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On Armstrong's Radical Absolutism

Julien Tricard

Sorbonne Université – SND

Abstract: Within the metaphysics of quantity, the debate rages between Absolutism and Comparativism. In retrospect, Armstrong appears to be an absolutist, for he claims that magnitudes like *being 1 kg in mass* are intrinsic properties of particulars, in virtue of which relations like *being twice as massive as* hold. More importantly, his theory is an instance of what I call 'Radical Absolutism' (and the only one on the market), for he doesn't merely argue *that* relations are grounded in magnitudes, but also (and quite admirably) tries to explain *how* they "flow from" (his words) the intrinsic features of magnitudes. The goal of the paper is not to support his theory, but to better understand why it fails, and why this must be of concern to contemporary absolutists.

Key words: Quantities; Armstrong; Absolutism; Comparativism; Magnitudes; Relations; Metrical Internalism

1. Introduction

Particular objects exhibit quantitative characteristics, like mass or charge. But they are not merely massive or charged, they are of determinate masses or charges. Let us call 'quantities' the determinable characteristics like being massive, and 'magnitudes' the determinates properties like being 1 kg in mass. Each physical quantity like mass is associated with a whole range of magnitudes. As it is common to express magnitudes by monadic predicates, it seems natural to think of magnitudes as being absolute, that is, intrinsic and non-relational properties of objects.¹

But massive particulars also stand in various determinate mass-relations like *being twice as massive as*. It is for instance the case of an object with a measured mass of 2 kilograms and the International Prototype Kilogram (IPK) stored in Paris. So, there are not only monadic, but also relational facts to be expressed about quantities, and one may wonder which of these two sets of facts is more fundamental than the other. The Absolutism vs. Comparativism debate has consequently been formulated in the following terms since (Dasgupta, 2013).

On one hand, **Absolutism** is the thesis that absolute masses are fundamental, and that mass relations hold among objects *in virtue of* these objects having absolute masses. The absolute mass-magnitudes ground the mass relations.

On the other hand, **Comparativism** is the opposite claim that the determinate mass relations aren't grounded in absolute masses. Thus formulated, Comparativism is merely the denial of Absolutism. But comparativists often support stronger thesis, and argue that absolute masses are actually grounded in the mass relations.²

There are of course intermediate positions between these two. Sider calls 'Mixed Absolutism' the view which also admits of fundamental (second-order) structuring relations (of order and summation³) on top of absolute masses (Sider, 2020, p. 136); more sophisticated – and maybe more adequate – positions were also formulated, as it became clear that this divide between Absolutism and Comparativism doesn't overlap with the

¹ Although monadic predicates can also express extrinsic characteristics (Martens, 2016, p. 9).

² Leaving aside the controversies attached to the notion, I take the term 'to ground' to mean 'to be true or to obtain in virtue of', and also 'to be metaphysically (although not causally) explained by'. Absolutism, then, is the claim that the mass relations are explained by the intrinsic masses. Others prefer to speak in terms of 'fundamental' or 'perfectly natural' properties or relations (Eddon, 2013). Here I choose the notions of grounding and metaphysical explanation because, as we shall see, Armstrong uses them.

³ Summation is a three-terms relation $*(m_1, m_2, m_3)$ which holds when m_1 and m_2 sum to m_3 .

opposition between Substantivalism and Relationism concerning the nature of space-time.⁴ Yet, it is to an extreme position that I will here be interested, that of Armstrong.

As we shall see, Armstrong is an absolutist who not only argues that the mass relations are grounded in the absolute masses, but also tries to explain *how* this metaphysical grounding obtains, that is, *in which way* these relations “flow from” (his words) the intrinsic properties among which they hold. His theory is attempt to see how relations trace their roots back into the internal structure⁵ of absolute magnitudes, and may therefore be called a ‘Radical Absolutism’:

Radical Absolutism: (i) the determinate absolute masses ground the determinate mass relations and (ii) they do so because they are structured in such and such a way.

Most of this paper (sections 2-5) is devoted to criticizing Armstrong’s Radical Absolutism. But what good could it bring to the contemporary debate, if no one today endorses his theory? Moreover, why should an absolutist be radical? After all, she may simply claim that the relational aspects of quantities are grounded in the absolute magnitudes, but that the demand for an explanation as to ‘how’ this happens doesn’t need to be met. She may deny that there is anything interesting to be gained by entering into a deep analysis like the one attempted by Armstrong. Nevertheless, I think there are good reasons to believe that the pressure towards radicalism must be felt even by the mildest absolutist (section 6).

Lots of objections against Armstrong’s theory have been voiced by (Eddon, 2007), (Eddon, 2013) and (Wolff, 2020). After a brief presentation of Armstrong’s theory of magnitudes as structural properties (section 2), this paper offers to classify these criticisms into two categories.

First, I show that many scattered attacks converge into a global problem, that of explaining how different magnitudes are nonetheless determinates of the same quantity (section 3). In this category, I add a new objection, based on the case of zero-valued magnitudes.

The second problem has to do with the relations which, according to an absolutist approach, are to be grounded in the magnitudes. (Eddon, 2013) and (Wolff, 2020) argues that Armstrong’s theory fails to account for the *metrical* relations which give structure to the set of magnitudes. His construal of magnitudes as structural properties successfully explains why they are ordered, but not why they are more or less ‘far apart’. I show that this objection falls short, because Armstrong actually supports a form of ‘Metrical Internalism’, that is the claim that magnitudes have intrinsic “sizes”, on which both ordering and distance relations supervene (section 4). Although his theory then avoids Eddon’s criticism, I show that it falls into a vicious regress (section 5).

2. Armstrong’s theory of magnitudes as structural properties

Armstrong clearly is an absolutist. He intends to “explain the existence of these [determinate] relations as internal relations which flow necessarily from properties of the related things” (Armstrong, 1988, p. 308). Consider for instance two stretches of time, one from a to b and another from b to c (a , b and c being point-events), which the latter *being twice as long as* the former. Armstrong claims that their durations are intrinsic properties of these stretches, and that the relation *being twice as long as* is “an internal relation, grounded in the intrinsic nature of the two stretches” (p. 310). The comparativist

⁴ See for instance Wolff’s structuralist account in (Wolff, 2020), chapters 8 and 10.

⁵ Here, ‘structure’ only means that magnitudes are given some intrinsic features. These may not be analyzed as ‘structural properties’, although it is the case in Armstrong’s theory.

attempt to explain the magnitudes by their relations is, “as a Marxist might put it, trying to explain the base by appealing to the superstructure” (p. 308).

His main motivation is that the quantitative aspects of particular objects don’t seem relational but intrinsic. Goliath’s size is intuitively thought of as an intrinsic property of Goliath, as he plausibly has his size whether or not he is twice as tall as David. Hence, Goliath may be *twice as tall as* David, but this relation doesn’t determine how tall Goliath (nor David) is, but “flows necessarily” from Goliath’s and David’s sizes (p. 308).

Another of Armstrong’s arguments is based on what (Eddon, 2013) named the “Necessity Assumption”: given two magnitudes *being two kilograms in mass* and *being one kilogram in mass*, it is a necessary truth that the latter stands in the relation *being twice as massive as* with the former. But if the latter relation is external, then it doesn’t hold between the two mass magnitudes in every possible world, which contradicts the Necessity Assumption (Armstrong, 1988, p. 308). Hence, one must take the mass relation as internal and thus adopt an absolutist stance.⁶

Anyhow, the focus here will not be on his arguments, but on his account of *how* the magnitudes’ intrinsic features ground their determinate relations. Indeed, Armstrong develops a precise construal of magnitudes as structural properties, which makes his theory the most complete – if not the only – Radical Absolutism on the market.⁷

Armstrong defines a structural property as type of states of affairs which proper parts have properties and stand in relations to each other (Armstrong, 1988, p. 312). For instance, *being a methane molecule* (CH₄) is the type of states of affairs which have one part instantiating the property *being a carbon atom* and four parts instantiating *being a hydrogen atom*, these parts being arranged in the proper way by bonding relations. The structural property *being a methane molecule* is thus said to be *constituted* by the simpler properties *being a carbon atom*, *being a hydrogen atom* and the *being bonded with* relation.

It is important to note that structural properties have other properties not as *parts*, but as *constituents*, because states of affairs have a non-mereological mode of composition (Armstrong, 1986). This means in particular that there is no obvious sense in which one could say that *being a methane molecule* has *being a hydrogen atom* “four times” as its constituents, because a property cannot “duplicate itself”.⁸

But then, how to distinguish between *being a methane molecule* (CH₄) and *being a methyle molecule* (CH₃), if they have the same constituents? Armstrong thinks that identity conditions for structural properties are thus given by the set of their constituents and *the number of times* they occur in the structure of their instances.⁹ This will cause problems later (see section 5).

Magnitudes are construed as structural universal. For instance, *being an aggregate of 19 electrons* is constituted by the properties *being an aggregate of 18 electrons* and *being*

⁶ This argument relies on the contentious assumption that every external relation is contingent. Armstrong certainly claims that internal relations are *logically* necessitated by the nature of their relata, see for instance (Armstrong, 1983, p. 84), and thus every external relation is *logically* contingent. But the relation *being twice as massive as* can still be *physically* necessary, if necessitated by a nomic connection. In fact, Armstrong considers and then rejects this possibility later on (Armstrong, 1988, p. 309).

⁷ (Swyer, 1987) and (Mundy, 1987) have also argued that magnitudes are monadic properties and not relations. Yet, they have offered no analysis of these properties nor any account of the way quantitative relations supervene on their intrinsic aspects (Wolff, 2020, p. 122). More recent absolutist defenders, like (Martens, 2016), are simply no *radical* absolutists (see section 6).

⁸ Unless one chooses the “pictorial” conception of structural properties, or an “amphibian” conception of universals, which Lewis has effectively argued against (Lewis, 1986), and which anyhow Armstrong doesn’t adopt (Armstrong, 1997, p. 37).

⁹ Here, I try to avoid Armstrong’s idiosyncratic vocabulary. For further details, see (Armstrong, 1997, p. 37).

an electron (called the “unit-property”).¹⁰ This gives him a way to recursively define discrete quantities:

We take these structures to be properties characterised recursively thus:

- (i) being the sum of one (distinct) part which is a P just is being a P;
- (ii) being the sum of N+I (distinct) parts which are Ps is being the sum of two distinct parts one of which has the property of being a P, the other of which has the property of being the sum of N (distinct) parts which are Ps. (Armstrong & Forrest, 1987, p. 172)

Thus defined, structural properties (of the same quantity) “contain” themselves in an orderly fashion, like Matryoshka nesting dolls.¹¹ Armstrong then generalizes his account to continuous extensive quantities, like mass. Similarly, magnitudes of mass are nested structural universals, with each being constituted by all the lesser mass-magnitudes. For instance, *being 5 kg in mass* is the structure of massive aggregates with two proper parts, one being 4 kg in mass and the other being 1 kg in mass, which recursively reduces to a sum of five proper parts each being 1 kg in mass.

However, unlike discrete aggregates, continuous magnitudes can be carved up in indefinitely many different ways. For instance, Goliath’s size of 2,90 meters not only has 29 parts which instantiate *being 10 centimeters long*, but is also described in the Bible as “six cubit and a span” (Samuel 17:4). So, *being of size 2,90 meters* is the property of a body which has six parts being the size of a cubit and one part being the size of a span. Thus, a magnitude has to be identified not with *one* structural property, but with a cluster of equivalent structural properties (more on that in section 5).

Importantly, magnitudes of the same quantity are supposed to have internal relations, like *being five times as massive as* between *being 5 kg in mass* and *being 1 kg in mass*. Indeed, one important feature of Armstrong’s theory is his take on resemblance among universals as partial identity. Mass relations are internal because they are resemblance relations, which admit of degrees. With respect of its mass, a 2 kg flour sack resembles more to a 1 kg philosophy book than to an electron of mass $9,109.10^{-31}$ kg. Yet, they stand in these resemblance relations because their mass-magnitudes do. But whereas particular objects can resemble while remaining numerically distinct, resemblance among universal is analyzed as *partial identity*. The resemblance between *being 2 kg in mass* and *being 1 kg in mass* means that the latter is identical to a non-mereological part – a constituent – of the former. And the more constituents two universals share, the more they resemble. This is why, for instance, *being 2 kg in mass* resembles more to *being 1 kg in mass* than to *being $9,109.10^{-31}$ kg in mass*. This last aspect of Armstrong’s theory has attracted most of the objections in the last years, so let us turn to these criticisms and see if they meet their target.

3. The ‘same quantity’ problem

Let us start with a first round of criticisms. All tend to show that Armstrong's theory fails to account for the fact that different magnitudes are nonetheless magnitudes of the same quantity. For instance, *being 2 kg in mass* and *being 1 kg in mass* are two distinct magnitudes, yet they are determinates of the same physical quantity, mass. They stand in the relation *being (a magnitude) of the same quantity as*, in a way that *being 2 kg in mass*

¹⁰ *Prima facie*, it seems odd to treat ‘the number of electrons’ as a quantity, because one usually takes quantities as dimensionful, like mass or charge. Here, the problem is not that “the number of electrons” is discrete (because there are discrete dimensionful quantities, like quantized electric charge), but that it is dimensionless. Yet, as strange as it may seem, Armstrong’s account of dimensionful quantities is based on a simpler analysis of discrete dimensionless quantities.

¹¹ This clarifying comparison is due to (Eddon, 2007, p. 387).

and *being 2 meters in size* do not, and one legitimately asks for what accounts for the holding of this relation.

The issue is of great importance to measurement, where different units can be chosen and used in different contexts, while the measurable quantity stays the same. For instance, distance can be measured in meters, using a rod which has the property *being one meter long*, or in light years, by taking as a unit a stretch of space having the property *being one light year long*. But one expects that what is measured in meters is the same quantity as what is measured in light years, namely *spatial distance*. Since every magnitude could be used (or referred to) as a unit-property, an account of the fact that they are magnitudes of the same quantity is needed.

Although Armstrong doesn't articulate this problem, he would most certainly contend that two magnitudes are of the same quantity (mass) because each *intrinsically* is a magnitude of mass. Moreover, let us remember that he sees magnitudes of the same quantity as tied up by a constituency relation. Each is constituted by all 'smaller' magnitudes and is a constituent of all 'bigger' magnitudes. This may give us a criterium to sort magnitudes into families: two magnitudes are of the same quantity, say mass, in virtue of the fact that *each* intrinsically is a magnitude of mass, because it is constituted by lower mass-magnitudes (Eddon, 2013, p. 640). Thus, the relation of *being of the same quantity* is internal, for it supervenes on intrinsic characteristics of the related magnitudes. Yet, this solution to the "same quantity" problem has three serious shortcomings.

First, if a property is a magnitude of the quantity X because its proper constituents are magnitudes of X, then it only works for infinitely divisible continuous quantities. In the case of mass, no matter how 'small' a magnitude is, it has constituents which are magnitudes of mass. But in the case of a discrete quantity, like the number of electrons, not every aggregate of electrons contains proper parts which are aggregates of electrons: the basic unit, an electron, has no proper parts which are aggregates of electrons.¹² As Armstrong's theory of quantities extends to, and indeed is built on, his account of discrete quantities, this constitutes a serious issue. Eddon makes a rather similar point regarding magnitudes of continuous quantities that are instantiated by point particles: "no object without proper parts can instantiate a structural universal. Now take a single electron. An electron is point-sized; it has no radius and no apparent proper parts. Yet it has a finite mass. How can this be?" (Eddon, 2007, p. 391).

Note that this problem cannot be solved by adding that a property is a magnitude of the quantity X because it is constituted by magnitudes of X *or is a constituent of* magnitudes of X. Indeed, remember that Armstrong's properties are nested in each other *in an orderly fashion*. So, *x's being constituted by y* describes an intrinsic feature of magnitude *x*, for it is a relation between *x* and one of its inner constituents. But *y's being a constituent of x* is not an intrinsic feature of *y*, for it is a relation (or a relational property) to a magnitude *x* "bigger" than *y*. Hence, *y's being of magnitude X* cannot supervene on that.

Second, the solution doesn't work for quantities which admit of positive and negative values, like charge (Eddon, 2007). Conceivably, all positive charges contain lower positive charges as proper parts, and all negative charges contain higher negative (but lower in absolute value) charges, but this doesn't explain why positive and negative charges are magnitudes of the same quantity. Armstrong's view seems to work only for magnitudes with absolute values, and to imply that positive charges and negative charges are qualitatively different quantities¹³.

¹² The same can be said with a discrete *dimensionful* quantity.

¹³ Armstrong is consequently unable to account for the *directional* aspects of vectors, as two vectors of opposite directions are nonetheless determinations of the same vectorial quantity (Eddon, 2007, p. 396). In other words, their difference of direction is not qualitative.

The last problem is actually closely related to a third: the account of zero-valued magnitudes. In the case of discrete aggregates, Armstrong asserts that *being an aggregate of 0 electron* is the structural property of an aggregate with no part instantiating *being an electron* (Armstrong & Forrest, 1987, p. 174). This aggregate is simply not an aggregate of electrons. How can this account be extended to continuous quantities? Let us try the following way: *being 0 kg in mass* is the structural property of a body with no part instantiating *being 1 kg in mass*. This obviously won't do, because every mass strictly lower than 1 kg will be treated as null. The same problem will emerge with any unit mass-property one can choose.

Another solution is to say that *being a null mass* is the structural property of a body with *no part* instantiating *any* mass-magnitude. But this won't do either, because following the above definition of what it is to be a mass-magnitude, having no part instantiating any mass-magnitude is not being a null mass but being no mass-magnitude at all. This is a serious problem for quantities with positive and negative values, like charge. A *charged* body can have a null (or neutral) charge and still have *a* charge. Furthermore, a body can have a neutral charge on the whole but parts with non-null charges, like an ionic solution. Its parts, the cations and anions, have opposite charges which cancel and *annul* each other out. But the solution is nonetheless a charged body.

But this is a troublesome issue even for absolute quantities like mass or distance. Say that the possession of a mass is what makes a body subject to a law of nature relating mass with other physical quantities. A body with a null-mass may enter the scope of this law as well as any other massive body. But this won't be the case if *being of null mass* is not a mass-magnitude at all. Think of a photon, which travels at the speed of light according to the laws of Special Relativity because it has a null mass.¹⁴ So, having no mass has to be treated as a certain magnitude of mass, so that bodies with zero mass enter the scope of such laws. Thus, a body with zero charge or zero mass *does* have a charge-magnitude or a mass-magnitude.¹⁵

Final attempt: why not say that there is a property of *being 0 kg in mass*, which has no other mass properties as constituents, but is a constituent of all other mass properties?¹⁶ First, without any internal structure, I see no means of accounting for the fact that this is a magnitude *of mass*. One might argue that every property is a constituent of itself, purportedly *being 0 kg in mass* is a magnitude of mass because it is constituted by a magnitude of mass, but this is evidently circular. There is no way around the unanalyzable fact that *being 0 kg in mass* primitively is a magnitude of mass.

Now, even admitting it as a primitive fact, it behaves poorly as a constituent of "bigger" mass magnitudes. Remember that the identity conditions for a structural property are given by its constituents and *the number of times* they occur in the pattern. If *being 0 kg in mass* is a constituent of any (strictly) positively valued mass magnitude, then how many times does it occur in its structure? Once? Or infinitely many times? I fear that the case is as undetermined as a division by zero in mathematics.

So, Armstrong's theory seems generally unsatisfactory, as it encounters difficulties with every type of quantity: discrete, continuous, negatively and positively valued. In each case, there is a point at which magnitudes quantitatively differ in such a way that the relation *being of the same quantity as* doesn't supervene of their intrinsic aspects.

¹⁴ This is ignoring the mass-energy equivalence, which implies that travelling photons possess energy and thus, mass.

¹⁵ See (Balashov, 1999) for further – and much more detailed – reasons to admit that zero-valued magnitudes are real determinate null-properties, and not just "privations" thereof.

¹⁶ I must thank an anonymous referee for suggesting this idea.

4. Eddon's objection from the metric

Let us now turn to another objection, which has been raised by (Eddon, 2013). It is a fact that magnitudes of the same quantity don't form any unstructured set, for (i) they are ordered and (ii) they are more or less 'far apart' from one another. For instance, *being 1 kg in mass* is less than *being 2 kg in mass*, but is also closer to it than to *being 5 kg in mass*. In an absolutist approach, these (second-order) relations of ordering and distance are to be grounded in their relata. Now, Armstrong's *radical* absolutist stance brings us one step further: his theory of magnitudes as structural properties is supposed to explain *how* this metaphysical grounding obtains. Eddon argues that it fails to do so.

Admittedly, the constituency relation imposes an order among magnitudes: for any two mass-magnitudes m_1 and m_2 , m_1 is *less than* m_2 ($m_1 < m_2$) if and only if m_1 is a constituent of m_2 (whereas m_2 is not a constituent of m_1). But Armstrong seems unable to account for the "closeness structure" of magnitudes of the same quantity: it doesn't tell us if m_1 is closer to m_2 than to m_3 , because it doesn't capture the distance between them. At first glance, Armstrong has no choice but to treat the distance between two magnitudes as a relation of resemblance, which admits of degrees: the more two magnitudes resemble, the closer they are – this is a "similarity metric" (Wolff, 2020, p. 118). For any two mass-magnitudes m_1 and m_2 , with $m_1 < m_2$, the distance between them should be fixed by the number of different intermediate magnitudes m_i which are constituents of m_2 , but not of m_1 .¹⁷ But Eddon shows that this fails, both in a discrete or a continuous case.

First, take three magnitudes of a discrete (dimensionful) quantity, a , b and c , with the following structure: a has no constituents, a is the only constituent of b , and a and b are the only constituents of c . Then, "on Armstrong's account, b is equidistant from a and c , no matter what mass values these universals correspond to. But if a is the *two grams mass* universal, b is the *ten grams mass* universal, and c is the *eleven grams mass* universal, then b is not equidistant from a and c . The trouble is this: we've no guarantee that the gaps between the constituents of a universal are all the same 'size'" (Eddon, 2013, p. 86). The mere differences between numbers of constituents underdetermine the metric.

Second, to avoid the problem, one may argue that any mass magnitude has *all* the "lesser" magnitudes as its constituents, for mass is a continuous quantity. Then, the *ten grams mass* has not only the *two grams mass*, but all the continuum of intermediate magnitudes as its constituents. But now, any two mass magnitudes have uncountably infinite numbers of constituents in common as well as not in common. So, Armstrong's account fails to capture the metrical relations between magnitudes of continuous quantities, because "the constituency structure that Armstrong posits is not rich enough to recover the closeness structure of quantity" (Eddon, 2013, p. 641).

Yet, I think that this criticism misses its target, for two reasons. First, Eddon misinterprets Armstrong's views on metrical relations and their relative ontological priority with respect to ordinal aspects of quantities, like constituency. As a matter of fact, he claims that when quantities are involved, ordinal aspects do *not* ground, but *are grounded* in the metrical aspects:

Point-events a , b and c are in a temporal order, with a before b and b before c . That is an ordinal fact, the *before* relation is external, and quantity is not involved. But suppose that a is before b by twice as long as b is before c . It seems natural to take the *stretch of time* from a to b as having a certain duration, with the duration a non-relational property of that stretch. Similarly for b to c . These stretches stand in a certain relation, the proportion 2:1. This is an internal relation, grounded in the intrinsic nature of the two stretches. (Armstrong, 1988, p. 310)

¹⁷ Eddon seems to take issue with the fact that this implies that every magnitude lesser than any instantiated magnitude M is supposed to be instantiated, but this is a fact that Armstrong recognizes and even claims to account for (Armstrong, 1988, p. 312).

Relations of order sometimes are external, like the *before* relation between point-events. But when quantities are involved, Armstrong says, ordinal aspects supervene on intrinsic *metrical* aspects of the relata, and become internal. That is why it makes no sense in his framework to ask for the constituency relation, which is a relation of order, to be a ground for the metric: because without the metric, the constituency relation becomes perfectly external, and simply doesn't supervene on intrinsic aspect of its relata anymore.

Take two magnitudes m_1 and m_2 with m_1 bearing the relation *being a constituent of* to m_2 . And suppose, like Eddon, that any metrical relation between m_1 and m_2 can only obtain in virtue of this constituency relation. But this relation clearly is external, because no intrinsic feature of m_1 plays any role in its being a constituent of m_2 . Since it doesn't supervene on intrinsic features of *both* its relata, the constituency relation (much like the 'parthood relation' in mereology) is external. If the metrical relations were grounded on it, then they would also be external, and we would end up admitting external (second-order) relations, and thus totally departing from Armstrong's Radical Absolutism.

This alone invalidates Eddon's criticism: there is no sense in blaming the constituency structure for failing to ground the metric, because he sees any relation among magnitudes, be it of ordering constituency or of distance, as *internal*, supervening on *intrinsic metrical* features of the magnitudes. This leads Armstrong directly to a claim which I call 'Metrical Internalism' and examine in the next section. Now, if we go back to Eddon's first case, this gives Armstrong an easy way out. Suppose that mass is in fact a discrete dimensionful quantity, which can be properly quantized in quanta of, say, one m . Then, the "gap" between any mass magnitude and its immediate successor in the constituency structure always has the same "size": *being (n+1) m in mass* is being a body with one part instantiating *being (n-1) m in mass* and another part instantiating *being 1 m in mass*. Not only their constituency relation – the fact that the former is a constituent of the latter – but also their distance relation *being 1 m apart* supervene on both their intrinsic "sizes".

Second, and on a wider scale, her objection is voiced from the perspective of the *representational theory of measurement*, which is quite orthogonal to Armstrong's approach. In empirical sciences, a lot of quantities are measurable, which means that numbers are attributed to their magnitudes and adequately express their relations. A problem, then, is to explain why numbers are so useful – if not indispensable – in measurement. The *representational* understanding of measurement is one way to approach this issue. A numerical scale is applicable to a range of magnitudes whenever these magnitudes exhibit a structure of ordering and summation relations, which can be represented by a certain structure of numbers through the proper homomorphism (according to *representation* and *uniqueness* theorems).¹⁸ Then, when the proper relations cannot be found or defined among the magnitudes, one fails to explain why numbers are applied to them. This is precisely what happens in Armstrong's case according to Eddon: the degrees of resemblance among properties with shared constituents don't give magnitudes enough structure to be represented by real numbers (for instance, by a *ratio scale*).

But this is omitting that the representational approach is not the only way to understand why numbers apply to quantities. As underlined by Joel Michell, one can also posit that numbers are *actually instantiated* relations among magnitudes of the same quantity. "If they are understood in this way then the logic of the application of arithmetic to reality is that of *instantiation* rather than that of *representation*" (Michell, 1997, p.

¹⁸ A summation relation is required only for additive quantities, which are represented by *ratio scales*. For non-additive quantities, like temperature, which are represented by *interval scales*, a more complex four-terms relation of comparison has to be defined, see (Swoyer, 1987, p. 249). See (Krantz, et al., 1971) for a nominalist representational account of measurement, and (Swoyer, 1987) or (Mundy, 1987) for a realist representational approach.

270). Under this assumption, the question cannot be whether magnitudes exhibit the adequate qualitative structure of ordering and summation to be represented by a numerical scale anymore, because, as a matter of fact, they already instantiate the desired *numerical* structure. If numbers, or more precisely, numerical scales, are not mere representational tools, but genuine relations which magnitudes instantiate, then magnitudes have a chance to be *ipso facto* structured in the proper (numerical) way.

Armstrong precisely holds such a view on numbers. Along with Forrest, he identifies them as *internal numerical* relations among magnitudes. For the purpose of this article, a detailed presentation of this (convoluted) aspect of his ontology is not required.¹⁹ We just need to understand that, under this account, magnitudes cannot be blamed for lacking structure to be *represented* by numbers, because as a matter of fact, they already stand in the proper numerical relations. In our case, these are metrical relations, which internally hold among magnitudes, because of their intrinsic “sizes”. Armstrong’s radical absolutism then culminates in his “Metrical Internalism”.

5. Armstrong’s Metrical Internalism and the structural regress

As a Radical Absolutist, Armstrong tracts “intrinsicness” down to the core, and this especially applies to the metrical aspects of magnitudes. Metrical Internalism is the claim that magnitudes are intrinsically “measured” or have intrinsic “sizes”.

It is a much stronger thesis than the absolutist claims encountered so far. Take two bodies, one *being of mass 5 kg* and the other *being of mass 2 kilograms*. The first stands in the relation *being 2.5 times more massive than* to the other. A standard absolutist takes this mass relation to be grounded in intrinsic aspects of the two bodies, namely their absolute magnitudes. The two bodies are so related because of the intrinsic “sizes” of their masses, captured by their absolute magnitudes. As the explanation of the relation is now deferred to the level of mass properties, a *radical* absolutist asks for what it is about these magnitudes that makes this relation obtain. Armstrong answers that *being of mass 5 kg* and *being of mass 2 kilograms* stand in the metrical relation of *being 2.5 times more than*, and that this relation is internal, because it supervenes on the intrinsic *metrical* aspects of these magnitudes.²⁰ Metrical relations thus don’t build a structure upon an unmeasured set of magnitudes, but they are a supervenient structure flowing from the intrinsic characteristics of its elements. Metrical Internalism captures the core of Armstrong’s theoretical proposal. Although I claim that it leads to a vicious regress, let us see how it can be worked out.

First, to speak of intrinsic metrical aspects of magnitudes is quite unclear. Usually speaking, “metrical” implies “numerical”, for a metric is a function associating pairs of elements of a set with (real positive) numbers. Does Metrical Internalism endow the magnitudes with intrinsic *numerical* features? Does Armstrong think that there is

¹⁹ Armstrong and Forrest offer to identify real numbers with topic-neutral, internal, second-order proportion relations holding among aggregates or magnitudes (Armstrong & Forrest, 1987). Thinking of numbers as relations is particularly counter-intuitive. *Prima facie*, a number is a thing, not a property nor a relation. But Armstrong rejects “abstract”, causally inert entities. So, if numbers exist and aren’t spatiotemporal concrete particulars, then they are (instantiated) universals (Armstrong, 1997, p. 5). They are second-order dyadic universals (that is, relations) because they hold between first-order universals: structural properties and in our particular case, magnitudes. They are topic-neutral, because the same number can relate two mass-magnitudes or two charge-magnitudes. And most importantly, they are *internal* relations, because they supervene on intrinsic features of structural properties (Armstrong & Forrest, 1987, p. 172).

²⁰ Note that these metrical relations are now topic-neutral, because the same relation can hold between two mass-magnitudes or two charge-magnitudes. They lack the *intensive* nature of the mass-relations which a comparativist must take as fundamental, simply because it is not needed anymore. *Being of mass 5 kg is 2.5 times more massive* (rather than charged) than *being of mass 2 kilograms*, simply because both intrinsically are magnitudes of mass (rather than of charge) – see section 3 for this issue.

something intrinsically ‘5-ish’ in the magnitude *being of mass 5 kg*? This would nicely explain why it stands in the relation *being 2.5 times more than to being of mass 2 kilograms*: the relation would supervene on the ‘5-ishness’ of the first and the ‘2-ishness’ of the second.

Yet, this conflicts with the dimensionful nature of mass. The ‘5-ishness’ of *being of mass 5 kg* depends on the conventional choice of a measurement unit, and shouldn’t be seen as a fundamental property of this magnitude. And even if it is, the choice of the unit is largely arbitrary: a body of mass 5 kg also measures 11.02 pounds. Thus, its mass-magnitude is both ‘5-ish’ and ‘11.02-ish’. But if a magnitude has as many fundamental numerical properties as there are measurement units, this leads to a massive redundancy in the fundamental properties. Failing to distinguish between conventional numerical aspects and intrinsic features of magnitudes, Armstrong’s theory would suffer from the same “flat-footedness” as other most naïve accounts of quantity.²¹

But I don’t think it is the case. Armstrong claims that magnitudes have intrinsic metrical features, but sees them not as *numerical* but as *structural*. Each magnitude is intrinsically “measured” by its internal structure, and metrical relations among magnitudes supervene on their internal structures. This solution admittedly avoids the “flat-footedness” problem.

Take for instance the magnitude of *being of mass 2 kg*. Armstrong perfectly admits that there is nothing particularly ‘2-ish’ in a body’s *being of mass 2 kg*. Sure, it has *two* parts each instantiating the magnitude *being of mass 1 kg*. But recall that a massive body can be carved up in infinitely many different ways, for mass is a continuous quantity. Hence, *being of mass 2 kg* corresponds not to *one* structural property, but to a class C_2 of infinitely many structural properties: “all those non-relational structural properties formed by taking the innumerable pairs, triples, etc. [of non-overlapping individuals less than two kilograms in mass], where the mass of the members of each N-tuple sums to two kilograms. (Armstrong, 1988, p. 312)”.²² This gives Armstrong a nice way to explain how the intrinsic metric of a magnitude is determined by its structural aspects: the intrinsic metric of *being of mass 2 kg* is fixed by the equivalence class C_2 ; *being of mass 2 kg* and *being of mass 5 kg* are metrically different because the classes C_2 and C_5 of all the ways they can be carved up are different.

This also nicely applies to discrete (dimensionful) quantities. Suppose again that in fact mass is a quantized quantity, with the quantum $1 m$ as a privileged unit. Because mass is dimensionful, any mass magnitude can be expressed by various numbers depending on which unit is used, but because mass is discrete, one of these magnitudes is a natural unit-property: *being of mass 1 m*. The mass of any massive particular can still be carved up in various ways, but this time, they are of finite number. A body of mass $n m$ has all those structural properties formed by taking the numerable pairs, triples, etc. [of non-overlapping individuals less than $n m$ in mass], where the mass of each member of each N-tuple is $k m$ (with $0 < k < n$), and the mass of all the members of each N-tuple sums to $n m$. Thus, *being of mass n m* has an intrinsic measure, fixed by this class C_n of equivalent structural properties. Only here, in contrast to what happens with continuous quantities, the class is finite.

But here lies the rub. Indeed, both in continuous and discrete cases, the different structural properties need to be equivalent, in order to correspond to a unique magnitude. But how to explain this equivalence? Armstrong suggests that they all amount to the same

²¹ Sider calls “flat-footed” the theory which naïvely attributes numbers to objects. Having a mass-magnitude is analyzed as bearing a “mass-in-a-unit” relation to a number (Sider, 2020, p. 120). Thus, the flat-footed theory sees as fundamental an aspect of magnitudes which depend on a conventional choice, and leads to a massively redundant ontology, see (Eddon, 2013, pp. 83-84). See also (Martens, 2016) for the need to “distinguish between physical magnitudes and the numerical quantities used to represent them” (p. 14).

²² Due to identity conditions for structural properties, all these properties are different (see section 2).

magnitude because they are various partitions which all *sum* to the same mass, two kilograms. In other words, two structural properties correspond to the same magnitude because their *instances* are of equal mass. But this won't do, because it reverses the "direction of explanation" to which Armstrong constantly sticks: what holds for properties (or universals) explains what holds for their instances, and not the reverse²³. So, he cannot explain a relation between properties by appealing to particulars.

There is no obvious way to explain how different structural properties are actually different ways to carve up the *same* amount of mass. One may admit *primitive* equivalence classes. They are the classes of structural properties which correspond to the same magnitude, hence which all have the relation *being equal to* one to another. But then this relation ought to be external, for it doesn't supervene on any intrinsic aspect of these structural properties. This would be a big rock in the absolutist shoe, because now the metrical relation isn't "ontological free lunch" anymore, and has to be postulated on top of first-order structural properties.

Or, one may insist that this metrical relation be internal, supervening on structural features of structural properties. But now, this clearly leads to a vicious regress, for now one has to postulate structural properties of structural properties.

Recall indeed that a structural property captures the way that a *particular* object or aggregate is structured. A particular's *being of mass 2 kg* is the fact that it is structured in certain way, for instance, its having two parts instantiating *being of mass 1 kg*. This gives the property a "constituency structure", because *being of mass 2 kg* is constituted by *being of mass 1 kg*. But this doesn't give the property a "mirrored" structure corresponding to the structure of its instance.²⁴ To say that magnitudes have intrinsic structure, one has to postulate structural properties of structural properties; but then a threat of regression appears.

Take the discrete case (the same easily applies in the continuous case). One might want to say that the properties of C_n are equivalent because they all have the same structure: that of having n parts each instantiating the unit-property. But this may be true of the particular body, not of the property, because no part of the property has a mass. A mass property simply isn't massive. Then, one may say that equivalent properties all have the following structure: that of being composed of n times the (first-order) unit property *being of mass 1 m*. That way, one gives these properties a structure mirroring that of the particular aggregates. But this begins the regress, for now one has to explain how *being of mass $n m$* and *being of mass 1 m* stand in the metrical relation *being n times as many as*. As account of how a numerical relation supervenes on two structures is constantly deferred to the next-order, the regress is vicious.

Let's take stock. Let's admit that if two massive *particulars* have a metrical internal relation, one *being x times as massive as* the other, it is so because of their respective *intrinsic* structure. But then the burden of grounding the relation is transferred to the structural properties. If now these properties have the proper relation in virtue of their own *intrinsic* structure, then we are no better off. Chasing the ground of metrical relations down into the intrinsic features of the relata is an analysis which never ends. And the regress is vicious, unless at some point the relations aren't grounded in *intrinsic* characteristics of the related magnitudes anymore. But it is too relationist a solution for a radical absolutist to accept.

²³ See for instance (Armstrong, 1989, p. 28).

²⁴ See section 2 and note 8.

6. ‘Mild’, ‘Mixed’ or ‘Radical Absolutism’?

So, Armstrong’s Radical Absolutism fails. So what? After all, his theory is only one of many possible Radical Absolutisms. Yet, it is the only Radical Absolutism actually developed and available on the market. But more importantly, an absolutist can perfectly *not* be a radical one. She may simply claim that the relational aspects of quantities are grounded in the absolute magnitudes, but that the demand for an explanation as to ‘how’ this happens doesn’t need to be met. Let us call her a ‘mild absolutist’. Why bother being radical, when one can perfectly deny the need to explain how relations trace their roots back into the internal structure of absolute magnitudes? However, the two problems that Armstrong’s theory trips over are of concern to the contemporary mild absolutist, because they raise disquieting questions that, once asked, cannot be unasked. Here, I argue that these questions irresistibly push the absolutist either towards a more Radical Absolutism, or towards a form of ‘Mixed Absolutism’.

Take the ‘same quantity’ problem to explain how different magnitudes are nonetheless determinates of the same determinable quantity. The mild absolutist claims that *being of mass 1 kg* and *being of mass 1 pound* are magnitudes of the same quantity in virtue simply of the fact that each intrinsically is a mass-magnitude, and that there is no need to further explain it. Yet, these magnitudes are *prima facie* *different* properties, and there is simply no obvious reason to think that *being of mass 1 kg* should be stored with *being of mass 1 pound* rather than *being of charge 1 coulomb*. Compare with *being apple-green*, *being bottle-green*: they are *qualitatively* different because they are different determinates. So, one may ask what it is that makes them determinates of the same determinable, *being green*.

One may retort that, unlike determinate colors, magnitudes of the same quantity differ only quantitatively, and not qualitatively. They are qualitatively identical, or genuinely resemble in some way. But resemblance usually means the sharing of at least one (natural) property, which give the resembling items a common qualitative aspect. If the mild absolutist wants the resemblance among magnitudes to remain unaccounted for, then saying that they resemble is not saying anything illuminating. She simply finds herself with plainly different properties. If instead she consents to account for the qualitative identity (or genuine resemblance) of magnitudes, she quickly ends up giving them some internal ‘structure’²⁵ and thus explaining that in virtue of which the resemblance obtains: either by analyzing qualitative identity as partial identity, which is the armstrongian way, or by attributing to the magnitudes a second-order perfectly natural property, that of *being a determinate of mass*. Either way, the slope toward some form of Radical Absolutism is very slippery.

There remains a way to resist the analysis, and account for the genuine resemblance among magnitudes without giving them some internal structure. In a Lewisian fashion, it is to posit that magnitudes form *primitive* natural classes, and to identify, for instance, the perfectly natural determinable property *being a determinate of mass* with the natural class of all mass magnitudes. But anyhow, this conflicts with the fact that determinables are generally considered as grounded in determinates, and thus, not as perfectly natural properties.

Despite its importance, this problem has not been straightforwardly addressed by recent advocates of absolutism. For instance, Martens defines a quantity as a set of monadic properties on which the adequate structures (of order and summation) are placed. Then, strangely enough, he adds: “as it stands, these structures render all absolute masses qualitatively identical” (Martens, 2016, p. 15). If the role of grounding qualitative identity among magnitudes is played by second-order relations, then this is not a ‘Simple’ but a

²⁵ In the loose sense of the term, see note 5.

‘Mixed’ form of Absolutism. If qualitative identity of magnitudes isn’t grounded in the magnitudes, but in their second-order relations, then these relations ought now to be perfectly natural.²⁶

All this shows that there are strong reasons for the absolutist either to stop being ‘mild’, and adopt a form a ‘Radical Absolutism’, or to stop being ‘simple’ and drift toward a form of ‘Mixed Absolutism’.

Now, take the second problem of explaining how the mass-relations, and especially the metrical relations, ‘flow from’ the absolute magnitudes. The absolutist can easily claim that these relations are (metaphysically) explained by the absolute magnitudes, and yet refrain from further clarifying *how* this explanation relation works. But there are good reasons to find this ‘mild’ position unsatisfactory.

First, one might doubt that an explanation of *Y* by *X* that is not an explanation of *how* *X* gives rise to *Y*, or *why* *Y* obtains in virtue of *X*, really is a genuine explanation, because it is not illuminating at all. Simply stating that ‘*X* explains *Y*’ is nothing more than saying that there is an explanation of *Y* by *X*, which only prompts us to ask *what* this explanation is. Yet, this objection may only reflect the limits of the notion of grounding, and may be avoided by preferring other notions, like ‘fundamentality’ of ‘perfect naturalness’.²⁷

A second reason is fairness in the debate. It is helpful to see Absolutism and Comparativism as opposite deviations from an intermediate ‘Mixed’ position, which admits of *equally fundamental* absolute magnitudes and structuring relations (of order and distance). Then, opposite to the (simple) absolutist, let the (simple) comparativist claim that absolute magnitudes obtain in virtue of the relations holding among them. Now, lots of objections blame Comparativism for being unable to recover facts about absolute magnitudes based solely on facts about determinate relations.²⁸ It is then only fair to ask the absolutist the same in return: it is not enough to argue *that* relations aren’t fundamental, but she must also explain *how* she can recover relational facts from facts about absolute intrinsic properties?

Third, even if one refuses this reason of fairness, the mere fact that intrinsic and thus, non-relational properties give rise to relations is nonetheless puzzling. Even Lewis, the great advocate of Humean Supervenience (i.e. the thesis that everything supervenes on point-sized particulars with perfectly natural intrinsic properties), makes an exception for spatial or spatiotemporal relations. So, one can naively ask: what makes quantitative relations, unlike spatiotemporal ones, supervene on the intrinsic magnitudes of objects? Goliath has the size magnitude *being 2,90 meters tall* and David *being 1,45 meters tall*. The first has the relation *being twice as tall as* to the second. The absolutist claims that this relation holds simply in virtue of what David’s and Goliath’s sizes intrinsically are is extremely plausible indeed. But much of this *prima facie* plausibility comes from the intuitive idea that this relation flows from the ‘how much’ David’s and Goliath’s sizes are. If this intuition is to be taken seriously, then one attributes intrinsic *metrical* aspects to David’s and Goliath’s sizes: this is the ‘radical absolutist’ road that Armstrong has explored, leading him towards Metrical Internalism. If one refuses to go down this road, and takes the magnitudes to be unanalyzable ‘blobs’, then the absolutist claim loses much of its apparent credibility.

²⁶ Eddon makes a similar case: perfectly natural second-order relations are needed to account for resemblance among magnitudes (Eddon, 2013, p. 95). Note that a Comparativist also faces the same problem: how does she account for the qualitative resemblance between her fundamental determinate mass-relations? This suggests that a ‘Mixed’ position between Absolutism and Comparativism should be favoured.

²⁷ For more detailed objections against ‘grounding’ as explanation, see (Wilson, 2014).

²⁸ See for instance arguments based on (active) Leibniz Scaling in (Dasgupta, 2013), or the ‘Ozma games’ and the ‘Comparativist Buckets’ in (Martens, 2016).

Fourth and last, the absolutist might be tempted to say that a mass relation is an ‘ontological free lunch’ because it is an *internal* relation, which supervenes on the intrinsic features of its relata, i.e. of the absolute magnitudes. But now, as Sider notes, the absolutist must now posit nontrivial second-order intrinsic properties of the first order intrinsic magnitudes (Sider, 2020, p. 137). The slope is very slippery, and the absolutist quickly finds herself drifting towards a form of Radical Absolutism, as she now gives the absolute magnitudes some internal structure. The danger is then great to follow Armstrong in a vicious regress (section 5). On the other hand, the road of *external* relations seems barred: if mass relations are construed as external relations, then they don’t supervene on their relata and are as fundamental as the magnitudes, and the absolutist drifts toward a form of “Mixed Absolutism” again. The simple and mild absolutist could finally try to dismiss the problem, claiming that mass relations are neither internal nor external, or that the question doesn’t make sense. But the burden is now on her to explain why the question doesn’t arise, or that such relations can be neither internal nor external.

For all these reasons, Mild Absolutism is not a quiet position. If the absolutist does not want to make a step towards the comparativist, by admitting perfectly natural relations, then strong reasons pushed down the road of ‘Radical Absolutism’.

7. Conclusion

Armstrong’s Radical Absolutism is the most ambitious attempt to explain *how* absolute mass magnitudes can ground determinate mass relations. Yet, as argued, it fails on two counts. First, it is unable to account for different magnitudes’ being determinates of the same quantity. The relation *being a magnitude of the same quantity as* is treated as internal, but the theory fails to specify on which intrinsic features of magnitudes it supervenes.

Second, it proves unable to conclusively explain how determinate metrical relations are grounded on the magnitudes’ intrinsic metrical features. Armstrong’s theory tries to materialize the absolutist intuition that having a magnitude is being intrinsically a ‘certain amount of’ something. But his attempt to explain the metrical by the structural fails, because it also needs to avoid a ‘flat-footed’ version of absolutism which attributes numerical aspects to magnitudes. Thus, it embarks in a vicious regress, where metrical relations of order n are explained by positing structural properties of order $n+1$, which in turn need to stand in determinate metrical (equivalence) relations of order $n+1$, and so on.

Now, the fact that Armstrong’s theory, the only Radical Absolutism on the market, fails, should be of concern for absolutists in general if, as argued in the last section, they can’t afford to stay ‘mild’ for long.

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