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RATIONAL HEURISTICS? EXPECTATIONS AND BEHAVIORS IN EVOLVING ECONOMIES WITH HETEROGENEOUS INTERACTING AGENTS

GIOVANNI DOSI, MAURO NAPOLETANO, ANDREA ROVENTINI, JOSEPH E. STIGLITZ and TANIA TREIBICH* 

We analyze the individual and macroeconomic impacts of heterogeneous expectations and action rules within an agent-based model populated by heterogeneous, interacting firms. Agents have to cope with a complex evolving economy characterized by deep uncertainty resulting from technical change, imperfect information, coordination hurdles, and structural breaks. In these circumstances, we find that neither individual nor macroeconomic dynamics improve when agents replace myopic expectations with less naïve learning rules. Our results suggest that fast and frugal robust heuristics may not be a second-best option but rather “rational” responses in complex and changing macroeconomic environments. (JEL C63, D8, E32, E6, O4)

I. INTRODUCTION

In this work we study the impact of heterogeneous expectations and action rules on individual performance and macroeconomic dynamics by means of an agent-based model (ABM) populated by heterogeneous, interacting firms. In the model we explore, agents have to cope with an environment characterized by deep uncertainty resulting from technical change, imperfect information coordination problems, and, as a result, endogenous structural breaks.

Expectations have long been central in macroeconomics, from the seminal distinction between risk and uncertainty suggested by Knight (1921), to the description of “animal spirits” playing an important role in generating multiple equilibria and coordination failures in Keynes (1936, 1937), all the way to the rational expectations (RE) hypothesis (Lucas and Prescott 1971; Muth 1961). Note, however, that before the “rational expectations (RE) revolution”, there were many different views about the nature of expectations themselves, their origin and their accuracy.¹ Only with the RE assumption

1. Indeed, there was considerable exploration of the consequences of different ways of forming expectations (e.g., in

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ABBREVIATIONS

ABM: Agent-Based Models
 ADA: Adaptive Expectations
 MSFE: Mean Squared Forecast Error
 RE: rational expectations
 RLS: Recursive Least Squares
 STR: Strong Trend Rule
 WTR: Weak Trend Rule

has (a significant part of) the profession taken expectations to be uniform among agents (take or leave some noise) and on average “true.”² Correspondingly, the “action” has been assumed to be “right,” that is the one maximizing some objective, conditional on the true expectation on the future (with those expectations themselves conditional on all the information available). Still, the claims on expectation or action are supported neither by empirical evidence (see e.g., Carroll 2003; Coibion and Gorodnichenko 2012, 2015; Gennaioli, Ma, and Shleifer 2016) nor by experimental studies (see e.g., Anufriev and Hommes 2012; Kahneman 2003; Schweitzer and Cachon 2000; Tversky and Kahneman 1974). Indeed, it is not even clear what “rational” expectations means, even in principle, in the presence of Knightian uncertainty, when there are radical changes in policies (Stiglitz 2011, 2015) and/or structural breaks in the underlying distributions on which agents form their forecasts (Hendry and Mizon 2010).³

Tentative ways forward have entailed exploring macroeconomic models with learning (e.g., Evans and Honkapohja 2001) and a somewhat parsimonious use of *bounded rationality*.⁴ However, both routes continue to take Olympic

cobweb cycles). Knight himself was skeptical of the accuracy of expectations, *even on average*. Discussions of subjective probabilities made it clear that in many of the critical instances in which expectations were being formed there was no way of even assessing the accuracy of expectations.

2. There is a large literature on the (restrictive) conditions that have to be satisfied in order for this assumption of common knowledge to be plausible. Yet the RE literature typically ignores this issue—partially because much of the literature is framed in the context of a representative agent. Even as more recent advances have tried to break out of the representative agent assumption, they have typically not departed from the assumption of common knowledge, which becomes less plausible in the presence of significant heterogeneity.

3. The large differences in expectations even among experts in say expectations about inflation, growth, or employment are hard to reconcile with the assumption of common knowledge. For an assessment of risk versus some form of uncertainty in an econometric perspective, see also Rossi, Sekhposyan, and Soupre (2016).

4. Since the Great Recession, an increasing number of bounded-rationality DSGE models have appeared. See Dilaver, Calvert Jump, and Levine (2018) and Fagiolo and Roventini (2016) for surveys from different theoretical perspectives. Other models (“Internally rational NK Models”) also aim at bridging the gap between DSGE and ABMs by modifying the standard NK model to accommodate some of the criticisms addressed to DSGE (see e.g., Catalano and Di Guilmi 2019; Dilaver, Calvert Jump, and Levine 2018; Gobbi and Grazzini 2019, for a discussion). We believe that, in complement to these models, it is important to assess the role of expectations for macroeconomic dynamics when equilibrium and symmetry are not imposed.

rationality either as something to be achieved, possibly through learning, or at the very least as the benchmark against which actual expectations ought to be assessed out of the “wilderness of bounded rationality” (Sims 1980). The “behavioral” approach does introduce meaningful restrictions, but still invokes cognitive limitations, insufficient information, computing power and time as giving rise to expectations and actions which are to be assessed against the yardstick of “full rationality.” Rationality is typically then invoked to explain observed behaviors as would then resulting from a trade-off between accuracy and effort (a general discussion is in Kahneman 2003).⁵

Here, we explore an alternative route grounded in the seminal contributions of Simon (1955), March and Simon (1993), and Cyert and March (1992). First, in complex evolving environments, expectations and behaviors cannot be neatly distinguished,⁶ and, second, behavioral patterns are adequately accounted for by *heuristics*, which under Knightian uncertainty and non-stationarity of the fundamentals of the economy, may well be not only “reasonable” on prima facie grounds, but also may be rational in a particular sense to be explored below. When individuals all use these heuristics, the accuracy of their predictions and their welfare may both be higher than if they had followed seemingly more sophisticated rules. A heuristic is “a strategy that ignores part of the information, with the goal of making decisions more quickly, frugally and/or accurately than more complex methods” (Gigerenzer and Gaissmaier 2011, 454). The heuristics we explore below exhibit the so-called “less-is-more” effect, which emerges as a response to the “bias-variance dilemma”—basically the possible trade-off between the accuracy in the interpolation of past observations and the

5. Even if individuals were to have sufficient information on the basis of which to make such a decision “rationally,” the question would arise: how do they make a decision about the allocation of effort to obtain the information necessary to make that decision. Somewhere in this infinite regression, the assumption of rationality has to break down. Incidentally, notice that the accuracy-effort trade-off is also present in recent rational-expectations models with information frictions, see for example, Mankiw and Reis (2002), Woodford (2003), and Sims (2003).

6. This was, of course, also true in standard subjective utility when we drop the independence axiom. It is only with that axiom that we can infer beliefs, represented by subjective probabilities (Savage 1972). There is a large literature explaining the deficiencies in the independence axiom and extending the expected utility framework (see e.g., Machina 1982).

accuracy of the prediction—also well known in machine-learning and statistical inference (see e.g., Alpaydin 2004; Geman, Bienenstock, and D. R. 1992; Hastie, Tibshirani, and Friedman 2001). Note that heuristics are not “biases” based on “fast thinking” yielding suboptimal behaviors (as one would gather from Kahneman 2003 and from a good deal of behavioral economics), but might well be robust strategies that result in performance that is better than in purported “rational” choices in changing worlds characterized by substantive and procedural uncertainty (i.e. in all interesting crises, like the financial crisis of 2008 and the Coronavirus crisis of 2020, see also Dosi and Egidi 1991). The reason for this seemingly anomalous result is that how expectations are formed affects how the system performs; more “rational” expectations may lead to poorer system performance, where the difficulties of forecasting increase, in some sense, faster than the sophistication of the forecasting methodology. (Our results have one further implication, reflecting the well-known effect that outside the extreme assumptions of the Arrow-Debreu model, what is individually rational may not be systemically rational: given that all other agents use naïve expectations formation processes, it may pay a single agent to employ a more sophisticated expectations process. But when all do so, system performance decreases. In this sense, it may be lucky that (most) agents are not as “rational” as standard economics assumes; or it may be a result of “collective learning.”)⁷

We study individual and aggregate performances under different rules of expectation formation and behavior in an agent-based framework. ABMs represent the economy as a complex, evolving system populated by heterogeneous, interacting agents (Dosi 2012; Farmer and Foley 2009; Kirman 2010; LeBaron and Tesfatsion 2008; Tesfatsion and Judd 2006).⁸

To study particularly the effects of different rules concerning expectation formation in a

concrete general equilibrium dynamic model, we extend the Keynes + Schumpeter (K + S) model (Dosi et al. 2013, 2015, 2016, 2017b; Dosi, Fagiolo, and Roventini 2010), the structure of which is particularly well suited for accounting for heterogeneous expectation rules and adaptive learning because of the “deep uncertainty” which is embedded in it. It is designed so that agents cannot easily learn from observing outcomes about the underlying structure, and especially so because outcomes are a function of actions which themselves are based on beliefs about the economy, which are heterogeneous and evolving. The K + S model is a bridge between Keynesian theories of demand generation and Schumpeterian theories of innovation and economic growth, with “Minskian” financial dynamics (Greenwald and Stiglitz 1993). It represents an economy characterized by endogenous and persistent novelty, imperfect information, where Knightian uncertainty is pervasive and coordination failures are the norm. As imperfect information is ubiquitous, the economy is never in a (constrained) Pareto equilibrium (Greenwald and Stiglitz 1986) and agents’ current behaviors are conditioned by (beliefs concerning) future constraints (Neary and Stiglitz 1983). In turn, large endogenous innovation and imperfect coordination on the demand side entail nonlinearities, positive feedbacks and, most importantly, structural breaks in the dynamics of the system. The microeconomic foundations of the model are genuinely “behavioral” (Akerlof 2002): heterogeneous firms and banks behave in tune with what we know from micro-empirical evidence, and they interact without resorting to any *ex ante* commitment to the reciprocal consistency of their actions, thus implicitly addressing the call by Solow (2008) for genuine micro-heterogeneity.

Naturally, the very nature of the K + S model rules out the existence of a rational-expectation equilibrium on which fictitious representative agents could coordinate, or even more importantly, a dynamics which could give rise to and be generated and supported by common knowledge. Still, we can compare the impact of heterogeneous, more or less “sophisticated,” expectations and learning rules on heterogeneous agents’ performance, as well as on macroeconomic dynamics. In addition, we evaluate the robustness of our results to alternative heuristic-based rules. The K + S model is, of course, a highly simplified model, and not everyone may agree that it provides a good description of the economy. But that is not the point. Rather, the point is to explore

7. That is, typically, when one agent decides to try a more sophisticated expectation formation process, others do so as well—ideas quickly spread. Individuals do not fully understand the complex structure of the economy; they simply observe results; and what they observe is that as many try these more sophisticated expectations process, performance does not increase. The lesson they learn from this “experiment” is to revert to more naïve processes.

8. The literature on agent-based macroeconomics has been blossoming in the recent years, see for example, Fagiolo and Roventini (2012, 2016) or Dawid and Delli Gatti (2018) for recent surveys. See also Sinitzskaya and Tesfatsion (2015) and Salle (2015) for two investigations of non-RE rules in agent-based frameworks.

how, even in a very simplified context, different behavioral hypotheses about expectations interact with system dynamics, and to assess whether, in this plausible set-up, seemingly more sophisticated behaviors lead to better performance and more accurate forecasts. If they do not in this highly idealized world, there is little reason to believe that they would in the much more complex real world. We are fully aware that our interpretative strategy is largely at odds with what has gone on in most of the economic discipline, wherein very simple, behaviorally ungrounded models, are decked against the dynamics of enormously more complex real worlds. The epistemological strategy here is to bring microbehaviors nearer to what agents appear to actually do in complex environments. How simple should the model be? This is another crucial challenge. We believe that the discipline for the models comes from both the realism of micro behaviors and the number of micro and macrostatistical regularities that the model is able to robustly account for. We begin by introducing in the K + S model five simple expectation rules (based on the experimental findings of Anufriev and Hommes 2012, concerning indeed much simpler environments), allowing firms to switch among them according to their past forecasting performance (Brock and Hommes 1997). In such a framework, expectations are heterogeneous and evolve over time. We then allow agents to learn by employing more “sophisticated” expectations grounded on recursive least squares (RLS, see e.g., Evans and Honkapohja 2001), and compare the individual and system-level performances.

Simulation results show that in line with the K + S tradition, the model with simple expectation formation processes can account for endogenous growth and business cycles, where mild fluctuations are punctuated by deep downturns (Fagiolo, Napoletano, and Roventini 2008), as well as for a wide ensemble of macro and micro empirical regularities (Dosi et al. 2017b). Moreover, we find that compared to simple (benchmark) myopic expectations, somewhat more complex heuristics increase the forecast errors of the agents and do *not* substantially improve the performance of the system (see also Dosi, Fagiolo, and Roventini 2006).

Moreover, both individual and aggregate performances, both in the short run and in the long run, considerably deteriorate when firms abandon “fast and frugal” heuristics and start estimating their future demand via RLS. The volatility of business cycles and the average value of the

unemployment rate increase, while the growth potential of the economic system is reduced, as innovation is curbed. These results are explained by the fact that the forecast errors of RLS agents are amplified by the positive feedbacks induced by income constraints in the model. Moreover, we find that whenever agents are allowed to choose between RLS-learning and simple invariant rules, they “rationally” adopt the latter. And the more frequent are structural breaks, the more pronounced is this property.⁹

Our results support the *ecological rationality* of heuristics: in complex, evolving economies characterized by pervasive uncertainty and perpetual structural change, heuristics may provide a more accurate and robust tool for inference and action than more sophisticated forecasting techniques; changing expectation formation processes changes the system dynamics; more sophisticated heuristics may result in systematically less accurate forecasts (Luan, Reb, and Gigerenzer 2019; Todd and Gigerenzer 2012). Of course the agents with an infinite amount of data and highly sophisticated learning techniques might possibly do better. However, even if the system may be in principle “discoverable” by the agents, the complexity of the system itself increases with the complexity of the individual learning techniques. Heterogeneity among agents in what information they receive and in their learning processes further complicates the ability of each to learn about the underlying structure of the economy. Moreover, notice that the aggregate volatility observed in the model is an essential feature of high-dimensional decentralized economies painstakingly coordinating at varying levels of activity, and with large, endogenous technology shocks, the occurrence of which may

9. We do not perform a “horseshoe” among the appropriately calibrated different specifications, for a number of reasons. First, we are not convinced of the usefulness of such an exercise in models which are so underidentified. Second, and related to the observation made earlier, better fits in the sample period may systematically be related to worse predictions. Third and last, we do not believe that the standard metrics used (“goodness of fit”) fully reflect what is important. We do not care about how well a model does on average, but how well it does at critical times, for example, a slight increase in accuracy in normal times is of little value compared to a better prediction of turning points and economic performance in extreme events. Rather, we focus on the question of whether, given a hypothesized structure of the economy, increasing the sophistication of the expectations formation process leads, systemically, to increased accuracy in predictions. (In assessing accuracy of predictions, we use standard metrics, in order to demonstrate that even if using these imperfect metrics increases sophistication in forecasting, it does not systemically pay off.)

generate complex system dynamics. In short, the theory of the second best applies here as it does elsewhere in economics: a seeming improvement in forecasting techniques does not lead to better forecasts.

The rest of the paper is organized as follows. In Section II, we discuss the impact of expectations and agents' interactions on macroeconomic dynamics. In Section III, we describe the K + S model. We then empirically validate it in Section IV. The impact of heterogeneous expectation rules is studied in Section V, while learning is introduced in Section VI and is further investigated in Section VII. Section VIII discusses in general the properties of heuristic-driven decisions. Finally, our concluding remarks are in Section IX.

II. EXPECTATIONS, INTERACTIONS AND MACROECONOMIC DYNAMICS: THE GENERAL PROBLEM

In the most general terms, the dynamics of any economy can be seen as an enormously high-dimensional system of difference equations. They describe the "laws of motion" of the system itself and of its multiple constituent agents, driven by the behavioral (and, relatedly, expectational) adjustments of the agents themselves, their interactions, and some (endogenous or exogenous) shocks. In such a "meta-model," agents' individual outcomes depend on (a) their expectations based on both their individual and the aggregate histories, (b) their individual histories and the value of their current state variables, (c) the aggregate history, and (d) the individual and aggregate shocks:

$$(1) \quad \mathbf{x}(t) = F(\underbrace{\mathbf{f}[\mathbf{x}(t-1), \dots, \mathbf{x}(t-\tau); X(t-1), \dots, X(t-\tau)]}_{\text{Individual expectations}}; \underbrace{\mathbf{x}(t-1), \dots, \mathbf{x}(t-\tau)}_{\text{Individual histories}}; \underbrace{X(t-1), \dots, X(t-\tau)}_{\text{Aggregate history}}; \underbrace{\epsilon(t), \epsilon(t)}_{\text{Shocks}});$$

where $\mathbf{x}(t) = [x_1(t), \dots, x_n(t)]^T$ is a matrix comprising the state variables for all heterogeneous $i = 1, \dots, n$ agents populating the economy (e.g., individual capital stocks, net worths, sales,¹⁰ prices, etc.), $X(t)$ is the vector of macroeconomic variables (e.g.,

GDP, total investment, unemployment rate, etc.)¹¹; $\mathbf{f}(t) = [f_1(t), \dots, f_n(t)]^T$ is a vector of individual expectation functions which map the individual and system-level histories into forecasts and actions by the agents; and finally $\epsilon(t) = [\epsilon_1(t), \dots, \epsilon_n(t)]^T$ is the vector of idiosyncratic shocks hitting agents (e.g., their productivity or their demands), while $\epsilon(t)$ are system-wide shocks (affecting e.g., the technological frontier of the economy). F maps past history and current (exogenous) shocks into this period's state variables.

In turn, macroeconomic outcomes (e.g., GDP, total investment, etc.), which are just aggregates of the corresponding agent-level variables, are determined simultaneously by individual and aggregate prior histories and state variables, and by the result of the behavioral interaction of agents, given the shocks experienced by the economy. There is also a feedback loop from the macroeconomic aggregates to agents' forecasts and decisions. In such a framework, agents ought to form their expectations based on the observation of the past, that is, they are extrapolative, adaptive agents.¹²

Clearly, for each individual to come to a full understanding of this dynamical system is an extraordinarily difficult task—especially because the behavior of the system may be very sensitive to how each individual forms his expectations. Even neglecting the possibility of changing fundamentals of the economy (due to e.g., technological change), interactions generically entail endemic externalities and nonlinearities. And with that come unimaginably high informational demands on the part of decision-makers.

11. As the discussion below will clarify, in our model, aggregate variables are themselves the outcome of individual state variables and behaviors: $X(t) = G(\mathbf{x}(t))$.

12. RE models are, in a sense extrapolative: the agent makes inferences about the future from the past. But he is assumed to know fully the structure of the economy. He can therefore make perfect inferences about future probability distributions of all the relevant variables given the observed past history. Most of the models are Markovian, and the distribution of the relevant driving exogenous shocks are assumed stationary, so that knowing only a limited information about the past (and/or having enough information about the relevant state variables) is all that is required.

10. In the specific model formulated here, with prices adjusting slowly, sales are a state variable. In other specifications, sales could be an endogenous variable.

A. Rational Expectations: Avoiding the Central Issues

Facing all this, the prevailing response of macroeconomic theory has been to eliminate complexity at its roots by:

1. eradicating interaction altogether and assuming a representative agent. (Some go slightly beyond this by exploring very limited forms of heterogeneity.) The fallacies of such a reduction have been shown in Kirman (1992, 2014) at the level of theory, and by Forni and Lippi (1997, 1999) at the level of econometric aggregation.

2. eliminating the possibility of “deep uncertainty,” for example, structural breaks arising from episodic innovations.¹³

Now, assuming a representative-agent economy, leaving aside all the objections, greatly simplifies the analysis. In particular, one does not have to cope with the problem of aggregation, implying that macroeconomics shrinks to microeconomics and we have a much lower dimensional system of the form:

$$(2) \quad X(t) = F(f[X(t-1), \dots, X(t-\tau)]; \\ X(t-1), \dots, X(t-\tau); \varepsilon(t)).$$

where the aggregate state variables only depend on the aggregate expectation, the aggregate history and the aggregate shocks. This is akin to the basic sketch of example Evans and Honkapohja (1999), where the reduced-form model is a vector of endogenous variables (X), depending on their lagged values, on expectations of next period's values, $f[X]$, and on a vector of exogenous shocks ε .

The “rational expectation (RE) revolution” further suggests that actual expectations¹⁴ correspond to the “true” statistical conditional expectations. The forward-looking representative agent—like the macroeconomic theorists—know the “true” model of the

13. The models might have made sense when the only source of variability was weather, and climate was not varying. By the same token, it is not possible to embrace the possibility of climate change, with its deep uncertainty (Weitzman 2009), within a RE model: it should be obvious that there are widely varying expectations among different groups of individuals about the future of the global climatic system with profound implications for uncertainty about the future course of the economy.

14. Behavior may depend on more than the expected value of a distribution, but on its entire shape. RE equilibria require that in such a situation, the individuals' beliefs about the shape of the distribution coincide with the observed (conditional) relative frequencies.

economy, and hence $f[X(t-1), \dots, X(t-\tau)] = E[X(t+1)]$ so the system further simplifies to:

$$(3) \quad X(t) = F(E[X(t+1)]; X(t-1), \dots, X(t-\tau); \varepsilon(t)).$$

However, even in this reductionist framework, there can be multiple stationary RE equilibria: self-fulfilling expectations can affect the optimal choice of the representative agent and sunspot equilibria can arise (among a vast literature, see the seminal contribution of Woodford 1990 and the survey in Benhabib and Farmer 1999).

Given such a multiplicity of RE equilibria, the natural question is then “where do these expectations come from”? In the case of a unique equilibrium, agents, believing that the world is in a RE equilibrium, and believing that others believe similarly, and act accordingly, could ascertain the actions which maximize their expected utility. But when there is more than one possible equilibrium, how are they to know what others will do? They ought plausibly learn what others are doing. Yet the literature presents mixed results, with the most general analyses giving rise to skepticism concerning the generality of results on convergence. In a nutshell, “it is not rational to have rational expectations” (Kirman 2016, 8).¹⁵

Equally important, whether under plausible general conditions economies converge to a RE equilibrium or whether a RE model can be calibrated with plausible values of the relevant parameters consistent with microeconomic evidence to generate observed macroeconomic behavior, there is considerable direct evidence against the RE hypothesis; it simply goes against (the little) we know about actual expectation formation by actual economic agents. For instance, using the survey of professional forecasters, Coibion and Gorodnichenko (2012, 2015) reject the RE hypothesis. Similarly, employing survey data on the investment plans of the chief financial officers of large U.S. corporations, Gennaioli, Ma, and Shleifer (2016) find evidence against the RE benchmark, while supporting extrapolative expectations. Other researchers have identified systematic differences between expectations of different groups (e.g., men and women forecasters), casting doubt on the hypothesis of common knowledge (Guzmán 2012). Finally, the recent evidence stemming from laboratory experiments show robust and persistent deviations from RE (see Assenza et al. 2014, for a review).

15. Within a wide literature, see Bray (1982), Bray and Kreps (1987), Marcet and Sargent (1989), and the discussions in Evans and Honkapohja (1999, 2001).

In the following, we explore a different route. The system dynamics of our model maintains the complexity of the evolving systems as sketched in the “meta-model” of Equation (1). Modeling a system similar to Equation (1) fully acknowledges the deep macroeconomic externalities associated with heterogeneous beliefs which in turn influence the state variables of the system: it includes what Soros has called reflexivity (Soros 2013). At the same time, we explore the conjecture that the orderly system-level properties that are exhibited are an emergent collective property of relatively simple, inertial behaviors whereby agents learn how to repeatedly swim in a Heraclitus’ river in which one is literally unable to ever step in twice—and not the outcome of highly sophisticated individuals with anything approaching RE.¹⁶

III. THE EXPECTATION-ENHANCED K+S MODEL

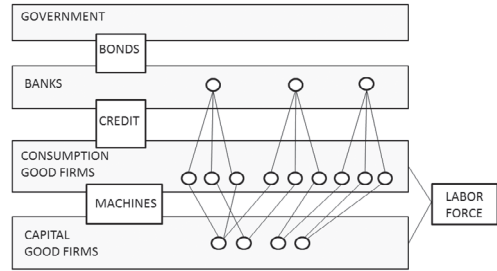
This work extends the Keynes + Schumpeter (K + S) family of models that have already been extensively explored (Dosi, Fagiolo, and Roventini 2010; Dosi et al. 2013, 2015, 2016, 2017b) by introducing different expectation formation rules. The barebone structure of the model is portrayed in Figure 1.¹⁷

The economy is composed of F_1 capital-good firms (labeled with index i), F_2 consumption-good firms (denoted by the index j), L^S consumers/workers, B commercial banks (denoted by the index k), a Central Bank and the Government sector. The presence of a

16. Recall the infinite regress involved into any process of RE formation: how do we decide about how much information to gather for the decision about how much information to gather ... Our work has some (superficial) point of contact with an increasing stream of research which introduces information frictions in RE models. For instance, Mankiw and Reis (2002) assume that the information available to agents are sticky and agents update them infrequently, while Sims (2003) and Woodford (2003) build noisy-information models, where agents continuously update their beliefs facing a signal extraction problem. However, differently from us, such works assume a fully rational agent (with bounds on their information or information processing capacities) and do not account for the “deep” Knightian uncertainty and possible coordination failures occurring in the presence of multiple heterogeneous interacting agents. In those models, agents rationally understand the limits of their computational capacities, and heuristics are rational solutions responding to the constraints, like well-designed algorithms. All modeling involves trade-offs, in which sophistication in one direction precludes the ability to introduce complexity in others. Focusing on the optimization problem for well-designed heuristics necessitates simplification of the model in other dimensions.

17. The full code is available from the authors upon request.

FIGURE 1
The Structure of the Keynes + Schumpeter Model



capital-good sector and a consumption-good industry¹⁸ introduce an important source of coordination failures.¹⁹

Capital-good firms invest in R&D to increase the productivity of their heterogeneous machine-tools (with product innovation/imitation) and their own production techniques (with process innovation/imitation). Consumption-good firms combine machines bought from capital-good firms and labor in order to produce a homogeneous product for consumers. The banks provide credit to consumption-good firms and buy Government bonds. The public sector levies taxes on firms’ and banks’ profits, pays unemployment benefits and bails banks out in case of banking crises. The Government can run deficits by issuing bonds, which are bought by the banking sector. Finally, the Central Bank fixes the baseline interest rate in the economy and the macroprudential regulatory framework.

Let us now sketch the main characteristics and dynamics of the expectation-enhanced K + S model. A detailed description of the model is provided in Dosi et al. (2015).

A. The Timeline of Events

In any given time period (t), the following microeconomic decisions take place in sequential order:

1. Policy variables (e.g., banks’ capital requirement, tax rate, Central Bank interest rate, etc.) are fixed.

18. This model structure is nowadays widely adopted in the endogenous growth literature: see for example, Acemoglu and Guerrieri (2008) and Acemoglu et al. (2012).

19. In particular, as savings in this model are equal to firm profits, a decision to save does not automatically map into a corresponding decision to invest and this gives rise to a potential deficiency in aggregate demand.

2. Banks determine the potential supply of credit.

3. Capital-good firms perform R&D, trying to discover new products and more efficient production techniques and to imitate their competitors.

4. Consumption-good firms decide how much to produce and invest according to different expectation rules. They apply for bank credit (and may be rationed) if their internal funds are not enough.

5. The capital-good market opens. Given the presence of imperfect information, capital-good firms advertise their products to an evolving subset of consumption-good firms, which in turn choose their supplier.

6. Firms in both industries hire workers according to their production plans and start producing.

7. The imperfectly competitive consumption-good market opens. Pervasive imperfect information implies that the market shares of firms evolve according to their price competitiveness.

8. The firms in both sectors compute their profits and pay back their bank loans.

9. Entry and exit take place. In both sectors, firms with near zero market shares or negative net liquid assets exit and are replaced by new ones.

10. Banks compute their profits and net worth. If the latter is negative they fail and are bailed out by the Government.

11. The Government computes its surplus or deficit, the latter being financed by sovereign debt which is bought by banks.

12. Machines ordered at the beginning of the period are delivered and become part of the capital stock of consumption-good firms at time $t + 1$.

At the end of each time step, aggregate variables (e.g., GDP, total investment, unemployment) are computed, summing over the corresponding microeconomic variables. As in its direct ancestor (Dosi et al. 2015), the model is stock-flow consistent. Note that throughout this sequence the “Schumpeterian” and “Keynesian” engines are deeply intertwined. So, for example, endogenous supply shocks (the Schumpeterian engine) continuously arrive on the efficiency of capital techniques (step 3), but their probability of arrival depends also on R&D investments, determined by past demand of capital goods (the Keynesian engine). Similarly, the diffusion of new vintages via capital investments crucially depends on demand dynamics (step 4).

B. The Capital- and Consumption-Good Sectors

In both capital- and consumption good markets, information is imperfect and firms’ prices are heterogeneous. As a consequence, the economy is *never* in a constrained Pareto state (Greenwald and Stiglitz 1986, 1988) and the current behavior of firms is conditioned by various constraints. The *capital-good industry* is the locus of endogenous machine-embodied innovation. The current technology mastered by a capital-good firm is defined by A_i , the labor productivity of the machine it sells to the downstream sector, and by B_i , the efficiency of its production technique. Capital-good firms develop new technologies or imitate their competitors in order to produce and sell more productive and cheaper machines that are in turn supplied to consumption-good firms. Capital-good firms invest IN_i , a fraction of their past sales in R&D in order to discover new machines or copy existing ones. The innovation process has two steps: first a random draw from a Bernoulli distribution with parameter $\theta_i^{in}(t) = 1 - e^{-\zeta_1 IN_i(t)}$ determines whether firm i innovates or not. Therefore, the frequency of innovations (whether successful or not) depends on $\zeta_1 \leq 1$, the firms’ *search capabilities*, and the specific amount of R&D they have invested. If an innovation occurs, the firm obtains a new technology, whose labor productivity levels are given by $A_i^{in}(t) = A_i(t)(1 + x_i^A(t))$ and $B_i^{in}(t) = B_i(t)(1 + x_i^B(t))$, where x_i^A and x_i^B are two independent draws from a Beta(α_1, β_1) distribution.²⁰ Therefore, α_1 and β_1 define the extent of *technological opportunities* available to firms, that is, the magnitude of the innovation leaps. Capital-good firms employ only labor and set prices with a fixed mark-up over unit costs of production.

In the *consumption-good industry*, firms produce a homogeneous consumption good using capital (composed of different vintages of machines) and labor under constant returns to scale employing a Leontief technology. Desired production Q_j^d is based on expected demand and desired increases in inventories (N_j^e). Demand expectations (D_j^e) (given prices) are adaptive and given by:

$$(4) \quad D_j^e(t) = f(D_j(t-1), D_j(t-2), D_j^e(t-1), Y(t-1)),$$

20. The draws $x_i^A(t)$ and $x_i^B(t)$ may well be negative (i.e., the innovation fails) in that case the firm continues to offer the “old” machine.

where D_j is firm's demand and Y is the gross domestic product. Note that well in tune with the evidence on organizational behavior, quantities and prices are not set simultaneously, but are subject to different heuristics of adjustment. Moreover, in our formulation here, as in some other areas of macroeconomics, they are assumed to adjust slowly and at discrete time, (later, we shall describe the dynamics of price adjustment). The detailed characterization of firms' expectation formation and dynamics is discussed below (cf. Section C).

The desired increase in inventories in turn is just the difference between the desired level of inventories (N^d) and the inherited stock:

$$N_j^e(t) = N_j^d(t) - N_j(t - 1),$$

with $N_j^d(t) = \iota D_j^e(t)$, the parameter $\iota \in [0;1]$ identical for all firms, and the desired level is simply a multiple of expected demand.

We can now define desired production by:

$$(5) \quad Q_j^d(t) = D_j^e(t) + N_j^e(t).$$

Given the actual stock of inventories, if the capital stock constrains the production plans of the firm, it invests in new machines in order to expand its production capacity.²¹ Thus firms' investment choices are affected by their demand expectations.²² Moreover, firms also invest to acquire state-of-the-art technologies: they replace old and obsolete machines with new ones when it is profitable to do so.²³

21. More specifically, firms invest if the desired capital stock is higher than the current one (more details in Dosi et al. 2015). The results of the models do not substantially change if firms cope with uncertain future demand by investing only if the difference between desired and actual capital stock is above a given precautionary threshold (in line with Dosi, Fagiolo, and Roventini 2006).

22. It is important to emphasize that individual agents form expectations on the state variables which are going to affect their performance (e.g., their demand), and building on such expectations, they determine their control variables, for example, planned production and investments, in a genuine "Keynesian" perspective. Conversely, they do *not* care about system-level variables which might have the utmost importance for the modeler, but exert only a very indirect influence on individual agents (e.g., economy-wide levels of productivity). The two types of forecasts, unfortunately, get confounded when one assumes the representative agent, who is also the Central Planner, who is also the modeler.

23. In line with a widespread business practice, firms scrap machines according to a payback period heuristics (more details in Dosi et al. 2015). Notice that in an equilibrium where the agent could correctly anticipate future profits, the payback rule and the internal rate of return maximization criteria would yield identical outcomes (Terborgh 1949).

The capital-good market is characterized by imperfect information and "Schumpeterian" competition (Nelson and Winter 1982). Upstream firms signal the price and productivity of their machines to their current customers as well as to a set of potential new ones. Consumption-good firms choose their supplier comparing the price and the production costs entailed by the subset of machines they are aware of. Capital-good firms produce on demand according to the orders received from their customers.

As we mentioned above, demand expectations play a key role in determining the desired production and investment plans of the firms. At the same time, their actual levels of production and inventory accumulation may differ from the desired ones, as firms can face constraints in the availability of external financing.²⁴ More precisely, in the model, consumption-good firms have to advance worker wages as well as pay for the machines they have ordered. Thus they may need external financing. Because capital markets are imperfect (e.g., Greenwald and Stiglitz 1993; Hubbard 1998; Stiglitz and Weiss 1981), internal and external sources of finance are imperfect substitutes. To fund their production and investment plans, firms first use their stock of liquid assets, and then they ask credit from banks. Firms pay an interest rate on their loans, which depends on the Central Bank interest rate (r), as well as on their credit rating (more on that in Dosi et al. 2015). However, if banks are unwilling to provide loans, firms can end up being credit constrained. In that case, they first cut their investment and then downscale their production plans. Imperfect capital markets and the possibility of credit rationing represent a first important source of income constraints in our model, one of the important ways that our model differs from those where substitution effects drive the dynamics.

Imperfect information is pervasive also in the consumption-good market (see Rotemberg 2008, for a survey on consumers' imperfect price knowledge). As a consequence, consumers do not (and in fact, cannot) instantaneously switch to the most competitive producer, and this is true even if the good is homogenous. Consumption-good firms fix their prices applying a *variable* idiosyncratic mark-up on their production costs, that is, there is a variable idiosyncratic ratio between

24. At the same time, it should be clear that say shortfalls in inventory accumulation in one period will be made up for in subsequent periods, provided that the desired level of inventories remains high.

the nominal wage and the average labor productivity resulting from the machines employed in the production process.²⁵ Mark-up dynamics are driven by the evolution of firms' market shares (in line with "customer market" models originally described by Phelps and Winter 1970 and more recently explored in Greenwald and Stiglitz 2003): firms increase their margins whenever their market share is expanding. In turn, market shares evolve in a natural way, with more competitive firms expanding while firms with a relatively lower competitiveness level shrinking. In turn the competitiveness of consumption-good firms depends on the price they charge, as well as in their ability to satisfy consumer demand captured by the level of unfilled orders (more details are spelled out in the Appendix, Equation (A1)).²⁶

Price dynamics are based on a variable mark-up over costs, with mark-ups adjusting in relation to market shares (f_j):

$$(6) \quad \mu_{j,t} = \mu_{j,t-1} \left(1 + v \frac{f_{j,t-1} - f_{j,t-2}}{f_{j,t-2}} \right),$$

with $0 \leq v \leq 1$.

At the end of every period, capital- and consumption-good firms compute their profits, pay taxes, and update their stock of liquid assets. If the latter is positive, they increase their bank deposits (consumption-good firms repay their debt first). If a firm's stock of liquid assets is negative or if its market share shrinks to zero, then the firm goes bankrupt and exits the market.²⁷ As we assume that the number of firms is fixed over time, each dead firm is replaced by a new entrant. In line with the empirical literature on firm entry (Caves 1998), we assume that entrants are on average smaller than incumbents, with the stock of capital of new consumption-good firms and the stock of liquid assets of entrants in both sectors being a fraction of the average stocks of the incumbents. To be precise, in our simulated model, each entrant draws a random value between 0.1 and 0.9 which then multiplies the average capital and liquid assets of incumbents in the previous period.

25. In this model, labor is the only variable cost.

26. While in our model we use a single measure of competitiveness determining the evolution of shares, the evolution of shares could as well depend separately on price and unfilled orders/excess capacity.

27. Note that at the beginning of the period, firms may use all their liquid assets before borrowing. However, as the period ends they receive payments from their sales and "healthy" ones pay first a fixed fraction of their debt (1/3) and then replenish their liquid assets.

Note that in the spirit of ABMs, the agents in the model have access roughly to the same information as the agents in the real world do. And, of course, they are not anointed with some special knowledge on "how the world truly works." So, concerning for example demand expectations, agents form expectations about own-demand *levels*, making inferences based on their knowledge—basically, their own demand history and the history of some aggregate variables, and not much more.

C. An Ecology of Expectation Heuristics

In the presence of imperfect information and deeply uncertain environments, we assume that agents follow behavioral rules, or *heuristics*, to form their demand expectations.²⁸ More specifically, in line with the experimental evidence provided by Anufriev and Hommes (2012), firms can choose among the following repertoire of rules.²⁹

First, firms may follow naïve demand expectations, according to which the past is the best proxy for the future³⁰:

$$(7) \quad D_{naive,j}^e(t) = D_j(t-1).$$

This is the common expectation assumption in the K+S model and it represents our benchmark case.

Second, under adaptive expectations (ADA), firms correct for their past demand forecast mistakes:

$$(8) \quad D_{ada,j}^e(t) = D_j^e(t-1) + \omega_{ada}(D_j(t-1) - D_j^e(t-1)),$$

In our simulations, we assume that $\omega_{ada} = 0.65$, in accordance with the experimental evidence provided by Hommes (2011) and Anufriev and Hommes (2012) (as in the other expectations rules). The results appear, however, to be robust across a reasonable range of parameter values.

28. The use of heuristics in decision-making under uncertainty is largely documented in the psychology literature, see in particular applications to exchange rate forecasts by banks using an adaptive heuristic in Gigerenzer (2015) and other management and economic decisions in Luan, Reb, and Gigerenzer (2019) and Boneva et al. (2019). See also Dosi, Faillo, and Marengo (2018a) for a more general review on bounded rationality.

29. See also Dosi, Fagiolo, and Roventini (2006), Hommes (2011), Assenza et al. (2019), and Colasante et al. (2015). Coibion, Gorodnichenko, and Kumar (2018) find empirical evidence supporting heterogeneity of beliefs among firms.

30. This rule belongs to the class of recency heuristics from psychology, as introduced in Brown (1838).

Third, in the weak (WTR) and strong (STR) trend expectation rules, firms behave like “chartist” traders, trying to ride demand patterns:

$$(9) \quad D_{wtr,j}^e(t) = D_j(t-1) + \omega_{wtr}(D_j(t-1) - D_j(t-2));$$

$$(10) \quad D_{str,j}^e(t) = D_j(t-1) + \omega_{str}(D_j(t-1) - D_j(t-2)).$$

The only difference between the WTR and STR expectation rules is the value of the parameter weighing past demand changes, that is, $\omega_{wtr} = 0.4$ and $\omega_{str} = 1.3$.

Finally, firms may react to both their past demand dynamics and to some aggregate “anchor,” the GDP. The “anchor and adjustment” expectation rule (AA, see Tversky and Kahneman 1974) is thus:

$$(11) \quad D_{aa,j}^e(t) = [1 + w_{aa}\Delta GDP(t-1) + (1 - \omega_{aa})\Delta D_j(t-1)]D_j(t-1),$$

with $\omega_{aa} = 0.5$.³¹

We then consider a richer model of expectation formation (labeled “SWITCH” in the attached tables) wherein rules are selected by agents on the basis of their predictive performance (Anufriev and Hommes 2012). Starting from a uniform distribution of the five expectation rules described above, firms switch across heuristics according to their past performance.³² Firms, of course, have strong incentives to forecast future demand correctly so as to avoid costly accumulation of inventories or conversely to avoid missing sales and profit opportunities.³³

Furthermore, in line with the experimental evidence in Schweitzer and Cachon (2000), firms update the performance (U) of each heuristic

31. Our anchor and adjustment rule is different from that employed by Anufriev and Hommes (2012), who, in a financial market framework employ either the fundamental price or the sample average of past prices. In order to keep the “spirit” of their rule in a macroeconomic model, we employ the GDP, which can be considered as a proxy of the state of the whole economy.

32. Entrant firms copy the expectation rule of an incumbent and their probability to adopt any one of them is proportional to its diffusion in the system. Simulation results presented in Section V are robust to the assumption that the entrants start with a uniform distribution of expectation rules.

33. The effects of the two types of forecasting errors are indeed roughly symmetric.

$h \in \{naïve, ada, wtr, str, aa\}$ according to the last demand forecast error:

$$(12) \quad U_{h,j}(t) = -\left(\frac{D_j(t-1) - D_{h,j}^e(t-1)}{D_{h,j}^e(t-1)}\right)^2 + \eta U_{h,j}(t-1),$$

where $0 \leq \eta \leq 1$ is a memory parameter measuring the relative weight attributed by agents to past errors. Firms adopt a given expectation rule with a probability $n_{h,j}(t)$, which is updated in each period as follows:

$$(13) \quad n_{h,j}(t) = \delta n_{h,j}(t-1) + (1 - \delta) \frac{\exp(\beta U_{h,j}(t))}{Z_j(t)},$$

with $0 \leq \beta, \delta \leq 1$, and $Z_j(t) = \sum_{h=1}^H \exp(\beta U_{h,j}(t))$ being a normalization factor. The parameter δ captures the persistence of expectation-formation rules (the higher is δ , the higher is the path dependency in terms of the weight of each expectation rule), while the parameter β measures the impact of past performance on the choice of expectation rule (the higher is β , the higher is the impact of the relative success of an expectation rule on its probability to be chosen).³⁴

In the SWITCH scenario, the expectation formation rules are heterogeneous and evolve over time. In Section VI, we experiment with a more “sophisticated” expectations rule which introduces *learning* and thus involves higher degrees of “rationality.” More specifically, firms will behave as econometricians, estimating the parameters of the expectation rules via RLS.

D. The Banking Sector

In the model, the credit supply is determined endogenously by banks (among a vast body of literature, see e.g., Stiglitz and Greenwald 2003; Godley and Lavoie 2007; McLeay, Amar, and Ryland 2014). The number of banks and the network linking firms and banks are fixed over time.³⁵

Banks’ supply of credit is a function of their equity and is constrained by capital adequacy requirements inspired by Basel-framework rules (see e.g., Ashraf, Gershman, and Howitt 2017;

34. We choose in our baseline parametrization the values of η, β and δ which are aligned with the empirical evidence in Anufriev and Hommes (2012, 51) (cf. Table 1). Our results do not change when we make the three parameters vary between 0 and 1.

35. See Dosi et al. (2015).

Delli Gatti et al. 2010; Popoyan, Napoletano, and Roventini 2017; Raberto, Tegli, and Cincotti 2012). Moreover, banks, which gather deposits before they provide credit, maintain a buffer over the mandatory level of capital, whose magnitude is altered over the business cycle according to their financial fragility (Becker and Ivashina 2014; Bikker and Metzmakers 2005), proxied by the ratio between accumulated bad debt (i.e., loans in default) and bank assets (Adrian and Shin 2010). Credit supply is thus influenced by changes in a bank's balance sheet, which itself is affected by bank profits net of loan losses. This creates a feedback loop from loan losses to reductions in the amount of credit supplied to firms in the next period, via changes in banks' equity.

Credit demand stems from consumption-good firms' financing needs for investment and production, net of their internal funds (see Section B). Banks allocate credit among their clients by ranking the applicants in terms of their creditworthiness, defined by the ratio between net worth and sales. Banks provide credit up to their credit supply ceiling. Credit availability is the lesser of what banks are willing to supply, based on their equity,³⁶ and what they are allowed to supply, based on regulatory (capital adequacy) standards.

Banks fix the interest rate on loans applying a mark-up on the Central Bank interest rate (r),³⁷ which is set in each period according to a Taylor rule (Howitt 1992; Taylor 1993). Loan rates are changing over time, but they are also heterogeneous across borrowers, as they incorporate a spread linked to firms' idiosyncratic credit risk.³⁸ Banks experience loan losses whenever one of their clients goes bankrupt and exits the market. Loan losses represent an (endogenous) negative

shock to bank profits, which may become negative. The net worth of the bank is updated in each period with positive or negative profits. Positive profits are retained by the bank and act as a buffer for future negative shocks. If the net worth of the bank is not sufficient to cover such losses, the bank goes bankrupt. Whenever a bank fails, the Government steps in and bails it out providing fresh capital; we can think of the recapitalized bank as a new bank.³⁹

E. The Labor Market, Consumption and the Government Sector

In our model, we assume there may be unemployment (the labor market may not clear). The labor supply L^S is fixed and inelastic to the nominal or real wage rate. The nominal wage (w) is determined by institutional and market factors.⁴⁰ As a consequence, both involuntary unemployment and labor rationing may emerge. Nominal wage dynamics depend on the gap between actual and targeted inflation (see Table 1),⁴¹ and on the dynamics of average productivity and of the unemployment rate:

(14)

$$\frac{\Delta w(t)}{w(t-1)} = \pi^T + \psi_1(\pi(t-1) - \pi^T) + \psi_2 \frac{\overline{\Delta AB}(t)}{AB(t-1)} - \psi_3 \frac{\Delta U(t)}{U(t-1)},$$

where \overline{AB} is the average labor productivity, U the unemployment rate, and $\psi_{1,2,3} > 0$.

Unemployed workers receive a subsidy (w^u) which is a fraction of the current nominal wage, that is, $w^u(t) = \varphi w(t)$, with $\varphi \in [0, 1]$. Given the total labor demand L^D , the total amount of

36. In a more general model, banks' willingness to supply credit also depends, of course, on their risk perceptions. See Stiglitz and Greenwald (2003).

37. In this sense, banks are a particular class of firms—they supply credit: and like other firms, do not instantaneously adjust their prices (interest rates charged for any risk class of borrowers). And because prices (interest rates) are not adjusted instantaneously to clear the credit market, there will be credit rationing—even though higher risk borrowers are assumed to pay higher interest rates, the adjustment in interest rates is assumed not to fully compensate for the additional risk, so lower risk borrowers are still preferred, as described above. Of course, even without such constraints on adjustment, there will often be credit rationing, arising out of asymmetries of information and costs of enforcement of credit contracts. See for example, Stiglitz and Weiss (1981).

38. Such credit risk is assessed by the net worth to sales ratio of the firm, which defines its ranking in the bank's portfolio of clients, see Dosi et al. (2015).

39. As in the case of firms, the net worth of the new bank is a random fraction of the smallest incumbent's equity, provided it respects the capital adequacy ratio as set by the regulator (see also Dosi et al. 2015). Here for simplicity we assume simply that the Government supports the creation of the new bank with *fiat* money. However, in Dosi et al. (2015) we examine the complex but more realistic case in which public bailouts are an explicit burden on Government finance.

40. We assume here that all workers get the same wage, ignoring, for simplifying reasons, the impact of firm heterogeneity on wages. Instead, a detailed microfoundation of the labor market in the K + S models is provided in Dosi et al. (2017c, 2018b).

41. This term reflects the effect of monetary policy, which is assumed to tighten when inflation exceeds the target rate, and to loosen otherwise. Inflation is computed as the variation of the price of consumption-good firms weighted by their market shares.

unemployment subsidies paid by the Government (G) is $G(t) = \max\{w^u(t)(L^S - L^D(t)), 0\}$.

We assume that workers fully consume their income (which is equivalent to assuming that workers are credit constrained and therefore cannot engage in standard consumption smoothing),⁴² while capitalists do not, but only save and invest. Accordingly, aggregate consumption (C)⁴³ depends on the income of both employed and unemployed workers:

$$(15) \quad C(t) = w(t)L^D(t) + G(t).$$

Finally, the public deficit (or surplus) in each period is the difference between Government expenses (unemployment subsidies, public debt cost, and bank bailout costs) and its revenues (taxes are levied on firm and bank profits at the fix rate tr).⁴⁴ Public deficit is then equal to:

$$(16) \quad Def_t = Debt_t^{cost} + G_{bailout_t} + G_t - Tax_t.$$

If $Def_t > 0$, the Government issues new bonds, bought by banks with their net profits.⁴⁵

The tight relation between the dynamics of consumption and income is the second main source of *income constraints* in our model (the other one being the effect of credit constraints of firms' investments, see Section B). Notice that our model is very different from other macro-models (e.g., DSGE ones), where consumption is instead determined by an inter-temporal allocative decision driven by the difference between the interest and inter-temporal discount rates.

To repeat, the explicit microfoundation of the dynamics for all aggregate variables of interest (e.g., output, investment, employment, etc.) is nested in the decisions of a multiplicity of heterogeneous, adaptive agents and in their interaction mechanisms.⁴⁶ The model satisfies

42. The above assumption is also in line with the microeconomic empirical evidence suggesting that the consumption of most households tracks their income as their wealth is close to zero (see e.g., Wolff 1998). Notice that the conclusions of the paper qualitatively hold as long as, in standard Keynesian fashion (see e.g., Kaldor 1955), the propensity to consume out of profits is lower than that out of wages.

43. Note that there are no taxes on consumption in the model.

44. See Dosi et al. (2015) for more details on the computation of the public budget as well as a discussion on the role of fiscal rules on macroeconomic outcomes.

45. In the rare cases in which the supply of bonds exceeds their demand, the residual quantity is bought by the Central Bank.

46. While each of these decisions is made in a plausible way, consistent with observed behavior, the decisions themselves may or may not be based on the solution to a complex intertemporal dynamic optimization problem. There are

the standard national account identities: the sum of value added of capital- and consumption goods firms (GDP) equals their aggregate production. Total production in turn coincides with the sum of aggregate consumption, investment and inventories.

Table 1 presents the parameters used in the benchmark specification of the model.

IV. EMPIRICAL VALIDATION

The K+S model can jointly account for a large number of *macro* and *micro* stylized facts. The ability of the model to reproduce *at the same time* a wide set of empirical regularities, with a plausible set of parameters, is a procedure that both empirically corroborates the model, and disciplines the parametrization used in the simulation experiments. The stylized facts are tested and shown below in the case of our baseline with Naïve expectations; however, they are also valid in the other scenarios presented in Section V. It is also one of the major advantages of agent-based vis-à-vis DSGE models (Fagiolo and Roventini 2016), which cannot account for many stylized facts such as those reported by Haldane and Turrell (2019).⁴⁷

We briefly recall the micro- and macroregularities reproduced by the K+S model in Table 2. On the macroeconomic side, self-sustained growth is endogenously generated by the model (see left plot in Figure 2) together with emergent business cycles (see the bandpass-filtered GDP, right plot in Figure 2). Mild economic fluctuations are punctuated by *deep downturns*. As a consequence, the GDP growth-rate distribution generated by the model exhibits fat tails (cf. Figure 3) strikingly similar to those observed empirically (Castaldi and Dosi 2009; Fagiolo,

complex trade-offs in modeling. Our analysis focuses on lags in adjustments, the complexity of interactions among agents, and the role of income and regulatory constraints, all of which are not well captured in standard DSGE models. The richness of the dynamics our model generates suggests that ABMs should be viewed as, at a minimum, an important complement to these standard models.

47. Some more recent DSGE models (see for instance Kaplan, Moll, and Violante 2018) attempt to artificially introduce heterogeneity by introducing dispersion in income through idiosyncratic shocks. Yet, the way this is done does not address the issues raised here of complexity, does not provide an adequate description of the full consequences of this heterogeneity, which depends importantly on the role of constraints, such as that on credit availability, and does not address issues of firm heterogeneity, which play a central role in the analysis here and in the economy.

TABLE 1
Parameters

Description	Symbol	Value
<i>Benchmark parameters</i>		
Monte Carlo replications	MC	50
Time sample	T	600
Number of firms in capital-good industry	F_1	50
Number of firms in consumption-good industry	F_2	200
Number of banks	B	10
Capital-good firms' mark-up	μ_1	0.04
Consumption-good firm initial mark-up	$\bar{\mu}_0$	0.20
Banks deposits interest rate mark-up	r^D	-1
Banks reserve interest rate mark-up	μ^{res}	-0.33
Bond interest rate mark-up	μ^{bonds}	0
Loan interest rate mark-up	μ^{deb}	0.30
Bank capital adequacy rate	τ^b	0.08
Wage setting ΔAB weight	ψ_1	1
Wage setting Δcpi weight	ψ_2	0.05
Wage setting ΔU weight	ψ_3	0.05
Shape parameter of bank client distribution	$pareto_a$	0.08
Desired inventories	l	0.10
Beta distribution parameters (innovation process)	(α_1, β_1)	(3, 3)
Firm search capabilities parameters	$\zeta_{1,2}$	0.30
<i>Policy parameters</i>		
Inflation adjustment parameter (TR_π)	γ_π	1.10
Target interest rate	r^T	0.025
Target inflation rate	π^T	0.02
Tax rate	tr	0.10
Unemployment subsidy rate	φ	0.40
<i>Expectation parameters</i>		
ADA adjustment	w_{ada}	0.65
WTR adjustment	w_{wtr}	0.4
STR adjustment	w_{str}	1.3
LAA adjustment	w_{aa}	0.5
Memory parameter	η	0.7
Intensity of choice	β	0.4
Inertia parameter	δ	0.9

Napoletano, and Roventini 2008).⁴⁸ At business cycle frequencies, the relative volatility of fluctuations between output, investment and consumption and the comovements between GDP and the main macroeconomic time series are in line with the empirical evidence (for the empirics and discussion cf. Stock and Watson 1999; Napoletano, Roventini, and Sapio 2006).

48. Note that DSGE models are not able to match such empirical regularities even if they are fed with fat-tailed shocks (Ascari, Fagiolo, and Roventini 2015). This implies that they cannot jointly account for mild recessions and deep downturns (Stiglitz 2018).

In particular, aggregate R&D investment is pro-cyclical (see e.g., Walde and Woitek 2004).

Furthermore, the model also matches the major business cycle stylized facts concerning *credit* (Bikker and Metzmakers 2005; Mendoza and Terrones 2014) and *banking crises* (Laeven and Valencia 2008; Reinhart and Rogoff 2009). In particular, credit booms lead to higher firm default rates, which often trigger banking crises. The impact of banking crises on the public budget is severe, much higher than those of “standard” recessions, and not limited to bailout costs (Reinhart and Rogoff 2013).

The model is also able to replicate several *microeconomic* empirical regularities. Note that the properties described emerge directly from the simulations, and are not imposed by initial conditions; all firms are initialized in the first period of the model with the same size and productivity level. As an outcome from the simulations, firms are extremely heterogeneous in terms of size, growth rate, and productivity: firm size distributions are right skewed; firm growth-rate distributions are fat tailed; productivity differentials among firms are persistent over time (see e.g., Bartelsman and Doms 2000; Dosi 2007). Moreover, firms invest in a lumpy fashion (Doms and Dunne 1998). Discontinuities in investment follow from the large demand and technological shocks emerging in the model.

Finally, the causal relations between macroeconomic variables within the K + S model have been externally validated by Guerini and Moneta (2017), comparing the simulated series from Dosi et al. (2015) to empirical time series in a structural vector auto-regression framework.

V. THE IMPACT OF HETEROGENEOUS EXPECTATION FORMATION RULES

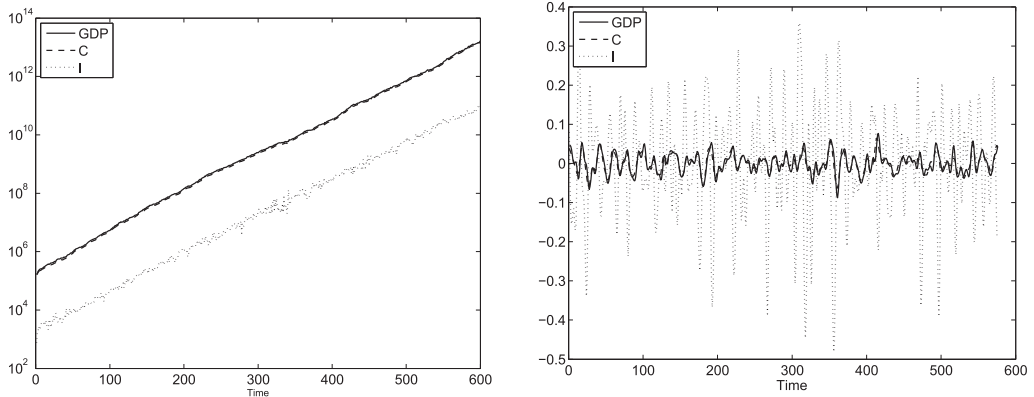
After having showed the explanatory capabilities of the K + S model in the baseline scenario with naïve expectations in the previous section, let us compare the performance of the economy under alternative expectation formation scenarios. More specifically, we assess the impact of different expectation heuristics on variables capturing the long-run performance of the economy (average GDP growth), as well as short-run fluctuations (output volatility, average unemployment rate, economic crises—defined here as the likelihood of GDP drops higher than 3%). We also study the forecast mistakes of firms in alternative expectation regimes, measured as

TABLE 2
Stylized Facts Replicated by the K + S Models

Stylized Facts	Empirical Studies (Among Others)
<i>Macroeconomic stylized facts</i>	
SF1 Endogenous self-sustained growth with persistent fluctuations	Burns and Mitchell (1946), Kuznets and Murphy (1966), Zarnowitz (1985), Stock and Watson (1999)
SF2 Fat-tailed GDP growth-rate distribution	Fagiolo, Napoletano, and Roventini (2008)
SF3 Recession duration exponentially distributed	Ausloos, Miskiewicz, and Sanglier (2004), Wright (2005)
SF4 Relative volatility of GDP, consumption and investment	Stock and Watson (1999), Napoletano, Roventini, and Sapio (2006)
SF5 Cross-correlations of macro variables	Stock and Watson (1999), Napoletano, Roventini, and Sapio (2006)
SF6 Pro-cyclical aggregate R&D investment	Walde and Woitek (2004)
SF7 Cross-correlations of credit-related variables	Lown and Morgan (2006), Leary (2009)
SF8 Cross-correlation between firm debt and loan losses	Mendoza and Terrones (2014), Foons, Norden, and Weber (2010)
SF9 Banking crises duration is right skewed	Reinhart and Rogoff (2009)
SF10 Fiscal costs of banking crises to GDP distribution is fat-tailed	Laeven and Valencia (2008)
<i>Microeconomic stylized facts</i>	
SF11 Firm (log) size distribution is right-skewed	Dosi (2007)
SF12 Fat-tailed firm growth-rate distribution	Bottazzi and Secchi (2003, 2006)
SF13 Productivity heterogeneity across firms	Bartelsman and Doms (2000), Dosi (2007)
SF14 Persistent productivity differential across firms	Bartelsman and Doms (2000), Dosi (2007)
SF15 Lumpy investment rates at firm-level	Doms and Dunne (1998)
SF16 Firm bankruptcies are counter-cyclical	Jaimovich and Floetotto (2008)
SF17 Firm bad-debt distribution fits a power-law	Di Guilmi, Gallegati, and Ormerod (2004)

FIGURE 2

Model-Generated GDP, Consumption and Investment Time Series (Left) and Bandpass-filtered Series (Right).



follows:

$$(17) \text{Error}_j(t) = \left[\frac{D_j(t) - (D_j^e(t) + N_j^e(t))}{D_j^e(t) + N_j^e(t)} \right]^2,$$

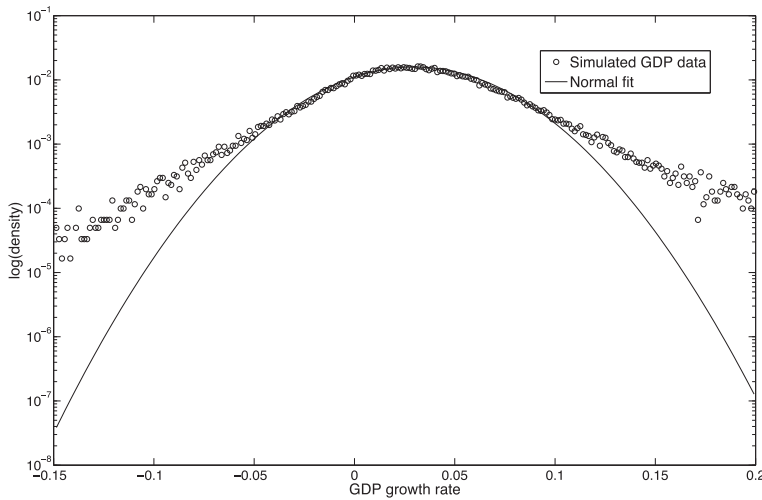
which include also expected inventories (N^e).⁴⁹ Then, we compute the *mean squared forecast*

49. The expectation mistakes of consumption-good firms are computed at the end of the period, once realized demand is

error (MSFE), built by aggregating the individual mistakes in consumption-good firms' demand

observed. Expectational errors are normalized to be independent from firms' size. The *MSFE* is then the Monte Carlo average of the mean over all agents' squared errors for all periods in each independent run. However, note that shocks to large firms have significant aggregate consequences in our model. Indeed, as shown in Dosi et al. (2019), aggregate fluctuations in our K + S model display the "granularity" property highlighted by Gabaix (2011).

FIGURE 3
GDP Growth-Rate Distribution



Note: Simulated data versus normal fit.

TABLE 3
Correlation between Squared Forecast Errors
and Profits of Individual Firms

	Avg. Correlation	Std. Dev.
Unconditional	-0.147	0.023
Conditional on firms with negative profits	-0.758	0.021

Note: Average over 50 Monte Carlo runs.

forecasts. We consider MSFEs for sake of comparability with the incumbent literature (see e.g., Anufriev and Hommes 2012). However, it is crucial to note that MSFEs directly map into firms' profitability, thus affecting their evolution and survival probability. Indeed, the correlations between MSFEs and firms' profit margins are significantly negative, especially when they accumulate losses (see Table 3). If firms underestimate their demand, they lose competitiveness and market shares, while in case of overproduction, they have to pay wages and accumulate inventories without earning revenue.

The results of our Monte Carlo simulation analyses are presented in Table 4, where we report, for all the variables, the ratio between alternative expectation rules and the baseline heuristic with naïve expectations, and mean-difference t tests. The first four scenarios (ADA, WTR, STR, and AA) assume that all the firms in the economy follow the same expectation rule.

This allows us to understand the forecast errors of each rule as well as their impact on the economic system, *independently* of other heuristics.

The MSFEs are reported in Table 4. The MSFEs of naïve expectations are significantly lower than those of most other heuristics (WTR, STR, AA), with one exception: the MSFE is lower with the ADA regime, although the difference is not significant.

The reason that the forecasting performance does not improve when other rules are employed is, as we have already noted, that the behavior of the system itself changes in ways which make forecasting more difficult. In a sense, the adverse effect on system volatility outweighs any improvement that might have occurred *within* a fixed system.

Results show that the quality of the forecasts of alternative expectations rules does not necessarily map into macroeconomic performances (cf. Table 4). This is as such another piece of evidence on the lack of isomorphism between micro expectations/behaviors and system-level dynamics. On one side, with strong trend heuristics (STR), higher MSFEs translate into lower long-run growth and higher short-run instability. This result is explained by the destabilizing role in model dynamics of additional positive feedbacks resulting from the STR (see e.g., Anufriev, Hommes, and Philipse 2013; Heemeijer et al. 2009). Similarly, when firms take into

TABLE 4
Expectation Heuristics and Macroeconomic Performance

Expectation Rules	Avg. GDP Growth	GDP Volatility	Unemployment Rate	Likelihood of Crises	Mean Squared Forecast Error
<i>Average value</i>					
Naïve	0.030	0.042	0.047	0.066	0.072
<i>Ratio wrt. naïve</i>					
ADA	0.996	0.858**	1.304	0.611**	0.960
WTR	1.005	1.060	0.691*	1.049	1.842**
STR	0.966**	2.879**	2.341**	3.082**	7.731**
AA	1.000	1.563**	0.890	1.775**	1.321**
SWITCH	1.008	0.947	0.395**	0.765*	1.773**

Note: Average values in the baseline (naïve) and ratio with respect to the baseline. * Significant difference wrt. baseline (naïve) at 1% level (**) and 5% level (*). Average over 50 Monte Carlo runs.

account both their own demand and GDP dynamics as in the AA case, MSFEs, output volatility and the likelihood of economic crises significantly *increase*.⁵⁰ On the other hand, the MSFEs of the weak trend rule (WTR) is higher than those of naïve expectations, but the unemployment rate seems somewhat lower⁵¹ (while the performance of other variables is not significantly different from the benchmark case). Finally, in comparison to the benchmark scenario, the ADA rule reduces GDP volatility and the likelihood of crises, yet MSFEs are of a similar range.

A. Choosing Expectation Rules

Let us now consider the scenario (labeled SWITCH), in which agents can switch across heuristics according to their past performance (cf. Section C), thus “learning” from experience. Because different firms will have different experiences, there will be heterogeneity in the rules they employ. Figure 4 depicts the evolution of the share of each heuristic followed by agents over time. With the exception of the STR, the share of the other heuristics is similar and fluctuates around a relatively stable value: firms do not converge to a single dominant expectation rule, but rather the system grows with an ecology of rules. Such a result is robust to different values of the parameters affecting firms’ choice

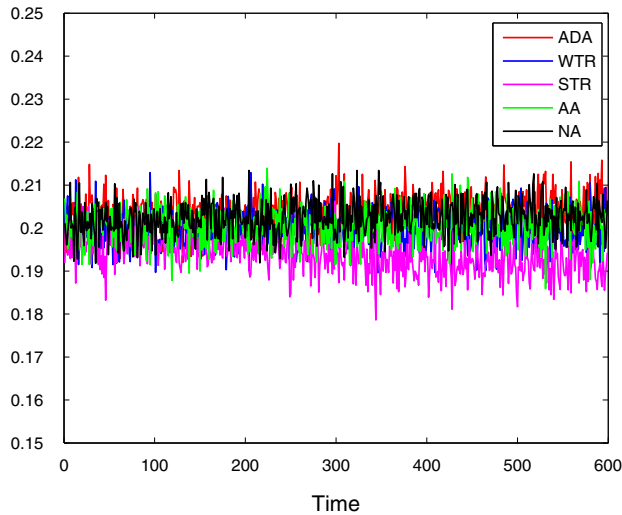
50. The worse performance of AA expectations is confirmed also when firms consider only GDP growth ($w_{aa} = 1$) in forecasting their demand. The results are available from the authors upon request.

51. The conjecture here is that even if the WTR induces a higher forecasting error of individual agents, the rule might better capture *on average for the whole economy* the fact that most of the times the economy grows, and in an autocorrelated fashion. See also the discussion below on the switching regime.

of the expectation heuristic (cf. η , δ , and β , in Equations (12) and (13)). In presence of such an ecology of expectation heuristics, the MSFEs are considerably and significantly higher than in the benchmark naïve case (cf. Table 4). If agents try to improve their forecast performance by switching more actively among different heuristics according to their past performance, they indeed *worsen* it—again because the behavior of the economic system itself *changes*. Yet, the performance of the economy is *not* worse than the one observed under the naïve rule: on the contrary, the unemployment rate and occurrence of crises are significantly lower (see Table 4). Again, higher MSFEs do not appear to significantly affect the performance of the economy (see Table 4). (Even GDP volatility is reduced.)

The first general conclusion from this battery of simulation exercises is that the *fast and frugal* naïve heuristics can (sometimes) forecast better than relatively more complex rules (in line with the results in Gigerenzer and Brighton 2009; Gigerenzer and Todd 1999)—and did so in the model investigated in this paper. Second, rules involving learning from experience (such as in the SWITCH regime) can yield *worse* forecasts. This is partly because in most instances, such rules lead to greater volatility: the ways expectations are formed affects dynamics, and not always in the way that might have been thought. In a second- or third-best world, as we know, improvements in individual markets may lead to Pareto-inferior outcomes; so too here, what might appear as an improvement in forecasting methodology leads to worst forecasting. But third, such worsened individual performance in forecasting, however, is not necessarily reflected by any deterioration of the performance of the system: on

FIGURE 4
Share of each Expectation Heuristic over Time



Note: Average over 50 Monte Carlo runs

the contrary, stochastic transitions within ecologies of rules seem to have somewhat stabilized the economy, *thus possibly improving macroeconomic dynamics*. Part of the reason for this seemingly anomalous result may be related to an observation made by Knight: excess optimism on the part of firms, he argued, had a salutary effect on economic growth, because it led firms to invest more than they otherwise would have. In modern parlance, there are macroeconomic externalities, resulting in under- or overinvestment, and these may be partially corrected by forecast errors.

It does not appear that the worsened forecasting performance in this case is the result of more economic volatility. Rather in a complex and changing environment with agents pursuing different expectation formation strategies, there is no simple link between even average forecasting error and systemic volatility.⁵² Conversely, in the opposite direction, from the micro to the system-level dynamics, it is the *ecology of different* (even if relatively poor) *learning rules* which appears to exert some stabilization effect. (Later experiments, described below, however question the

generality of these results.) Finally, while expectations do have some effect on the dynamics of the economy, some aspects of the behavior of the economy, in particular average growth rates, are not particularly sensitive to the assumptions of expectation formation, although others (such as the frequency of crises) are.

In the exercises so far, agents just switch between fixed parameter heuristics. Let us now explore how further increasing the sophistication of firms' expectation formation processes affect individual and macroeconomic performances.

VI. FROM HEURISTICS TO LEARNING EXPECTATIONS

Let us now relax the assumption of common and stable parameters in the expectation heuristics followed by firms, and make agents learn *as if* they were econometricians. We introduce a learning process which tries to capture "a boundedly rational model of how rational expectations can be achieved" (Evans and Honkapohja 1999, 452).⁵³ More specifically, agents are assumed to

52. This could easily be seen to be the case, for instance, if at least at certain times the forecast errors of those using different expectation rules were negatively correlated. Then while on average forecasts might be (more) accurate, an increase in variability of forecasts would lead to an increase in the mean forecast error.

53. RE models are, of course, forward looking, but they assume that individuals have been able to construct a structural model of the economy such that, looking at past data, they can "rationally" anticipate what that future looks like (stochastically).

TABLE 5
 Macroeconomic Performance under RLS-Learning ADA Expectations, $T_{rls}^{\min} = 5$

Expectation Rules	Avg. GDP Growth	GDP Volatility	Unemployment Rate	Likelihood of Crises	Mean Squared Forecast Error
RLS-learning experiment					
<i>Ratio wrt. to naïve</i>					
ADA + RLS learning	0.961**	1.242**	4.553**	1.623**	7.529**
<i>Ratio wrt. to ADA</i>					
ADA + RLS learning	0.965**	1.448**	3.492**	2.657**	7.847**

Note: Ratio with respect to the baseline (naïve, ADA). Significant difference wrt. baseline at 1% level (**) and 5% level (*). Average over 50 Monte Carlo runs.

predict their future demand estimating the parameter of their expectation rule via RLS.⁵⁴ We will introduce RLS learning in the ADA scenario (cf. Section V). In this setting expectations are based on an adaptive process according to which agents learn from their past, and the parameter (ω_{ada}) of the expectation heuristic now varies cross-sectionally and over time, according to firms' estimations over their own demand time series ($\hat{\omega}_{rls,j}$). Because entrants have no data on which to formulate RLS estimates, we assume that they follow simple heuristics. Those with enough data may switch and become *sophisticated* firms.⁵⁵ Thus young firms follow a heuristic, setting the parameter in Equation (8) as earlier (i.e., $\hat{\omega}_{rls,j} = \omega_{ada} = 0.65$). Once incumbent firms gather enough observations (T_{rls}^{\min}), they become “sophisticated” and start performing RLS (see below).

The (“adult”) sophisticated firms estimate Equation (8) by RLS:

$$(18) \quad D_j(t-1) - D_j^e(t-2) = const + w_{rls,j}(D_j(t-2) - D_j^e(t-2)) + \epsilon(t),$$

where the estimation sample size is between $T_{rls}^{\min} = 5$ and $T_{rls}^{\max} = 40$ observations. To account for agents' limited memory, when the sample reaches the maximum size T_{rls}^{\max} , the firm replaces the oldest observation with the newest one.⁵⁶

54. We also assume a simple linear structure. Below, we discuss some of the complexities that arise as we formulate nonlinear models.

55. As pointed out by Haltiwanger and Waldman (1985), when there is a fraction of agents which have no previous experience with a specific situation, learning does not converge to a RE equilibrium. Similarly, in our model, as we shall see, sophisticated learning does not jettison heuristic-guided firms from the economy.

56. Moreover, the changing economy may correctly lead individuals to believe that data from the distant past is of only

Not surprisingly, the relative share of heuristic versus sophisticated firms (which, as shown below, impacts both the micro- and the macroeconomic performance in the system) depends on entry and exit processes, and on the minimum number of observations required to perform RLS (T_{rls}^{\min}).

As the ADA was the regime with the lowest MSFE, we will also compare RLS with ADA when they both coexist. In that case we label RLS as “ADA + RLS learning” in the tables. In that way we assess the impact of learning on macroeconomic dynamics.⁵⁷

In Table 5, we compare our target indicators under RLS learning in the ADA scenario vis-à-vis the baseline (naïve expectations) as well as the simple heuristic ADA. Simulation results show that RLS learning has both short- and long-run *destabilizing* effects on macroeconomic dynamics, as it increases output volatility, the unemployment rate and the likelihood of economic crises, while reducing average GDP growth.

Why does the introduction of RLS-learning considerably worsen the performance of the economy? Overall, firms make considerably larger forecasting mistakes (cf. Table 5, last column).⁵⁸ Looking at what is going on more in detail, let us consider separately the mean squared demand forecast errors of heuristic-guided and sophisticated agents. Table 6 presents

limited relevance to today. After the estimation we bound the parameters such that $\hat{\omega}_{rls,j,t} \in [-2; 2]$. The presented results are robust also in the unbounded case.

57. Implementing instead RLS in the framework of the trend rule—as under the RLS-learning scenario, the “weak” and “strong” trend rules collapse into a unique one—yields similar results as in the case of the ADA framework shown here.

58. Note that when RLS-learning is introduced, the increases in MSFEs are much higher than the differences across alternative heuristic-expectation scenarios. And this comes together with worse macroeconomic performances.

TABLE 6

Mean Squared Demand Forecast Errors under the Different Expectation Scenarios

Expectation Rule	All Agents	Sophisticated Agents	Heuristic-Guided Agents
Naïve	0.072	—	—
ADA	0.069	—	—
ADA + RLS learning	0.544	0.685	0.082

Note: Average over 50 Monte Carlo runs.

such statistics. The surge in the economy-wide MSFEs is mainly driven by sophisticated agents, whose errors are eight times larger than heuristic ones. Moreover, the presence of sophisticated agents also inflates the forecast errors of heuristic-guided firms, from 0.069—in the ADA scenario, whereby all firms follow the heuristic, to 0.082—in the ADA + RLS scenario, in presence of sophisticated firms (the relation between the relative share of the two types of firms and their MSFEs will be further studied below).

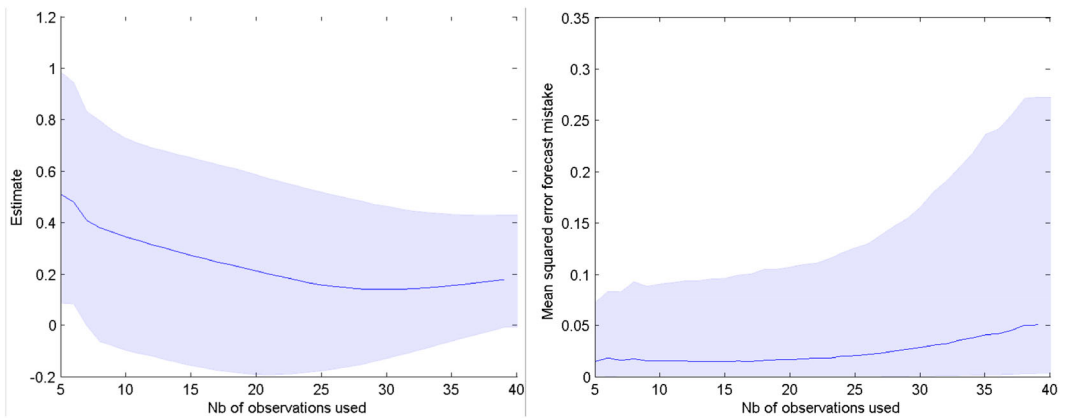
What can explain the huge mistakes of sophisticated firms, and the consequent lower performance of the RLS-learning scenario vis-à-vis the naïve and ADA ones? There are four alternative hypotheses. A straightforward explanation is simply that fast and frugal heuristic expectations outperform RLS-learning ones in an economic environment characterized by deep uncertainty and technical change. In such a framework, heuristics can allow one to get more accurate forecasts than complex procedures, because they are robust to changes in the fundamentals of the economy. This is the *less-is-more* principle emerging when agents must take decisions or form forecasts in complex environment. The second hypothesis is that the larger forecast errors of sophisticated agents are due to an insufficient number of observations employed in the estimations. The third is that heuristic-guided firms create noise, although it should be deeply troubling if “learning” only works when everyone is a sophisticated learner. The fourth is that in complex evolving economies, more sophisticated rules themselves contribute to greater volatility. In order to get insights into these alternative interpretations, we begin by exploiting the cross-sectional heterogeneity in the size of the samples employed by the sophisticated agents to estimate their expected demand. Indeed, depending on their age, firms rely on a variable number of observations bounded between T_{rls}^{\min} and T_{rls}^{\max} .

Figure 5 (left) shows that, as the size of the sample increases and approaches $T_{rls}^{\max} = 40$, the estimates become more and more similar across firms, but the demand forecast errors steadily *rise* (cf. Figure 5, right). This means that long-lasting incumbents make larger mistakes than novel RLS-learning firms. This is a first indication that more information does *not* yield more accuracy in such a setting.

We then consider whether the under-performance of sophisticated agents is due to the “noise” created by heuristic-based ones. By tuning the parameter T_{rls}^{\min} , which defines the minimum number of observations required for RLS estimation, we exogenously change the relative share of sophisticated and heuristic firms in the economy. Figure 6 (bottom, left) shows that the share of RLS learners decreases from 79% to 20% as T_{rls}^{\min} rises from 5 to 30. At the same time, as more heuristic followers populate the economy, output volatility and the unemployment rate steadily *fall* (see Figure 6, top row). Furthermore, the analysis of the MSFEs of the two types of agents suggests that two effects are responsible for such improvement in the performance of the economy (cf. Figure 6, bottom right). First, as heuristic agents make lower mistakes than sophisticated ones, the increase in their relative share automatically reduces the average forecast error, due to a sheer *composition effect*. At the same time, an *interaction effect* is at work, as both types of agents (and especially the sophisticated ones) reduce their mistakes when the fraction of sophisticated firms is lower, which is not surprising given that the economy behaves in a more stable way. The presence of firms following simple heuristics *stabilizes* the economy. The RLS-learning firms turn out to be the source of noise as they destabilize the forecasting performance of all agents. We shall come back below to why this might happen.

In the previous setup, heuristic and sophisticated agents differ in age, and as such, they are not really comparable. Further important insights can be gained instead by experimenting with the evolutionary competition between heuristic and RLS agents within the same environment. In order to do that, we assume that when firms become old enough to perform RLS regressions (i.e., 8-periods old in the benchmark case given the estimation lags), they continue to be heuristic with probability one-half, or conversely become of the RLS type and start estimating their adaptive parameter. (In the next section, we consider

FIGURE 5
 Cross-Sectional Performance across Agents Using Different Number of Observations Required to Perform RLS



Note: Binned plot relating sample size and estimated coefficients (Left). Binned plot relating sample size and demand forecast mistakes (Right). Average over 50 Monte Carlo runs.

the case where, once they are “old” enough and have sufficient data to engage in RLS forecasting, they do so if and only if their data suggests it would be more profitable to do so.) Such a “randomized” experiment allows us to compare the characteristics of the two types of firms. We focus below on their relative fitness proxied by, first, the revealed profitability of the two behavioral types, and, second, their survival rates.

In terms of profitability, the greater forecasting errors make more sophisticated firms less profitable (see also Table 3). In addition, both types display a mean of age at death that is much higher than the median—evidence of a fat right-tail of distribution for firm longevity. Firms that survive longer do so either because they are technologically more competent or simply luckier in their forecast and investment decisions. However, the sophisticated firms seem to live a more precarious and marginal life. The volatility of their size is much higher (see Figure 7, left vs. right) and they represent 97% of the firms in the bottom decile of the market share distribution (average over 50 Monte Carlo runs).⁵⁹

The poor performance of economies where expectations are formed with RLS learning boils

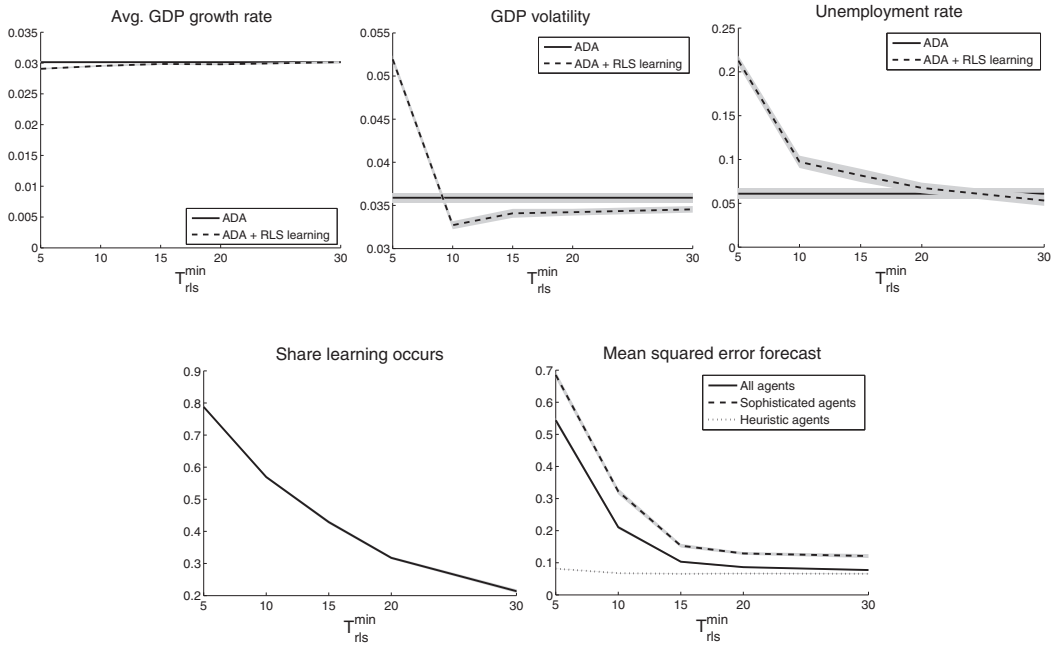
down to the fact that it is not possible to bend complex, nonlinear worlds into a linear framework. This is the case in this model, and it is also the typical situation in contemporary economies where the stream of innovations and the resulting perpetual structural change coevolve with Knightian uncertainty, making the typical econometric tools employed in standard macroeconomics of limited value or even misleading. In such a framework, the less-is-more principle holds and more “sophistication” of the kind described in this section deteriorates the quality of the forecasts. Indeed, in line with Box and Jenkins (1976), more complex time-series models, even when they fit better the (training) data, can lead to higher forecast errors than simpler ones.⁶⁰ Thus, in a complex evolving economy, the adoption of fast and frugal heuristics may be the “rational” response, not only of agents, but of regulators and policy makers too (Haldane 2012). Of course, there are many ways in which agents can become more sophisticated in “learning,” including by means of nonlinear models. But note that as nonlinearities are introduced, the firm must estimate a larger number of parameters, and thus the confidence in the correctness of the model, for a fixed data set, is lowered. But as the number of periods

59. We also experimented with different selection intensities (proxied by the parameter χ governing the replicator equation, Equation (A1) in the Appendix): the median and mean age at death of RLS agents falls faster than that of heuristic ones.

60. In time-series econometrics, simple univariate ARMA models can forecast better than larger structural macroeconomic models. See also Hendry and Clements (2019).

FIGURE 6

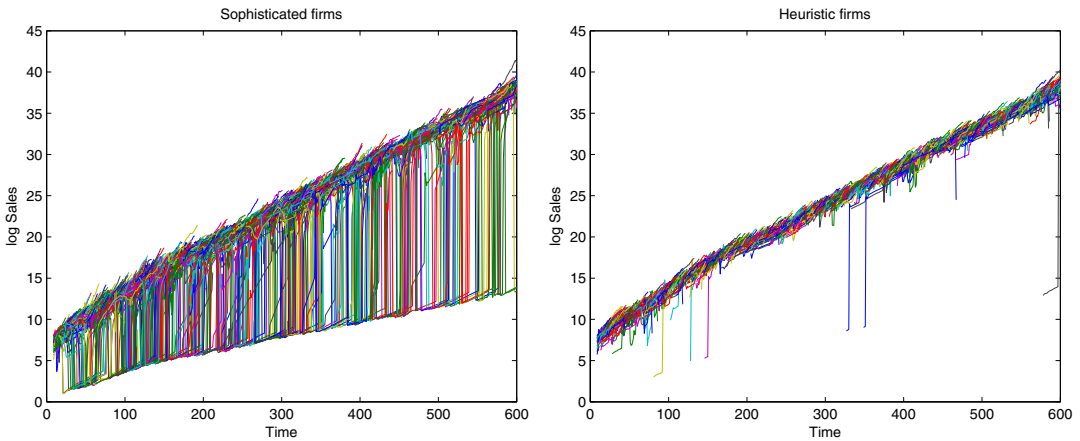
Effect of Changing the Minimum Number of Observations to Perform RLS, T_{rls}^{min}



Note: Average over 50 Monte Carlo runs

FIGURE 7

Evolution of the Logarithm of Sales of RLS Agents (Left) and Heuristic-Guided Agents (Right) over One Simulation Run



of data required before it can achieve a comparable degree of confidence is increased, so too is the likelihood of a structural break. This would not be a problem in a world without technological

change, or in which the pace of technological change never changed. But this is not the world we live in. We have deliberately constructed a model in which the agents populating the model

would not have an easy time uncovering the process generating the changes in technology. In the next section, we explore further the relationship between structural breaks, expectations, and macroeconomic performance.

VII. STRUCTURAL BREAKS, UNCERTAINTY AND EXPECTATIONS

In this section, we describe the results of two additional sets of experiments. First, we study how RLS-learning expectations fare in an environment presenting a lower level of uncertainty and complexity (*low-innovation* regime). This first robustness test will allow us to evaluate how the economic environment affects the performance of different expectation rules. As discussed above, the poor performance of the RLS-learning rule might be related to the complex evolving nature of the environment, as driven by innovation leaps. It follows that such result might be affected by an exogenous change in the size and frequency of innovation leaps.

We exogenously reduce the uncertainty and complexity of the environment by limiting the Schumpeterian engine of the K+S model, scaling down the frequency and magnitude of the micro-shocks that characterize the innovation process. More specifically, we consider a *low-innovation* scenario where both firm search capabilities and technological opportunities are lower with respect to the baseline parametrization (see also Dosi, Fagiolo, and Roventini 2010).⁶¹ We compare the performance of the economy under the baseline and the low-innovation regimes under the ADA + RLS expectation formation.

In the low-innovation regime, when firms adopt RLS-learning expectations, the short-run performance of the economy improves, in particular, rendering it less volatile. This is what we expected: in a less complex environment, RLS works relatively better, resulting also in significantly lower MSFEs.⁶²

61. The parameters impacting search capabilities in the innovation and imitation processes (ζ_1 and ζ_2), are reduced from 0.3 to 0.05, and the Beta distribution governing technological opportunities is modified from a Beta(3, 3) to a Beta(2.7, 3.3).

62. Compared to the baseline innovation regime, the low-innovation regime more than halves the growth rate in the economy, no matter the expectation rule. Yet, the demand forecast errors of sophisticated firms fall (from 0.69 to 0.58 on average).

The conclusion of the foregoing exercise is that the performance of RLS-learning expectations is improved if the economy is less subject to innovation shocks, and is thus more predictable. The second experiment addresses the question: What happens instead if firms try to check the relative accuracy of the heuristic and RLS-learning expectations, and if they account for structural breaks when they select their forecasting rules?

In what we call the *choosing-RLS* scenario, we allow firms to choose between the heuristic and sophisticated expectation rules on the basis of the comparison of the ex post squared errors (as defined in Equation (17)) of the two rules in the previous period (case a). We consider two alternative rules: the agent chooses RLS so long as the ex-post squared errors of the RLS rule is no greater than 20% higher than one of the heuristic rule (case b), or chooses RLS unless the ex-post squared errors of the RLS rule is more than twice the *average* ex post squared error of the heuristic rule (case c). Simulation results show that, not surprisingly, given a choice, agents choose to follow heuristics most of the time. Indeed, for instance in case (a), firms decide not to employ RLS 56% of the time, reducing the population of RLS-learning agents from 79% to 31%.⁶³ As a consequence, the MSFE considerably contracts and the performance of the economy improves (i.e., higher GDP growth, lower GDP volatility, unemployment rate and likelihood of crises than in the ADA + RLS scenario; cf. Table 7, first three rows). In this particular case, it turns out that what is individually rational is also collectively rational, but as we have noted, this is not necessarily the case.⁶⁴

As RLS learning assumes a linear relationship between past and future individual performance, it is inadequate if the relationship under study is characterized by sudden changes and breaks (more on that in Hendry and Mizon 2010). Thus in the *structural-break* scenario, we allow firms to decide whether or not to use RLS expectations after performing a Chow test for structural breaks. More specifically, once a firm has accumulated enough past observations ($T_{rls}^{chow} = 24$), it performs a Chow test, dividing the most recent T_{rls}^{chow} observations into two equal subsamples. If the test rejects the null hypothesis of structural stability, the agent reverts to the heuristic rule.

63. More detailed simulation results are available from the authors upon request.

64. As would be true in any model with macroeconomic externalities.

TABLE 7
Effect of Choosing between the Heuristic and Sophisticated Rules

Expectation Rules	Avg. GDP Growth	GDP Volatility	Unemployment Rate	Likelihood of Crises	Mean Squared Forecast Error	
Choosing-RLS experiment						
<i>Ratio wrt ADA + RLS</i>						
Direct comparison	Case (a)	1.038**	0.641**	0.185**	0.284**	0.119**
$Error_{RLS,t} > 1.2 \times Error_{h,t}$	Case (b)	1.031**	0.634**	0.282**	0.272**	0.064**
$Error_{RLS,t} > 2 \times Error_h$	Case (c)	1.020**	0.635**	0.677**	0.281**	0.429**
Structural break experiment						
<i>Ratio wrt to ADA</i>						
ADA + RLS and structural break test		0.978**	1.021	1.974**	1.209*	3.517**
<i>Ratio wrt ADA + RLS</i>						
ADA + RLS and structural break test		1.014*	0.705**	0.565**	0.455**	0.448**

Note: Ratio with respect to the ADA regime with RLS learning. Significant difference wrt. baseline at 1% level (**) and 5% level (*). Average over 50 Monte Carlo runs.

If no structural break is found, it keeps on with RLS-learning expectations.

We find that the Chow test does not accept the null hypothesis 25% of the times, resulting in a lower share of sophisticated agents in the system (on average, 56% of firms perform RLS, against 79% when the Chow test was not present). As a consequence, when firms can choose to switch to heuristics when they detect a structural break, the relative MSFE falls and all the macroeconomic indicators *improve* (see the last row in Table 7). However, in such a case, even if firms can employ sophisticated econometric procedures, and even when the use of RLS is endogenously determined, the short- and long-run performance of the economy are still worse than when adaptive heuristic rules prevail (see Table 7, and compare with the second row)!⁶⁵ Our findings, again, confirm that in an uncertain, complex world, characterized by frequent structural breaks, “less-is-more” procedures lead to more accurate forecasts and heuristic expectations are (ex post, and systemically) *rational*. Indeed, all this is quite in tune also with the results on organizational learning which, in turbulent environments, crucially involves *learning by forgetting* (Dosi et al. 2017a).

VIII. THE PROPERTIES OF HEURISTIC-DRIVEN EXPECTATIONS

It is crucial to emphasize that in complex evolving worlds, *even the analyst*, as well as any

agent with the same knowledge of the analyst, with the “true” model of the world, would not do any better than the heuristic agent. Consider the analyst who happens to be the constructor of the world, that is, us, the authors of the model: we know the true model and we can simulate it up to time t . Are we able to predict what a state variable will be, say demand, of agent i at time $t + 1$, better than any heuristic agent? The answer is in general negative. Of course we would be very good at predicting the past—as Balzac once wrote—that is in fitting the data to the model, but probably poor in forecasting.

To see this, recall that the model is a very high dimensional system: in its bare-bone structure, it has $N1 + N2$ firms, hit by endogenously generated idiosyncratic shocks (capital-embodied productivity improvements) which affect the competitiveness of the firms via their unit costs, and through that, the dynamics of the market shares and survival probabilities. Therefore, the minimum dimensionality of the system is $(N1 + N2) \times c$ (i.e., the number of control variables of each firm) $\times s$ (the system-level state variables). Furthermore, besides being high dimensional, the system is also highly nonlinear.

First, microtechnological shocks propagate in the economic system. Second, different degrees of competitiveness introduce system-level correlations in the dynamics of firms’ market shares. Third, of course, there is yet another Keynesian system-level correlation, because the individual demands are the market shares multiplied by the size of the whole market, but the latter (endogenously) sums up over all employed workers multiplied by their wages. Fourth, pervasive

65. Notice that our analysis shows that there is no tight link between switching rules based on forecasting errors and the detection of structural breaks.

financial imperfections imply that firms can be constrained in their production and investment decisions by the credit supply of banks, which endogenously evolve according to their equities, possibly leading to emergent banking crises, leading to deep downturns. The emerging outcome is a system which, at the level of the individual components—that is, the firms that make decisions—is a combination between some complex nonlinear dynamics and seemingly random walks. So notionally, if we knew all the functions (including functional forms) and had a long enough history, we might be able to infer parameter values (using nonlinear estimators). However, it is the very ontology of the worlds we are trying to model that prevents precisely this: “structural breaks” driven by technological changes and by the ensuing changes in the competitiveness of individual firms occur all the time. Hence, there is also no hope to observe long time series *between* two successive structural breaks, even for the external analyst, let alone the individual agent. And the more complex and nonlinear the (actual and hypothesized) economic structure, the longer the required time series.

Even in a stationary world, forming unbiased and accurate (low variance) estimates is difficult. These well-known problems are exacerbated by the intrinsic nonstationarity and nonlinearity of the world as a whole, and especially in environments with complex interactions such as explored here. We are not able to dissect nonlinear deterministic and possibly chaotic processes versus the seemingly stochastic components. The very nature of the economy explored here is markedly different from that where agents are assumed to have RE (see Marcet and Sargent 1989), presumably arrived at through some learning process. In the view presented here, the world is too complex, and too much changing, to be able to learn its fine structure, let alone its parameters. In these cases, no accuracy/efforts trade-offs in information gathering/complexity of forecasting appear: heuristics outperform RLS learning in forecasting because their forecast, while perhaps “biased” as compared to those which an omniscient Laplacian God would make, nonetheless have a lower variance than those which finite agents could make using seemingly sophisticated econometrics, even, as we have seen, attempting to account for structural breaks. In this world, it might be expected that quasi-rational agents would adopt some heuristics in forming expectations and as the basis of decision making. Understanding the heuristics that agents use, and the

systemic properties of economies with interacting agents using such heuristics in the presence of deep uncertainty then provides an alternative research agenda to that which has dominated macroeconomics over the past 40 years.

IX. CONCLUDING REMARKS

In this work we have extended the Keynes + Schumpeter (K + S) family of models to account for the impact of heterogeneous expectations and learning processes on the performance of the economy. In particular, firms can forecast their future demand either by choosing among an ensemble of different heuristics or via RLS estimations.

Simulation results show that alternative heuristics, generating significantly different MSFEs may *not* considerably affect macroeconomic performance. Furthermore, when firms switch among heuristics based on performance, none of the heuristic rules disappear from the market: the equilibrium entails an ecology of heuristics.

However, when “sophisticated” firms are allowed to estimate their future demand via RLS, expectations do matter: Soros’ reflexivity gains strength. And they matter for the worse: their forecast errors increase significantly and the performance of the economy significantly worsens as interdependencies amplify mistakes. Indeed, in models where agents are allowed to select among heuristics vis-à-vis RLS-learning of expectations they overwhelmingly “rationally” choose heuristics, and with fewer agents choosing RLS, the forecasts of individual agents are more accurate—and the economic system performs better. The conclusion is that, at least in models with high levels of nonlinearity and complexity, heuristics should *not* be considered as a second-best approximation, based on a trade-off accuracy with effort in the presence of cognitive limitations and biases. Instead, the less-is-more principle holds, and “[we] can rely on heuristics because they are more accurate, not because they require less effort at the cost of some accuracy” (Gigerenzer and Brighton 2009, 135).

Why does RLS learning fail in the model? We have suggested that the huge forecast errors made by RLS-learning firms come from the fact that it is not possible to bend complex, nonlinear worlds into a linear representation. Our analysis provides a commentary on prevailing macroeconomic analyses along two lines. First, concerning expectations: while earlier literature

(cited before in this paper) had questioned both the plausibility of the economy converging to a RE equilibrium, and the empirical relevance of the RE hypothesis, we have questioned the rationality of the RE hypothesis. In the presence of deep uncertainty, technical change and structural breaks, the best “evolutionary” response of firms seem to be the adoption of *heuristics*. They are broadly consistent with *ecological rationality* in complex evolving economies—even if they may or may not be individually rational, although given the pervasive macroeconomic externalities, almost surely the equilibrium proportions (when there is choice) which emerge are not systematically optimal.

We have suggested that the role of expectations may well have been overstated in contemporary macroeconomic theory. For sure, expectations matter in influencing business cycle dynamics—in the real world and also in the model analyzed here. We have shown that similar dynamics may emerge under quite different expectations processes. The economy’s dynamics seems to be driven by other mechanisms, such as firms’ heterogeneous innovation performance, productivity dynamics and financial conditions. These interact with demand expectations to trigger growth waves, avalanches of bankruptcies, as well as mild and deep recessions.⁶⁶ RE provided a mechanism whereby agents plans could be coordinated and made consistent with each other in the absence of a complete set of Arrow-Debreu state-contingent markets. (In the Arrow-Debreu framework, nothing is assumed by rationality of expectations, or even common knowledge.) Together with the representative agent model, the efficiency of the economy could be assured.⁶⁷

As we have noted, the hope was that increased sophistication of agents would lead to a world which might be well-approximated by a RE model, restoring the faith in the competitive market that had been undermined by the restrictive conditions that Arrow and Debreu had shown had

to be satisfied if markets were to be efficient. Our results cast further doubt on this hypothesis: simple and robust heuristics may not only be better in terms of performance of individual agents and the economy as a whole, but turn out to be also a source of predictability of behaviors (Heiner 1983), and a “collective stabilizer,” allowing for easier coordination among heterogeneous interacting agents.

There are different ways forward in this research path. We have explored the role of different expectations behavior in a model which is able to exhibit many of the key macro- and macrostylized facts. There are undoubtedly other specifications that describe well at least some of these, and it would be worthwhile to see if the findings here are reflected in other highly complex economic systems in which there is fundamental uncertainty, for example as here, associated to the arrival of big innovations. So too, we have shown that the problems we have identified do not disappear at least with simple nonlinear estimations, for example, involving structural breaks. Whether the problems increase or decrease as firms proceed to look for reasonable but still more sophisticated nonlinear estimation procedures is still an open question. If it were to turn out that plausible increases in complexity of the estimation procedures by any single firm do not lead to significantly higher profitability, it would suggest that it would not pay firms to continue to search for better estimation methodologies. There are more profitable ways that the firm can employ its scarce resources. The economy might appear “trapped” in an equilibrium in which firms are not as “rational” as they might be—there might be better estimation procedures to be discovered; but for the economy as a whole, this may be actually a good thing, for as we have seen, systemic performance may be better with less sophisticated estimation methodologies.⁶⁸

In practice, it should be obvious that there is not common knowledge: the most difficult economic challenges relate to assessing the future course of economic organization, politics, and technology, about which there is never likely to be agreement, no matter how much data we have available. Economic models should help us understand how economies behave in such circumstances. To assume an economic framework

66. Of course, even within a RE framework, once one departs from the infinite-lived representative agent model with a single homogenous fully flexible capital stock, with no financial constraints, one can generate complex dynamics that are markedly different from the simple convergence exhibited in that model. Indeed, there is a large literature showing how variations in any one of these dimensions (finite-lived individuals, multiple capital goods, putty-clay models, financial constraints) generate complex dynamics.

67. Nothing, of course, was said about the stability of the system. It was simply assumed that somehow, the economy was always in equilibrium.

68. It has long been known that RE equilibrium are not in general (constrained) Pareto efficient (see Newbery and Stiglitz 1982). Accordingly, it is not that much of a surprise that deviations from RE might be welfare enhancing.

in which none of these challenges arise is likely to be at most of limited help. Here, we have constructed a model where the underlying driver of technological change is sufficiently complex that it is not easily discoverable by economic agents. The behavior of the economic system as a whole is, in turn, based on their understandings of the workings of this complex system. The concerns we raise here are likely to become even greater as we move towards an innovation economy, where the stochastic nature of the innovation process itself becomes more central. At the very least, our analysis illuminates a central aspect and insufficiently appreciated aspect of second-best economics: seemingly improving on forecasting methodologies, using sophisticated econometrics, even working hard to identify structural breaks, may well not improve the systemic performance of the economy.

We also believe that our methodology may be able to contribute to the current debate about the robustness of macroeconomic policy across different expectation frameworks and under different hypotheses concerning the underlying economic structures.

APPENDIX A

DETERMINATION OF MARKET SHARES

Market shares are positively associated with consumption-good firms' competitiveness (E_j), which reflects both their price and their amount of unfilled demand (l_j) as inherited from the previous period⁶⁹:

$$(A1) \quad E_j(t) = -p_j(t) - l_j(t),$$

where the unfilled demand $l_j(t)$ is the difference between actual demand and production of the period. A firm's market share is then driven by its relative competitiveness compared to the weighted average (\bar{E}),⁷⁰ following a "quasi" replicator dynamics:

$$(A2) \quad f_j(t) = f_j(t - 1) \left(1 - \chi \frac{E_j(t) - \bar{E}(t)}{\bar{E}(t)} \right),$$

with $\chi = 1$.

69. We normalize the price and the level of unfilled demand according to the industry average to make them comparable.

70. It is computed using the market shares of the previous period: $\bar{E}(t) = \sum_{j=1}^{F_2} E_j(t) f_j(t - 1)$.

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