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► **To cite this version:**

Shahid Rahman, Farid Zidani, Walter Edward Young. Ibn Ḥazm on Heteronomous Imperatives and Modality. A Landmark in the History of the Logical Analysis of Norms. 2018. halshs-01808440

HAL Id: halshs-01808440

<https://shs.hal.science/halshs-01808440>

Preprint submitted on 5 Jun 2018

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Ibn Ḥazm on Heteronomous Imperatives and Modality. A Landmark in the History of the Logical Analysis of Norms.

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Zwei Dinge erfüllen das Gemüt mit immer neuer und zunehmender Bewunderung und Ehrfurcht, je öfter und anhaltender sich das Nachdenken damit beschäftigt: der bestirnte Himmel über mir und das moralische Gesetz in mir. Beide darf ich nicht als in Dunkelheit verhüllt oder im Überschwänglichen, außer meinem Gesichtskreise suchen und bloß vermuten; ich sehe sie vor mir und verknüpfe sie unmittelbar mit dem Bewusstsein meiner Existenz.

Two things fill the mind with ever new and increasing admiration and awe, the more often and the more steadily we reflect on them: the starry heavens above me and the moral law within me. I have not to seek or conjecture either of them beyond the horizon of my vision as if they were veiled in obscurities or in transcendent region; I see them before me and connect them immediately with the consciousness of my existence.

Immanuel Kant, *Kritik der praktischen Vernunft*, 1788, Beschluss.

Introduction.

The passionate and staunch defence of logic of the controversial thinker Ibn Ḥazm, Abū Muḥammad ‘Alī b. Aḥmad b. Sa‘īd of Córdoba (384-456/994-1064), had lasting consequences in the Islamic world.¹ Indeed his book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples* (*Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāz al-‘āmmiyya wa-l-amthila al-fiqhiyya*), composed in 1025-1029, was well known and discussed during and after his time; and it paved the way for the studies of his compatriots Ibn Bājjah (d. 1138), Ibn Ṭufayl (1185), and Ibn Rushd (1198), who each gave demonstrative reasoning a privileged place within the methods of attaining knowledge.²

Unfortunately, as too often in the history of science, Ibn Ḥazm’s innovative perspectives and contributions in logic have been overlooked or considered with an attitude of contempt. On the one hand, his work has been seen, at best, as promulgating the benefits of studying Aristotle’s logic, so that his contribution is assessed as more didactical than conceptual. And on the other hand, those who do examine his innovations often consider them to be mistaken.

As indicated by Chejne (1984, p. 2) contempt towards the logical work of Ibn Ḥazm was also present in its reception by the Eastern philosophers who accused him of deviating from

¹ For a recent comprehensive volume on his work, including detailed and updated bibliography see Adang, Fierro, and Schmidtke (2013).

² For an overview of the reception and reshaping of the Aristotelian Peripatetic work on logic see Hasnawi and Hodges (2016).

Aristotelian logic and of dabbling in things beyond his capability. However, a reassessment of his work on logic has since begun, by delving into the ways the thinker of Córdoba studied the links between deontic and modal qualification of propositions.³ In this context Lameer's (2013) paper on the logical sources of Ibn Ḥazm is worth mentioning; the author (p. 417, footnote 1) observes that, although, strictly speaking, it was al-Fārābī who first drew the parallelism between deontic and modal concepts, it was Ibn Ḥazm who developed it and worked it out in a more precise manner.

In fact, as Lameer elsewhere points out (1994, p. 240), though Al-Fārābī, while paraphrasing the *de Interpretatione*, speaks of the possibility for rephrasing a command in terms of necessity, the convertibility strategy he follows does not achieve the reduction he is after. Indeed, his example: converting "Zayd come over here" into "Zayd must come over here", does not seem to paraphrase away the deontic component of the command encoded by the expression "must".

Independently of the success of these attempts, it seems to be the case that these passages by al-Fārābī ground Lameer's remarks (1994, pp. 240-241, and 2013, p. 417) that both al-Fārābī's and Ibn Ḥazm's perspectives appear to be the earliest testimony on record of a transference from deontic to modal concepts.⁴ As a matter of fact, Lameer's observations targeted Knuuttila's (1981) well-known overview of the history of deontic logic, situating its origins around the 14th century, and Von Wright's (1981b, p. 3) assertion that deontic logic was born in Leibniz's *Elementa Juris Naturalis* of 1671.⁵ Von Wright's assertion is based on the fact that Leibniz explicitly states in that work that the *transference* between deontic and modal concepts can be carried out in the following way:

Modal	Deontic
possible, it is intelligible.	(<i>licitum</i>) permissible
necessary, its negation is not intelligible.	(<i>debitum</i>) obligatory
possibly not, its negation is intelligible.	(<i>indebitum</i>) omissible
impossible, it is not intelligible.	(<i>illicitum</i>) forbidden

It is certain that Leibniz's work on legal reasoning was inspired by his studies in Roman Law and Stoic Logic. However, a detailed study of the influence of these sources in reducing deontic to modal concepts, so far as we know, has not yet been undertaken, particularly so in the context of the Arabic tradition. This is despite the fact that there is work on the influence of Stoicism on Arabic thinkers in general, and on the moral classification of acts, including studies by van Ess (1964) and Jadaane (1968).⁶

Some further, more general, points as to a Stoic background for the emergence of the deontic-modal parallelism include:

- i) Roman Jurists, and Cicero in particular, transferred the different forms of

³ See also Guerrero (1997, 2013) and Lameer (2013).

⁴ Lameer acknowledges Gutas (1988, p. 270; 2014, p. 306) for the reference to Ibn Ḥazm.

⁵ In fact, Knuuttila (1993, p. 182) observes that Peter Abelard (1079-1144) and other early medieval philosophers often endorsed an inverted form of Leibniz's reduction by defining modal concepts by means of deontic concepts. According to this characterization, necessity is taken to be what nature demands, possibility is identified with what nature allows, and impossibility with what nature forbids.

⁶ Jadaane (1968, pp. 184-189) discusses and relativizes convincingly Van den Bergh's (1954, reprinted 1987, vol. II, p. 117 of the notes) strong assertion that the *obligatory*, *recommendable*, *reprehensible* and *forbidden* notions of Islamic jurisprudence correspond (respectively) to the Stoic notions of *recte factum*, *commodum*, *incommodum*, and *peccatum*. In the same footnote Van den Bergh (1954, vol. II, p. 118 of the notes) points out that Islamic theologians coupled the deontic notion of *permissible* with the modality *not logically impossible*. Van den Bergh does not develop the issue any further, however.

- natural causality into the realm of Law. This contributed to the inception of the notion of *ratio legis*, which is quite close to the notion of ‘*illa*, or “occasioning factor,” developed by Muslim jurists (see Young, 2017).
- ii) The notion of *conditional right* in Roman Law – the source of Leibniz’s *conditio moralis*, whereby an obligation, such as the obligation to pay some fixed amount of money, is made dependent upon some future contingent conditions (set by the benefactor in favor of a beneficiary) – seems to have set the background for the development of Leibniz’s parallelism.
 - iii) more general, and complementary with the two above, is the central role of the notion of *conditional* in Stoic logic, which seems to have provided the ground for the further sophisticated developments within the Islamic tradition of implications (including bi-implications), or *sharṭiyya muttasila*, and disjunctives, or *sharṭiyya munfaṣila*.⁷

To be certain, analogies between deontic and modal concepts have a long and rich history before their resurgence in the context of contemporary deontic logic. Important gaps are nevertheless present, however, though such gaps in the history of deontic logic have gone practically unnoticed, even in the most recent overviews.⁸ The time is now ripe to revisit the history of the logical analysis of deontic concepts, and, more generally, of legal reasoning.

A primary aim of this paper is to help fill some of these gaps by stressing the role of the work of Ibn Ḥazm in developing a notion of deontic necessity deeply rooted in legal normativity, and in explicitly discussing the transference between deontic and modal concepts. According to our view; the basic units of Islamic deontic logic are what we might call, indulging in terminological anachronism, *heteronomous imperatives*.⁹ As it turns out, the heteronomy of imperatives within Islamic legal systems contrasts with those of the purely moral realm, which seem to be closer to an *autonomous* conception of moral law. There it concurs again with Leibniz’s proposal to define obligatory as “what is necessary for a good person to do”. In the present paper we will focus on the heteronomous imperatives of legal systems rather than on the imperatives of the purely moral realm.

In this context the work of Ibn Ḥazm extends the parallelism between the necessity of events and that of human actions stressed by his predecessors. According to our understanding, Ibn Ḥazm’s parallelism can be rendered explicit formally by means of a *conditional* (or hypothetical) structure shared by both deontic and modal propositions. Moreover, this structure makes apparent that this parallelism displays an underlying system of “degrees”. Indeed, while in the domain of events, given some conditions, it makes sense to distinguish between an event that is more likely to happen than another. Ibn Ḥazm speaks of *near* and *distant possibility* (such as the higher degree of likelihood of rain, given the condensation of clouds in December, and its lower degree, when the condensation occurs in summer); and, in the domain of actions, the Islamic notion of weighting

⁷ Cf. Jadaane (1968, pp. 117-21). For a recent thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges (2016, section 2.4.3, pp. 63-65).

⁸ See, for example, the otherwise excellent essay by Hilpinen and McNamara (2013, p. 14), who, though they discuss the occurrence of deontic concepts in classical Islamic jurisprudence, do not mention the early testimonies of the parallelism between deontic and modal concepts in that tradition.

⁹ We refer to Kant’s distinction between an autonomous imperative, that prescribes to act virtuously for the sake of the virtue itself, and a heteronomous, or hypothetical, imperative, that prescribes to act virtuously in view of attaining some kind of benefit or reward. Von Wright (1981b, p. 34) has already suggested that “traditional deontic logic is not a genuine ‘logic of norms’ but a logic of structures resembling what Kant called hypothetical imperatives.” We agree with the last part of the quote and would add that this shows that the traditional logical analysis of norms is indeed quite different from current deontic logic.

actions determines degrees of virtue (or of legal and moral value).¹⁰

So whereas the degree of virtue of performing a forbidden act as determined by the distribution of sanction and reward is **0**, we obtain the same value by pondering the likelihood of an impossible event to take place. Furthermore, as discussed in section II.3.2, the system of values at work in the parallelism can be seen as opening the way to a more direct correspondence.¹¹ Thus, while in the realm of nature, *likelihood of occurrence of an event* is dependent upon the conditions specific to the occurrence of that event; while, in the realm of jurisprudence, *likelihood of performing an action* is dependent upon the distribution of reward and sanction specific to that action (being that, given the choice to perform or not perform a given action, those that will be rewarded are more likely to be performed than those that are not). According to this perspective, the point of the parallelism is that both deontic and modal qualifications measure the degree of an action or event to become *actual*; that is – indulging once more in anachronism (but this time from the Leibnizian background) – the degree measures how feasible (*facile*) an action or event may be.

This suggests that Ibn Ḥazm’s perspectives already herald the links to probability and possibility explored by Leibniz six centuries later – see for example Leibniz’s (1671, A VI, I, p. 424-26) use of probability in the context of conditional right.¹² Nevertheless, it also shows a crucial difference to the approach developed in the *Elementa Jura Naturalis*: whereas Leibniz’s studies seek to define what is to be just or virtuous, the logical system of deontic imperatives within Islamic jurisprudence presupposes that what is to be virtuous has already been settled. Determining what is to be virtuous is not achieved by logical reasoning within the system of legal jurisprudence, but by delving into the higher objectives of the Shari‘a.

While developing our point we will delve into the logical structure of the heteronomous imperatives. This distinguishes our contribution from the existing literature, such as the papers of Chejne (1984), Lameer (2014), and Guerrero (1997, 2010, 2014), which do not provide a logical analysis of the deontic concepts put into work by Ibn Ḥazm. The true antecedent to the present paper is the work of Farid Zidani (2007, 2015), who, so far as we know, was the first to undertake such a task.

Some of our own developments and general epistemological thoughts go beyond Ibn Ḥazm’s framework and motivations; mainly those contained in sections II.3.2 and IV. However, according to our view, these reflections suggest that Ibn Ḥazm’s approach has the substance for a broader and deeper exploration of the logic of norms.

The paper is structured as follows:

- I. **Ibn Ḥazm’s Logic of Heteronomous Imperatives.** After presenting some extracts of the relevant text, we proceed by providing a formal reconstruction of the five forms of deontic modalities.
- II. **A Landmark in the History of the Logical Analysis of Norms. Duties and Modalities.** In this section we study the transferences from deontic to modal necessity

¹⁰ As pointed by Puerta Vélchez (2013, p. 33), Ibn Ḥazm’s notion of moral beauty and virtue underlies his sophisticated system of degrees of beauty – though beauty does not reduce to the moral. In fact, Puerta Vélchez (2013) shows that degrees of virtue and its links with degrees of beauty is an important feature of Ibn Ḥazm’s epistemology. Moreover, as we will discuss below, the notion of degrees might be related to degrees of reward and sanction; for Ibn Ḥazm’s adoption of such a system of degrees of reward and sanction see Kaddouri (2013, p. 588).

¹¹ However, there is no textual evidence that Ibn Ḥazm explored this path.

¹² Notice that while studying the application of probability to conditional right, Leibniz (1671, A VI, I, p. 426) needs to add an axiom that is very close to the heteronomy-approach to obligations of the Islamic tradition. We will discuss this issue below.

and possibility. We briefly compare the deontic system of Islamic Jurisprudence with that of Leibniz.

III. **Leibniz and Hypothetical Imperatives in Law.**

IV. **Beyond Ibn Ḥazm: Conclusions and the Work Ahead.** We will conclude the paper by discussing briefly some conceptual points that distinguish the logic of heteronomous imperatives from contemporary deontic logic. Our final words will discuss deontic-modal parallelism in the context of Ibn Ḥazm's rejection of reasoning by conjecture and what we call "the internalization of nature."

I **Ibn Ḥazm's Logic of Heteronomous Imperatives.**

I.1 **The main definitions.**

Muslim jurists identified five deontic qualifications for an action. Ibn Ḥazm defines them as follows:¹³

- 1 ***wājib, farḍ, lāzim***. Obligatory action is the one which:
If we do it we are rewarded.
If we do not do it we are sanctioned.
- 2 ***ḥarām, mahzūr***. Forbidden action is the one which:
If we do it we are sanctioned.
If we do not do it we are rewarded.
- 3 ***mubāḥ mustaḥabb***. Recommended permissible action is the one which:
If we do it we are rewarded.
If we do not do it we are neither sanctioned nor rewarded.
- 4 ***mubāḥ makrūh***. Reprehended permissible action is the one which:
If we do not do it we are rewarded.
If we do it we are neither sanctioned nor rewarded.
- 5 ***mubāḥ mustawin***. Evenly permissible action is the one which:
If we do it we are neither sanctioned nor rewarded.
If we do not do it we are neither sanctioned nor rewarded.

Note that the classification assumes that *reward* and *sanction* are incompatible but not contradictory. Some actions can be neither rewarded nor sanctioned; and, as we will discuss in section 2, this latter point is crucial for the introduction of degrees. Notice too that whereas the notion of *sanction* corresponds to the vocabulary of contemporary European jurisprudence, the notion of *reward* at work in the classification of actions seems to pertain to the realm of theology.¹⁴

¹³ Ibn Ḥazm (1926-1930, vol. 3, p. 77); idem (1959, p. 86; 2003, pp. 83-4). Note that Ibn Ḥazm's extension of *mubāḥ*-permissibility into the categories of recommended and reprehended is atypical. To be certain, this innovative subdivision facilitates the comparison (see below) with nearly possible, distantly possible, and purely possible; but it also highlights the underlying values. All forms of "permissibility" have a value; that is, in terms of doing the recommended or not doing the reprehensible, both surpass the neutral value of the "evenly permitted," while not yet reaching the value of doing the obligatory and not doing the forbidden. At the same time, neither doing the reprehensible nor neglecting the recommended descends below the neutral value of the "evenly permitted," which latter, always above the status of doing the forbidden and neglecting the obligatory, remains steadfastly in the middle.

¹⁴ Cf. Hartmann (1992, pp. 74-75).

The point is simply that, as thoroughly developed by Hallaq (2009), the inseparable “groundwork” for the emergence of both Islamic morality and law is in the Qur’ān.

Interestingly, Ibn Ḥazm’s classification of actions varies in relation to others, such as that of the prominent Mu‘tazilite, the Qādī ‘Abd al-Jabbār (324-415/935-1025), in his *Mughnī* (vol 11-14) and in his *al-Uṣūl al-Khamsa*, pp. 79-96).¹⁵

- A is evil (*qabīḥ*) if and only if the doer deserves blame.
- An act A is an act of grace (*tafaḍḍul*) or recommended (*nadb*) if and only if the doer deserves praise, and the ommitter does not deserve blame.
- A is merely permissible [or optional] (*mubāḥ*) if and only if neither the doer nor the ommitter deserves blame or praise.
- A is obligatory (*wājib*) if and only if the ommitter deserves blame.

The last three kinds of actions are described as “good” (*ḥaṣan*) actions, and the set seems to be lacking the category of *reprehended*. However, ‘Abd al-Jabbār, instead of distinguishing a special category for the *reprehended*, introduces the category of *not obligatory* (*ghayr wājib*), characterizing all those actions for which the ommitter is not blamed – including the *evil*, the *permissible* and the *recommended*.

It is worth mentioning that Hilipinen and McNamara (2012, p. 14), who briefly discuss this classification of ‘Abd al-Jabbār, point out that it is very close to Alexis Meinong’s logic of norms, with the exception of a missing category of excusable actions—such being precisely the category which is included in Ibn Ḥazm’s classification mentioned above!

Be that as it may, ‘Abd al-Jabbār’s formulation has certain desirable properties in relation to Ibn Ḥazm’s, but also one which is less felicitous.

- ‘Abd al-Jabbār’s category of actions that can be omitted has the logical advantage that it allows logical inferences from categories of *the evil*, *the permissible* and *the recommended* to *the not obligatory*. Ibn Ḥazm’s definitions, given above, are more static.
- The use of “deserve” in ‘Abd al-Jabbār’s definitions is deontically charged. Already in his day, ‘Abd al-Jabbār’s formulation was qualified as circular, so he attempted to respond to the objection by introducing the notion of *correspondence*. What we have, therefore, is that such actions as are obligatory are those that, if omitted, correspond to evil (cf. Hourani, 1985, p. 102).

Obviously, Ibn Ḥazm’s definitions do not suffer from this form of circularity, mainly because *reward* and *sanction* – rather than *blame* and *praise* – are primitive incompatible notions. Notice too that one way to see the problem of ‘Abd al-Jabbār’s definitions is that they are purported to define what “good” is, from both the moral and theological point of view.

I.2 Freedom and Heteronomy: *Ought* presupposes *Can*.

The following approach is based on the insight that the most salient characteristics of deontic imperatives listed above are:

¹⁵ We owe the citation to Hourani (1985, pp. 99-102), who extracted these definitions from the cited texts.

Assumption of freedom of choice, or *takhyīr*: the fact that an action can be chosen to be performed or not.

The heteronomy of imperatives: the fact that the way actions are qualified by reward or sanction depends upon the choices made.

Both conditions are linked to the idea of responsibility that is at the core of Ibn Ḥazm's understanding of obligation. This point has been stressed by Hourani (1985, p. 175) as follows:

The fact that concerns us in a historical account is that in all ethical contexts [Ibn Ḥazm] regards man as responsible for his own actions and liable to Reward and Punishment accordingly.

In our understanding, responsibility manifests itself in the fact that a legally accountable individual can not only choose to do or not to do some kind of action, but he can also choose not to choose at all; the actions must be contingent on us: we need not necessarily accept the choice. On the other hand, reward and sanction are both dependent on the choices made.

In fact, Islamic Jurisprudence makes explicit the presuppositions for the application of a deontic qualification. Indeed, classifications such as obligatory, forbidden, and permissible, grounding a juridical decision (*ḥukm*) for a particular action (e.g., *it is forbidden to eat pork*), presuppose that:

- a) the person who performs an action is legally accountable (*mukallaf*);
- b) the action in question is one for which the liberty to choose between carrying it out or not has been given (the provision of this liberty of choice is called *takhyīr*).

Notice that this approach is quite different from current studies in deontic logic that include, as axiom, the implication $OA \supset MA$ – where “O” stands for “obligatory” and “M” for “possible,” known as the principle that *Ought implies Can*, and also dubbed *Kant's principle (Sollen-Können-Prinzip)*.¹⁶ According to our analysis of the Islamic conception, however, we find that:

- Every deontic qualification, and not only the obligatory, **presupposes rather than implies** that the qualified action is allowed to be chosen.¹⁷

So, *ought presupposes can*. However; if *can* is understood as some general form of *Permissibility*, then all actions qualified as obligatory are also permissible. We will come to this issue in I.3.4. Still; there is another sense of “*can*” involved in Kant's principle, namely, as *ability*

¹⁶ Cf. Prior (1958), von Wright (1963, pp. 108-116, 122-125), Hilpinen (1981a, pp. 14-15), Chellas (1974), al-Hibri (1978, p. 18-21), Hilpinen and McNamara (2013, p. 38).

¹⁷ Notice that Hintikka's (1981, p. 86) analysis of Kant's principle is quite close to our view of the role of *takhyīr* – though he speaks of **non logical consequence** rather than presupposition:

Our result is in itself very simple, and may even appear trivial – after it has been established. (It ought to be the case that all duties are fulfilled. Hence it ought to be possible to fulfil them). Some additional interest is in any case lent to our observations by the possibility that the ‘sollen-können’ principle was perhaps right from the beginning intended, however dimly and inarticulately, as an expression of a deontic consequence rather than a logical consequence. The principle was brought to prominence in moral philosophy by Kant. Hence we have to ask: how did he conceive of it? Kant's explanations are not distinguished by their lucidity, but an unmistakable and recurrent turn of thought in Kant is in any case a connection between the ‘ought implies can’ principle and the concept of freedom. (See e.g. Critique of Pure Reason A 807, Critique of Practical Reason, 1st ed., p. 54.) Moral freedom, for Kant, lies in the very fact that a man can act in the way he ought to act.

to fulfil the duty, that triggers some known puzzles of current deontic logic.¹⁸ We will briefly come back to this issue in the last section of our paper.

The logical upshot of all this is that the underlying structure is that of a hypothetical, such that if we accept to make the choice between performing or not performing a certain action, we are rewarded or sanctioned in relation to this choice.

In this context, let us recall that in the Arabic tradition propositional logic involves the study of *sharṭiyya* propositions, usually translated as *conditionals*. The compounds of judgments involving a *sharṭiyya* proposition are not asserted, but simply “connected”. This differentiates judgements involving *sharṭiyya* propositions from those involving attributives, called *ḥamliyya* propositions, whereby a predicate is asserted for a subject. *Conditionals* are subdivided into that type constituted by implication (or bi-implication), called *sharṭiyya muttaṣila*, and that type which is constituted by disjunctives (exclusive or inclusive), called *sharṭiyya munfaṣila*.¹⁹ If we take the stance that *connecting without asserting* amounts to *making the truth of the consequent dependent upon the truth of the antecedent*, we might formulate the subdivision as follows:

- The truth of the consequent of a *conditional* judgment involving an implication is dependent upon the truth of the antecedent, which is not (yet) determined to be true.
- The truth of the consequent of a *conditional* judgment involving a disjunction is dependent upon the truth of one of its sides, which is not (yet) determined to be true.

This already suggests the main idea behind our analysis of judgements involving deontic and modal concepts. In a nutshell, our point is to analyse such judgements as a conjunction of two implications, such that the truth of the antecedent of each of these implications is dependent upon a disjunction. Take the case of the *conditional* expressing an obligation. This conditional is constituted by the following implications:

If an action x of type A is performed, then it will be rewarded; and if it is omitted, then it will be sanctioned (omitting to perform A has been established by the legal system as triggering a sanction, i.e. the contrary of reward) – provided the choice of performing or not performing an action of the type A ; that is, provided the disjunction $A \vee \neg A$.

Similarly, for the case of necessary events:

If some cause z of the type C is present, then an event of the type E will occur; and if the cause is absent, then an event of that type will not be produced (the absence of C has been established as inhibiting the occurrence of events of the type E) – provided the cause can or can not take place; that is, provided the disjunction $C \vee \neg C$.

Following Per Martin-Löf’s (1984) analysis of judgements, we will not here employ the term *conditional* for judgements involving implications or disjunctions, but, rather, *hypothetical*, which stresses the point that the compounds of such judgements are not yet known to be true. Thus, according to this terminology, *sharṭiyya* denominates general *hypothetical* judgements, which can be constituted by implications, or bi-implications (*sharṭiyya muttaṣila*), and/or disjunctives (*sharṭiyya munfaṣila*). Let us not now furnish the main formal elements of Martin-Löf’s theory underlying our interpretation.

¹⁸ Cf. Hilpinen and McNamara (2013, pp. 67-69).

¹⁹ Cf. Rescher (1963, pp. 76-78), and Jadaane (1968, pp. 117-21). For a recent, thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges (2016, section 2.4.3, pp. 63-65).

I.3 Deontic Imperatives and the CTT-Analysis of Hypotheticals.

Per Martin-Löf's (1984) Constructive Type Theory (CTT) provides a thorough formal framework whereby categorical and hypothetical judgements can be explicitly distinguished at the object-language level without conflating judgements with the propositions that constitute them.

Since these distinctions are crucial for the formal reconstruction of traditional logic in general—and of the Arabic tradition in particular—we have chosen to employ the language of CTT for our logical study on the origins of deontic concepts. More precisely, the CTT-framework allows one to distinguish, at the language level, both the *taṣawwur* of a judgement, i.e., its conceptualization or (roughly) proposition, and its *taṣdīq*, or *assent*, i.e., the act of judgment itself, or, sometimes, the linguistic expression of that act.²⁰

Let us first briefly introduce the formal instruments we will make use of.

I.3.1 Categorical and Hypothetical Judgments in CTT.

On Categorical Judgments.

In the CTT framework it is possible to express at the object-language level

A true,

which, when asserted by some individual **g**, conveys the information that this individual is in possession of some proof-object for *A*. Moreover, it can be rendered explicit by means of the **categorical judgement**

$d : A,$

which reads, *there is a proof-object d of A* – or the individual **g** can bring forward the proof-object d in support of his claim that *A* is true.²¹

More generally, within CTT a proposition is interpreted as a set the elements of which represent the proofs of the proposition, the solution to a problem, the fulfilments of an expectation.²² Accordingly,

$d : A$

A true

can be read as

²⁰ Cf. Hasnawi and Hodges (2016, pp. 56-57).

²¹ See Martin-Löf (1984, pp. 9-10). For a short introductory survey see Rahman, McConaughy, and Klev (2018, chapter II).

²² This array of readings is due to combining the Curry-Howard correspondence between propositions and sets with Heyting's proof-theoretical interpretation of propositions. So within CTT a proposition is interpreted as a set whose elements represent the proofs of that proposition. It is also possible to view a set as a problem description in a way similar to Kolmogorov's explanation of the intuitionistic propositional calculus. In particular, a set can be seen as a specification of a programming problem, the elements of the set are then the programs that satisfy the specification – see Martin-Löf (1984, p.7). Furthermore in CTT sets are understood also as types so that propositions can be seen as data- (or proof-) types.

d is an element of the set A	A has an element
d is a proof of the proposition A	A is true
d is a solution to the problem A	A has a solution
d fulfils the expectation A	A is fulfilled

Ranta (1994, p. 54) combines CTT with Davidson's (1980, essays 6-10) idea that an individual action makes an action-proposition true. Accordingly the proposition

(that) Al-Fārābī read Aristotle's *Analytica Posteriora*

is made true by individual readings of Al-Fārābī performing that action-type. This interpretation is not far from the interpretation mentioned above of expectations as propositions and fulfilments as proof-objects.

- We will follow here Ranta's suggestion and assume that we have action-propositions that are made true by some evidence that some action of the type expressed by those propositions has been performed.
- Notice that this not only fits nicely with Ibn Ḥazm's original text (see appendix), where he uses the term *al-ashyā'*, "things," to include actions and events, but, as mentioned in the preface and discussed below, it is a consequence of the insight that deontic and modal concepts qualify both actions and events.

On Hypothetical Judgments

One of the characteristic features of CTT is that it also allows, at the object-language level, expression of **hypothetical judgements** as a form of statement distinguishable from the assertion of the truth of an implicational proposition. Hypothetical judgments give rise to dependency structures in CTT, such as

$B(x : A)$ true

or, in its explicit form:

$b(x) : B(x : A)$,

which reads: $b(x)$ is a (dependent) proof object of B , provided x is a proof object of the set A .

Or: the function b takes elements from the set A , and yields proof-objects for B .²³

In other words, in this frame the dependence of the truth of B upon the truth of A amounts to the dependence of the proof-object of B upon the proof-object of A . And the dependence of the proof object of B upon the proof-object of A is expressed by means of the function $b(x)$ (from A to B), where x is a proof-object of A and where the function $b(x)$ itself constitutes the dependent proof-object of B .

²³ For example, intuitively, if A is the set of natural numbers and B is the set of whole numbers, then the function takes one natural number and yields an element of the set of whole numbers B .

In our context, we have the set of (evidences of) performances of actions A , and the set R of rewards (or reward-actions). Thus, the expression

$b(x) : R(x : A)$, can be read as:

For any evidence of a performance of an action of the type A , the function b will provide a reward (for that performance).

Thus, if we have $b(x) : R(x : A)$ as a premise, and we have as a second premise the fact that indeed an action of the type A has been performed (i.e., if we have as premise $a : A$), then we can infer that performance a will be rewarded (i.e., $b(a) : R$).

In plain words, from the premises

some performance x of an action will be rewarded, provided it is the performance of an action of the type A ($b(x) : R(x : A)$);

and

a is such a performance ($a : A$);

we can infer:

performance a is rewarded ($b(a) : R$).

$$\frac{a : A \quad b(x) : R(x : A)}{b(a) : R}$$

What is still lacking is the idea that the reward is made dependent on the occurrence of a *future contingent action*. In order to implement this task, we will supply the antecedent of the hypothetical with a richer structure than the one discussed above. More specifically, we take it that the antecedents of hypotheticals underlying deontic imperatives have the form of a constructivist disjunction. That is, a disjunction such as whose proof-object amounts to indicating explicitly which of either disjunct obtains. Thus the head looks like:

$x : A \vee \neg A$

(where x stands for some piece of evidence for either carrying out an action of type A , or for omitting to perform it).

Since we are in the context of a constructive disjunction, its truth requires that we know which of either disjunct obtains. Different to classical logic, the disjunction $A \vee \neg A$ is not per se assumed to be true. This disjunction is to be conceived as a presupposition of the distribution of reward and sanction on actions set by the legal system. So in such a formal system, facing the choice of performing or not performing a given type of action can be rendered explicit in a quite straightforward manner.

In the context of our reconstruction, **omitting to perform an action of type A (i.e., not doing it) is conceived of as frustrating the performance of an action of that type**, e.g., *stopping (or inhibiting) eating or drinking when a day of fasting in Ramadan begins*. This interpretation is close to the notion of **aborting a process** found in the constructivist understanding of negation (see Martin-Löf 1984, p. 36).

What we now need is to express the dependence of the rewarding or sanctioning upon the choice made. More precisely, if we are describing an *obligatory action*, what we need to express is the following:

Obligatory action

*If there is some evidence that the individual g made the choice to perform an action of type A (i.e., if there is evidence that he made the choice for the **left side** of the disjunction) then he is rewarded (for this performance).*

*If there is some evidence that the individual g made the choice to omit performing an action of type A (i.e., if there is evidence that he made the choice for the **right side** of the disjunction) then he is sanctioned (for this omission).*

If we pull all this together we obtain:

$$b(x) : (((\exists y : A) \mathbf{left}^{\vee}(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \mathbf{right}^{\vee}(x) =_{\neg A} z) \supset S) (x : A \vee \neg A),$$

Where:

the expressions “ $\mathbf{left}^{\vee}(x)$ ” and “ $\mathbf{right}^{\vee}(x)$ ” stand for the injective functions that render, respectively, the left and right component of the disjunction true.²⁴ Whereas $\mathbf{left}^{\vee}(x)$, stands for the choice of performing an action of type of A , $\mathbf{left}^{\vee}(x)$ stand for the choice or not performing such a type of action.

And:

the expressions “ $\mathbf{left}^{\vee}(x) =_S y$ ” and “ $\mathbf{right}^{\vee}(x) =_{\neg S} z$ ” are identities defined within the sets A and $\neg A$, respectively.²⁵ This can be glossed as follows:

The piece of evidence that renders true the left (right) of the disjunction is identical to the evidence for carrying out (omitting to carry out) an action of type A .

Thus:

“($(\exists y : A) \mathbf{left}^{\vee}(x) =_A y) \supset R$, given the choice $x : A \vee \neg A$ ” reads:

*Assuming that, given the choice of performing or not performing an action of type A , performing it has been chosen (i.e., if one chose the **left side** of the disjunction), then, if there is a performance y within the set A that is identical to this choice, reward follows.*

However, given a particular performance a of action-type A , the formalization does not render explicit the fact that the reward or sanction applies to that particular performance a . Moreover, omitting to perform an obligatory action, or performing a forbidden one, is sanctioned **provided there is no valid excuse**; i.e., provided that $\neg E$ applies (the prescribed fasting during Ramadan is not obligatory, for example, while travelling). These points can be implemented as follows:

If we formulate as a subset of actions, we obtain:

$$\{ x : A \vee \neg A \mid ((\exists y : A) \mathbf{left}^{\vee}(x) =_A y \supset R(y)) \wedge ((\neg A \wedge \neg E) (\mathbf{right}^{\vee}(z) =_{\neg A \wedge \neg E} z) \supset S(z)) \} : set.$$

Which reads:

²⁴ We have slightly changed the notation, which is usually $i(x)$ and $j(x)$ – see Ranta (1994, pp. 47).

²⁵ Cf. Ranta (1994, pp. 52-53).

The set of obligatory actions is the subset of those action-types such that
 if action-type A is performed, then that performance is rewarded,
 if action-type A is not performed and its omission is not excused, then it is sanctioned.

NB: In order to avoid an overly cumbersome notation we will not always explicitly mention the proviso $\neg E$.

I.3.2 Ibn Ḥazm's Heteronomous Imperatives.

As already mentioned, deontic qualifications of actions presuppose that the performer is legally accountable and the performer has been given the liberty to choose (*takhyīr*) between two alternatives. The CTT-framework for hypotheticals provides the formal means to express

that the deontic qualifications assume such a choice,
 that the definition does not assume that such a choice has been made.

Notice that the notion of “allowance to choose” involved in *takhyīr* is different from the notion of *permissible*, which latter applies to an action already chosen. Permissibility, as with all other deontic qualifications, presupposes the *liberty to choose*.

In other words, the choice alternatives constitute the assumption of a hypothetical. Let us now elaborate this via logical analysis.

I.3.2.1 The Logical Analysis of Ibn Ḥazm's Heteronomous Imperatives.

1. **wājib, farḍ, lāzim**. Obligatory action is the one which:
 If we do it we are rewarded.
 If we do not do it we are sanctioned.

$$a(x) : (((\exists y : L) \text{left}^V(x) =_L y) \supset R) \wedge (((\exists z : \neg L) \text{right}^V(x) =_{\neg L} z) \supset S) (x : L \vee \neg L)$$

2. **ḥarām, maḥzūr**. Forbidden action is the one which:
 If we do it we are sanctioned.
 If we do not do it we are rewarded.

$$b(x) : (((\exists y : H) \text{left}^V(x) =_H y) \supset R) \wedge (((\exists z : \neg H) \text{right}^V(x) =_{\neg H} z) \supset S) (x : H \vee \neg H)$$

3. **mubāḥ mustaḥabb**. Recommended permissible action is the one which:
 If we do it we are rewarded.
 If we do not do it we are neither sanctioned nor rewarded.

$$c(x) : (((\exists y : M) \text{left}^V(x) =_M y) \supset R) \wedge (((\exists z : \neg M) \text{right}^V(x) =_{\neg M} z) \supset (\neg S \wedge \neg R)) (x : M \vee \neg M).$$

4. **mubāḥ makrūh**. Reprehended permissible action is the one which:
 If we do not do it we are rewarded.
 If we do it we are neither sanctioned nor rewarded.

$$d(x) : (((\exists y : \neg M^*) \text{right}^V(x) =_{\neg M^*} y) \supset R) \wedge (((\exists z : M^*) \text{left}^V(x) =_{M^*} z) \supset (\neg S \wedge \neg R)) (x : M^* \vee \neg M^*).$$

5. **mubāḥ mustawin**. Evenly permissible action is the one which:
 If we do it we are neither sanctioned nor rewarded.
 If we do not do it we are neither sanctioned nor rewarded.

$$e(x) : (((\exists y : M^\#) \mathbf{left}^\vee(x) =_{M^\#} y) \supset (\neg S \wedge \neg R)) \wedge (((\exists z : \neg M^\#) \mathbf{right}^\vee(x) =_{\neg M^\#} z) \supset (\neg S \wedge \neg R)) \quad (x : M^\# \vee \neg M^\#).$$

Let us also add ‘Abd al-Jabbār’s category of *not obligatory*.

- ghayr wājib**. Not obligatory action is the one which:
 If we do not do it we are not sanctioned.
 If we do it we are either rewarded or not rewarded.

$$f(x) : (((\exists z : \neg G) \mathbf{right}^\vee(x) =_{\neg G} z) \supset \neg S) \wedge (((\exists z : G) \mathbf{left}^\vee(x) =_G z) \supset (R \vee \neg R)) \quad (x : G \vee \neg G).$$

I.3.2.2 Ibn Ḥazm’s classification as subsets.

Strictly speaking, each of the deontic concepts determine a **subset** of a general set of actions in a straightforward manner. For instance,

- **Obligatory** is the set of all those actions rewarded when performed and sanctioned when omitted.

If we elaborate this for all deontic qualifications we obtain:

wājib, fard, lāzim:

$$\{ x : A \vee \neg A \mid (((\exists y : A) \mathbf{left}^\vee(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \mathbf{right}^\vee(x) =_{\neg A} z) \supset S) \} : set.$$

ḥarām, mahzūr:

$$\{ x : A \vee \neg A \mid (((\exists y : \neg A) \mathbf{right}^\vee(x) =_{\neg A} y) \supset R) \wedge (((\exists z : A) \mathbf{left}^\vee(x) =_A z) \supset S) \} : set.$$

mubāḥ mustaḥabb:

$$\{ x : A \vee \neg A \mid (((\exists y : A) \mathbf{left}^\vee(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \mathbf{right}^\vee(x) =_{\neg A} z) \supset (\neg S \wedge \neg R)) \} : set.$$

mubāḥ makrūh:

$$\{ x : A \vee \neg A \mid (((\exists y : \neg A) \mathbf{right}^\vee(x) =_{\neg A} y) \supset R) \wedge (((\exists z : A) \mathbf{left}^\vee(x) =_A z) \supset (\neg S \wedge \neg R)) \} : set$$

mubāḥ mustawin:

$$\{ x : A \vee \neg A \mid (((\exists y : A) \mathbf{left}^\vee(x) =_A y) \supset (\neg S \wedge \neg R)) \wedge (((\exists z : \neg A) \mathbf{right}^\vee(x) =_{\neg A} z) \supset (\neg S \wedge \neg R)) \} : set.$$

ghayr wājib:

$$\{ x : \neg A \mid (\exists z : \neg A) x =_{\neg A} z \supset \neg S \}$$

And now we can use the expression **OA**, as an abbreviation.

$$\mathbf{OA} =: \{ x : A \vee \neg A \mid (((\exists y : A) \mathbf{left}^\vee(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \mathbf{right}^\vee(x) =_{\neg A} z) \supset S) \}$$

etc.

Note that this does not make of **O** an operator, but only a notational device for abbreviating the definition of the set of obligatory actions.

I.3.3 An Extension: From *Ought* to the *Permissible*.

In his *al-Iḥkām fī uṣūl al-Aḥkām* (vol. 8, p. 101), Ibn Ḥazm seems to extend his deontic system with notions of *forbidden to do* and *obligatory not to do*, based only on what is *permissible to do* or *not to do*.

*Forbidden is all that is not permissible to do,
obligatory is all that is not permissible not to do.*

This suggests that forbidden, let us here call it **prohibited**, is based on a *positive notion* of *permissibility to do*. *Permissibility to do* includes all that is *recommended to do* and all that is *obligatory to do*, and excludes all that is indifferent. Moreover it assumes that those actions which are *not permissible to do* are sanctioned if they are carried out.

By the same line of thought we obtain the obligatory, let us here call it **mandatory**, from the negation of *permissible not to do*. *Permissible no to do* includes such as whose doing *should be avoided* as well as the *prohibited*, and it excludes all that is indifferent. It assumes, too, that those actions that are *not permissible not to do* are sanctioned if they are not carried out.

This yields a general notion of permissibility as a subset, including all such actions as are rewarded if carried out and all such actions as are rewarded if not carried out. It presupposes that “indifferent” actions are excluded. Thus...

Mubāḥ

$\{ x : A \vee \neg A \mid (((\exists y : A) \text{ left}^{\vee}(x) =_A y) \supset \neg S) \wedge (((\exists z : \neg A) \text{ right}^{\vee}(x) =_{\neg A} z) \supset \neg S) \}$: set – given that no *mustawin*-action is included in the resulting subset.

And the negation is now defined as all such actions as are sanctioned if carried out, and all such actions as are sanctioned if not carried out:

Prohibited to do. The set of such actions as are sanctioned when performed.

$\{ x : A \mid (((\exists y : A) x =_A y) \supset S) \}$: set.

Mandatory not to omit. The set of such actions as are sanctioned when not performed.

$\{ x : \neg A \mid (((\exists y : \neg A) x =_{\neg A} y) \supset S) \}$: set.

It is this general form of permissible that yields the second path to understanding Kant’s *Sollen-Können Prinzip*: all those actions that are obligatory are also permissible. However, many of the difficulties linked to the standard formalization can be avoided if we assume that we have a fully-interpreted language that pays attention to the content of the expressions involved.²⁶ This leads us to our next section.

²⁶ For a discussion of these difficulties see, among others, Chellas (1974), al-Hibri (1978, pp. 18-20), Hilpinen and McNamara (2013, pp. 67-69).

I.3.4 Ibn Ḥazm’s Interpreted Language and the Heuristic Role of *Jadal*.

One of the most salient features of the CTT language is that it allows formulating inferential rules that determine the content of non-logical expressions. For instance, formation rules plus introduction rules determine the set of natural numbers. This provides the framework with a method for checking whether or not some individual is a member of the set at stake.

This fully interpreted approach of CTT makes it an optimal formal framework for studying ancient logic, which in general is constituted by interpreted languages; and it is useful in particular for the work of Ibn Ḥazm, who explicitly avoids the use of purely syntactic variables or purely formal expressions. In fact, Ibn Ḥazm’s parallelism between deontic and modal notions emerged from the task of bridging the modal logic of Aristotelians and Post-Aristotelians with Islamic Jurisprudence. Indeed, while building that bridge Ibn Ḥazm employs an interpreted language wherein empirical content is made explicit.

In this context it is relevant to recall that our author was a proponent of the use of dialectic and disputation (*al-jadal wa-l-munāẓara*) for attaining truth.²⁷ Moreover, he endorsed true dialectic—wherein rules are determined by logic and the whole endeavor is seen as a cooperative interaction for advancing knowledge, rather than occupied solely with winning arguments—and he rejected *safsatah* or sophistic.²⁸ Walter Young (2017, p. 1) formulates this view as follows:

Ultimately, and most importantly, a truly dialectical exchange – though drawing energy from a sober spirit of competition – must nevertheless be guided by a cooperative ethic wherein truth is paramount and forever trumps the emotional motivations of disputants to “win” the debate. This truth-seeking code demands sincere avoidance of fallacies; it views with abhorrence contrariness and self-contradiction. This alone distinguishes dialectic from sophistical or eristic argument, and, in conjunction with its dialogical format, from persuasive argument and rhetoric. And to repeat: dialectic is formal – it is an ordered enterprise, with norms and rules, and with a mutually-committed aim of advancing knowledge.

In such a framework the precise content of the sets at stake is established by the intertwining of questions and answers on the formation units presupposed by the thesis. This is particularly so when it appears that a set needs to be extended. Consider the following problem question. From unequivocal verses in the *Qur’ān* we learn that it is forbidden to violate another’s privacy (physically).²⁹ What, then, is the correct ruling with regard to reading another’s email without their permission?

The *jadal* theory developed and practiced by Muslim jurists, coupled with a mainstream development of theories of *qiyās*, or “co-relational inference,” provided innovative methods for justifying or critiquing the extension of the scope of an original ruling.³⁰ But Ibn Ḥazm, of course, as key proponent and theorizer of the *Zāhirī madhhab*, disavowed the use of *qiyās*, since, according to his view, such inferences are based on parallelisms and analogies, and are thus departures from the more reliable methods of logical reasoning.³¹ Moreover, his rejection is coupled with a quite static view of language.³² One way Ibn Ḥazm deals with “new” legal cases is to acknowledge that

²⁷ Chejne (1984, pp. 68-69).

²⁸ Ibid.

²⁹ See, e.g., Q.24:27 and 49:12.

³⁰ Cf. Young (2017); Rahman and Iqbal (2018).

³¹ Cf. Ibn Ḥazm (1959, pp. 144, 160- 166), Chejne (1984, pp. 67-68).

³² See Arnaldez (1956, p. 222); see too the various papers on Ibn Ḥazm’s conception of language in Adang, Fierro, and Schmidtke, eds. (2013).

cases not explicitly mentioned in the sources are to be considered as legally indeterminate and thus, in principle, permitted.³³

In any case, if the pertinence of a deontic prescription for some specific case is at issue, then a disputation theory enriched with a fully interpreted language comes to the fore in order to elucidate the precise meanings of the terms involved. This does not require necessarily a non-monotonic approach, such as those which are employed in current deontic logic.³⁴ What it requires is that sets are not defined by extensionality (as in CTT), and that there is a level within the disputation where the constitution of the terms involved can be revised before settling a final decision on the thesis at stake.³⁵

The interpreted language of Ibn Ḥazm allows expression of degrees within the categories of permissibility. We will integrate this feature in the next section.

II A Landmark in the History of the Logical Analysis of Norms. Duties and Modalities.

As mentioned above, the Stoics produced a shift in the conception of necessity that grew out of their interest in propositional logic. Indeed, while it seems plausible that in Aristotle necessity and possibility are relations between terms,³⁶ the Stoics understood them as a qualification of events. Once this shift took place events and actions can be mirrored quite naturally. However, the Stoics, in order to allow some degree of liberty to human action, tried to keep the realm of actions and the realm of events apart.

Now, so far as we can see there are two distinctive approaches to modality in the Arabic tradition; namely: a) an approach closer to the relational, integrating modality or temporality in the quantified structures employed by Aristotle (such being by far the most widespread approach);³⁷ and b) one that is based on a propositional structure, which, besides in Ibn Ḥazm's landmark parallelism, might be seen at work in some of the modal approaches to causality.³⁸

Thinkers in the Arabic tradition proposed innovations to both approaches. We will focus on the second approach, which, according to our interpretation, is the one that allowed Ibn Ḥazm's paralleling of deontic and modal concepts. According to our view the insight developed by Ibn Ḥazm is to parallel the freedom assumed by legal responsibility with the contingency of events, given some natural conditions. This provided roots for transference from the realm of actions to the realm of events. In other words, the mirroring of deontic and modal concepts is a result of:

- 1) understanding modalities as affecting propositions, rather than the Aristotelian view of modalities as *modes of predication*;
- 2) mirroring the freedom of choice assumed by the legal system of sanctions and rewards within the contingency of natural events (or actions), given some natural conditions;

³³ For a thorough discussion on the subject see Sabra (2013).

³⁴ For an overview on defeasibility and deontic logic see Hilpinen and McNamara (2013).

³⁵ Cf. Rahman/Iqbal (2018, IV.2).

³⁶ For a discussion on Aristotle's relational view on modalities see Malink (2013).

³⁷ For an overview of work on the first approach, see the sections on modal logic in Hasnawi and Hodges (2016); Strobino and Thom (2016).

³⁸ Our suggestion is up to now only that, a suggestion. A thorough examination of the bearings of the notion of conditional modality for the Arabic studies on causality are still due.

- 3) the idea that both modal and deontic qualifications admit degrees. In our view this can be seen as one of the earliest sources for the introduction of probabilistic terms in order to analyze modal and normative expressions.

Witness to the first point is that when Ibn Ḥazm, like other Muslim thinkers before him, speaks about modalities, he refers to them as qualifications of all “things” (including, here, actions and events). All these “things” can be classified as necessary, possible, or impossible. The Arabic word employed for “things” is *ashyā* (s. *shay*’); and its usage appears to be close to that of “pragma.”

Chapter on Elements (‘ <i>anāṣir</i> ’)	باب العناصر
Know that the elements (‘ <i>anāṣir</i> ’) of all things (<i>ashyā</i>)—that is, their classes with regard to making assertions (<i>ikhbār</i>) about them—are of three classes, there being no fourth.	اعلم ان عناصر الأشياء كلها أي أقسامها في الاخبار عنها ثلاثة أقسام لا رابع لها
[They are] either necessary (<i>wājib</i>), being such as are necessary and manifest, or from among such as must be, like the rising of the sun each morning, and the like of that, such being called in God’s laws ‘obligatory’ (<i>farḍ</i>) and ‘binding’ (<i>lāzim</i>);	إمّا واجب وهو الذي قد وجب وظهر أو ما يكون مما لا بد من كونه كطلوع الشمس كلّ صباح وما أشبه ذلك وهذا يسمى في الشرائع الفرض واللازم
or possible (<i>mumkin</i>), being such as might be and might not be, like our anticipation that it will rain tomorrow, and the like of that, such being called in God’s law ‘lawful’ (<i>ḥalāl</i>) and ‘permitted’ (<i>mubāḥ</i>);	وإما ممكن وهو الذي قد يكون وقد لا يكون وذلك مثل توقعنا أن تمطر غداً وما أشبه ذلك وهذا يسمى في الشرع الحلال والمباح
or impossible (<i>mumtani</i>) being such as to which there is no path, like a human’s remaining under water for an entire day, or his living a month without food, or his walking in the air without some cunning artifice, and the like of that. And this is the type of thing that, if we saw it manifest in a human, we would know he is a prophet; and this class is called in God’s laws ‘forbidden’ (<i>ḥarām</i>) and ‘prohibited’ (<i>maḥzūr</i>).	وإما ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا حيلة وما أشبه ذلك وهذه التي إذا ظهرت من إنسان علمنا أنه نبي وهذا القسم يسمى في الشرائع الحرام والمحظور
Furthermore, the possible (<i>mumkin</i>) is divided into three classes, there being no fourth:	ثم الممكن ينقسم اقساماً ثلاثة لا رابع لها
the nearly possible (<i>mumkin qarīb</i>), like the possibility of occurrence of rain upon a condensing of clouds in the two months of	ممكن قريب كما مكان وقوع المطر عند تكاثف الغيم

<p><i>Kānūn</i>,³⁹ or the victory of a large number of the courageous over a small number of the cowardly;</p>	<p>في شهري كانون وغلبة العدد الكبير من الشجعان العدد اليسير من الجبناء</p>
<p>and the distantly possible (<i>mumkin ba'īd</i>), which is like the defeat of a large number of the courageous at the hands of a small number of the cowardly, and like a cupper (<i>hajjām</i>) [i.e., a practitioner of cupping] taking charge of the Caliphate, and the like of that;</p>	<p>وممكن بعيد وهو كانهزام العدد الكثير من الشجعان عند عدد يسير من جناء وكحجام يلي الخلافه وما أشبه ذلك</p>
<p>and the purely possible (<i>mumkin maḥḍ</i>), whose two extremes are equal, such being like one standing—either he will walk or he will sit—and the like of that.</p>	<p>وممكن محض وهو يستوي طرفاه وهو كالمراء الواقف إما يمشي وإما يقعد وما أشبه ذلك</p>

Extract from Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Alfāz al-‘Āmmiyya wa-l-Amthila al-Fiḥiyya*, ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī (Beirut: Manshūrāt Muḥammad ‘Alī Baydūn, Dār al-Kutub al-‘Ilmiyya, 2003), pp. 83-4. Translated by Walter Young (see appendix for the complete text on this subject).

Witness to the second point is that Ibn Ḥazm’s examples of modal possibilities are all variations of contingency – that is, they are neither necessary nor impossible. This coincides with what in the recent literature on Aristotle’s Modal logic is called two-sided (or two-way) possibility, although, as already mentioned, Aristotle’s modalities are relations between terms rather than propositional binding operators.⁴⁰

Notice also that, as pointed out by Jadaane (1968, pp. 29-33), according to Islamic thought absolute necessity is unknown even in nature, since the counterfactual is a prerogative of God’s absolute freedom. The whole parallelism with which we are concerned is based on the conception that God can shape the world of events at His will, and on the further stance that God provides freedom of choice to human actions.

As for the third point, we will discuss it in following sections.

II.1 Ibn Ḥazm’s Modalities.

Ibn Ḥazm’s examples of modalities and the denominations he employs make apparent one of the most innovative insights of the Arabic perspective on deontic concepts; namely, that deontic concepts admit degrees: degrees are what allow different forms of legal sanction. Ibn Ḥazm’s contribution is to parallel these degrees with degrees for modalities.

Nearly possible (*mumkin qarīb*). It is likely, but not necessary, that the event occurs.

Examples:

Rain, when clouds condense in December and January.

Victory for many courageous opponents over a few cowardly opponents.

³⁹ I.e., December and January.

⁴⁰ See Malink (2013, pp. 248-260).

Distantly possible (*mumkin ba'īd*). It is not likely, but not impossible, that the event occurs.

Examples:

Defeat of many courageous opponents at the hands of a few cowardly opponents.

A cupper taking charge of the Caliphate.

Purely possible (*mumkin mahḍ*). The event might occur or might not occur, equally; it is neither necessary nor impossible.

Example:

Whether a standing man walks or sits.

As for Necessity and Impossibility, they are clearly factual; i.e., they do not concern logical validity or contradictions.

Necessary (*wājib*).

Example:

The rising of the sun each morning.⁴¹

Impossible (*mumtani*).

Examples:

Remaining (alive) under water for an entire day.

Living a month without any food.

Walking in the air, without employing some kind of trickery.

II.2 The Parallelism.

As mentioned above, Ibn Ḥazm's parallelism between deontic and modal concepts, growing out of his studies on the *De Interpretatione*, is the product of his efforts towards translating logical terms from the Ancient Greek tradition into Arabic—and, more generally, in integrating logic into Islamic Jurisprudence. His manner of paralleling the nearly and distantly possible with the recommended and reprehended permissible is relatively straightforward:

In a similar way as, given the right conditions, an event is likely to occur, so will it also be the case that an action is likely to be performed, if performing it is rewarded.

In a similar way as, given the right conditions, an event is unlikely to occur, so will it also be the case that an action is unlikely to be performed, if omitting it is rewarded.

The third form of permissibility indicates that this parallelism is to be considered with a *tertium comparationis*; namely, degrees: the likelihood of an event (or action) to occur, given some natural conditions, compared with the likelihood of an action to be performed, given a juridical or ethical system of evaluation.

⁴¹ Elsewhere, Ibn Ḥazm brings as example the case wherein something has already come into existence: “Know that the obligatory (*al-wājib*) is before the possible (*al-mumkin*) because the obligatory is the existing (*al-mawjūd*), but as for the possible it is not yet come (*lam ya'ti*), and as for the impossible it is null / invalid (*bāṭil*) because it is not, and does not manifest itself” [واعلم ان الواجب قبل الممكن لأن الواجب هو الموجود وأما الممكن فلم يأت بعد وأما الممتنع فهو باطل لأنه لا يكون ولا يظهر] (Ibn Ḥazm 1959, p. 87; 2003, p.84). This example is, perhaps, less clear than the others, and it is difficult to make explicit its complete formulation. In commenting on an early version of the present paper, Ahmad Hasnawi suggested that this might allude to the case of Aristotle's conditional necessity: *What is, necessarily is, when it is...* (*De interpretatione* 9, 19a23).

Notice that the examples for necessity, possibility and impossibility are not restricted to events but also qualify actions, such as walking or sitting. Let us examine these actions in terms of their likelihood to happen.

From the modal point of view, it is just as likely that walking will take place as sitting, given the condition (i.e., the standing of the man).

From the deontic point of view, assuming that walking and standing are indifferent – i.e., they are neither rewarded nor sanctioned – the likelihood of the man choosing to walk is the same as his choosing to sit down.

Paralleling the pairs of necessary-obligatory and impossible-forbidden, given the context of Islamic jurisprudence, is more complicated. Indeed, this parallelism assumes that the conditions underlying the pair of modalities are *necessary and sufficient*. This might be seen as a shortcoming of the system, though von Wright (1981a, p. 160) himself attempted to develop a logic of norms based on necessary and sufficient conditions.⁴² In other words, the transferences as conceived by Ibn Ḥazm assume that

For the parallelism between the necessary and obligatory: If the causal conditions *C* are present then the event occurs, and if the contradictory of *C* is the case then the event does not occur.

For the parallelism between the impossible and the forbidden: If the causal conditions *C* are present then the event does not occur, and if the contradictory of *C* is the case then the event does occur.

The following table seeks to condense the parallelism. Note that on the deontic side, the degree assumes the ideal choices of the *just* or *virtuous* agent. Again, this is related to the degrees in that the high likelihood of an action to be performed if it is rewarded (i.e., has a high juridical or moral value), assumes the actor's willingness to do what is normatively best. For the sake of simplicity, we will mostly employ the expression *event* as a subject of modal qualification and the expression *action* as a subject of deontic qualification. However, as mentioned above, actions also admit modal qualification. Thus, *event* here and in the following sections denotes some general notion of states of affairs. Note that the paraphrases of the modalities in the table below employ modal expressions, which will be avoided in the formalization of the second table.

Modal	Deontic	Degree
Near possibility <i>mumkin qarīb</i> If the causal condition <i>C</i> occurs, it is likely that the event occurs. If not- <i>C</i> is the case, the event might or might not occur.	Recommended Permissibility <i>mubāḥ</i> <i>mustaḥabb</i>	Likely to happen, but not certain to happen.
Distant possibility <i>mumkin ba'īd</i> If the causal condition <i>C</i> occurs, it is likely that the event does not occur. If not- <i>C</i> is the case, the event might or might not occur.	Reprehended Permissibility <i>mubāḥ makrūh</i>	Unlikely to happen, but not certain not to happen.
Pure possibility <i>mumkin mahḍ</i> If the causal condition <i>C</i> occurs, it is equally likely the event does	Even Permissibility <i>mubāḥ</i>	Equally likely to happen or to not happen.

⁴² Notice, however that our formalization of necessity, based on CTT, does not collapse into a bi-conditional. Furthermore, the formation rules of CTT keep condition and conditioned apart by setting the rules *Event : prop (Conditions)*. Notice, too, that in the context of Islamic jurisprudence and also in current European Law – at least from the time of Leibniz's *De Conditionibus* – this assumption is part and parcel of the notion of conditional obligation. See Armgardt (2001, 2008, 2010), Magnier (2015), Rahman (2015).

or does not occur.	<i>mustawin</i>	
Necessity <i>wājib</i> If the causal condition C occurs, the event occurs. If not- C is the case, the event does not occur.	Obligatory <i>wājib, farḍ, lāzim</i>	Certain to happen.
Impossibility <i>mumtani</i> ⁴³ If the causal condition C occurs, the event does not occur. If not- C is the case, then the event occurs ⁴³	Forbidden <i>ḥarām</i>	Certain not to happen.

Notice that the modal qualification of events is established under the assumption of some set of conditions. This suggests that heteronomous imperatives and modalities share a similar hypothetical structure. For the sake of simplicity we will employ only one set of conditions; for this we will use the notation “ C ”, with “ E ” denoting the set of events.

Since contingency is a crucial feature for the introduction of degrees, we will simply assume that the intersection of each of the subsets of the set $A \vee \neg A$ characterizing the five deontic categories is empty (similarly for their modal counterparts):

<i>wājib, farḍ, lāzim.</i> $\{ x : A \vee \neg A \mid (((\exists y : A) \text{ left}^V(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \text{ right}^V(x) =_{\neg A} z) \supset S) \} : set.$	Deontic
<i>wājib.</i> $\{ x : C \vee \neg C \mid (((\exists y : C) \text{ left}^V(x) =_C y) \supset E) \wedge (((\exists z : \neg C) \text{ right}^V(x) =_{\neg C} z) \supset \neg E) \} : set.$	Modal

<i>ḥarām.</i> $\{ x : A \vee \neg A \mid (((\exists y : A) \text{ left}^V(x) =_A y) \supset S) \wedge (((\exists z : \neg A) \text{ right}^V(x) =_{\neg A} z) \supset R) \} : set.$	Deontic
<i>mumtani</i> ⁴³ . $\{ x : C \vee \neg C \mid (((\exists y : C) \text{ left}^V(x) =_C y) \supset \neg E) \wedge (((\exists z : \neg C) \text{ right}^V(x) =_{\neg C} z) \supset E) \} : set.$	Modal

<i>mubāḥ mustahabb.</i> $\{ x : A \vee \neg A \mid (((\exists y : A) \text{ left}^V(x) =_A y) \supset R) \wedge (((\exists z : \neg A) \text{ right}^V(x) =_{\neg A} z) \supset (\neg S \wedge \neg R)) \} : set.$	Deontic
<i>mumkin qarīb.</i> $\{ x : C \vee \neg C \mid (((\exists y : C) \text{ left}^V(x) =_C y) \supset E) \wedge (((\exists z : \neg C) \text{ right}^V(x) =_{\neg C} z) \supset (E \vee \neg E)) \} : set.$	Modal

<i>mubāḥ makrūh.</i> $\{ x : A \vee \neg A \mid (((\exists y : \neg A) \text{ right}^V(x) =_{\neg A} y) \supset R) \wedge (((\exists z : A) \text{ left}^V(x) =_A z) \supset (\neg S \wedge \neg R)) \} : set.$	Deontic
<i>Mumkin ba’id.</i> $\{ x : C \vee \neg C \mid (((\exists y : C) \text{ left}^V(x) =_C y) \supset \neg E) \wedge (((\exists z : \neg C) \text{ left}^V(x) =_{\neg C} z) \supset (E \vee \neg E)) \} : set.$	Modal

<i>mubāḥ mustawin.</i> $\{ x : A \vee \neg A \mid (((\exists y : A) \text{ left}^V(x) =_A y) \supset (\neg S \wedge \neg R)) \wedge (((\exists z : \neg A) \text{ right}^V(x) =_{\neg A} z) \supset (\neg S \wedge \neg R)) \} : set.$	Deontic
<i>Mumkin maḥḍ.</i> $\{ x : C \vee \neg C \mid (((\exists y : C) \text{ left}^V(x) =_C y) \supset (E \vee \neg E)) \wedge (((\exists z : \neg C) \text{ right}^V(x) =_{\neg C} z) \supset (E \vee \neg E)) \} : set.$	Modal

So far so good; but the second table does not make the underlying system of degrees explicit. We will tackle this issue in the next section.

II.3 Quantifying Degrees and Empirical Quantities: A Unifying Structure.

As pointed out by Oscar Sheynin (2017, p. 12), there are traces of probability concepts in logic at least from the time of Aristotle (e. g., *Physics* 196b 30), who speaks of the randomness of

⁴³ See our comment on necessary and sufficient conditions above.

the meeting of two friends; and there are several other studies concerning probability and statistics as found in other ancient and medieval sources, including the Talmud and the Stoics. Ibn Ḥazm's contribution to the subject is to employ degrees of possibility and deontic force as an instrument for comparing the logical structures of modal and deontic judgements.

Thus, if we provide some numbers for the preceding table, we obtain:

Modal	Deontic	Likelihood of Happening <i>h</i>
Near possibility <i>mumkin qarīb</i>	Recommended Permissibility <i>mubāḥ mustaḥabb</i>	$h > \frac{1}{2}$ $h \neq 1$
Distant possibility <i>mumkin ba'īd</i>	Reprehended Permissibility <i>mubāḥ makrūh</i>	$h < \frac{1}{2}$ $h \neq 0$
Pure possibility <i>mumkin maḥḍ</i>	Even Permissibility <i>mubāḥ mustawin</i>	$h = \frac{1}{2}$ $h \neq 1$ $h \neq 0$
Necessity <i>wājib</i>	Obligatory <i>wājib, farḍ, lāzim</i>	$h = 1$
Impossibility <i>mumtani'</i>	Forbidden <i>ḥarām</i>	$h = 0$

In the preceding sections we followed Ranta's (1994, pp. 54-57) proposal to include performances of actions as elements of action-types, but this formalization does not include degrees. Another possibility, then, is to employ the notion of *empirical quantity*. This notion allows the integration, within the hypothetical structure, of numerical parameters that reflect the degree. Moreover, with the help of the notion of empirical quantity we can formulate a single general hypothetical frame, such that each of the modal and deontic qualifications will be distinguished simply by the values of the parameters. And in fact this same frame can be used for both the deontic and the modal qualifications; it is only at the empirical level of the method used for determining the value of the empirical quantity that deontic concepts are distinguished from modal ones.

II.3.1 Values and Degrees of Modal and Deontic Force.

The notion of empirical quantity was introduced by Martin-Löf in applying CTT to the empirical realm (Martin-Löf, 2014): whereas quantities of mathematics and logic are determined by computation, empirical quantities are determined by experiments and observation. An example of a mathematical quantity is $2 + 2$; it is determined by a computation yielding the number 4. An example of an empirical quantity is the colour of some object. This is not determined by computation; rather, one must observe the object under normal conditions.⁴⁴

We can consider empirical quantities as answers to a question. For instance, given the question

Are the waters of the Guadalquivir blue?

⁴⁴ Cf. Rahman, McConaughey, Klev, and Clerbout (2018, chapter X.4).

The yes or no answer, obtained through some manner of empirical procedure received in a given context, can be defined over the set **Bool**; namely, as being equal to **yes** or **no**. The following question, however, might involve many different answers:

What is the colour of the waters of the Guadalquivir?

If X stands for the empirical quantity *colour of the waters of the Guadalquivir*, we might define the possible answers over some finite set \mathbb{N}^n of natural numbers:

$X = 1: \mathbb{N}^n$ if the colour of the waters of the Guadalquivir are brown

$X = 2: \mathbb{N}^n$ if the colour of the waters of the Guadalquivir are green

$X = 3: \mathbb{N}^n$ if the colour of the waters of the Guadalquivir are blue

...
 $X = n: \mathbb{N}^n$ if the Guadalquivir's waters are...

Certainly the question *Are the waters of the Guadalquivir blue?* can also be defined over a larger set, if several degrees of colours are to be included as an answer, or, alternatively, degrees of certainty (definitely blue, quite blue, slightly blue...). Let us assume, then, another set \mathbb{N}^j for the *degree* of colour:

$Y = 0_1: \mathbb{N}^j$ if the colour of the waters of the Guadalquivir are dark blue

$Y = 0_2: \mathbb{N}^j$ if the colour of the waters of the Guadalquivir are light blue.

$Y = 0_3: \mathbb{N}^j$ if the colour of the waters of the Guadalquivir are green-blue.

...
 $Y = 0_j: \mathbb{N}^j$ if the colour of the waters of the Guadalquivir are...

Recall that determining the value of the empirical quantity is an empirical procedure specific to that quantity; the result of carrying out such a procedure is determined by the rules set for determining that quantity. Moreover, the value of two different empirical quantities might be the same: the quantities only indicate that *the way of determining* the answer to the question might be the same. Take for example these two enquiries:

(1) Is Ibn Ḥazm the author of the *Taqrīb*?

(2) Is Ibn al-Haytham the author of the *Shukūk 'alā Baṭlamīyūs (Doubts Concerning Ptolemy)*?

These two enquiries involve determining the value of the empirical quantity X for (1) and Y for (2), which can be the same: they can both be **yes** for instance if the underlying set is **Bool**.

II.3.2 Introducing Dependent Empirical Quantities: Values and Degrees.

A crucial feature underlying Ibn Ḥazm's parallelism is that it not only employs a scale for the degree of occurrence, but that this degree is set in relation to a previous set of values. Being rewarded for performing an action indicates the value ascribed. More precisely, the values of two actions, both of which are rewarded if performed, might differ in relation on how sanction and reward are defined if not performed. If not performing an action is sanctioned, rather than just not rewarded, it indicates that performing it has the highest value.

It is at this point that the parallelism becomes more delicate. Let us take the examples of

*A condensing of clouds in the two months of Kānūn (December and January),
and
A condensing of clouds in May.*

The condensing of clouds is a condition for rain; however, the first (clouds in December and January) seems to have some kind of stronger, causal force than the second (clouds in May). Thus, while in the deontic domain the degree of virtue is dependent upon certain values (as determined by the distribution of reward and sanction), in the realm of events it is the empirical circumstances under which the condition is fulfilled that determine the empirical value of that condition: the measurement of its causal force.

This might suggest, as we will discuss in the next section, that the parallelism has a strong bias in favor of the deontic over the moral realm. Perhaps this is related to Knuuttila's (1993, p. 182) remark on the inverse parallelism of Peter Abelard, who spoke of nature as *allowing, forbidding or commanding, events to occur*.

Be that as it may, if we wish to express this interdependence of values and degrees within a CTT-framework, we will require the setting of some scale, and a function that takes us from values to degrees. And in order to work this out for both the deontic and natural realms, we need to have at our disposal empirical quantities that relate choices with the occurrence of future events.

In other words, we need to single out those empirical quantities that involve quantities of future events, the indicator then being whether or not the event occurs. Martin-Löf used such empirical quantities of future events for dealing with Aristotle's sea-battle puzzle: the question

Will a sea-battle take place tomorrow?

can have two answers,

X = yes tomorrow a sea-battle will take place (the event will occur), or
X = no tomorrow a sea-battle will not take place (the event will not occur).

If we replace the empirical quantity **X** by a variable **x**, we obtain the following hypothetical (in the form of a hypothetical):

$$(Id(\mathbf{Bool}, \mathbf{x}, \mathbf{yes}) \vee Id(\mathbf{Bool}, \mathbf{x}, \mathbf{no})) (\mathbf{x} : \mathbf{Bool})$$

Now, since every element of **Bool** is either identical to **yes** or to **no**, this also applies to empirical quantities **X**, which are to be understood as non-canonical elements of **Bool**.

A first interpretation

The idea is that we can express the full parallelism with only one hypothetical, such that:

If some empirical quantity is evaluated in relation to some scale, by discovering whether some event occurred or not, or if it is evaluated within that same scale by discovering whether some action is rewarded / sanctioned in a specific manner, then there is another empirical quantity, dependent upon the first, that renders the degree of likelihood either as an event to happen or the degree of virtue (that is the moral and legal value) ascribed to that kind of actions.

In order to implement this idea, let us now go a step further and introduce the set **D** constituted by the canonical elements **1, -1, 2, -2, 3, -3, 4, -4, 5, -5** and some empirical quantities **X, Y, U, V, W, X*, Y*, U*, V*, W***. These empirical quantities represent non-canonical elements of **D**. Their evaluation requires an empirical procedure specific to that quantity, such that if some condition (say, C_1) happens, then there is an evaluation of the corresponding quantity (say, **X** is **1**); if, for example, some action A_4 is rewarded when not performed, then the evaluation of **V*** is **4**.

Let the evaluation of **X** be

1 if some event C , conditioning event E , took place under some specific circumstances N ,
and **-1** if not.

Let the evaluation of **Y** be

2 if some event C , conditioning event E , took place under some specific circumstances I ,
and **-2** if not.

Let the evaluation of **U** be

3 if some event C , conditioning event E , took place under some specific circumstances P^+ ,
and **-3** if not.

Let the evaluation of **V** be

4 if some event C , conditioning event E , took place under some specific circumstances P^- ,
and **-4** if not.

Let the evaluation of **W** be

5 if some event C , conditioning event E , took place under some specific circumstances P_3 ,
and **-5** if not.

Let the evaluation of **X*** be

1 if action A has been performed, and performing action A is rewarded,
and **-1** if action A has been omitted, and A is sanctioned when not performed.

Let the evaluation of **Y*** be

2 if action A has been omitted, and A is rewarded when not performed,
and **-2** if action A has been performed, and A is sanctioned when performed.

Let the evaluation of **U*** be

3 if action A has been performed, and A is rewarded when performed,
and **-3** if action A has not been performed, and if A is neither rewarded nor sanctioned when
not performed.

Let the evaluation of **V*** be

4 if action A has been omitted, and if action A is rewarded when not performed,
and **-4** if action A has been performed, and A is neither rewarded nor sanctioned when
performed.

Let the evaluation of **W*** be

5 if action A has been performed, and A is neither rewarded nor sanctioned when performed,
and **-5** if action A has been omitted, and action A is neither rewarded nor sanctioned when
not performed.

We will now associate with these empirical quantities another list of ten empirical quantities: **Z₁, Z₂, Z₃, Z₄, Z₅, Z*₁, Z*₂, Z*₃, Z*₄, Z*₅** that are elements of the finite set $\mathbb{N}^{\{0, \dots, 9\}}$ (containing 10 canonical elements). Each of these fresh empirical quantities represents the likelihood (measured by canonical elements of $\mathbb{N}^{\{0, \dots, 9\}}$) of some specific event E to occur, or the degree of virtue of some action A when performed:

\mathbf{Z}_1 measures the likelihood of event E_1 to occur, or not to occur, depending upon the occurrence / non-occurrence of action A_1 .

\mathbf{Z}^*_3 measures the degree of virtue (i.e. the moral and legal value) of A_3 , depending upon the distribution of sanction and reward.

The association is carried out by the function $g(\mathbf{x})$, from **Bool** to $\mathbb{N}^{\{0, \dots, 9\}}$, such that:

If $\mathbf{X} = \mathbf{1}$, then $g(\mathbf{X}) = \mathbf{Z}_1 = 9$ (it is certain that the event E_1 occurs)

If $\mathbf{X} = -\mathbf{1}$, then $g(\mathbf{X}) = \mathbf{Z}_1 = 0$ (it is certain that the event E_1 does not occur).

Similarly for the other cases of events. As for actions, let us take the case of recommended permissibility:

If $\mathbf{U}^* = \mathbf{3}$, then $g(\mathbf{U}^*) = \mathbf{Z}^*_3 > 4$ and $\mathbf{Z}^*_3 < 9$

If $\mathbf{U}^* = -\mathbf{3}$, then $g(\mathbf{U}^*) = \mathbf{Z}^*_3 = 4$ (it is as virtuous for action A_3 to be performed as not).

- Notice that the scale allows the system to quite easily be extended in order to introduce a notion of “permissible to do” that also includes obligatory actions. It is sufficient to define a \mathbf{Z}^* bigger than 4 and smaller or equal to 9.

Further notational convention:

Now let *yes* stand for some positive element \mathbf{D} (for instance $\mathbf{2}$), and *no* for some negative element of that set (for instance $-\mathbf{2}$).⁴⁵ So, if \mathbf{d} is a variable for some empirical quantity in \mathbf{D} , and the function $g(\mathbf{d})$ is identical to some fixed positive (canonical) element of the set, then the evaluation of the function depends on the positive number which *yes* stands for, according to the setting of the function described above.

This renders the following hypothetical that makes use of the degrees in order to unify the realms of events and actions:

$$(\mathbf{Id}(\mathbf{D}, \mathbf{d}, \mathbf{yes}) \supset \mathbf{Id}(\mathbb{N}^{\{0, \dots, 9\}}, y, g(\mathbf{d}))) \wedge (\mathbf{Id}(\mathbf{D}, \mathbf{d}, \mathbf{no}) \supset \mathbf{Id}(\mathbb{N}^{\{0, \dots, 9\}}, y, g(\mathbf{d}))) (\mathbf{d} : \mathbf{D})$$

For instance:

Assume that we have evidence that C occurred, and that in the scale it evaluates with one of the positive numbers, say, $\mathbf{3}$ – i.e., assume that the variable \mathbf{d} is identical to the empirical quantity \mathbf{U} , and its evaluation yielded the value $\mathbf{3}$. Then the function will yield that some corresponding event, E , will be likely to happen – i.e., the function will yield as degree a number bigger than 4 and smaller than 9.

Assume that we have evidence that A has been omitted, and that in the scale it evaluates with one of the positive numbers, say, $\mathbf{2}$ – i.e., assume that the variable \mathbf{d} is identical to the empirical quantity \mathbf{Y}^* , and its evaluation yielded the value $\mathbf{2}$. Then the function will yield that some corresponding action, A , has the highest possible degree of virtue – i.e., the function will yield as degree the highest number 9.

⁴⁵ Strictly speaking, *yes* and *no* should be introduced as codes within universes, but we will not employ this notion in the present paper.

It is true that this formalism goes beyond Ibn Ḥazm's text. However, according to our point of view, it renders in a compact form how the notion of degree constitutes the bridge from the possible to the permissible. Moreover, as mentioned in the introduction, if we dare to go a step further than the textual evidence, there is a more direct way to unify both systems of ascribing degrees; namely, by paralleling the likelihood of an event to become actual with the likelihood for an action to be performed.

A second interpretation: Likelihood for events or actions to become actual

If not performing an action is sanctioned, rather than just not rewarded, it indicates that performing it has the highest moral and legal value; and one may induce from this the assessment that it is very likely—in fact, certain—that the just agent will choose to perform it if given the choice. From this perspective, that an action is set as being more likely or less likely to be performed rests on the juridical value ascribed to this action. Thus, if we follow this interpretation, the preceding system yields a direct form for establishing a parallelism; namely, the likelihood for an event to occur finds its counterpart in the likelihood for an action to be performed. This yields the following reformulation of the underlying hypothetical, although the resulting formalism remains the same:

If some empirical quantity is evaluated in relation to some scale, by discovering whether some event occurred or not, or if it is evaluated within that same scale by discovering whether some action is rewarded / sanctioned in a specific manner, then there is another empirical quantity, dependent upon the first, that renders the degree of likelihood either as an event to happen or some action to be performed.

In fact, our formalization also shows the shortcomings involved in setting such a system of degrees. Indeed, while the “prior” system of values for actions can be easily set, since it is based on the distribution of reward and sanction fixed by the legal system, setting the “prior” values which measure the causal force of a condition is left unexplained. The degree-system does not provide a way to calculate the conditional probability of an event. It just sets a fixed number for the likelihood of an event to happen, given the fulfilment of a condition set as specific to that event: the degree system employed by Ibn Ḥazm is no substitute for Bayes' rule. Nevertheless, the following is worth mentioning:

1. For the sake of drawing the parallelism, Ibn Ḥazm does not need to render a theory of how the prior values have been established in the realm of events. It is sufficient that both deontic and modal qualification share one system of values.
2. Moreover, as we can learn from the history of Bayes' rule, the difficulty of identifying and evaluating the adequate condition for an event to occur is a problem for any theory of conditional probability.

Both points seems to involve the notion of measuring the passage from the potential to the actual. This background seems to be quite close to Leibniz's notion of *facilius*, which we will now briefly discuss.

III. Leibniz and Hypothetical Imperatives in Law.

Assume an action is performed by an individual agent, let us call him the *benefactor*, in favour of another individual, the *beneficiary*, such that the action performed by the benefactor makes some legal claim accessible to the beneficiary.⁴⁶

Such actions can be performed through contracts. For instance, a contract by the means of which the benefactor, let us call him *Primus*, commits himself to pay 100 dinars to the beneficiary, *Secundus*. The contract can establish that the legal claim (in our example the obligation to pay) is dependent upon the fulfilment of some condition (such as *the arrival of a ship from Asia*). This presupposes the legal validity of the contract, and that benefactor and beneficiary agree on the content of the legal claim and on the terms in which this claim has been made dependent upon the condition fixed by the contract. In our example this amounts to:

*Primus must pay 100 dinars to Secundus,
provided that a ship arrives from Asia.*

Now, given such a contract where an obligation is made dependent upon the fulfilment of a condition, what is its legal effect?

Does it mean that the new legal claims provided by the contract will only be accessible to the benefactor in the future?

Or does the contract make these legal claims already accessible to the benefactor?

According to the first view, the effect of the condition is a “*suspension*”, so that as long as the condition is not fulfilled, no new legal claims are accessible to the benefactor. According to the second view, however, as long as the condition is not fulfilled (but still can be fulfilled), the suspension already provides the benefactor with new legal claims. A conditional right, in other words, is more than the mere hope of a future entitlement.

At the early age of 19 Leibniz launched a logical analysis of the notion of conditional right which favours the view that the legal claim provided by a conditional right is already accessible to the beneficiary at the moment of the contract, despite the fact the condition is suspensive.⁴⁷

On Leibniz’s view what makes of a conditional right a conditional is its logical structure, which takes the form of a hypothetical judgement. According to this analysis, the dependence of condition and conditioned amounts to the dependence of the truth of the **jus** (the consequent of the hypothetical, the legal obligation) on the truth of the **fact** (the antecedent of the hypothetical).

⁴⁶ The present section is based on certain previous publications on Leibniz’s legal reasoning, such as Rahman (2015), and some work in progress by Rahman and Nordtveit Kvernenes (2018).

⁴⁷ More precisely, during the period 1664-1669, the young Gottfried Wilhelm Leibniz (1646-1716) studied the theory of law with the prolific creativity that made him famous. It is during this period that Leibniz developed his theory of conditional right in two main texts that furnished the content of two academic dissertations:

- (1) The *Disputatio Juridica (prior) De Conditionibus* (A VI, I, pp. 97-150), which was defended in July and August 1665. At that time Leibniz was a 19-year-old student, bearing the title of Master of Philosophy since February 1663 thanks to his *Disputatio Metaphysica De Principio Individui*, defended in December 1662.
- (2) The *Disputatio Juridica (posterior) De Conditionibus* (A VI, I, pp. 97-150), which is part of Leibniz’s *Specimina Juris* (1667-1669).

A modified version is developed in *The Specimen Certitudinis Seu Demonstrationum In Jure, Exhibitum In Doctrina Conditionum* (A VI, I, pp. 367-430), which is also part of Leibniz’s *Specimina Juris* (1667-1669), a compilation and reformulation of three of his already held disputations: the *Disputatio Inauguralis De Casibus Perplexis In Jure*, that granted Leibniz the doctoral degree in November 1666. The prior and posterior disputations constitute the main source of the present discussion.

Furthermore, what distinguishes the hypothetical structure underlying conditional right from other forms of hypothetical judgement is that the dependence of the head and the tail of the hypothetical is not defined by some kind of natural causality or purely logical necessity, but it has been introduced by the will of the *arbiter* (the benefactor) and has thus also a *moral* feature – that is why, when he refers to the structure underlying conditional rights, he calls them *moral conditionals*.

Therefore, and coming back once more to our ship-example, the logical structure of conditional rights requires only the following:

The arrival of the ship must be assumed to be contingent and unknown at the moment of the contract,

but also,

If we know that a ship does not come, it should be inferred that it is not the case that *Primus* must pay.

In order to implement this double task, Rahman (2015) proposed the following logical analysis of Leibniz's approach, by means of which the obligation is made dependent upon a future contingent condition:

*If there is some evidence for a ship arrival, and this arrival solves the uncertainty (S or not S) underlying the conditional right (i.e., if the ship arrival provides an evidence for the **left side** of the disjunction), then the beneficiary is entitled to the payment agreed by the terms of the contract.*

*If there is some evidence for no ship arrival, and this solves the uncertainty (S or not S) underlying the conditional right (i.e., if the evidence for no ship arrival provides an evidence for the **right side** of the disjunction), then the beneficiary is not entitled to the payment agreed by the terms of the contract.*

If we pull all this together, we obtain:

$$b(x) : (((\exists y : S) \mathbf{left}^{\vee}(x) =_S y) \supset P) \wedge (((\exists z : \neg S) \mathbf{right}^{\vee}(x) =_{\neg S} z) \supset \neg P) (x : S \vee \neg S).$$

It is apparent that this expression is, structurally speaking, very close to the hypothetical imperatives for obligatory actions and those for necessary events described in the preceding sections.

That is:

wājib, farḍ, lāzim (deontic):

$$a(x) : (((\exists y : L) \mathbf{left}^{\vee}(x) =_L y) \supset R) \wedge (((\exists z : \neg L) \mathbf{right}^{\vee}(x) =_{\neg L} z) \supset S) (x : L \vee \neg L)$$

wājib (modal):

$$b(x) : (((\exists y : C) \mathbf{left}^{\vee}(x) =_C y) \supset E) \wedge (((\exists z : \neg C) \mathbf{right}^{\vee}(x) =_{\neg C} z) \supset S) (x : C \vee \neg C)$$

More generally; this structural similarity is independent of the logical framework employed by the formalization. This might confirm von Wright's (1981b, p. 34) dictum that the logical analysis of norms of the philosophical tradition is a logic of hypothetical imperatives, rather than a

deontic one. In any case, it is clear that the emergence of the parallelism in Leibniz is the context of jurisprudence and the employment of Stoic logic within it.

At this point in the discussion it might not be much of a surprise that both Ibn Ḥazm's and Leibniz's parallelism are based on hypothetical imperatives. A closer look at the structure of the parallelism indicates that the similarity between Leibniz's and Ibn Ḥazm's approaches to the logical analysis of norms runs deeper yet.

Recall that, according to our analysis of Ibn Ḥazm's parallelism, in the realm of jurisprudence *likelihood of performing an action* is dependent upon the distribution of reward and sanction specific to that action, while in the realm of nature *likelihood of occurrence an event* is dependent upon the degrees of causal force of the conditions specific to that event. The point of the parallelism is that both deontic and modal qualifications measure the degree of an action or event to become *actual*, the degree measures how *feasible (facile)* an action or event is.

Now, the expression denoting this, *feasibility*, or *realizability*, is part of Leibnizian terminology. The precise term used by Leibniz is *facile* and the comparative *facilius*. He developed this in several works, but mainly in the *Elementa Juris Naturalis* (1671), the *De legum interpretatione, rationibus, applicatione, systemate* (1678/1679), and the *Nouveaux Essais sur l'entendement humain* (1704, published posthumously in 1765). Ian Hacking (1975; pp. 124-127), points out that Galileo spoke of events happening with *equal ease (aeque facile)*, and that Leibniz defines *equally easy* as *equally possible*.

In fact, Leibniz explains in a letter to Bourget in 1714 that *facile* stems from *faciendo*. Thus, what is more easy, means what is more possible, and this in the end is more *feasible*. Armgardt (2015) shows how the notion of *more easy* relates to probability, and, more specifically, to Leibniz's (1678) remarkable dictum: *Probabilitas est gradus possibilitatis*. Relevant to our reflections is the fact that, in this context, Leibniz associates the comparative *facilius* (more easy, more feasible) with the causal power of various conditions to trigger various events.

Summing up, one might wonder if the notion of feasibility of the 16th century, at the origins of the emergence of modern probability, with its difficulties and early hesitations between an epistemic and objective probability, is not an offspring of transferring the system of values from the deontic to the natural realm.

Indeed, such transference is made possible because the degree in the causal power to achieve events mirrors the degree in the legal and moral value of actions. In fact, in our view, this is exactly what is transpiring in Leibniz's (1671) different attempts to study just actions by means of what is more feasible.⁴⁸

If, in this context, it is correct to say that the origin of conditional probability stems from Leibniz's interest in legal reasoning,⁴⁹ then we might say that, perhaps unknown to Leibniz himself, Ibn Ḥazm had already pointed out the way.

Notice that, according to Leibniz's own observation, *facile* stems from doing, and thus feasibility/realizability has a normative character: *what you can do*. The very notion of possible is associated with what *can be performed*, and this is already, so we claim, in the roots of the traditional view on inference in particular, and of logic in general.

⁴⁸ See Armgardt (2015) for a thorough study of the interplay between probability, *facilius*, and just actions.

⁴⁹ Cf. Hacking (pp. 135-142).

The deontic concepts of both Ibn Ḥazm and Leibniz are rather more *Tun-sollen* (ought to do) than *Sein-sollen* (should be).⁵⁰ The parallelism involves mirroring the logic of ought to do, concerned with human behaviour, in states that are not in principle influenced by human actions. In other words, the contingency of our choices to carry out, or not to carry out, some action, is mirrored by the actualization of a future contingent event. The contingency of the passage from the potential to the actual is what grounds the parallelism and opens the way to a suitable theory of measurement.

Closely related to the *ought-to-do* perspective is the emphasis on practical knowledge displayed in the *Taqrīb*. Indeed, as thoroughly discussed by Puerta Vílchez (2013), Ibn Ḥazm's conception of science (*al-ilm*) involves a notion of human knowledge that includes not only all that can be grasped by the senses and the intellect, but also the crafts. Moreover, Puerta Vílchez (2013, pp. 254-256) further points out that, in relation to the way of integrating practical knowledge within his conception of science, Ibn Ḥazm goes considerably beyond the tradition of Islamic commentators on Aristotle, by stressing the role of practical knowledge for learning and understanding, and by developing at the same time a conception of language suitable for this role.⁵¹

More generally, in our view Ibn Ḥazm's parallelism not only highlights the fact that one form of normativity mirrors the other, but it opens the door to a general notion of normativity, if we are prepared to understand causal necessity as involving an epistemic dimension: giving reasons for our assertions. This brings us to our final section.

IV Beyond Ibn Ḥazm: Conclusions and the Work Ahead.

I ought not to seek or conjecture either of them as if they were veiled obscurities or extravagances beyond the horizon of my vision. I see them before me and connect them immediately with the consciousness of my existence.

Immanuel Kant, *Kritik der praktischen Vernunft*, 1788, Beschluss.

IV.1 Brief Remarks on Ibn Ḥazm's Heteronomous Imperatives and Deontic Logic.

As mentioned more than once, we share von Wright's (1981b, p. 34) qualification of traditional logical analysis of norms concerning

[...] *structures resembling what Kant called hypothetical imperatives*

though we certainly take exception to the remark that such approaches do not constitute a *genuine deontic logic* – if, that is, the remark cannot be reduced to the trivial assertion that the traditional logic of norms is different from the analysis delivered by contemporary formal semantics. Our approach does not employ obligation as an operator, but rather as a special kind of inference. In our framework, the embedding of obligations can have sense, but only in contexts where rewards and sanctions are structured by dependences. For instance, the reward for performing action *A* is made dependent upon performing action *B*, too.

⁵⁰ For a thorough discussion regarding a contemporary view on the differences between these two, see Hansson (2013, p. 197).

⁵¹ Puerta Vílchez (2013) acknowledges throughout his paper that this point has already been stressed by Arnaldez (1956). The difficulty with this perspective is that, on the one hand, practical knowledge suggests dynamics, but on the other, Ibn Ḥazm's conception of language is quite static. For a classical text on this issue see Arnaldez (1956, p. 222); and for new studies on the static notion of language see Adang, Fierro, and Schmidtke, eds. (2013, part III).

And it is true that our incipient exploration does not yet deliver a logic of norms, but only a logical analysis of relevant notions.⁵² Notice however, that in such a framework some known paradoxes of current standard deontic logic do not arise.

For example, Ross's paradox (1941) becomes harmless in the logic of deontic imperatives. The paradigmatic example of this paradox is based on the fact that we can infer it is obligatory to send a letter, or to burn it, from the premise that it is obligatory to send it. But in the framework of heteronomous imperatives this paradox does not arise.

Recall our full formalization of obligatory:

$$\{ x : A \vee \neg A \mid ((\exists y : A) \text{left}^\vee(x) =_A y \supset R(y)) \wedge ((\neg A \wedge \neg E) \text{right}^\vee(z) =_{\neg A \wedge \neg E} z) \supset S(z) \} : \text{set.}$$

If action of type A has been chosen to be performed, and we know that this performance is rewarded (and we also know that omitting to perform it [without a valid excuse] is sanctioned), the weakening of the consequent is harmless (if well formed). Either the weakening to the right adds the reward to the performance of a new action, or it is a second performance of the first. The latter case is unproblematic. For the first case, however, if the fresh reward-assertion is well formed, then it must already be present in the disjunction that constitutes the hypothetical. In other words, the new reward must in this case presuppose that carrying out actions of that type is rewarded. So weakening of the consequent assumes that the disjunction has been extended, but this is a non-sequitur.

And notice that weakening the antecedent is also harmless. If carrying out action A is rewarded, adding action of type B does not support the inference that B will be rewarded.

In fact, in our reconstruction of heteronomous imperatives the so-called *inheritance principle* from standard deontic logic⁵³ does not hold. According to this principle, if it is a theorem that A implies B , we can infer that this implication is obligatory. But in our framework logical inference does not influence the distribution of reward and sanction. Something similar applies to the *equivalence rule*:⁵⁴ if it is a theorem that a certain bi-implication holds, according to our reconstruction, this does not have bearings on the distribution of rewards and sanctions involving that bi-implication.

Finally, to mention one more paradox: Tautologies are not obligatory, since the very definition of hypothetical requires a disjunction yet without proof-object; and the disjunction must be a disjunction of future contingents. Moreover, *every action* presupposed by the formation of a hypothetical expressing obligation is contingent.⁵⁵

IV.2 Beyond Ibn Ḥazm's Parallelism: *Qiyās* and the inferential structure of Imperatives.

⁵² As pointed out by Hilpinen and McNamara (2013, pp. 25-31), contemporary Deontic logicians have often made a distinction between two interpretations of deontic sentences. It has been suggested that a deontic sentence p can be interpreted normatively (or prescriptively) as expressing a mandatory norm, or descriptively as a statement that it is obligatory that p , according to some unspecified system of norms or law (see von Wright 1963, viii, pp. 104-5), Stenius (1963, pp. 250-1), Alchourrón (1969, pp. 243-5), Alchourrón and Bulygin (1971, p. 121). It might be argued that our framework is closer to the descriptive interpretation – though perhaps our distinction between the type of action and its performance might offer some middle-way.

⁵³ See Hilpinen and McNamara (2013, p. 38).

⁵⁴ Ibid.

⁵⁵ For an early treatment of these discussions, see, among others, Chellas (1974) and al-Hibri (1978, pp. 18-20). For a contemporary survey see Hilpinen and McNamara (2013, pp. 67-69).

It is interesting that Ibn Ḥazm's parallelism fits almost perfectly with the general structure of inferences via parallel reasoning and analogy subsumed by the term *qiyās* and developed with skill and sophistication by the *Shāfi'īs* and other schools of Islamic jurisprudence.⁵⁶ It is particularly significant because, despite having adhered to the Shāfi'ī *madhhab* earlier in his career, he eventually converted to the Zāhirī *madhhab*—in fact becoming its most famous proponent—and rejected with vehemence any form of reasoning by parallelism or analogy, a stance in keeping with the radically literalist Zāhirī approach to the interpretation of the sacred sources.

Nevertheless, let us recall that his main motivation for the parallelism was to translate into juridical terms the logic of the Greek tradition. He did not mean to employ parallelism as a form of reasoning to interpret the sacred texts, but as a way to bring together the approaches to necessity and normativity of the Greek tradition and those of Islamic jurisprudence. It might also be argued that the parallelism which is the subject of our paper occupies a completely different level from the sort of parallelism which he rejects. Still, when Ibn Ḥazm defends the study of logic, he defends it for its fruitfulness in Jurisprudence.

On the other hand, Ibn Ḥazm's literalist approach is developed within a framework where, as already mentioned, language is conceived quite statically, and where epistemological fictionalism—and in particular the use of *conjecture* for attaining knowledge—is rejected.⁵⁷ This would seem to disqualify our suggested approach to understanding the parallelism as achieved by some manner of parallel reasoning.

If we seek to locate an understanding of the parallelism within Ibn Ḥazm's own epistemology, we should say that it is *made apparent by a sole act of consciousness*. In fact, we do not need to employ Kantian terminology—Ibn Ḥazm's epistemology includes the notion of *rational perception* (*idrāk al-'aql*), an immediate intellectual act of knowledge, which is closer to the concept of *immediate inference* of contemporary constructivists than it is to Cartesian intuition.⁵⁸ This immediate act of knowledge produces rational understanding (*fahm*).⁵⁹

Now, despite its rational and logical nature, *rational perception* is not devoid of sense perception. Arnaldez (1956, p. 128) has pointed out that within epistemology there is always reasoning at the level of the senses. Rational perception is, accordingly, a kind of immediate act of abstraction.⁶⁰

In such an epistemological framework, the correspondence between the world of events and the world of actions seems to have a natural place. Does Ibn Ḥazm's parallelism support a kind of *naturalization* of philosophy, of the sort that is currently spreading everywhere? We do not think so. Or at least, this is not the only way to understand it.

IV.3 On Normativity the Other Way Round and The Internalization of Nature.

⁵⁶ For a comprehensive study of the *qiyās* theory of the Shāfi'ī Abū Ishāq al-Shīrāzī, see Young (2017). Rahman and Iqbal (2018a,b) developed a formalism based on Young's study, already employing the structure of hypothetical imperatives.

⁵⁷ See Puerta Vélchez (2013, pp. 298-300).

⁵⁸ Ibid. (pp. 297; 300).

⁵⁹ Notice that our reconstruction of Ibn Ḥazm's epistemological perspective goes beyond what is suggested by our opening quote from Kant. Kant states that the laws of nature and the moral law are grasped in an immediate act of consciousness; however, he does not suggest, as Ibn Ḥazm's notion of *idrāk al-'aql* does, that with this act we also grasp how natural and moral necessity mirror each other.

⁶⁰ See Puerta Vélchez (2013, p. 301).

It is hoped the aim of the present study to suggest an alternative perspective to the logic of norms has been achieved. As already mentioned, our proposal has not yet been developed into a logical system; we will now briefly mention the philosophical concepts grounding that project.

What at least the first author of the present paper is ready to claim is that the logic of norms should be built on a conception where logic itself is understood as normatively constituted. This is the reason why traditional logic involving deontic concepts is so close to inference itself. The point is that traditional logic, from Plato and Aristotle, through the Stoics and the Islamic tradition to Leibniz, Kant, Bolzano and even Frege, is about inference, and inference is understood as being normative. Let us briefly elaborate on this point that, it is believed, should be part of future research.

Logical approaches to meaning subsequent to the work of Frege and Tarski are based on what we might call, in a general manner, the *semantization of pragmatics* (SP) – contexts are formalized with the help of some specific indexes, usually in the metalanguage, upon which the truth-value is made dependent. According to this view, a propositional kernel is complemented by “modalities” expressing necessity, commands, temporality and knowledge. These modalities are defined then as truth-functional operators conceived as certain kinds of logical connectives.

As pointed out by Per Martin-Löf (2017, p. 9), standard approaches to the layers underlying logic got the order of priority between deontic and epistemic notions the wrong way round. Martin-Löf’s remark is motivated by his fine analysis of the dialogical conception of logic, particularly the one proposed by Paul Lorenzen (1958), whose point is that—different to other standard logical approaches—dialogical logic got the order right.⁶¹

On the other hand, deontic logic deals only with assertions, just like in standard logic, it’s only assertions whose content contains the deontic operations. This puts an opposite order between the deontic notions and the epistemic ones than the one that I have been advocating here, namely that the deontic notions make their appearance in the analysis of what it means to know how to do something: that’s where the deontic notions have their place. So, the deontic notions are at a more basic level than the epistemic notion of knowledge-how, whereas in deontic logic it’s the other way around: by putting the deontic notions in the content and retaining the ordinary assertoric force, which is the epistemic reading here, the deontic notions are subordinated instead to the epistemic notions. What I have advocated here, you could say, is a deontic basis of ordinary assertoric logic, rather than basing deontic logic on ordinary logic. Maybe this is the explanation for the difficulties of deontic logic: I mean, you have formal rules, but to convince yourself that these formal rules make good sense is a notorious problem, and maybe it can be explained in this way, that one has got the order of priority between the deontic notions and the epistemic notions the wrong way around.

Per Martin-Löf (2017, p. 9).

According to the dialogical approach, knowledge and meaning emerge as a fine intertwining of having the *right to ask for reasons* and the *duty to give them*, to put it in the words of Brandom (2000). From this perspective, not only are inferences understood as the duty to give reasons for supporting a conclusion, the interface right-duty is constitutive of the judgements that structure an inference.

⁶¹ For a comprehensive presentation of the dialogical conception of logic see Rahman, McConaughy, Klev, and Clerbout (2018); and for a discussion on normativity and deontic force within this framework, see chapter XI.5 in the same work.

Thus, the traditional view on the logic of norms as related to hypothetical imperatives is only a consequence of this overall normative view on knowledge and reasoning. The lesson of the parallelism between the *laws governing the starry heavens above me and the moral law within me*, is that in order to achieve knowledge both have to be internalized by the same act: they must be *placed in the space of reasons*.⁶² This internalization is one way to understand the inverse parallelism that ascribes moral features to nature, and is part and parcel of Ibn Ḥazm's notion of *rational perception (idrāk al- 'aql)* mentioned above.

To know about our world requires to know *why* the events in the world are linked the way they are. But *knowing why* comes always together with *knowing how*: we need to bring to the fore, to the tribunal of public criticism, how those reasons fulfil our obligation to justify our assertions. And this is no different to the way we provide reasons for the guiding of our actions.

Still, the logical structure of Ibn Ḥazm's heteronomous imperatives is hypothetical. Moreover, the parallelism is based on degrees of knowledge rather than on certainty. This is a puzzle within Ibn Ḥazm's own epistemology to which our paper does not provide an answer, besides the modest observation that if rational perception is targeted at the parallelism Ibn Ḥazm has bequeathed us, we might grasp in a sole act of consciousness that epistemology admits degrees after all.

Acknowledgments: We would like to thank Ahmad Hasnawi (CNTRS, SPHERE) and Michel Crubellier (Lille, Philosophie) for their wise comments and advice during fruitful exchanges with S. Rahman. Many thanks, too, to Muhammad Iqbal (Lille, Philosophie, UMR-STL) and Mohammad Shafiei (Shahid Beheshti University, Teheran) for contributing fruitful discussions.

⁶² This beautiful notion of the *space of reasons* stems from Sellars (1991, pp. 129-194).

Appendix.

Extract from Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Alfāz al-‘Ammiyya wa-l-Amthila al-Fiqhiyya*, ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī, (Beirut: Manshūrāt Muḥammad ‘Alī Baydūn, Dār al-Kutub al-‘Ilmiyya, 2003), pp. 83-84.

Translated by Walter Edward Young.

Chapter on Elements (‘ <i>anāṣir</i> ’)	باب العناصر
Know that the elements (‘ <i>anāṣir</i> ’) of all things (<i>ashyā</i>)—that is, their classes with regard to making assertions (<i>ikhbār</i>) about them—are of three classes, there being no fourth.	اعلم ان عناصر الأشياء كلها أي أقسامها في الاخبار عنها ثلاثة أقسام لا رابع لها
[They are] either necessary (<i>wājib</i>), being such as are necessary and manifest, or from among such as must be, like the rising of the sun each morning, and the like of that, such being called in God’s laws ‘obligatory’ (<i>farḍ</i>) and ‘binding’ (<i>lāzim</i>);	إمّا واجب وهو الذي قد وجب وظهر أو ما يكون مما لا بد من كونه كطلوع الشمس كلّ صباح وما أشبه ذلك وهذا يسمى في الشرائع الفرض واللازم
or possible (<i>mumkin</i>), being such as might be and might not be, like our anticipation that it will rain tomorrow, and the like of that, such being called in God’s law ‘lawful’ (<i>ḥalāl</i>) and ‘permitted’ (<i>mubāḥ</i>);	وإما ممكن وهو الذي قد يكون وقد لا يكون وذلك مثل توقعنا أن تمطر غداً وما أشبه ذلك وهذا يسمى في الشرع الحلال والمباح
or impossible (<i>mumtani</i>) being such as to which there is no path, like a human’s remaining under water for an entire day, or his living a month without food, or his walking in the air without some cunning artifice, and the like of that. And this is the type of thing that, if we saw it manifest in a human, we would know he is a prophet; and this class is called in God’s laws ‘forbidden’ (<i>ḥarām</i>) and ‘prohibited’ (<i>maḥzūr</i>).	وإما ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا حيلة وما أشبه ذلك وهذه التي إذا ظهرت من إنسان علمنا أنه نبي وهذا القسم يسمى في الشرائع الحرام والمحظور
Furthermore, the possible (<i>mumkin</i>) is divided into three classes, there being no fourth:	ثم الممكن ينقسم اقساماً ثلاثة لا رابع لها
the nearly possible (<i>mumkin qarīb</i>), like the possibility of occurrence of rain upon a condensing of clouds in the two months of <i>Kānūn</i> , ⁶³ or the victory of a large number of the courageous over a small number of the	ممكن قريب كإمكان وقوع المطر عند تكاثف الغيم في شهري كانون وغلبة العدد الكبير من الشجعان

⁶³ I.e., December and January.

cowardly;	العدد اليسير من الجبناء
and the distantly possible (<i>mumkin ba 'īd</i>), which is like the defeat of a large number of the courageous at the hands of a small number of the cowardly, and like a cupper (<i>hajjām</i>) [i.e., a practitioner of cupping] taking charge of the Caliphate, and the like of that;	وممكن بعيد وهو كانهزام العدد الكثير من الشجعان عند عدد يسير من جناء وكحجام يلي الخلافة وما أشبه ذلك
and the purely possible (<i>mumkin maḥḍ</i>), whose two extremes are equal, such being like one standing—either he will walk or he will sit—and the like of that.	وممكن محض وهو يستوي طرفاه وهو كالمراء الواقف إما يمشي وإما يقعد وما أشبه ذلك
And likewise we find that this middle class [i.e., the <i>mumkin</i> , corresponding to the <i>mubāḥ</i>] is, in God's laws, divided into three classes: recommended-permitted (<i>mubāḥ mustaḥabb</i>); reprehended-permitted (<i>mubāḥ makrūh</i>); and evenly permitted (<i>mubāḥ mustawin</i>) having no tendency towards one of the two sides.	وكذلك نجد هذا القسم المتوسط في الشرائع ينقسم أقساماً ثلاثة فمباح مستحب ومباح مكروه ومباح مستوي لا ميل له إلى أحد الجهتين
As for recommended-permitted (<i>mubāḥ mustaḥabb</i>), it is such that when you do it you are rewarded (<i>ujirta</i>), but if you neglect it you do not sin (<i>lam ta'tham</i>) and you are not rewarded; like praying two supererogatory prayer-cycles, voluntarily.	فأما المباح المستحب فهو الذي إذا فعلته أجزت وإذا تركته لم تأثم ولم تؤجر مثل صلاة ركعتين نافلة تطوعاً
And as for reprehended-permitted (<i>mubāḥ makrūh</i>), it is such that when you do it you do not sin and you are not rewarded, but if you neglect it you are rewarded; and that is like eating while reclining, and the like.	وأما المباح المكروه فهو الذي إذا فعلته لم تأثم ولم تؤجر وإذا تركته أجزت وذلك مثل الأكل متكئاً ونحوه
And as for evenly permitted (<i>al-mubāḥ al-mustawī</i>), it is such that when you do it or you neglect it you do not sin and you are not rewarded; and that is like dyeing your garment whichever colour you please, and like your riding whichever beast of burden you wish, and the like.	وأما المباح المستوي فهو الذي [إذا] فعلته أو تركته لم تأثم ولم تؤجر وذلك مثل صبغك ثوبك أي لون شئت وركوبك أي حمولة شئت ونحوه

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