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# Approximations of the mean synodic month according to Apollonius and ‘Hermes’

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**Abstract.** This note offers two emendations concerning the length of the mean synodic month according to Apollonius and ‘Hermes’.

Emanuel Zingg recently published in this journal new material from manuscripts of John the Lydian’s *De Mensibus*<sup>1</sup>. Among this material, we find two unparalleled references to the length of the synodic month (Zingg considers these to be interpolations in John’s text):

ὁ μὲν Ἀπολλώνιος εἰς ἡμέρας καθ’ ὥρας β’ ἡμισυ λέγει ἀποκαθίστασθαι τὴν σελήνην· ὁ δὲ Ἑρμῆς εἰς ἡμέρας καθ’ ἡμισυ τρίτον τριακοστόν. (p. 97 Zingg)

“Apollonius says that the moon returns to its position in 29 days and 12 and a half hours, Hermes in 29 days and a half, a third and a thirtieth (*sc.* of a day, *i.e.*  $29 + 1/2 + 1/3 + 1/30 = 29 + 26/30$  days).”

This sentence is only present in the manuscript Angelicus 29, in a section written by Eleutherius Elius after 1388. As they stand, the figures are so inaccurate that they appear to have suffered textual corruption. In order to compare them with commonly accepted ancient values of the synodic month, it is convenient to convert the unit fractions used in the text into time units. The length of the synodic month according to Apollonius can thus be written as  $29^d 12^h 30^m$ , its length according to Hermes as  $29^d 20^h 48^m$ .

Already in the 4<sup>th</sup> c. BCE, the 76-year intercalation cycle of Callippus implicitly posited a mean synodic month of approximately  $29^d 12^h 44^m 26^s$  (this results from dividing the 27 759 days in the Calippic cycle by its 940 months). By the second century BCE, Hipparchus had taken over from Babylonian sources a mean length comprised between  $29^d 12^h 44^m 3^s$  and  $29^d 12^h 44^m 4^s$ , which was accurate to within less than a second, and which is best expressed, as in Claudius Ptolemy’s *Almagest*, in sexagesimal fractions<sup>2</sup>: 29;31,50,8,20 days. This figure was accepted by Ptolemy and others and was widely used in astronomical texts<sup>3</sup>. In other terms, the figures as transmitted in the new text are far removed from the degree of accuracy expected for any period of the history of Greek astronomy.

Fortunately, plausible emendations suggest themselves if we assume that, at an earlier stage, the figures were entirely written with numerals rather than partially spelled out:

ὁ μὲν Ἀπολλώνιος εἰς ἡμέρας καθ’ ὥρας β’  $\zeta^4$   $\langle \delta \rangle$  λέγει ἀποκαθίστασθαι τὴν σελήνην· ὁ δὲ Ἑρμῆς εἰς ἡμέρας καθ’  $\zeta$   $\lambda \gamma$ .

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<sup>1</sup> E. Zingg, Aus einer späten neuplatonischen Schrift. Echte und angebliche Exzerpte aus Iohannes Lydos, *De mensibus* 3,8; 3,12; 4,7 und 4,36–38 in drei Handschriften des 14. Jh. (Ang. gr. 29, Par. gr. 2381, Scorial. Φ.III.11), *RhM* 163, 2020, 79-124.

<sup>2</sup> In sexagesimal fractions, the first number after the semi-colon, comprised between 0 and 59, denotes ‘sixtieths’ (Greek ἐξηκοστά or πρῶτα ἐξηκοστά) or ‘minutes’ (λεπτά); the one after that, separated by a comma from the one before, denotes ‘seconds’ (δεύτερα ἐξηκοστά), *i.e.*  $1/60^2 = 1/3600$ ; and so on. For an overview of sexagesimal fractions, see T.L. Heath, *A History of Greek Mathematics*, Oxford 1921, I 44-45 (notation) and 58-63 (calculations). Many Greek texts contain detailed expositions of how to use sexagesimal fractions: see references in A. Tihon, *Les sciences exactes à Byzance*, *Byzantion* 79, 2009, 380-434. Calculations can now be performed easily with Baptiste Méléès’ freeware MesoCalc (<http://baptiste.meles.free.fr/site/mesocalc.html>).

<sup>3</sup> O. Pedersen, *A Survey of the Almagest*, rev. by A. Jones, Berlin <sup>2</sup>2010, 162-163 and 465-468.

<sup>4</sup>  $\zeta$  is the special sign for the fraction 1/2.

“Apollonius says that the moon returns to its position in 29 days and  $12 + 1/2 + 1/4$  (=  $12 + 3/4$ ) hours, Hermes in  $29 + 1/2 + 1/33$  (=  $29 + 35/66$ ) days.”

The first conjecture supposes omission of one letter in a state of the text where the figures were written with numerals (most likely in uncial script, where the sequence  $\Delta\Lambda$  was especially prone to omission). The second one supposes the intervention of a scribe who, disturbed by the unusual fraction  $1/33$ , rearranged the numerals  $\lambda\gamma'$  into  $\gamma'\lambda'$ . Both of the corrected figures are supported by further textual evidence.

$29 + 1/2 + 1/33$  days is the approximate value of the synodic month according to Geminus (VIII, 2-3 and 37; XVIII, 1). It is approximately equal to  $29^d 12^h 43^m 38^s$ . It is the best possible approximation of the synodic month in short unit fractions of days (the next fraction that should be added to improve this approximation would be smaller than  $1/3300$ ). Its association with Hermes here is presumably due to its use in a Hermetic compilation.

$29$  days and  $12 + 1/2 + 1/4$  hours is equal to  $29^d 12^h 45^m$ . This figure seems to have been assumed for the mean synodic month in a fragmentary procedure text in P.Oxy. 4163<sup>5</sup>. It is the best possible approximation of the synodic month in simple fractions of equinoctial hours. Rather than the 3rd c. BCE mathematical author Apollonius of Perga, who is not known to have worked on astronomical computation, the Apollonius quoted in this text is likely to have been the author of lunar tables mentioned by Vettius Valens, IX, 11, 339, who is perhaps the astronomer elsewhere called Apollonius of Mynda<sup>6</sup>.

An alternative solution for the figure associated with Apollonius is to bracket  $\acute{\omega}\rho\alpha\varsigma\ \iota\beta'$  as a gloss of  $\acute{\eta}\mu\iota\sigma\upsilon$ , all the while supplementing  $\lambda'$  before  $\lambda\acute{\epsilon}\gamma\epsilon\iota$  (supposing haplography of one letter). We would then have “ $29 + 1/2 + 1/30$  days” =  $29^d 12^h 48^m$ . This less accurate figure (Ptolemy’s value, rounded to one sexagesimal place) is given as an approximation in the anonymous *Commentary to the Almagest*, § 14 and 24 Jones. In P. Rylands I 27, we also find half this duration for half a synodic month (written  $14;46$  with sexagesimal fractions – corrected by A. Jones from  $14;26$ )<sup>7</sup>. However, this alternative solution is less plausible: not only does it involve more drastic corrections than the first one, but it would seem odd for such a crude approximation to have been associated with a specific author.

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<sup>5</sup> A. Jones, *Astronomical Papyri from Oxyrhynchus*. P. Oxy. 4133-4300a, Philadelphia 1999, I 152. My thanks go to Prof. Jones for answering my questions about this number.

<sup>6</sup> On this Apollonius, cf. A. Jones, *Ptolemy’s First Commentator*, Philadelphia, 1990, 13-14.

<sup>7</sup> A. Jones, *The Development and Transmission of 248-Day Schemes for Lunar Motion in Ancient Astronomy*, *Archive for History of Exact Sciences* 29, 1983, 1-36, here: 15.