“Keep £20” and “Lose £30”, respectively) and the gamble as $(k_{50}, \frac{1}{3}; l_{50}, \frac{2}{3})$ its only description. The modal choice from decision makers can be represented straightforwardly in expected utility theory:

⁷For simplicity, we do not distinguish between the notation of a consequence (e.g., “ s_{200} ”) and the degenerate associated lottery (e.g., “ $(s_{200}, 1)$ ”).

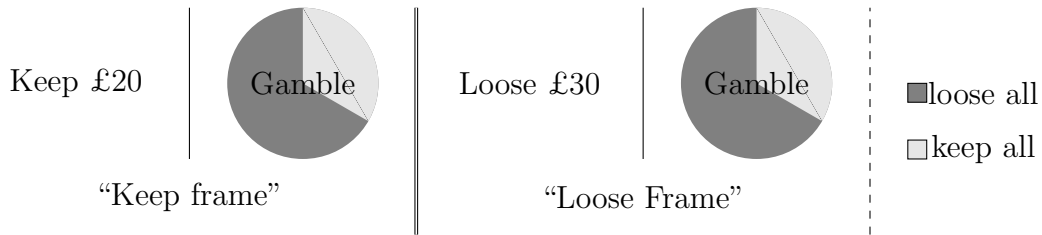


Figure 2: DeMartino et al.'s (2006) framing experiment.

$$u(k_{20}) > \frac{1}{3}u(k_{50}) + \frac{2}{3}u(l_{50})$$

$$\frac{1}{3}u(k_{50}) + \frac{2}{3}u(l_{50}) > u(l_{30})$$

Notice the formal representation of the distinction between, on the one hand, a difference in descriptions of a same alternative (for the sure one), and, on the other hand, an identity of descriptions of a same alternative (for the probabilistic ones).

In a sense, our framework is more general than cumulative prospect theory because we do not need to specify positive and negative valences in our choice set in order to generate description-dependent choices. It would however not be hard to modify our framework to make it equivalent to cumulative prospect theory. On the one hand, we could specify a neutral description within each equivalence classes of description and impose a structure so that each other description is either positive or negative with respect to that neutral element. On the other hand, we could weaken our reformulation of the independence axiom into a reformulation of comonotonic independence so that different utilities from descriptions would impact the associated decision weights.

6 Conclusion

Several contributions make the case that description-dependent choices are not necessarily irrational (see, e.g. [Sher and McKenzie, 2006, 2008, 2011](#); [Keren, 2011](#); [Bourgeois-Gironde and Giraud, 2009](#)). The main argument is that there are cases where the fact that an object of choice is described in one way instead of another is a choice-relevant information about the decision situation, which is “leaked” by the one who poses the decision problem and inferred by the decision maker. For instance, [van Buiten and Keren \(2009\)](#) experiments suggest that the two frames of the Asian Disease leak opposite choice-relevant information about the implicit goal of the one who describes the problem, and that if decision makers correctly infer these information, then the preference reversal is not necessarily irrational. Of course, it can still be argued that even if tacit-information is being exchanged, the preference reversal is irrational if manipulation is taking place (i.e., a leakage of choice-irrelevant information). But in some other cases, that a given institution has

a goal about a decision situation can be choice-relevant information for the decision-maker, especially when the latter is unsure of his or her preferences but has a positive attitude towards the institution. The theory that we proposed in this paper offers choice-theoretic foundations for such phenomena. However, fully modeling it would require to make explicit the decision maker's beliefs about the set of frames that was available to the one who decided to describe the decision problem under one frame rather than another. Because the formalism of choice correspondences is not very well suited to model the decision maker's beliefs, we prefer to leave that task for applications in further work.

To conclude, Schotter (2008, pp.86-7) notes that frame-sensitive choices are problematic when we just have choice data without information about the frame (e.g., descriptions) under which choices are elicited. Of course, our theoretical contribution by itself does not resolve this problem. However, it offers a framework to formally take such information into account when available for cases of description-dependent choices. Although this may be difficult to obtain in the field (e.g., data about insurance choices are silent about how insurance brokers orally describe their policies during a given deal), it is very easy to obtain in the lab because descriptions are in the instructions.

7 Proofs

Proof of Proposition 1

statement (i) We first show necessity. Suppose that there exists $x, y \in \mathcal{X}$ such that (x, y) is not revealed to be a non-sensitive pair. If $x \in c(f)$ but $y \notin c(f_y)$, then there must exist $z \approx x$ such that $z \in c(f_y)$ since $c(f) \approx c(f_y)$ and $x \in c(f)$. But then $[z] \cap f_y \subseteq c(f_y)$, and consequently $y \in c(f_y)$, a contradiction. The proof uses the same argument for the two remaining cases: $x \notin c(f)$ but $y \in c(f_y)$, and $c(f) \neq c(f_y) \setminus \{y\}$.

Conversely, let $f, g \in \Omega_{\mathcal{X}}^f$ such that $f \approx g$. There exists a finite sequence $\{f^1, \dots, f^{n+1}\}$ such that $f^1 = f$, $f^{n+1} = g$, and for all $i = 1, \dots, n$, either $f^{i+1} = f_y^i$ or $f^i = f^{i+1}$ for some $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. Let $i \in \{1, \dots, n\}$, by hypothesis, (x, y) is revealed to be a non-sensitive pair. Then, in the former case, it holds that $x \in c(f^i)$ iff $y \in c(f^{i+1})$ and that $c(f^i) = c(f^{i+1}) \setminus \{y\}$, hence $c(f^i) \approx c(f^{i+1})$. The proof is similar in the latter case. Therefore, by transitivity of \approx , $c(f) \approx c(g)$. We now prove the second property. Let $f \in \Omega_{\mathcal{X}}^f$ and $x_0, \dots, x_n \in f$ such that $x_0 \in c(f)$. For $i = 1, \dots, n$, set $g_i = (f \setminus [x]) \cup \{x_i\}$. Since $f \approx g_i$, it holds that $c(f) \approx c(g_i)$, hence $x_i \in c(g_i)$. Since $g_i = (f \setminus [x]) \cup \{x_i\}$, there exists a finite sequence $\{f^1, \dots, f^{p+1}\}$ such that $f^1 = g_i$, $f^{p+1} = f$, and for all $k = 1, \dots, p$, $f^{k+1} \neq f^k$ and $f^{k+1} = f_y^k$ for some $y \in [x] \setminus \{x_i\}$. By hypothesis, for all $k = 1, \dots, p$, $x_i \in c(f^k)$ and in particular $x_i \in c(f)$. It follows that $[x] \cap f \subseteq c(f)$.

statement (ii) Suppose that c satisfies RIP, let $f, x \in f$ such that $x \in c(f)$. If $[x] \cap f = \{x\}$, then the result is trivial. Let $y \in [x] \cap f$ such that $x \neq y$. Set $g = f \setminus \{y\}$, so that $f = g_y$. Since $x \in c(f) = c(g_y)$, then $y \in c(f)$. Hence $[x] \cap f \subseteq c(f)$. Conversely, suppose that for all $f, x \in f$, $x \in c(f)$ implies $[x] \cap f \subseteq c(f)$. Let

$x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$, x is not revealed to be identical to y . There exists a finite set f with $x \in f$, $y \notin f$, such that $x \in c(f_y)$ but $y \notin c(f_y)$ or $x \notin c(f_y)$ but $y \in c(f_y)$. That is, $[x] \cap f_y \not\subseteq c(f_y)$, a contradiction.

statement (iii) Suppose that for all $f, g \in \Omega_{\mathcal{X}}^f$, $f \approx g$ implies $c(f) \approx c(g)$ and that there exist $[x] \in X$ such that $[x]$ is not revealed to be a non-sensitive alternative. If $\exists x' \approx x$, $x' \in c(f)$ and $\forall y' \approx y$, $y' \notin c(f_y)$, then $c(f) \not\approx c(f_y)$ a contradiction. If $\forall x' \approx x$, $x' \notin c(f)$ but $\exists y' \approx y$, $y' \in c(f_y)$, then again $c(f) \not\approx c(f_y)$ a contradiction. Finally, if $c(f) \setminus [x] \not\approx c(f_y) \setminus [x]$, it is clear that $c(f) \not\approx c(f_y)$, again a contradiction.

Conversely, let $f, g \in \Omega_{\mathcal{X}}^f$ such that $f \approx g$. There exists a finite sequence $\{f^1, \dots, f^{n+1}\}$ such that $f^1 = f$, $f^{n+1} = g$, and for all $i = 1, \dots, n$, either $f^{i+1} = f_y^i$ or $f^i = f_y^{i+1}$ for some $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. Let $i \in \{1, \dots, n\}$, by hypothesis c satisfies WDIP_f . That is, there exists $x' \approx x$, such that $x' \in c(f)$ iff there is $y' \approx y$ such that $y' \in c(f_y)$ and $c(f) \setminus [x] \approx c(f_y) \setminus [x]$. Then, in the former case, it holds that $[x] \cap c(f^i) = \emptyset$ iff $[x] \cap c(f^{i+1}) = \emptyset$ and hence $c(f^i) \approx c(f^{i+1})$. The proof is similar in the latter case. Therefore, by transitivity of \approx , $c(f) \approx c(g)$.

statement (iv) Consequence of the above statements.

Q.E.D.

Proof of Theorem 1

Since \mathcal{X} is a non-empty finite set, c a choice correspondence on $\Omega_{\mathcal{X}}^f$, and WARPD_f reduces to “WARP”. By the fundamental theorem of revealed preference theory, it is readily seen that c satisfies WARPD_f if, and only if, there exists a unique complete preference relation \succsim on \mathcal{X} such that for all $f \in \Omega_{\mathcal{X}}^f$

$$c(f) = \{x \in f \mid x \succsim y, \forall y \in f\}.$$

We need to prove (i) and (ii). (i) Let $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. By Proposition 1, if it holds that for all f , for all $x \in f$, $x \in c(f)$ implies $[x] \cap f \subseteq c(f)$, then necessarily, $c(\{x, y\}) = \{x, y\}$, hence $x \sim y$. By Proposition 1-(iv) SDIP_f implies RIP . If c satisfies WARPD_f , then RIP implies that for all $x, y \in \mathcal{X}$, $x \approx y$ implies $x \sim y$. Hence c satisfies SDIP_f . (ii) Suppose that c satisfies WDIP_f and let $x, y, x', y' \in \mathcal{X}$ s.t. $x \approx x'$, $y \approx y'$, and $x \not\approx y$. By proposition 1-(iii) If $x \in c(\{x, y\})$, then $x' \in c(\{x', y'\})$ because $\{x, y\} \approx \{x', y'\}$. It follows that

$$x \succsim y \Leftrightarrow x' \succsim y'.$$

The converse is trivial.

Q.E.D.

Proof of Proposition 2

We first show necessity. Suppose that there exists $x, y \in \mathcal{X}$ such that $x \approx y$, $x \neq y$ and (x, y) is not revealed to be a non-sensitive pair. By hypothesis $c(f) \approx c(f_{x,-y})$. If $x \in c(f)$ but $y \notin c(f_{y,-x})$, then $c(f) \cap [x] = \{x\}$ and $c(f_{x,-y}) \cap [y] = c(f_{x,-y}) \cap [x] = \emptyset$. It follows that $c(f) \not\approx c(f_{x,-y})$, a contradiction and similarly if $x \notin c(f)$ but $y \in c(f_{y,-x})$. So suppose that $c(f) \setminus \{x\} \neq c(f_{y,-x}) \setminus \{y\}$. Since $\Omega_{\mathcal{X}}$ is the set of

all nonempty finite subsets of \mathcal{X} that contains one and only one description of an alternative, it follows that $c(f) \not\approx c(f_{x,-y})$, again a contradiction.

Conversely, let $f, g \in \Omega_{\mathcal{X}}$ such that $f \approx g$. There exists a finite sequence $\{f^1, \dots, f^{n+1}\}$ such that $f^1 = f$, $f^{n+1} = g$, and for all $i = 1, \dots, n$, $f^{i+1} = f_{y,-x}^i$ for some $x, y \in \mathcal{X}$ such that $x \approx y$ and $x \neq y$. Let $i \in \{1, \dots, n\}$. By hypothesis (x, y) is not revealed to be a non-sensitive pair., that is, $x \in c(f^i)$ iff $y \in c(f^{i+1})$ and $c(f^i) \setminus \{x\} = c(f^{i+1}) \setminus \{y\}$. It follows that $c(f^i) \approx c(f^{i+1})$. We conclude by transitivity of \approx . Q.E.D.

Proof of Theorem 2

Necessity is trivial, we only prove sufficiency. Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Suppose that c satisfies WARPD_r and WDIP_r . We denote by $\Omega_X \subseteq 2^X \setminus \{\emptyset\}$ the choice space on X in bijection⁸ with $\Omega_{\mathcal{X}} / \approx$. Slightly abusing notation (and terminology), we denote by $S \subseteq X$ the equivalence class of $f \subseteq \mathcal{X}$ when S is the image of f by this bijection. Let C be the choice correspondence, induced by c on Ω_X , defined by

$$\forall S \in \Omega_X, C(S) := [c(f)], \text{ if } \exists f, [f] = S,$$

By Proposition 2, C is well-defined. Let $S \in \Omega_X$, $[x] \in S$, and suppose that there exists an $[y] \in C(S)$ such that $[x] \in C(T)$ for some $T \in \Omega_X$ with $[y] \in T$. If $[x] = [y]$, the result is trivially true. So, suppose that $[x] \neq [y]$ and choose $x \in [x]$, $y \in [y]$, f such that $[f] = S$ and g such that $[g] = T$. WARPD_r applies and $x \in c(f)$, hence $[x] \in C(S)$. Hence C satisfies WARP . Therefore, there exists a unique and complete preference relation \succsim on X such that for all $S \in \Omega_X$

$$C(S) = \{[x] \in S \mid [x] \succsim [y], \forall [y] \in S\}.$$

Let \tilde{c} be the choice correspondence, induced by C on $\Omega_{\mathcal{X}}$, defined by

$$\forall f \in \Omega_{\mathcal{X}}, \tilde{c}(f) := \{x \in f \mid [x] \in C([f])\}.$$

Let $f \in \Omega_{\mathcal{X}}$, if $x \in c(f)$, then $[x] \in C([f])$ and hence, by definition, $x \in \tilde{c}(f)$. Conversely, if $x \in \tilde{c}(f)$, then $[x] \in C([f])$. Since $\Omega_{\mathcal{X}}$ is the set of all nonempty finite subsets of \mathcal{X} that contains one and only one description of an alternative, we have $[x] \cap f = \{x\}$ and consequently $x \in c(f)$. We have proven $\tilde{c} = c$, and hence that for all $f \in \Omega_{\mathcal{X}}$

$$c(f) = \{x \in f \mid [x] \succsim [y], \forall y \in f\}.$$

Q.E.D.

⁸If \mathcal{X} is non-empty set and \approx an equivalence relation on \mathcal{X} (which is naturally extended on $2^{\mathcal{X}}$), then there is a natural correspondence between $2^{\mathcal{X}} / \approx$ and $2^{\mathcal{X}/\approx}$. To see this, define $r : 2^{\mathcal{X}/\approx} \rightarrow 2^{\mathcal{X}} / \approx$ by

$$r(S) := \left[\bigcup_{[x]_{\approx} \in S} [x]_{\approx} \right]_{\approx}.$$

It is straightforward to show that r is one-to-one and onto.

Proof of Theorem 3

Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. Necessity of WARPD_r is trivial, we only prove necessity of WARPAF . Suppose that there exists a unique (not necessarily complete) preference relation \succsim on \mathcal{X} such that for all $f \in \Omega_{\mathcal{X}}$, \succsim restricted to f is complete, and

$$c(f) = \{x \in f \mid x \succsim y, \forall y \in f\},$$

where the incomparability part of \succsim is given, for all $x, y \in \mathcal{X}$, by

$$x \bowtie y \Leftrightarrow (x \approx y \text{ and } x \neq y) \text{ and } (\forall z \not\approx x, z \succ x, y \succ z).$$

Let $f \in \Omega_{\mathcal{X}}$ and $x, z \in f$ and suppose that there exist an $y \in c(f_{y,-x})$, a t such that $t \approx z$, $t \in c((f_{y,-x})_{t,-z})$, and $x \in c(f_{t,-z})$. We have that $x \succsim a$ for all $a \in f_{t,-z}$, $t \succsim b$ for all $b \in (f_{y,-x})_{t,-z}$, $y \succsim c$ for all $c \in f_{y,-x}$, and that $(x, y), (z, t), (x, z) \notin \bowtie$. It follows that $x \succsim d$ for all $d \in f$. Hence $x \in C(f)$.

We now prove sufficiency. Suppose that c satisfies WARPD_r and WARPAF . Define R_c on \mathcal{X} thus:

$$xR_c y \Leftrightarrow \exists f \in \Omega_{\mathcal{X}}, x, y \in f, x \in c(f),$$

and \succsim_c on \mathcal{X} as follows:

$$\begin{aligned} x \succsim_c y &\Leftrightarrow x = y, xR_c y \\ &\text{or } x \neq y, x \not\approx y, xR_c y \\ &\text{or } x \neq y, x \approx y, \exists t \in \mathcal{X}, t \not\approx x, xR_c t, tR_c y. \end{aligned}$$

Before presenting the main arguments of the proof, we provide a useful fact.

Claim 1. *Let \mathcal{X} be a non-empty finite set and c a choice correspondence on $\Omega_{\mathcal{X}}$. If c satisfies WARPD_r , then for all $x, y \in \mathcal{X}$ such that $x = y$ or $x \not\approx y$,*

$$xR_c y \Leftrightarrow x \in c(\{x, y\}).$$

Proof. Necessity is obvious. Conversely, if $x = y$, the result is trivial since c is non-empty. If $x \not\approx y$, either $y \in c(\{x, y\})$ or $y \notin c(\{x, y\})$. In the latter case, since c is non-empty, $x \in c(\{x, y\})$. In the former case, $y \in c(\{x, y\})$, $y \in f$ and $x \in c(f)$ for some $f \in \Omega_{\mathcal{X}}$ and WARPD_r applies. Hence $x \in c(\{x, y\})$. \square

By assumption, c is non-empty; it follows that for all $x \in \mathcal{X}$, $xR_c x$, hence \succsim_c is reflexive. Next, we show that \succsim_c is transitive. Assume $x \succsim_c y$ and $y \succsim_c z$. (We omit the trivial cases where transitivity follows from $x = y$ or $y = z$.)

Case 1. If $x \not\approx y$, $xR_c y$, $y \not\approx z$ and $yR_c z$, then either $x = z$, or $(x \neq z \text{ and } x \approx z)$, or $x \not\approx z$. The two former cases are straightforward. So suppose that $x \not\approx z$, by Claim 1, $x \in c(\{x, y\})$, and $y \in c(\{y, z\})$. Since $c(\{x, y, z\})$ is non-empty, there are three cases: if $x \in c(\{x, y, z\})$, then it holds that $x \succsim_c z$. If $y \in c(\{x, y, z\})$, since $x \in c(\{x, y\})$ and $x \not\approx y$, WARPD_r applies and then $x \in c(\{x, y, z\})$. Hence $x \succsim_c z$. Finally, if $z \in c(\{x, y, z\})$, since $y \in c(\{y, z\})$ and $y \not\approx z$, WARPD_r applies and it follows that $y \in c(\{x, y, z\})$, which implies that $x \succsim_c z$.

Case 2. If $x \not\approx y$, $xR_c y$, $y \approx z$, $y \neq z$, there exists $t \in \mathcal{X}$ such that $t \not\approx y$, $yR_c t$, and $tR_c z$. By hypothesis $x \not\approx z$ since $x \not\approx y$ and $y \approx z$.

Assume that $x \not\approx t$, by the preceding argument as in Case 1, $x \not\approx y$, $xR_c y$, $y \not\approx t$, $yR_c t$ and Claim 1 give $x \in c(\{x, t\})$. It follows that $x \not\approx t$, $xR_c t$, $t \not\approx z$ and $tR_c z$. Again, by use of Case 1 and the observation just made above, we have $x \in c(\{x, z\})$, hence $x \succsim_c z$. Suppose now that $x \approx t$. Setting $f = \{x, z\}$, it holds that $t \in c(f_{t,-x})$, $y \approx z$, $y \in c((f_{t,-x})_{y,-z})$, and $x \in c(f_{y,-z})$. WARPAF applies and it follows that $x \in c(\{x, z\})$, hence $x \succsim_c z$.

Case 3. If $x \approx y$, $x \neq y$, there exists $t \in \mathcal{X}$ such that $t \not\approx x$, $xR_c t$, $tR_c y$, $y \not\approx z$, $yR_c z$. The proof is similar to the Case 2.

Case 4. If $x \approx y \approx z$, $x \neq y$, $y \neq z$, there exist $s, t \in \mathcal{X}$, $s, t \not\approx x$, $xR_c s$, $sR_c y$, $yR_c t$, and $tR_c z$. If $x = z$, there is nothing to prove. Suppose then that $x \neq z$. We need to show that there exists $r \in \mathcal{X}$ such that $r \not\approx x$, $xR_c r$, and $rR_c z$. We can take either s or t and the proofs are similar to the cases 1 or 2.

We have just shown that \succsim_c is transitive. By construction, it is readily seen that

$$x \bowtie y \Leftrightarrow (x \approx y \text{ and } x \neq y) \text{ and } (\forall z \not\approx x, z \succ x, y \text{ or } x, y \succ z).$$

Since $\Omega_{\mathcal{X}}$ is the set of all nonempty finite subsets of \mathcal{X} that contains one and only one description of an alternative it follows that for any $f \in \Omega_{\mathcal{X}}$, \succsim restricted to f is complete. It is straightforward to prove that for all $f \in \Omega_{\mathcal{X}}$,

$$c(f) = \{x \in f \mid x \succsim y, \forall y \in f\},$$

and that \succsim is unique. Q.E.D.

Proof of Theorem 4

Necessity is straightforward. Conversely, assume that c is a choice correspondence on $\Omega_{\Delta(\mathcal{X})}$ that satisfies WARP D_f , and the (C) and (I)-axioms, and let \succsim be the preference relation defined on $\Delta(\mathcal{X})$ as follows:

$$\forall l, m \in \Delta(\mathcal{X}), l \succsim m \Leftrightarrow l \in c(\{l, m\}).$$

We easily show that \succsim satisfies Weak Order, Risk Independence, Archimedean, and by [Kreps \(1988, Theorem 5.11\)](#), it follows that there exists a real-valued function $u : \mathcal{X} \rightarrow \mathbb{R}$ such that $l \succsim m$ if and only if $\mathbb{E}(l, u) \geq \mathbb{E}(m, u)$. Finally, since $\Delta(\mathcal{X})$ is a metric space and \succsim a continuous preference relation on $\Delta(\mathcal{X})$, for all f non-empty closed (hence compact) subset of $\Delta(\mathcal{X})$,

$$c(f) = \{l \in f \mid \mathbb{E}(l, u) \geq \mathbb{E}(m, u), \forall m \in f\},$$

is non-empty. Q.E.D.

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