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Why Is Exclusivity in Broadcasting Rights Prevalent and Why Does Simple Regulation Fail?*

DAVID MARTIMORT[†] JEROME POUYET[‡]

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Abstract

Pay-TV firms compete both downstream to attract viewers and upstream to acquire broadcasting rights. Because profits inherited from downstream competition satisfy a *Convexity Property*, allocating rights to the dominant firm maximizes the industry profit. Such an exclusive allocation of rights emerges as a robust equilibrium outcome but may fail to maximize welfare. We analyze whether a ban on resale and a ban on package bidding may improve welfare. These corrective policies have no impact on the final allocation but lead to profit redistribution along the value chain.

JEL CODES. L13, L42.

KEYWORDS. Broadcasting rights; upstream and downstream competition; exclusivity.

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1. INTRODUCTION

MOTIVATION. The pay-television (pay-TV henceforth) market is characterized by fierce competition all along the value chain. In the upstream market for contents, TV firms compete through bidding procedures to obtain must-have contents (sports events or premium movies) from content owners (sport leagues, major film studios, etc.). In the downstream market for viewers, TV firms compete in price and quality to attract viewers.

Upstream and downstream competition are deeply intertwined. Downstream, viewers choose which channels to purchase by comparing the price and quality of their respective offers, with the understanding that quality depends on the distribution of broadcasting rights inherited from the upstream stage. Upstream, TV firms express a willingness to pay for contents that depends on how those contents might help them gain market shares downstream.

A remarkable feature of those markets is that most often, a single TV firm ends up obtaining all the broadcasting rights for a class of premium contents. That such exclusivity emerges as a robust outcome raises a number of important questions. First, on the positive side, we may wonder which fundamental economic forces drive such concentration. Why would a content owner choose to pick a single winner and not spread out such rights across rivals to *in fine* touch a larger viewership? Is exclusivity a property that prevails across all equilibrium outcomes and for various modes of downstream competition? Is it robust to the auction format? Second, and taking a more normative stance, we may also ask whether such exclusivity harms welfare and, under those circumstances, whether simple regulatory constraints might have a role in improving this outcome.

To answer these questions, we present a simple model of two-sided competition that blends key features of the upstream and downstream markets. Building such an integrated model is a necessary step to assess the origins and consequences of exclusive agreements in those markets. On the one hand, a partial focus on the upstream bidding market would not suffice to understand bidding strategies since TV firms evaluate their willingness to pay for broadcasting rights with an eye on how the downstream allocation of viewers across channels impacts downstream profits. Acquiring exclusive rights certainly gives a winning firm a competitive edge downstream by attracting more viewers. It also hurts this firm's competitors by reducing their own market shares, which is a standard feature of auctions with externalities.¹ On the other hand, focusing on the downstream market alone would abusively reduce the analysis to a two-stage game with firms choosing the price and quality of their own programming² without taking into account that the quality of contents actually depends on how the upstream market is cleared, which itself depends on downstream profits and thus on the quality of programs.

CONVEXITY OF PROFIT FUNCTIONS. This circularity is at the core of our analysis. Its key implication is that there are strong reasons why exclusive agreements emerge and dominate other non-exclusive modes of distribution from the industry's viewpoint. The key property beyond this result is the *convexity* of the profit functions in the downstream market. Under a broad range of actual modes of downstream competition, a dominant firm downstream gains more from obtaining exclusive rights upstream than what it would lose if those rights were instead given to a weaker competitor.

¹See Jéhiel and Moldovanu (2000), Varma (2002), Brocas (2013), Assef and Chade (2008), Molnar and Virag (2008) and Martimort and Pouyet (2020) among many others.

²See Shaked and Sutton (1982, 1983) for some contributions on this front.

This *Convexity Property* ensures that granting exclusivity for broadcasting rights is *constrained-efficient*; *i.e.*, it maximizes the industry's overall profit. Moreover, if firms are *a priori* different, because, for instance, one of them is better able to market premium programs or has already benefited from a captive viewership, then all rights should be given to this firm, which might reinforce its dominance in a dynamic context.

CONSTRAINED EFFICIENCY AND MONOTONIC BIDDING EQUILIBRIA. To avoid making any *ad hoc* restrictions on the auction procedure, we view competition in this market as a menu auction in the spirit of Wilson (1979) and Bernheim and Whinston (1986a). Downstream firms submit nonnegative bidding schedules that stipulate bids for all possible distributions of rights that could be chosen by their upstream owner. That the equilibrium allocation is *constrained-efficient* is a robust finding. This finding holds throughout the whole set of equilibria of the bidding procedure that can be reached by means of monotonic bidding schedules.³

Of course, those equilibria differ in terms of the distribution of profits they induce across the industry. Although the weaker firm always obtains the same payoff, profits may be redistributed across equilibria from the dominant firm to the upstream seller of rights. Among all possible equilibria, the highest profit to the dominant firm is achieved by means of *truthful bidding schedules* (Bernheim and Whinston, 1986a). Those bidding schedules are undominated strategies that perfectly reflect the preferences of firms across the various possible allocations of rights. In such a *truthful equilibrium*, the dominant firm bids a Vickrey-Clarke-Groves payment that compensates the seller for the foregone opportunity of not having sold to the weaker firm. In other equilibria (based on weakly dominated strategies for the weaker firm), the dominant firm may end up paying even more to obtain exclusivity. This property of the truthful equilibrium thus echoes the celebrated Chicago School argument, which posits that if any exclusive agreement is ever signed, the seller has certainly been compensated for the foregone opportunity of not having chosen a more even distribution of rights across downstream competitors.

IS REGULATION WARRANTED? The presumption often made in related antitrust cases is that the gains of exclusivity that pertain to the industry can be redistributed to viewers. However, exclusive agreements might also be considered, perhaps more pessimistically, as reducing downstream competition for viewers and thus hindering consumer surplus. That the industry equilibrium may fail to be welfare-maximizing thus *a priori* calls for some sort of public intervention.

As a result of this tension between the possible efficiency gains of exclusivity for the industry and its negative consequences on viewers, competition authorities throughout the world have taken different postures. In North America, case law specific to sports media views exclusive agreements as *a priori* legal. In the recent *Spinelli v NFL* case, the Court held that “*because the benefits of exclusive licensing agreements are well-recognized,*” the arrangements were presumptively legal.⁴ In 2015, the Canadian Competition Bureau approved a 12-year exclusive distribution agreement between the NHL and Rogers Broadcasting, arguing that the deal would not foreclose competition and that pro-competitive gains in the form of quality investments would benefit viewers.⁵ The attitude of the

³Monotonicity means that a firm should bid more for an allocation that increases its profit, which is quite a weak restriction.

⁴*Spinelli v NFL*, 96 F Supp 3d 81 (SDNY 2015), available at <https://law.justia.com/cases/federal/appellate-courts/ca2/17-0673/17-0673-2018-09-11.html>.

⁵See Bachelor et al. (2020).

EU competition authority toward such exclusivity agreements differs, as exemplified by the landmark UEFA Champions League decision.⁶ There, the joint-selling arrangement initially notified by the UEFA implied that all broadcasting rights for this elite soccer clubs competition were sold to a single broadcaster in each Member State and on an exclusive basis for periods up to four years. The European Commission negotiated several important changes to the initial agreement, including that open, transparent and nondiscriminatory tenders must be used to sell those rights, that several packages have to be offered and no single bidder can acquire all packages exclusively, and that exclusivity is limited in time (up to three years as a general rule).⁷

To the extent that corrective policies are certainly called for, policy recommendations should be based on a full characterization of equilibrium payoffs. An important take-away of our analysis is that such characterization actually shows that the light-handed tools available to competition authorities to prevent dominance have no impact on the allocation of rights; they only impact the distribution of profits across the industry. Thus, exclusivity seems hard to avoid.

LIGHT-HANDED REGULATORY CONSTRAINTS. To motivate a regulatory view on how resale and package bidding may impact welfare, the following case is in order. In 2018, the French soccer league sold the broadcasting rights for the next four seasons. Mediapro ended up getting five lots over the seven on sale. Other bidders, like beIN and the telecom operator Free each obtained a package while the historical distributor Canal+ returned empty-handed. After the tender, Canal+ and beIN promptly sealed an exclusive distribution and sublicensing deal for the broadcasting of the lots obtained by beIN. Mediapro subsequently failed to fulfill its payment obligations to the French soccer league, thereby putting the financial health of the league itself at risk. The rights initially obtained by Mediapro were then reallocated in a separate tender, with Amazon obtaining all the rights against a consortium of bidders formed by Canal+ and beIN at a much discounted price relative to Mediapro's initial offer.

Resale of Rights. Pay-TV markets often feature the resale of rights, sometimes even between firms that were earlier competitors at the bidding stage. Resale has often been viewed by practitioners as a means for a dominant firm to increase its monopoly power by buying back rights sold earlier on (maybe wrongly so) to a competitor. The possibility of resale should certainly be anticipated by bidders earlier on. They actually bid for getting an *interim* allocation of rights that might be renegotiated towards a constrained-efficient allocation. Such renegotiation transforms the bidding game into a constant-sum game. Whoever finally gets the good ends up paying whoever gets it at the *interim* stage. However, bidding head-to-head for an *interim* allocation erodes much of the dominant firm's profit. This fierce competition now benefits the initial owner of rights. Henceforth, any ban on resale has a redistributive impact along the supply chain, although it does not change the final allocation of rights that remains constrained-efficient.

Package Bidding and Multiple Rights. Most procedures taking place for broadcasting sports events feature the simultaneous sale of multiple rights. In those circumstances,

⁶See 2003/778/EC: Commission Decision of 23 July 2003 relating to a proceeding pursuant to Article 81 of the EC Treaty and Article 53 of the EEA Agreement (COMP/C.2-37.398 – Joint selling of the commercial rights of the UEFA Champions League), available at <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32003D0778>.

⁷See Toft (2003) for a detailed account of the negotiations between the UEFA and the European Commission.

competition authorities have sometimes suggested that the dominance of key players could be undermined by splitting upstream auctions into smaller tender procedures in which broadcasting rights of lesser size would be for sale. The welfare consequences of such design should be assessed with a careful comparison between the outcomes that follow from splitting and the scenario of a package auction where all rights are on sale at once.

Our analysis starts with the case of package bidding, where firms are allowed to freely bid for any combination of the rights for sale. Again, convexity of the downstream profit functions prevails when multiple rights are at stake and constrained efficiency calls for giving all rights to the dominant firm. This outcome is achieved in all monotonic equilibria, although different distributions of profits can be sustained. Among those possible distributions, the one that most favors the dominant firm is achieved with truthful bidding schedules. The incremental value for the dominant firm of obtaining a second set of rights, when already holding one set of rights, is large. This dominant firm is thus eager to make an aggressive bid for the whole package of exclusivity rights.

We then consider the outcome when auction markets for each set of rights are split. Accordingly, bidding schedules are required to be additively separable across separated auctions. Under those circumstances, we characterize equilibria on all markets at once. The additivity constraint *a priori* seems to limit the expression of the willingnesses to pay for bundles, which are so attractive in this environment (thanks again to the convexity of profit functions). As such, one may expect that this would make it difficult for the dominant firm to gain exclusivity in all markets. This intuition is again wrong. We demonstrate that *all* distributions of profits available in any monotonic equilibrium with package auctions remain feasible when the additivity restriction is imposed. This conclusion holds because, with the discrete allocation of rights (each set of rights being allocated to either downstream firm), there is much leeway in finding bids that force the dominant firm to pay what it is worth obtaining exclusivity on all rights while also preventing the initial owner from choosing other allocations. Split auctions are not an effective remedy against exclusivity, and the set of equilibrium payoffs under split auctions is the same as under package bidding.

LITERATURE REVIEW. This paper touches on several trends in the literature. On the more applied side, models of pay-TV downstream competition have most often relied on the Hotelling-based approach established by Gabszewicz et al. (2001, 2002, 2004), Dukes and Gal-Or (2003), Gal-Or and Dukes (2003), Anderson and Coate (2005) and Peitz and Valletti (2008). These contributions examine a number of issues, including content differentiation, advertising intensities and comparisons of welfare under pay-TV and free-to-air (advertising-financed) television. None of these articles considers content exclusivity. The literature on exclusivity in the pay-TV market is actually thin. Earlier works were mainly informed by British experiences on the subject. For instance, Armstrong (1999) discusses the possibility that two pay-TV networks could be implicitly colluding by exchanging content. Harbord and Ottaviani (2001) analyze contractual agreements, particularly in the context of the resale of broadcasting rights. Stenneck (2014) shows how exclusive distribution can stimulate specific investments by distributors and benefit viewers. Sonnac (2012) explains the strategic nature of exclusivity clauses in the pay-TV market. Weeds (2016) examines incentives for exclusivity in a dynamic setting with switching costs.

Taking a broader theoretical perspective, this paper contributes on several fronts. Aghion and Bolton (1987), Martimort (1996), Bernheim and Whinston (1998), Choné

and Linnemer (2015) and Calzolari and Denicolò (2015) show that exclusive dealing, when it arises, maximizes the profit of the vertical structure, eventually subject to contractual frictions, as in our analysis below. While Esö, Nocke and White (2010) study a general model of competition for a scarce resource, Bedre-Defolie and Biglaiser (2022) study, as we do, competition for the exclusive use of a superior input, but in a context where this input lowers the cost of improving the quality of a firm's product. We are interested in a different set of issues related specifically to how broadcasting rights should be allocated.

To assess the possible consequences of regulatory constraints on the auction formats, we need a full characterization of equilibria in menu auction games. On this front, Bernheim and Whinston (1986a) and Laussel and Lebreton (2001) focus on truthful equilibria in a context of public common agency while Chiesa and DeNicolò (2009) do not restrict a priori contribution schedules and provide a full characterization of equilibrium payoffs in a context of private common agency.⁸ To provide such characterization in our context, we rely on some general techniques developed by Martimort and Stole (2012) in their study of aggregate games.

ORGANIZATION OF THE PAPER. Section 2 presents the different players (sellers of rights in the upstream market, pay-TV firms, viewers) and describes the upstream and downstream markets in which they interact. Section 3 discusses the structure of downstream profits and their important *Convexity Property*. In Section 4, we characterize monotonic equilibria and the corresponding distributions of profits and surplus. Section 5 considers the possibility of resale. Section 6 extends our framework to allow for multiple broadcasting rights and the possible complementarities between those rights from the point of view of downstream firms. Section 7 briefly recaps our findings.

2. THE MODEL

STRUCTURE OF THE INDUSTRY. Two pay-TV firms, F_0 and F_1 , compete in the downstream market for viewers. They offer program packages that include some basic service (free-to-air TV or classic movies, for instance) plus possibly some premium programs (sports events or blockbusters). Competition in the downstream market thus entails both a quality (the content of those packages) and a price component. In the upstream market, F_0 and F_1 bid to acquire premium broadcasting rights from a supplier A . Those rights may entail either an exclusive or a joint distribution of the premium contents by service providers.

ALLOCATIONS OF RIGHTS AND CONSEQUENCES ON DOWNSTREAM PROFITS. Let the set of feasible allocations of rights be denoted by $\mathcal{A} = \{0, 1, c, \emptyset\}$, where 0 (resp. 1) stands for exclusivity to F_0 (resp. F_1), c stands for joint distribution, and \emptyset stands for no allocation of rights at all, which is an issue that is of course weakly dominated but included in our description for the sake of completeness.

An allocation of rights a is parameterized by the quality attributes (α_0, α_1) that respectively pertain to firm F_0 or F_1 . To simplify notations below, let the quality differential be denoted by $\Delta\alpha \equiv \alpha_0 - \alpha_1$. We express the firms' respective profits as $\tilde{\Pi}_0(\alpha_0, \alpha_1)$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1)$. The parameter α_i describes F_i 's quality on the downstream market for viewers which is inherited from upstream competition for the acquisition of broadcasting rights.

⁸Martimort (2007) defines public common agency games as one where all principals contract with their common agent on the same set of contracting variables (hereafter, the final allocation of the asset) while, under private agency, each principal contracts with a specific set of variables.

If rights are jointly allocated to both firms, firms have the same quality parameter α and earn the same profit, which we denote by $\Pi(c) \equiv \tilde{\Pi}_0(\alpha, \alpha) = \tilde{\Pi}_1(\alpha, \alpha)$. If rights are allocated exclusively to F_0 (resp. F_1), then profits are given by $\Pi_0(0) \equiv \tilde{\Pi}_0(\alpha + \Delta_0, 0)$ and $\Pi_1(0) \equiv \tilde{\Pi}_1(\alpha + \Delta_0, 0)$ (resp. $\Pi_0(1) \equiv \tilde{\Pi}_0(0, \alpha + \Delta_1)$ and $\Pi_1(1) \equiv \tilde{\Pi}_1(0, \alpha + \Delta_1)$). The parameter $\Delta_i > 0$ captures how firm F_i benefits from obtaining exclusivity on the broadcasting rights compared to the scenario of a joint distribution of those rights. The following string of inequalities hold for F_0 's and F_1 's profit functions respectively, $\Pi_0(0) > \Pi(c) > \Pi_0(1)$ and $\Pi_1(1) > \Pi(c) > \Pi_1(0)$.

DOMINANCE. Dominance on the downstream market by firm F_0 is modeled by assuming that $\Delta_0 > \Delta_1$. In words, F_0 benefits more than F_1 from obtaining exclusivity over the broadcasting rights.

THE CONVEXITY PROPERTY AND EFFICIENCY NOTIONS. The next assumption turns out to be useful for our analysis.

ASSUMPTION 1. *Firms' profits are such that*

$$(2.1) \quad \sum_{i=0,1} \Pi_i(0) > \sum_{i=0,1} \Pi_i(1) > 2\Pi(c).$$

Assumption 1 essentially states that the industry profit is maximized when broadcasting rights are allocated exclusively to either firm rather than when they are jointly allocated without any exclusivity clause to both firms. In that sense, the industry profit is convex in the allocation of broadcasting rights. Exclusive distribution creates an asymmetry between firms, since one of them now offers viewers a higher-quality product. Granting exclusivity to either firm is an extreme allocation that thus dominates a joint distribution of rights from the point of view of the industry when Assumption 1 holds. Moreover, this effect is more important when the dominant F_0 has exclusivity since the quality differential with its rival is then greater. This explains the first inequality in (2.1).

Assumption 1 means that moving from joint to exclusive distribution of broadcasting rights increases more the high-quality firm's profit than it decreases the low-quality firm's, leading to an overall higher industry profit. We postpone to the next section a more in-depth discussion of the plausible market structures that support Assumption 1.

In the sequel, we are interested in the efficiency properties of the equilibrium allocations of rights. A *constrained efficiency* criterion only considers the industry's viewpoint, thus maximizing the total profit of the grand coalition $A - F_0 - F_1$. An *efficiency* criterion instead assesses the consequences of such an allocation on overall welfare, including consumer surplus; which is an objective that is in line with that of competition authorities.

BIDDING STRATEGIES. Pay-TV firms approach A and bid for acquiring broadcasting rights. Following a methodology that was initiated by Bernheim and Whinston (1998), F_0 and F_1 compete by means of bidding schedules.⁹ Those schedules are commitments that stipulate how much these firms are ready to pay A for each configuration of rights. Formally, a bidding schedule is thus a nonnegative mapping T_i on \mathcal{A} .¹⁰ $T_i(a)$ stands for

⁹Hagiu and Lee (2011) adopt a similar approach in the context of competition between platforms.

¹⁰Non-negativity of the bidding schedules is without loss of generality. Indeed, A would never accept a bidding schedule from F_i with some negative payments and choose an allocation a corresponding to such a negative payment, *i.e.*, $T_i(a) < 0$. A could also increase its payoff by simply choosing the same allocation and excluding F_i from the auction. Beyond this, no further restrictions are made on the bidding strategies.

the share of F_i 's profit that is left to A when an allocation of rights a is chosen by A . Whether exclusivity or joint distribution emerges is actually an equilibrium outcome.

TIMING. The overall game of bidding and downstream competition unfolds as follows:

1. *Upstream market.* F_0 and F_1 simultaneously offer bidding schedules T_0 and T_1 to A . Since bidding schedules are nonnegative, A accepts those offers.
2. *Allocation of rights.* A chooses a distribution of rights $a \in \mathcal{A}$ and accordingly pockets the bids $T_i(a)$ ($i = 0, 1$) from downstream firms.
3. *Downstream market.* F_0 and F_1 simultaneously and non-cooperatively choose their prices for their own TV-offers.

The equilibrium concept is subgame-perfect equilibrium. We sometimes rely on refinements (based on monotonicity, truthfulness or/and dominance) to pin down a unique outcome.

3. MARKET STRUCTURES AND THE *Convexity Property*

The *Convexity Property* stated in Assumption 1 actually applies under a broad range of scenarios for the kind of downstream competition that prevails.¹¹

Example 1: Hotelling Competition with Single-Homing Viewers. There is a population of viewers with unit mass uniformly located on the $[0, 1]$ segment. Firms are located at the extreme points of this segment. A viewer located in $x \in [0, 1]$ derives a utility $v + \alpha_i - p_i - td_i(x)$ when buying from firm F_i , where t is the per-unit of distance transportation cost, $d_i(x)$ is the distance to F_i and v a parameter that represents the innate valuation for pay-TV services. The viewer who is indifferent between buying from either firm is located at $\hat{x} = \frac{1}{2t}(t + \Delta\alpha - (p_0 - p_1))$. Assuming that the market is fully covered, the total demand for product 0 (resp. 1) is thus given by $D_0(\Delta\alpha, p_0, p_1) = \hat{x}$ (resp. $D_1(\Delta\alpha, p_0, p_1) = 1 - \hat{x}$).¹² Routine computations allow then to determine Nash equilibrium prices as $p_0^*(\Delta\alpha) = t + \frac{\Delta\alpha}{3}$ and $p_1^*(\Delta\alpha) = t - \frac{\Delta\alpha}{3}$ and F_0 's and F_1 respective market shares as $\hat{x}^*(\Delta\alpha) = \frac{1}{2} + \frac{\Delta\alpha}{6t}$ and $1 - \hat{x}^*(\Delta\alpha) = \frac{1}{2} - \frac{\Delta\alpha}{6t}$. Equilibrium profits at the Nash equilibrium only depend of the quality differential $\Delta\alpha$, namely $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \pi(\Delta\alpha)$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \pi(-\Delta\alpha)$ where $\pi(\Delta\alpha) = \frac{1}{2t}(t + \frac{\Delta\alpha}{3})^2$. Assumption 1 holds, when F_0 is dominant (*i.e.*, $\Delta_0 > \Delta_1$), since we have

$$(3.1) \quad \pi(\alpha + \Delta_0) + \pi(-\alpha - \Delta_0) > \pi(-\alpha - \Delta_1) + \pi(\alpha + \Delta_1) > 2\pi(0) = t.$$

Example 2: Hotelling Competition with Both Single- and Multi-Homing Viewers. This example introduces the possibility for some viewers to buy from both pay-TV firms rather than from only one supplier. In the case of sports broadcasting rights, an often-heard argument is that splitting rights across different suppliers somewhat forces sport fans to subscribe to several pay-TV firms. Moving beyond the realm of sport broadcasting, anecdotal evidence suggests that some viewers subscribe to more than one pay-TV offer.

¹¹Details for the various market structures we discuss below can be found in the Appendix.

¹²Some conditions are needed to ensure that all viewers buy one unit in equilibrium (full market coverage).

A share $1 - \varepsilon$ of viewers are now single-homers, that is, they buy at most one unit of pay-TV services as in *Example 1*. The complementary share ε are multi-homers who may buy at most two units, one from each supplier. Both types of viewers are again uniformly distributed on the $[0, 1]$ segment. Multi-homing viewers are modeled following Doganoglu and Wright (2006). If such a viewer buys one unit of service only from firm F_i , he obtains a utility given by $v + \alpha_i - p_i - td_i(x)$. Buying one unit from each supplier yields a utility $v + \frac{1}{2}(\alpha_0 + \alpha_1) + \omega t - (p_0 + p_1) - t$, where $\omega > 0$ describes the multi-homing viewer's gain from diversity. We can now define the marginal multi-homing viewer who is indifferent between buying two units and buying only one unit from F_i as being located respectively at $\tilde{x}_0 = \frac{1}{t}(\Delta\alpha/2 + p_1) + 1 - \omega$ and $\tilde{x}_1 = \frac{1}{t}(\Delta\alpha/2 - p_0) + \omega$. The total demand for product 0 (resp. 1) is thus given by $D_0(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)\hat{x} + \varepsilon\tilde{x}_1$ (resp. $D_1(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)(1 - \hat{x}) + \varepsilon(1 - \tilde{x}_0)$). Generalizing the analysis of the single-homing scenario, equilibrium profits for F_0 and F_1 are respectively $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \pi(\Delta\alpha)$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \pi(-\Delta\alpha)$, for some function $\pi(\cdot)$.¹³ It can be readily verified that (3.1) holds here also; which ensures that Assumption 1 holds.

Example 3: Discrete Choice Model. Consider now a discrete choice model in which a viewer gets utility $\alpha_i + \varepsilon_i - p_i$ when buying from F_i . The taste shocks ε_i s are i.i.d. variables according to the cumulative distribution function $G(\cdot)$. Let $z = \varepsilon_0 - \varepsilon_1$ be the difference in taste shocks and let $F(\cdot)$ be the corresponding distribution function. A customer chooses F_0 if and only if $z \geq p_0 - p_1 - \Delta\alpha$. At the Nash equilibrium in prices, profits depend only on the quality differential $\Delta\alpha$ and the distribution of z . The market is actually split at $z^*(\Delta\alpha)$ that solves $z^*(\Delta\alpha) + \Delta\alpha = \frac{1-2F(z^*(\Delta\alpha))}{f(z^*(\Delta\alpha))}$. Equilibrium profits for F_0 and F_1 are again functions of the quality differential $\Delta\alpha$ only and are respectively given by $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \frac{(1-F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \frac{(F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$. Further assuming that ε_i follows the Gumbel distribution with location 0 and scale $1/\lambda$ (with $\lambda > 0$), z follows itself a logistic distribution with cumulative distribution $F(z) = \frac{e^{\lambda z}}{1+e^{\lambda z}}$. Equilibrium profits again depend on $\Delta\alpha$ and write as $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \pi(\Delta\alpha)$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \pi(-\Delta\alpha)$ for some function $\pi(\cdot)$. Condition (3.1) again holds; which ensures Assumption 1.

Example 4: Duopoly with Vertical Demand Shifters. Suppose that F_0 and F_1 compete in prices and face demands respectively given by $D_0(\Delta\alpha, p_0, p_1) = a + \Delta\alpha - p_0 - \frac{b}{2}(p_0 - p_1)$ and $D_1(\Delta\alpha, p_1, p_0) = a - \Delta\alpha - p_1 - \frac{b}{2}(p_1 - p_0)$ with $a > 0$ large enough and $b > 0$.¹⁴ An allocation of rights that is favorable to the dominant firm F_0 increases (resp. reduces) F_0 's (resp. F_1 's) demand. Equilibrium profits again write as in *Examples 2 and 3* above and Condition (3.1) again holds; which ensures Assumption 1.

Example 5: Cournot with Linear Demands. Assume that firms are Cournot competitors. Inverse demand functions are given by $P_i(\alpha_i, q_i, q_{-i}) = a + \alpha_i - q_i + bq_{-i}$, for $i = 0, 1$ and $-1 < b < 0$. In contrast with *Examples 1, 2, 3, 4*, equilibrium demands and prices for both goods now not only depend on the difference of quality levels $\Delta\alpha = \alpha_0 - \alpha_1$ but also on the average quality $\frac{\alpha_0 + \alpha_1}{2}$. It can be checked that Assumption 1 now holds whenever Δ_1 (and thus Δ_0 since $\Delta_0 > \Delta_1$ when F_0 is dominant) is large enough.

To summarize, *Examples 1, 2, and 3* are such that total demand is kept unchanged as F_0 offers more quality. Improving its own quality raises its own demand while it reduces

¹³Some conditions are needed to ensure that, in equilibrium, all viewers buy at least one unit (full market coverage), and a strictly positive mass of multi-homers buy two units. These conditions are detailed in the Appendix.

¹⁴In an earlier version of this paper, we considered the case of more general functional forms for demands and show the robustness of our results.

that of its competitor by the same amount. Had prices being kept constant, this shift in demand would also increase F_0 's profit and reduce F_1 's profit by the same amount but the industry's overall profit would be kept unchanged. However, increasing its own product quality also lowers its own demand price elasticity which allows in turn F_0 to increase its own price more than F_1 reduces its own. Industry profit thus increases; illustrating the *Convexity Property*. In *Examples 4 and 5*, improving its own quality again raises its own demand while it reduces less that of its competitor. Yet, it also allows F_0 to raise its own price and increase its profit more than the decrease in its competitor's profit; which again ensures the *Convexity Property*.

4. EQUILIBRIA CHARACTERIZATION

4.1. Generalities

At equilibrium, F_i chooses its own bidding schedule T_i to induce an allocation \bar{a} that maximizes its individual profit, *i.e.*, $\Pi_i(a) - T_i(a)$. Given a pair of offers (T_0, T_1) , A implements such allocation if it also maximizes its own payoff $\sum_{i=0,1} T_i(a)$. For future reference, we shall denote F_i 's (resp. A 's) equilibrium profit by $\bar{\Pi}_i$ (resp. $\bar{\Pi}_a$).

DEFINITION 1. *An equilibrium of the continuation game for stage 2 onward is a triplet $(\bar{T}_0, \bar{T}_1, \bar{a})$ that satisfies the following conditions.*

- *Profit maximization for A. A chooses an allocation within the best-response correspondence $\bar{\mathcal{A}}(\bar{T}_0, \bar{T}_1)$ where $\bar{\mathcal{A}}(T_0, T_1)$ is more generally defined as follows:*

$$(4.1) \quad \bar{\mathcal{A}}(T_0, T_1) = \arg \max_{\bar{a} \in \mathcal{A}} \sum_{i=0,1} T_i(\bar{a}) \quad \forall (T_0, T_1).$$

- *Profit maximization for F_i , $i = 0, 1$. \bar{T}_i satisfies the following:*

$$(4.2) \quad \Pi_i(\bar{a}) - \bar{T}_i(\bar{a}) = \max_{\substack{\tilde{a} \geq 0 \\ \tilde{a} \in \bar{\mathcal{A}}(T_i, \bar{T}_{-i})}} \Pi_i(\tilde{a}) - T_i(\tilde{a}),$$

where $\bar{a} \in \bar{\mathcal{A}}(\bar{T}_0, \bar{T}_1)$.

We can draw a few immediate consequences from conditions (4.1) and (4.2). First, at a best response to its rival's strategy \bar{T}_{-i} , F_i always minimizes its own payment $\bar{T}_i(\bar{a})$ to induce its most preferred allocation \bar{a} .

Second, this bidding game appears to be a *delegated common agency game*, as defined by Bernheim and Whinston (1986a). Martimort and Stole (2012) showed that those games are *aggregate games*. More specifically, A 's behavior on the equilibrium path only depends on the aggregate bidding schedule $\sum_{i=0,1} \bar{T}_i$. This property, together with the fact that payoffs are quasi-linear, then allows us to sum up the equilibrium conditions for each firm to obtain a more compact set of necessary conditions that must be satisfied by any equilibrium allocation. Such an allocation is actually a solution to a *self-generating* optimization problem in the vocable coined by Martimort and Stole (2012). Taking stock of these properties, the next proposition provides a set of necessary conditions that allows us to restrict the search for equilibrium allocations in a sharp way.¹⁵

¹⁵These conditions are not necessarily sufficient because the aggregate game is not bijective in the

PROPOSITION 1. *At any equilibrium $(\bar{T}_0, \bar{T}_1, \bar{a})$, the following conditions hold:*

$$(4.3) \quad \bar{a} \in \arg \max_{a \in \mathcal{A}} \sum_{i=0,1} \Pi_i(a) + \sum_{i=0,1} \bar{T}_i(a),$$

$$(4.4) \quad \bar{a} \in \arg \max_{a \in \mathcal{A}} \sum_{i=0,1} \bar{T}_i(a),$$

$$(4.5) \quad \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a) = \max_{a \in \mathcal{A}} \bar{T}_{-i}(a), \quad \forall i = 0, 1,$$

$$(4.6) \quad \bar{T}_i(a_{-i}) = 0, \quad \forall i = 0, 1,$$

where a_{-i} is any A 's best choice if F_i were to offer a null payment, i.e.,

$$(4.7) \quad a_{-i} \in \arg \max_{a \in \mathcal{A}} \bar{T}_{-i}(a).$$

The optimality condition (4.3) showcases the self-generating nature of equilibria. An equilibrium allocation maximizes the sum of the bidders' payoff $\sum_{i=0,1} \Pi_i(a)$ plus the seller's profit $\sum_{i=0,1} \bar{T}_i(a)$, which is itself an equilibrium object. Condition (4.4) simply characterizes the fact that this equilibrium allocation also belongs to the seller's best-response correspondence. More interesting are conditions (4.5) and (4.6) that offer an anchor for A 's payment and F_i 's bid schedule, respectively. The first one expresses the fact that A should be indifferent between choosing the equilibrium allocation given the aggregate bid $\bar{T}_i + \bar{T}_{-i}$ and its next best option, which is to choose an optimal allocation when dealing only with one bidder. The second condition shows that F_i 's bid is zero had any such off-path allocation been chosen by A . Formally, everything happens as if F_i had to make its own bidding schedule attractive enough to A to avoid being excluded from the market.¹⁶

We now specialize this general approach to determine what types of equilibria may emerge and discuss their constrained-efficiency properties. To this end, we first restrict the set of bidding strategies in a quite natural way, focusing on *monotonic bidding schedules* such that a bidder bids more for an action that yields a greater payoff, namely,

$$\Pi_i(a) \geq \Pi_i(a') \text{ (resp. } >) \Rightarrow T_i(a) \geq T_i(a') \text{ (resp. } >) \quad \forall (a, a') \in \mathcal{A}^2.$$

Accordingly, an equilibrium is said to be *monotonic* if it is implemented by means of *monotonic bidding schedules*. This focus has important implications that we now present.

PROPOSITION 2. *The following properties hold at any monotonic equilibrium:*

1. *The equilibrium allocation is constrained-efficient, i.e., $\bar{a} = (0)$.*

sense defined in Martimort and Stole (2012). More specifically, because bids must remain nonnegative, not all aggregate bids can be undone by a given firm with its own offer when this offer is restricted to remain nonnegative as requested in a delegated common agency game.

¹⁶Readers accustomed to the literature might have recognized the similarity between Proposition 1 and Lemma 2 in Bernheim and Whinston (1986a). However, there are a couple of differences that deserve to be stressed. First, the aggregate optimality condition (4.3) replaces their Condition *iii*), which captures the fact that an equilibrium allocation maximizes the bilateral payoff of a coalition made of any one bidder and the seller. Second, conditions (4.5) and (4.6) altogether replace and supersede their Condition *iv*) but are more precise, thereby making it explicit for which allocation a bidder's bidding schedule is zero, while Condition *iv*) only provides the existence of an allocation which entails a zero bid.

2. The set Σ of feasible profits $(\bar{\Pi}_0, \bar{\Pi}_1, \bar{\Pi}_a)$ in monotonic equilibria is defined by the following conditions:

$$(4.8) \quad \begin{aligned} \Pi_0(1) &\leq \bar{\Pi}_0 \leq \Pi_0(0) + \Pi_1(0) - \Pi_1(1), \\ \bar{\Pi}_1 &= \Pi_1(0), \\ \Pi_1(1) - \Pi_1(0) &\leq \bar{\Pi}_a \leq \Pi_0(0) - \Pi_0(1). \end{aligned}$$

The distribution of payoffs that arise at equilibrium can be readily explained. First, since F_0 is a dominant firm in the market, the constrained-efficient allocation $a^* = (0)$ is also the most preferred outcome for that firm, while at the same time it remains the worst for F_1 . Overall, and even though A could choose to sell rights non-exclusively to both downstream firms, everything happens as if those firms were bidding head-to-head for exclusivity.

From condition (4.6), F_1 bids zero for the equilibrium allocation $\bar{a} = (0)$ since giving rights to F_0 also minimizes F_1 's own payment. However, A can improve its bargaining position vis-à-vis F_0 by threatening to sell exclusive rights to F_1 . F_1 could thus be ready to pay up to $\bar{T}_1(1) = \Pi_1(1) - \Pi_1(0)$ to acquire such exclusivity. This gives the lowest bound on A 's equilibrium profit. In fact, F_1 might bid even more for exclusivity, say $\bar{T}_1(1) > \Pi_1(1) - \Pi_1(0)$, as long as, at such an equilibrium, A still chooses to sell exclusively to F_0 . Such a strategy is weakly dominated since F_1 's payoff $\Pi_1(1) - \bar{T}_1(1)$ is less than its payoff $\Pi_1(0)$ obtained when bidding zero. However, such a large bid also improves A 's bargaining position vis-à-vis F_0 since it makes switching to F_1 more attractive. To defeat such a bid, F_0 should pay at least $\bar{T}_0(0) = \bar{T}_1(1) > \Pi_1(1) - \Pi_1(0)$, and the highest such payment that F_0 is willing to make is thus $\Pi_0(0) - \Pi_0(1)$ to gain exclusivity. With such large bids, A grasps a larger share of the overall industry's profit obtained by granting exclusivity to F_0 .

Finally, we also stress that an immediate corollary of Proposition 2 is that a joint distribution of rights, *i.e.*, $a = (c)$, never arises at equilibrium. Proposition 2 thus shows the need for some kind of corrective intervention aiming to implement an allocation that could be preferred from a social welfare perspective, namely, joint distribution, even though this allocation is never implemented by free competition.

4.2. Truthful Equilibrium

In their general investigation of menu auctions, Bernheim and Whinston (1986a) exhibited an important class of equilibria sustained with so-called *truthful strategies* which are of the following form:

$$T_i(a) = \max\{\Pi_i(a) - \Pi_i; 0\} \quad \forall a \in \mathcal{A}$$

where Π_i is some constant.

These strategies have attractive properties. First, those payments schedules are monotonic. From Proposition 2, truthful equilibria thus necessarily implement a constrained-efficient allocation, *i.e.*, $\bar{a} = (0)$. Second, F_i keeps a constant profit Π_i over all possible allocations for which it makes a positive bid. In other words, truthful bidding schedules align the preferences of F_i and A over possible allocations that are found attractive for F_i . Third, each downstream firm always has a truthful strategy in its best-response

correspondence.¹⁷ Focusing on truthful bidding schedules thus amounts to imposing a refinement in the equilibrium correspondence. This refinement allows a sharp characterization of payoffs in truthful equilibria. In our context, this refinement even pins down a unique outcome.¹⁸

PROPOSITION 3. *There exists a unique truthful equilibrium with equilibrium profits given by:*

$$(4.9) \quad \begin{aligned} \bar{\Pi}_0^t &= \Pi_0(0) + \Pi_1(0) - \Pi_1(1) > 0, \\ \bar{\Pi}_1^t &= \Pi_1(0) > 0, \\ \bar{\Pi}_a^t &= \Pi_1(1) - \Pi_1(0) > 0. \end{aligned}$$

Together with Proposition 2, Proposition 3 shows that the constrained-efficient truthful equilibrium corresponds to an extremal point of the set of possible equilibrium profits. This equilibrium is actually sustained with strategies that are not weakly dominated. Indeed, at any other equilibrium described in Proposition 2 above, F_1 would possibly pay more for acquiring rights than its incremental benefit of doing so, since for those equilibria $\bar{T}(1) > \Pi_1(1) - \Pi_1(0)$. This dominance criterion thus provides another argument to focus on the unique truthful allocation.

For future reference, it is worth noting that in this unique truthful equilibrium, what F_0 bids for exclusivity, namely, $\bar{T}^t(0) = \Pi_1(1) - \Pi_1(0)$, is the Vickrey-Clarke-Groves payment, *i.e.*, what it takes for F_0 to avoid A choosing his or her next best option, namely, giving exclusivity rights to F_1 .

5. RESALE

Resale is an important feature of the pay-TV market. From a theoretical viewpoint, resale nevertheless remains a double-edged sword. First, the renegotiation of any allocation of rights between downstream firms F_0 and F_1 could, *a priori*, correct any constrained inefficiency that might arise in the downstream market and improve the industry's overall profit. Given that constrained efficiency may conflict with social efficiency, the possibility of resale could go counter welfare maximization. It should thus be viewed with an eye of caution by competition authorities.

In fact, this negative stance is incomplete. When anticipated, the possibility of resale may actually change the downstream firms' bidding strategies, which may in turn lead to payoffs rather different than those found in Proposition 3. In other words, banning resale has no impact on whoever *in fine* holds the rights. The dominant firm F_0 should always get those rights. However, it might impact the distribution of profits and how much the dominant firm pays for exclusivity.

To study how it can be so, consider adding a renegotiation stage in between stages 2 and 3 of the game form so far studied. Suppose that A has already chosen an arbitrary allocation \tilde{a} and assume that the corresponding bids $T_0(\tilde{a})$ and $T_1(\tilde{a})$ have been sunk. F_0 and F_1 can renegotiate away from the allocation \tilde{a} toward another allocation, say a . We model such renegotiation as a Nash bargaining game with outside options being determined by the chosen allocation \tilde{a} , and with parties having equal bargaining power.

¹⁷See Bernheim and Whinston (1986a, Theorem 1).

¹⁸Laussel and Le Breton (2001) provide general conditions for the uniqueness of a truthful equilibrium.

Renegotiation determines a final allocation a^* , as well as a compensatory payment z^* , between F_0 and F_1 that altogether solve the Nash-bargaining problem, *i.e.*,

$$(a^*, z^*) \in \arg \max_{(a, z)} \left(\Pi_0(a) - z - \Pi_0(\tilde{a}) \right) \left(\Pi_1(a) + z - \Pi_1(\tilde{a}) \right).$$

Renegotiation thus implements the constrained-efficient allocation $a^* = (0)$ since it indeed maximizes the joint profit of F_0 and F_1 once bids are sunk. Inserting the value of the compensatory payment $z^*(\tilde{a}) = \frac{1}{2}(\Pi_0(0) - \Pi_1(0) - \Pi_0(\tilde{a}) + \Pi_1(\tilde{a}))$ obtained into the firms' payoffs allows us to rewrite the firms' net profits once renegotiation is taken into account as $\tilde{\Pi}_i(\tilde{a}) - T_i(\tilde{a})$, where

$$(5.1) \quad \tilde{\Pi}_i(\tilde{a}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \left(\Pi_i(\tilde{a}) - \Pi_{-i}(\tilde{a}) \right).$$

The possibility of resale has modified the game between F_0 and F_1 . Remarkably, it is now a constant-sum game. By its choice of a pre-resale allocation, A determines how much should be transferred from one firm to the other through *ex post* bargaining.¹⁹

We may now apply the general methodology of Proposition 1 to the new payoff functions so-defined. It is easy to check that any *interim* allocation \tilde{a} can be part of an equilibrium of the game form extended by renegotiation. The two downstream firms could just offer the nil contracts $T_i = 0$, let A randomly choose an initial allocation \tilde{a} and then renegotiate away any constrained inefficiency so-obtained by trading rights to move toward the constrained-efficient allocation $\bar{a} = (0)$.

Among all possible equilibria that may arise with resale, there is an interesting class in which A chooses upfront a constrained-efficient allocation that is thus *not* renegotiated. Those equilibria are in a sense *resale-proof*. As soon as resale entails some (even slightly) positive transaction costs, such resale-proof equilibria may be particularly attractive.

Following a logic that is by now familiar, the constrained-efficient allocation $\bar{a} = (0)$ can be implemented as a *truthful resale-proof* equilibrium in the bidding game modified by the possibility of resale by means of the following (monotonic) truthful schedules:

$$(5.2) \quad \tilde{T}_i(a) = \max \left\{ \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \left(\Pi_i(a) - \Pi_{-i}(a) \right) - \tilde{\Pi}_i; 0 \right\}.$$

Resale has, of course, some consequences on the distribution of equilibrium payoffs, as explained in the next proposition.

PROPOSITION 4. *There exists a unique truthful resale-proof equilibrium. Profits in this equilibrium are given by the following:*

$$(5.3) \quad \begin{aligned} \bar{\Pi}_0^{rp} &= \frac{1}{2} \left(\Pi_0(0) + \Pi_1(0) + \Pi_0(1) - \Pi_1(1) \right), \\ \bar{\Pi}_1^{rp} &= \Pi_1(0), \\ \bar{\Pi}_a^{rp} &= \frac{1}{2} \left(\Pi_0(0) + \Pi_1(1) - \Pi_0(1) - \Pi_1(0) \right). \end{aligned}$$

¹⁹Readers accustomed with the incomplete contracts literature may have recognized a familiar feature of implementation when renegotiation is allowed. See Maskin and Moore (1999) for instance.

A (resp. F_0 , F_1) always earns more (resp. less, the same) profit in the unique truthful resale-proof equilibrium than in the unique truthful equilibrium without resale.

With resale, A can obtain a greater share of the industry profit while that of F_0 decreases and that of F_1 remains unchanged. Intuitively, A can now threaten each downstream firm to sell the rights to its rival as a base for future resale; this is a very unattractive outcome that forces the downstream firm that values the most those rights, *i.e.*, F_0 , to pay a lot for exclusivity.

It is straightforward to check that the payoff vector $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp}, \bar{\Pi}_a^{rp})$ belongs to Σ . This result comes as no surprise. The unique truthful resale-proof equilibrium implements the constrained-efficient allocation, gives to the dominated firm its reservation payoff and, by construction, satisfies all incentive constraints that must hold in equilibrium, as all equilibrium payoffs that belong to Σ must do.

Our results suggest that the sole role of banning resale is to change the distribution of profits between the dominant firm F_0 and the producer of rights A . This distribution is of course neutral for both the allocation of rights that arises at equilibrium and consumer welfare. Hence, the only justification that can be found for such a policy is that it might have an indirect impact on investment. Suppose indeed that F_0 's promotional effort is endogenized. With resale, F_0 's profits are lower, and we may expect lower incentives to exert such effort. In turn, F_0 and F_1 may become more symmetric, and a non-exclusive distribution of rights may become more attractive. In that sense, banning resale may have a role in preventing exclusivity; this, however, seems to be a very indirect role.

6. PACKAGE BIDDING FOR MULTIPLE RIGHTS

We now extend our previous approach to analyze settings in which firms in the downstream market can acquire several sets of premium contents from the upstream seller. To fix ideas, suppose that two sets of rights are available. Exclusivity can be given to either firm. Much as before, F_0 is the dominant player in both markets, as it is able to improve the quality of contents more than what F_1 would do if given these rights. Viewers now choose their favorite channel by comparing prices and the overall quality of the contents, taking into account how rights have been allocated to downstream firms. For simplicity, we assume that these rights have a symmetric impact on profits and surpluses across markets. Under those conditions, the model is rather similar to that presented in our previous setup.

The only difference is that we hereafter suppose that a joint distribution of either set of rights is no longer possible. There are two reasons for this assumption. First, it simplifies the analysis by limiting the number of configurations under scrutiny. However, this is only a minor restriction since the possibility remains that each downstream firm enjoys exclusively one set of rights, giving rise to a rather balanced market structure. Second, most real-world auctions for the broadcasting of premium sporting events involve exclusive distribution only. The main concern expressed by some competition authorities is to prevent the dominant firm from obtaining exclusivity over all sets of rights to maintain a long-term competitive balance in the market.

We now present the model and the structure of the equilibria with such *package bidding*.²⁰ We then analyze the consequences of a ban on package bidding on the equilibrium allocations.

²⁰Milgrom (2007) offers an exhaustive review of the literature on package auctions.

6.1. Preliminaries

Formally, we denote by $b = (a^1, a^2)$ any arbitrary allocation of rights, where a^k (for $k = 1, 2$) belongs to $\mathcal{A}_* = \mathcal{A}/\{c\}$. In other words, in each market, A can give exclusivity to either firm or refuse to distribute its rights. The set of possible allocations is thus the cross-product \mathcal{A}_*^2 . Assume that the value-enhancing parameter that applies to firm F_i 's services can be written as $\beta_i = \sum_{k=1,2} \alpha_i^k$, where the superscript k indices the set of rights with, as in our baseline model, $\alpha_i^k \in \{0, \alpha + \Delta_i\}$ depending on how rights are allocated across firms.²¹ From there, we can derive the expressions of F_i 's profit for a given allocation of rights b and the induced value-enhancing parameters. To illustrate, if F_0 acquires exclusivity over the bundle of rights, *i.e.*, $b = (0, 0)$, then $\beta_0 = 2(\alpha + \Delta_0)$ and $\beta_1 = 0$, so that $\Pi_0(0, 0) = \tilde{\Pi}_0(2(\alpha + \Delta_0), 0)$ and $\Pi_1(0, 0) = \tilde{\Pi}_1(2(\alpha + \Delta_0), 0)$. If rights are split across pay-TV firms, then $b = (0, 1)$, $\beta_0 = \alpha + \Delta_0$ and $\beta_1 = \alpha + \Delta_1$, so that $\Pi_0(0, 1) = \tilde{\Pi}_0(\alpha + \Delta_0, \alpha + \Delta_1)$ and $\Pi_1(0, 1) = \tilde{\Pi}_1(\alpha + \Delta_0, \alpha + \Delta_1)$. Because rights are symmetric, we have $\Pi_i(0, 1) = \Pi_i(1, 0)$ for $i = 0, 1$. Since $\Delta_0 > \Delta_1$, F_0 is dominant in both markets and we also have $\Pi_0(0, 0) + \Pi_1(0, 0) > \Pi_0(1, 1) + \Pi_1(1, 1)$. Of course, F_0 's and F_1 's profits can be ranked, respectively, as $\Pi_0(0, 0) > \Pi_0(1, 0) = \Pi_0(0, 1) > \Pi_0(1, 1)$ and $\Pi_1(1, 1) > \Pi_1(1, 0) = \Pi_1(0, 1) > \Pi_1(0, 0)$.

As our analysis will soon unveil, with multiple rights, we need the following version of the *Convexity Property*.

ASSUMPTION 2. *Firms' profits are such that*

$$\Pi_i(i, i) + \Pi_i(-i, -i) > 2\Pi_i(i, -i), \quad \forall i \in \{0, 1\}.$$

Assumption 2 simply states that, for each firm, the incremental value of a right increases with the number of rights already obtained, or, equivalently, that firms' individual profits are strictly convex in the allocation of rights. That assumption is maintained throughout this section.

The different market structures considered in Section 3 can be easily extended to account for multiple rights. In the Appendix, we show that, for all these modes of downstream competition, Assumption 2 always holds.

Assumption 2 and dominance, when taken in tandem, immediately imply that the allocation $b^* = (0, 0)$ is the *constrained-efficient* allocation, or

$$(6.1) \quad \sum_{i=0,1} \Pi_i(0, 0) = \max_{b \in \mathcal{A}_*^2} \sum_{i=0,1} \Pi_i(b).$$

An immediate consequence of (6.1) is that the allocation $(0, 0)$ is also *market-by-market constrained-efficient*; *i.e.*, F_0 should be given the second set of rights if it already owns the first one. In other words, rights are *strict complements* for the dominant firm. Formally, condition (6.1) indeed implies the following:

$$(6.2) \quad \sum_{i=0,1} \Pi_i(0, 0) = \max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(a, 0) = \max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(0, a).$$

²¹Introducing some complementarity or substitutability in the valuation of multiple rights by viewers does not change our analysis as long as the ranking of profits and Assumption 2 continue to hold.

This property is key to understanding that when F_0 is a dominant firm for both sets of rights, selling those rights as a bundle or separately in different auctions has no impact on the constrained efficiency of the equilibrium allocation. Of course, whether rights are sold as a bundle or separately may have an impact on bids and payoffs. We consider those two scenarios in the following.

6.2. Package Bidding Equilibria

Suppose first that both sets of rights are simultaneously on sale by means of the same tender procedure. Downstream firms are unrestricted in their bidding strategies on those packages. Formally, a bidding schedule T_i is thus any arbitrary nonnegative mapping on \mathcal{A}_*^2 . Such mapping determines F_i 's bid $T_i(b)$ to A for each feasible package $b \in \mathcal{A}_*^2$. Still relying on our definition of monotonicity, it is straightforward to recast our analysis of Proposition 2. We again denote by $(\bar{T}_0, \bar{T}_1, \bar{b})$ any arbitrary equilibrium pair of bidding schedules and the ensuing allocation of rights.

PROPOSITION 5.

1. All monotonic equilibria are constrained-efficient, i.e., $\bar{b} = (0, 0)$.
2. The set Σ of profit levels $(\bar{\Pi}_0, \bar{\Pi}_1, \bar{\Pi}_a)$ that can be achieved in any monotonic equilibrium is defined by the following conditions:

$$(6.3) \quad \begin{aligned} \Pi_0(1, 1) &\leq \bar{\Pi}_0 \leq \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1), \\ \bar{\Pi}_1 &= \Pi_1(0, 0), \\ \Pi_0(0, 0) - \Pi_0(1, 1) &\geq \bar{\Pi}_a \geq \Pi_1(1, 1) - \Pi_1(0, 0). \end{aligned}$$

Proposition 5 echoes, in a multimarket context, our earlier findings found in Proposition 2. There still exists a whole range of equilibrium profits corresponding to different distributions of the maximal industry profit between A and the dominant firm F_0 . Indeed, A obtains a greater share of this profit when it can threaten F_0 to sell exclusivity rights in both markets to F_1 at price $\bar{T}_1(1, 1) \geq \Pi_1(1, 1) - \Pi_1(0, 0)$. Offering such an attractive option in its bidding schedule remains, of course, a weakly dominated strategy for F_1 .

To eliminate those weakly dominated strategies and focus on the distribution of profits that is the most favorable to F_0 , we again consider truthful bidding schedules of the following form:

$$(6.4) \quad T_i(b) = \max\{\Pi_i(b) - \Pi_i; 0\}.$$

Those bidding schedules are of course monotonic, and any truthful equilibrium is thus constrained-efficient, thus implementing $\bar{b} = (0, 0)$. Echoing our earlier findings in Proposition 3, such an equilibrium determines a unique distribution of profits that again corresponds to an extremal point of the set described in Proposition 5. Then, F_0 gets the highest possible profit.

PROPOSITION 6. *There exists a unique equilibrium in truthful schedules with package bidding. Equilibrium profits are given by the following expressions:*

$$(6.5) \quad \begin{aligned} \bar{\Pi}_0^b &= \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1) > 0, \\ \bar{\Pi}_1^b &= \Pi_1(0, 0) > 0, \\ \bar{\Pi}_a^b &= \Pi_1(1, 1) - \Pi_1(0, 0) > 0. \end{aligned}$$

6.3. Split Auctions: Additive Bidding

Consider now a scenario where each set of rights is sold separately by means of a different auction procedure. A final allocation \bar{b} is thus determined *market by market*. To tackle the analysis of such a complex web of bilateral relationships and provide some predictive insights, we return to the mere definition of schedules for package bidding and impose additional constraints on those schedules. These constraints capture the fact that auctions for each set of rights are run separately. Instead of allowing arbitrary bidding schedules contingent on global allocations, as with the scenario of package bidding, we might view the requirement of *split bidding in each market* as an additivity constraint of the following kind:

$$(6.6) \quad T_i(a^1, a^2) = \sum_{k=1,2} T_i^k(a^k) \text{ for } i = 1, 2.$$

The benefit of such additivity restrictions on bidding schedules is that the gain of a given deviation away from any putative equilibrium can be assessed market by market. The cost of such a restriction is instead that a downstream firm's overall bidding strategy might no longer reflect the payoff complementarity that appears when this firm owns both sets of rights, contrary to what arises with a truthful schedule in the scenario of package bidding. In particular, one might expect that F_0 would bid less than its incremental value to obtain exclusivity on only any single set of rights. Under such circumstances, intuition might suggest that imposing the additivity constraint (6.6) on feasible strategies could *a priori* lessen competition between downstream firms.²²

Of course, the additive bidding schedule (6.6) is monotonic when each component $T_i^k(a^k)$ is itself monotonic. From Proposition 2, any equilibrium in additive and monotonic schedules thus implements the constrained-efficient allocation $\bar{b} = (0, 0)$. The next proposition characterizes such equilibrium profits. Surprisingly, and in contrast with the intuition just sketched, the additivity constraint does not restrict the set of equilibrium outcomes in comparison with the scenario where package bidding is possible.

PROPOSITION 7. *Any profit levels $(\bar{\Pi}_0^s, \bar{\Pi}_1^s, \bar{\Pi}_a^s)$ in Σ that can be achieved with monotonic equilibria when package bidding is allowed can also be achieved with additive bidding schedules of the form (6.6) when Assumption 2 is satisfied.*

An immediate corollary of our findings is that the extreme payoffs obtained in (6.5) by means of truthful schedules can also be achieved when auctions are split and firms

²²In the literature on package auctions, this phenomenon is referred to as the risk of *demand exposure*. A bidder who cannot express his willingness to pay for a bundle reduces his bid on each component because he is exposed to the risk that a local competitor, who is interested in only one item, prevents him from acquiring both items. In such contexts, not only is competition lessened, but there is also a risk that the final allocation will be inefficient. See Kagel and Levin (2005) and Milgrom (2007).

use additive bidding schedules. Let us see how this can be so. By construction, the truthful equilibrium schedules (6.4) are not additive. Indeed, F_0 benefits from some complementarity in obtaining exclusivity in both markets and, in a truthful equilibrium, is ready to pay the large price $\bar{T}_0^b(0, 0) = \Pi_1(1, 1) - \Pi_1(0, 0)$ that prevents A from selling both rights to F_1 . When restricted to additive bidding schedules, F_0 can still prevent A from selling both sets of rights to F_1 with a bid $\bar{T}_0(0)$ that would replicate what the truthful payment does, namely,

$$2\bar{T}_0(0) = \bar{T}_0^b(0, 0) = \Pi_1(1, 1) - \Pi_1(0, 0).$$

To better understand the large magnitude of this payment, it is useful to think of a putative setting where F_0 and F_1 would bid on a given market, anticipating the allocation that would arise for the other lot. Following Crémer and Riordan (1987), Hart and Tirole (1990), Horn and Wolinsky (1988) and O'Brien and Shaffer (1992), this approach selects a so-called *pairwise-proof* allocation. Because F_0 is dominant, the minimal payment it must deliver to prevent F_1 getting the lot on market k , assuming that F_0 has already won the lot on market $-k$ is actually

$$\tilde{T}_0(0) = \Pi_1(0, 1) - \Pi_1(0, 0).$$

Because of the convexity of F_1 's profit as defined in Assumption 2, we thus have

$$\bar{T}_0(0) = \frac{1}{2}(\Pi_1(1, 1) - \Pi_1(0, 0)) > \Pi_1(0, 1) - \Pi_1(0, 0) = \tilde{T}_0(0).$$

In other words, a *pairwise-proof* allocation has F_0 pay too little for winning on both markets. Actually, it can be shown that such a pairwise-proof allocation fails to be an equilibrium since F_1 could outbid those low bids with a multilateral deviation on both markets at once.²³

The flip side of the large bid $\bar{T}_0(0)$ is that one may wonder if F_0 could not simply prefer to acquire exclusivity in only one market rather than duplicating those bids in each market. This is not the case since F_0 's net benefit of acquiring the right on the second market, when already having the right on the first one, are always positive when

$$\bar{T}_0(0) \leq \Pi_0(0, 0) - \Pi_0(0, 1);$$

which is an inequality that again follows from the convexity of F_0 's profit as defined in Assumption 2.²⁴ Therefore, a restriction to split auctions does not change the distribution of profits and surplus.

7. CONCLUSION

Motivated by the prevalence of exclusivity in broadcasting rights in pay-TV markets, we have developed a model in which firms compete both upstream for the acquisition of broadcasting rights and downstream to attract viewers. Profit functions in the downstream market exhibit a fundamental *Convexity Property*. This property implies that

²³Proof available upon request. See McAfee and Schwartz (1994), Rey and Vergé (2004) and Segal and Whinston (2003) for similar insights.

²⁴We show in the Appendix that, in equilibrium, $\bar{T}_0(0)$ must be lower than $\frac{1}{2}(\Pi_0(0, 0) - \Pi_0(1, 1))$ and, moreover, the inequality $\frac{1}{2}(\Pi_0(0, 0) - \Pi_0(1, 1)) < \Pi_0(0, 0) - \Pi_0(0, 1)$ holds from Assumption 2.

giving exclusive broadcasting rights to one firm, namely, a dominant player, maximizes the industry profit. We characterize all equilibria when firms can freely bid for any possible allocation of the broadcasting rights and show that all monotonic equilibria implement this profit-maximizing allocation. Light-handed regulation such as banning resale or limiting the lot size under tenders in the case of multiple rights does not change this outcome. At best, such regulation might modify the distribution of profits that is achieved. As a result, and if anything, the dominant firm's incentives to further invest in enhancing the quality of services might be modified (and possibly diminished); however, this appears to be a very indirect way of promoting downstream competition.

Therefore, our analysis suggests that if exclusivity is to be avoided because it fails to maximize welfare, more heavy-handed tools may prove useful. As an example, the European Commission required the UK Football Association Premier League Limited to ensure that at least one package of media rights to the Premier League matches would go to an operator other than the dominant one, thereby forcing *de facto* a competitive outcome.²⁵ Perhaps unexpectedly, such a '*no single buyer rule*' is now almost always implemented by major European soccer leagues, even though competitive and regulatory landscapes differ. One may wonder, however, whether systematically forcing a non-exclusive outcome is warranted. We believe that this issue should be resolved on a case-by-case basis.

We have also remained silent on another issue that is salient in pay-TV markets and sports events broadcasting, namely, whether sports leagues that coordinate the selling of broadcasting rights on behalf of teams form a cartel and thus harm competition. Again, competition authorities differ in their assessments. Our model could be extended to take into account both the joint and individual selling of broadcasting rights and analyze how this impacts the vertical agreements that emerge between sellers of rights and pay-TV firms. This investigation is left for future research.

APPENDIX

MARKET STRUCTURES AND ASSUMPTIONS 1 AND 2. We detail here the analysis of Assumptions 1 and 2 under the different market structures described in Section 3.

Examples 1 and 2: Hotelling with Single- and Multi-Homing Viewers. The special case where all viewers single-home corresponds to $\varepsilon = 0$. For single-homers, the framework is similar to a standard Hotelling model. A viewer located in $x \in [0, 1]$ derives utility $v + \alpha_0 - p_0 - tx$ if he buys from F_0 , and $v - \alpha_1 - p_1 - t(1 - x)$ if he buys from F_1 , where t is the per-unit of distance transportation cost. Denote by \hat{x} the single-homing viewer who is indifferent between buying from either firms: $\hat{x} = \frac{1}{2t}(t + \Delta\alpha - (p_0 - p_1))$ with $\Delta\alpha = \alpha_0 - \alpha_1$.

Multi-homing viewers are modeled following Doganoglu and Wright (2006). If a multi-homing viewer located in $x \in [0, 1]$ buys only one unit from either F_0 or F_1 , he obtains $v + \alpha_0 - p_0 - tx$ or $v + \alpha_1 - p_1 - t(1 - x)$ respectively. If he buys one unit from both firms, he obtains $v + \frac{\alpha_0 + \alpha_1}{2} + \omega t - (p_0 + p_1) - t$, where $\omega \geq 1/2$ is a parameter reflecting a gain from diversity and $t = t(x + 1 - x)$ is the total transportation cost. Let \tilde{x}_0 (resp. \tilde{x}_1) be the viewer who is indifferent between buying two units and one unit only from F_0 (resp. F_1): $\tilde{x}_0 = \frac{1}{t}(\Delta\alpha/2 + p_1) + 1 - \omega$ (resp. $\tilde{x}_1 = \frac{1}{t}(\Delta\alpha/2 - p_0) + \omega$). We assume from now on that $\tilde{x}_0 < \tilde{x}_1$, or $p_0 + p_1 < (2\omega - 1)t$, a condition which has to be checked at equilibrium and ensures

²⁵See Commission Decision of 22/3/2006 Case COMP/38.173 – Joint selling of the media rights to the FA Premier League, available at <https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX%3A52008XC0112%2803%29>.

that some multi-homing viewers actually buy two units from (one from each firm) rather than one unit only from either firms. Equipped with these notations, it comes immediately that a multi-homing viewer with $x \leq \tilde{x}_0$ (resp. $x \geq \tilde{x}_1$) buys one unit only from F_0 (resp. F_1), and a multi-homing viewer with $x \in [\tilde{x}_0, \tilde{x}_1]$ buys one unit from both firms.

The total demand for product 0 (resp. 1) is given by $D_0(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)\hat{x} + \varepsilon\tilde{x}_1$ (resp. $D_1(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)(1 - \hat{x}) + \varepsilon(1 - \tilde{x}_0)$). Firms' profits are then given by $p_0 D_0(\Delta\alpha, p_0, p_1)$ and $p_1 D_1(\Delta\alpha, p_0, p_1)$ respectively. Assuming best responses are uniquely defined by first-order conditions, routine computations allow then to determine the Nash equilibrium prices that depend only on $\Delta\alpha$, namely $p_0^*(\Delta\alpha) = \frac{\Delta\alpha}{3+\varepsilon} + \frac{t}{1+3\varepsilon}(1 + \varepsilon(2\omega - 1))$ and $p_1^*(\Delta\alpha) = \frac{-\Delta\alpha}{3+\varepsilon} + \frac{t}{1+3\varepsilon}(1 + \varepsilon(2\omega - 1))$. From this, we can deduce the firms' profits at the Nash equilibrium respectively as $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \pi(\Delta\alpha) = \frac{1+\varepsilon}{2t} \left(\frac{\Delta\alpha(1+3\varepsilon) + t(3+\varepsilon)(1+\varepsilon(2\omega-1))}{(3+\varepsilon)(1+3\varepsilon)} \right)^2$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \pi(-\Delta\alpha)$. Let us now define

$$\varphi(\Delta\alpha) = \pi(\Delta\alpha) + \pi(-\Delta\alpha) = \frac{1 + \varepsilon}{t} \left(\left(\frac{\Delta\alpha(1 + 3\varepsilon)}{3 + \varepsilon} \right)^2 + \left(\frac{t(1 + \varepsilon(2\omega - 1))}{1 + 3\varepsilon} \right)^2 \right).$$

Notice that φ is increasing for $\alpha_0 > \alpha_1$. Hence, Assumption 1 holds since we thus have

$$(A.1) \quad \begin{aligned} \varphi(\alpha + \Delta_0) &= \tilde{\Pi}_0(\alpha + \Delta_0, 0) + \tilde{\Pi}_1(\alpha + \Delta_0, 0) \\ &> \varphi(\alpha + \Delta_1) = \tilde{\Pi}_0(0, \alpha + \Delta_1) + \tilde{\Pi}_1(0, \alpha + \Delta_1) > \varphi(0) = \tilde{\Pi}_0(\alpha, \alpha) + \tilde{\Pi}_1(\alpha, \alpha). \end{aligned}$$

The condition $p_0^*(\Delta\alpha) + p_1^*(\Delta\alpha) < (2\omega - 1)t$ amounts to $\omega \geq \frac{3+\varepsilon}{2(1+\varepsilon)}$, *i.e.*, the proportion of multi-homers is large enough or the gain from diversity is high enough. We must also check that the market is covered and that the utility of marginal customers is positive at the Nash equilibrium: $v + \alpha_0 - p_0^* - t\hat{x}(p_0^*, p_1^*) \geq 0$ amounts to $v + \frac{\alpha_0 + \alpha_1}{2} \geq \frac{t}{2} + t \frac{1+\varepsilon(2\omega-1)}{1+3\varepsilon}$; $v + \alpha_0 - p_0^* - t\tilde{x}_0(p_0^*, p_1^*) \geq 0$ and $v + \alpha_1 - p_1^* - t(1 - \tilde{x}_1(p_0^*, p_1^*)) \geq 0$ amount to $v + \frac{\alpha_0 + \alpha_1}{2} \geq t(1 - \omega) + 2t \frac{1+\varepsilon(2\omega-1)}{1+3\varepsilon}$. These conditions are satisfied for v sufficiently large for instance.

To check that Assumption 2 holds, it must be that

$$(A.2) \quad \pi(2\alpha + 2\Delta_0) + \pi(-2\alpha - 2\Delta_1) > 2\pi(\Delta_0 - \Delta_1)$$

and

$$(A.3) \quad \pi(2\alpha + 2\Delta_1) + \pi(-2\alpha - 2\Delta_0) > 2\pi(\Delta_1 - \Delta_0);$$

two conditions that hold thanks to the fact that π is convex.

Example 3: Discrete Choice Model. Let $z = \varepsilon_0 - \varepsilon_1$ and denote by $F(\cdot)$ its distribution obtained by convolution. $F(\cdot)$ is symmetric, *i.e.*, $F(x) + F(-x) = 1$ for all x . Profits for F_0 and F_1 are respectively written as $p_0(1 - F(p_0 - p_1 - \Delta\alpha))$ and $p_1 F(p_0 - p_1 - \Delta\alpha)$. Assuming log-concavity of $1 - F$ and F , best responses are uniquely defined by the following first-order conditions: $p_0 = \frac{1 - F(p_0 - p_1 - \Delta\alpha)}{f(p_0 - p_1 - \Delta\alpha)}$ and $p_1 = \frac{F(p_0 - p_1 - \Delta\alpha)}{f(p_0 - p_1 - \Delta\alpha)}$. Subtracting these two equations, it can be shown that the equilibrium prices $p_0^*(\Delta\alpha)$ and $p_1^*(\Delta\alpha)$ are such that $p_0^*(\Delta\alpha) - p_1^*(\Delta\alpha) - \Delta\alpha = z^*(\Delta\alpha)$ where $z^*(\Delta\alpha)$ is the unique solution to

$$z + \Delta\alpha = \frac{1 - 2F(z)}{f(z)}.$$

Observe that (i) $z^*(\Delta\alpha) < 0$ since $\Delta\alpha > 0$, (ii) $z^*(\Delta\alpha)$ is decreasing when $1 - F$ and F are log-concave, and (iii) $z^*(-\Delta\alpha) = -z^*(\Delta\alpha)$ by symmetry. Equilibrium profits for F_0 and F_1 are thus respectively given by $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \frac{(1 - F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))} = \pi(\Delta\alpha)$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \frac{(F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))} = \pi(-\Delta\alpha)$.

Assume from now on that ε_i s follow the Gumbel distribution with location 0 and scale $1/\lambda > 0$. Then, z is distributed according to the logistic distribution with cumulative $F(x) = \frac{e^{\lambda x}}{1+e^{\lambda x}}$ which is such that $1-F$ and F are log-concave. In this case, we have $\pi(\Delta\alpha) = \frac{e^{-\lambda z^*(\Delta\alpha)}}{\lambda}$. Define again $\varphi(\Delta\alpha) = \pi(\Delta\alpha) + \pi(-\Delta\alpha)$. We check that φ is increasing since $\dot{\varphi}(\Delta\alpha) = \dot{z}^*(\Delta\alpha) (e^{\lambda z^*(\Delta\alpha)} - e^{-\lambda z^*(\Delta\alpha)}) > 0$ and $z^*(\Delta\alpha)$ is decreasing and negative. Then, Assumption 1 holds since, for $\alpha_0 > \alpha_1$, condition (A.1) is again satisfied.

Finally, Assumption 2 is also satisfied since (A.2) and (A.3) both hold thanks to the fact that π is convex. Indeed, tedious computations yield

$$\dot{\pi}(\Delta\alpha) = -\frac{1}{1+2\lambda \cosh(\lambda z^*(\Delta\alpha))} > 0 \text{ and } \ddot{\pi}(\Delta\alpha) = \frac{\lambda e^{-\lambda z^*(\Delta\alpha)}(1+2\lambda e^{-\lambda z^*(\Delta\alpha)})}{(1+2\lambda \cosh(\lambda z^*(\Delta\alpha)))^3} > 0.$$

Example 4: Duopoly with Vertical Demand Shifters. Profits for F_0 and F_1 are respectively written as $p_0 D_0(\Delta\alpha, p_0, p_1)$ and $p_1 D_1(\Delta\alpha, p_0, p_1)$. Assuming that best responses are uniquely characterized by first-order conditions, the Nash equilibrium prices $(p_0^*(\Delta\alpha), p_1^*(\Delta\alpha))$ are given by $p_0^*(\Delta\alpha) = \frac{2a}{4+b} + \frac{2\Delta\alpha}{4+3b}$ and $p_1^*(\Delta\alpha) = p_0^*(-\Delta\alpha)$, while equilibrium demands are $D_0(\Delta\alpha, p_0^*(\Delta\alpha), p_1^*(\Delta\alpha)) = \frac{a(2+b)}{4+b} + \frac{(2+b)\Delta\alpha}{4+3b}$ and $D_1(\Delta\alpha, p_0^*(\Delta\alpha), p_1^*(\Delta\alpha)) = \frac{a(2+b)}{4+b} - \frac{(2+b)\Delta\alpha}{4+3b}$. Observe that total demand is kept unchanged as $\Delta\alpha$ varies. F_0 's and F_1 's equilibrium profits are thus given by $\tilde{\Pi}_0(\alpha_0, \alpha_1) = \pi(\Delta\alpha) \equiv \frac{2(2+b)(a(4+3b)+(4+b)\Delta\alpha)^2}{(4+b)^2(4+3b)^2}$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = \pi(-\Delta\alpha)$. Let us now define

$$\varphi(\Delta\alpha) = \pi(\Delta\alpha) + \pi(-\Delta\alpha) = \frac{4(2+b)(a^2(4+3b)^2 + (4+b)^2(\Delta\alpha)^2)}{(4+b)^2(4+3b)^2}.$$

Assumption 1 follows from the fact that $\varphi(\Delta\alpha)$ is increasing for $\alpha_0 > \alpha_1$ when $b > 0$.

Finally, Assumption 2 again holds since (A.2) and (A.3) are true thanks to the fact that π is convex.

Example 5: Cournot with Linear Demands. Profits for F_0 and F_1 are respectively written as $q_0 P_0(\alpha_0, q_0, q_1)$ and $q_1 P_1(\alpha_1, q_0, q_1)$. Assuming that best responses are uniquely characterized by first-order conditions, the Nash equilibrium quantities $(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1))$ are $q_0^*(\alpha_0, \alpha_1) = \frac{1}{2-b}(a + \frac{\alpha_0 + \alpha_1}{2}) + \frac{\Delta\alpha}{2-b}$, $q_1^*(\alpha_0, \alpha_1) = \frac{1}{2-b}(a + \frac{\alpha_0 + \alpha_1}{2}) - \frac{\Delta\alpha}{2-b}$. Equilibrium profits are given by $\tilde{\Pi}_0(\alpha_0, \alpha_1) = (\frac{a(2+b)+2\alpha_0+b\alpha_1}{4-b^2})^2$ and $\tilde{\Pi}_1(\alpha_0, \alpha_1) = (\frac{a(2+b)+2\alpha_1+b\alpha_0}{4-b^2})^2$. We denote $\alpha_i = \alpha + \Delta_i$ for $i = 0, 1$ with the condition $\Delta_0 > \Delta_1$ since F_0 is dominant. Simple manipulations show that, for $\Delta_0 > \Delta_1$,

$$\tilde{\Pi}_0(\alpha + \Delta_0, 0) + \tilde{\Pi}_1(\alpha + \Delta_0, 0) > \tilde{\Pi}_0(0, \alpha + \Delta_1) + \tilde{\Pi}_1(0, \alpha + \Delta_1).$$

Assumption 1 amounts to $f(\Delta_1) \equiv \tilde{\Pi}_0(0, \alpha + \Delta_1) + \tilde{\Pi}_1(0, \alpha + \Delta_1) - (\tilde{\Pi}_0(\alpha, \alpha) + \tilde{\Pi}_1(\alpha, \alpha)) > 0$. f is strictly convex in Δ_1 and admits two real roots, with the smallest one being negative and the highest $\bar{\Delta}_1$ one being positive. Hence, Assumption 1 holds if and only if $\Delta_1 > \bar{\Delta}_1 = \frac{(2+b)\sqrt{a^2(2+b)^2+4a\alpha(4+b^2)+2\alpha^2(4+b^2)}-a(2+b)^2-\alpha(4+b^2)}{4+b^2}$.

Last, consider the case of multiple rights. The condition in Assumption 2 for F_0 is

$$\tilde{\Pi}_0(2(\alpha + \Delta_0), 0) + \tilde{\Pi}_0(0, 2(\alpha + \Delta_1)) > 2\tilde{\Pi}_0(\alpha + \Delta_0, \alpha + \Delta_1).$$

After simple computations, this condition rewrites as $2(\frac{2\Delta_0-b\Delta_1+\alpha(2-b)}{4-b^2})^2 > 0$, which always holds. A similar result obtains for F_1 . Assumption 2 is again always satisfied in this example. \square

PROOF OF PROPOSITION 1. Because $\bar{T}_i(a) \geq 0$ for all $a \in \mathcal{A}$, we have the following:

$$(A.4) \quad \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) = \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a) \geq \max_{a \in \mathcal{A}} \bar{T}_{-i}(a).$$

The first equality writes as (4.4).

CLAIM 1. (4.6) holds.

PROOF OF CLAIM 1. Suppose to the contrary that $\bar{T}_i(a_{-i}) > 0$. Consider the bidding schedule

$$(A.5) \quad \tilde{T}_i(a) = \max\{\bar{T}_i(a) - \bar{T}_i(a_{-i}); 0\}.$$

By construction, we have

$$\tilde{T}_i(a_{-i}) = 0.$$

Since $\bar{T}_i(a) \geq 0$ for all $a \in \mathcal{A}$, we also have

$$\tilde{T}_i(a) \leq \max\{\bar{T}_i(a); 0\} = \bar{T}_i(a).$$

Moreover, the following string of inequality holds:

$$\begin{aligned} \max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max\{\bar{T}_i(a) - \bar{T}_i(a_{-i}) + \bar{T}_{-i}(a); \bar{T}_{-i}(a)\} \right\}, \\ &= \max \left\{ \max_{a \in \mathcal{A}} \bar{T}_i(a) - \bar{T}_i(a_{-i}) + \bar{T}_{-i}(a); \max_{a \in \mathcal{A}} \bar{T}_{-i}(a) \right\}, \\ &= \max \left\{ \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \bar{T}_i(a_{-i}); \bar{T}_{-i}(a_{-i}) \right\}, \\ &= \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \bar{T}_i(a_{-i}), \\ &\leq \tilde{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}), \end{aligned}$$

where the last equality follows from the definition of \bar{a} and the last inequality from the definition of $\tilde{T}_i(a)$ given in (A.5). From this, we deduce that \tilde{T}_i implements \bar{a} and does so at a weakly lower cost for F_i than \bar{T}_i ; which ends the proof. \square

CLAIM 2. (4.5) holds.

PROOF OF CLAIM 2. By definition, we have $\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) = \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a)$ and $\bar{T}_{-i}(a_{-i}) = \max_{a \in \mathcal{A}} \bar{T}_{-i}(a)$. Suppose to the contrary that

$$\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) > \bar{T}_{-i}(a_{-i}).$$

There thus exists $\varepsilon > 0$ small enough such that $\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) > \varepsilon + \bar{T}_{-i}(a_{-i})$. Consider the bidding schedule

$$\tilde{T}_i(a) = \max\{\bar{T}_i(a) - \varepsilon; 0\}.$$

Proceeding as in the Proof of Claim 1, we obtain the following:

$$\begin{aligned} \max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max\{\bar{T}_i(a) - \varepsilon + \bar{T}_{-i}(a); \bar{T}_{-i}(a)\} \right\}, \\ &= \max \left\{ \max_{a \in \mathcal{A}} \bar{T}_i(a) - \varepsilon + \bar{T}_{-i}(a); \max_{a \in \mathcal{A}} \bar{T}_{-i}(a) \right\}, \\ &= \max \left\{ \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \varepsilon; \bar{T}_{-i}(a_{-i}) \right\}, \\ &= \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \varepsilon, \\ &\leq \tilde{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}). \end{aligned}$$

From this, we deduce that \tilde{T}_i again implements \bar{a} and does so at a weakly lower cost for F_i than \bar{T}_i ; which ends the proof. \square

Claims 1 and 2 taken together allow to prove Proposition 1. \square

PROOF OF PROPOSITION 2.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that $a_{-0} = (1)$ (since $\Pi_1(1) > \Pi_1(0)$) and $a_{-1} = (0)$ (since $\Pi_0(0) > \Pi_0(1)$). From (4.6), it immediately follows that

$$(A.6) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0.$$

From (4.5), we have

$$(A.7) \quad \bar{T}_0(\bar{a}) + \bar{T}_1(\bar{a}) = \bar{T}_1(1) = \bar{T}_0(0).$$

From (4.3), any equilibrium allocation must satisfy

$$(A.8) \quad \sum_{i=0,1} \Pi_i(\bar{a}) + \sum_{i=0,1} \bar{T}_i(\bar{a}) \geq \sum_{i=0,1} \Pi_i(0) + \sum_{i=0,1} \bar{T}_i(0).$$

Inserting (A.6) and (A.7) into (A.8) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\bar{a}) \geq \sum_{i=0,1} \Pi_i(0).$$

Since (0) is constrained-efficient, we necessarily have $\bar{a} = (0)$; which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria. We start with the following lemma.

LEMMA A.1. *The set of equilibrium bids in monotonic equilibria is defined by the following conditions:*

$$(A.9) \quad \bar{T}_0(0) \in [\Pi_1(1) - \Pi_1(0), \Pi_0(0) - \Pi_0(1)],$$

$$(A.10) \quad \bar{T}_0(0) = \bar{T}_1(1) \geq 0,$$

$$(A.11) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0,$$

$$(A.12) \quad \bar{T}_0(c) \in [0, \bar{T}_0(0) + \Pi_1(0) - \Pi_1(c)],$$

$$(A.13) \quad \bar{T}_1(c) \in [0, \Pi_0(0) - \Pi_0(c)],$$

with

$$(A.14) \quad \bar{T}_0(c) + \bar{T}_1(c) \leq \bar{T}_0(0).$$

PROOF OF LEMMA A.1. *Necessity.* Since in any equilibrium $\bar{a} = (0)$, F_1 should not want to deviate to induce the alternative allocation $a = (1)$ by offering a payment $T_1(1) \geq \bar{T}_0(0)$. The corresponding incentive constraint is as follows:

$$(A.15) \quad \Pi_1(0) \geq \max_{T_1(1) \text{ s.t. } T_1(1) \geq \bar{T}_0(0)} \Pi_1(1) - T_1(1) = \Pi_1(1) - \bar{T}_0(0),$$

which gives the lower bound in (A.9).

Additionally, F_1 should not want to deviate from $\bar{a} = a_{-1} = (0)$ to induce an outcome with joint distribution, *i.e.*, $a = (c)$, by offering a payment $T_1(c)$ such that $\bar{T}_0(c) + T_1(c) \geq \bar{T}_0(0)$. The corresponding incentive constraint becomes as follows:

$$(A.16) \quad \Pi_1(0) \geq \max_{T_1(c) \text{ s.t. } \bar{T}_0(c) + T_1(c) \geq \bar{T}_0(0)} \Pi_1(c) - T_1(c) = \Pi_1(c) + \bar{T}_0(c) - \bar{T}_0(0).^{26}$$

From which, we obtain the upper bound in (A.12).

Second, turning to F_0 's incentives to abide to the equilibrium strategy rather than letting $a = (1)$ emerges, which is simply obtained by not paying since $a_{-0} = (1)$, it must be that

$$(A.17) \quad \Pi_0(0) - \bar{T}_0(0) \geq \Pi_0(1).$$

which gives the upper bound in (A.9).

Additionally, F_0 should not be willing to induce $a = (c)$ either, which requires the following:

$$(A.18) \quad \Pi_0(0) - \bar{T}_0(0) \geq \max_{T_0(c) \text{ s.t. } T_0(c) + \bar{T}_1(c) \geq \bar{T}_0(0)} \Pi_0(c) - T_0(c) = \Pi_0(c) + \bar{T}_1(c) - \bar{T}_0(0),$$

where the constraint in the maximand of the right-hand side follows from (A.6). Simplifying yields (A.13).

Putting together (A.15) and (A.17) yields (A.9).

Using (4.4) yields the following:

$$\bar{T}_0(c) + \bar{T}_1(c) \leq \bar{T}_0(0) + \bar{T}_1(0).$$

Taking into account that $a_{-1} = (0)$ (and thus $\bar{T}_1(0) = 0$) yields (A.14). The right-hand side inequality follows from bid being non-negative.

Using (4.5), the fact that $\bar{a} = a_{-1} = (0)$ and thus $\bar{T}_1(0) = 0$ (from (4.6)) implies (A.10).

Last, (A.11) follows from (4.6).

Sufficiency. Any pair of bidding schedules (\bar{T}_0, \bar{T}_1) that satisfy conditions (A.9) to (A.14) also satisfy, by construction, all the incentive constraints for F_0 , F_1 and A that must hold at an equilibrium. \square

Gathering everything and noting that the overall profit of the industry must be $\Pi_0(0) + \Pi_1(0)$ yields (4.8). \square

PROOF OF PROPOSITION 3. First, constrained efficiency follows from Proposition 2. Second, we remind that monotonicity implies $a_{-0} = (1)$ and $a_{-1} = (0)$, and we note that truthful schedules of the form $T_i(a) = \max\{\Pi_i(a) - \bar{\Pi}_i, 0\}$ satisfy those monotonicity conditions. Following Laussel and Le Breton (2001), we define the value of any arbitrary coalition $S \subseteq N = \{F_0, F_1, A\}$ as $W(S) = \max_{a \in \mathcal{A}} \sum_{i \in S} \Pi_i(a)$. The cooperative game with transferable utility defined through these values is *strongly subadditive* if, for all $S \subseteq N$, $T \subseteq N$ such that $S \cup T = N$, $W(N) \leq W(T) + W(S) - W(S \cap T)$; this is a condition that can be readily verified since we have the following:

$$W(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W(\{0\}) = \Pi_0(0), \quad W(\{1\}) = \Pi_1(1), \quad W(\emptyset) = 0$$

and thus

$$W(\{01\}) < W(\{0\}) + W(\{1\}) \Leftrightarrow \Pi_1(0) < \Pi_1(1).$$

²⁶Because \bar{T}_0 is monotonic and $\Pi_0(0) \geq \Pi_0(c)$, we have $\bar{T}_0(0) \geq \bar{T}_0(c)$ and the considered deviation requires $T_1(c) \geq 0$.

Following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\bar{\Pi}_0, \bar{\Pi}_1)$ must lie on the Pareto frontier of the set defined by the three following constraints:

$$(A.19) \quad \bar{\Pi}_0 \leq W(\{01\}) - W(\{1\}) = \Pi_0(0) + \Pi_1(0) - \Pi_1(1),$$

$$(A.20) \quad \bar{\Pi}_1 \leq W(\{01\}) - W(\{0\}) = \Pi_1(0),$$

$$(A.21) \quad \bar{\Pi}_0 + \bar{\Pi}_1 \leq W(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Because of strong sub-additivity, the following inequality holds:

$$W(\{01\}) - W(\{1\}) + W(\{01\}) - W(\{0\}) < W(\{01\}).$$

It implies that (A.21) necessarily holds when (A.19) and (A.20) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0, \bar{\Pi}_1) = (\Pi_0(0) + \Pi_1(0) - \Pi_1(1), \Pi_1(0)).$$

This finally gives us the expressions of profits for the whole industry in (4.9). \square

PROOF OF PROPOSITION 4. We adapt the methodology developed in Proposition 3 to a scenario with renegotiation. For any arbitrary coalition $S \subseteq N = \{F_0, F_1, A\}$, we may define the coalitional payoff with renegotiation as $W^{rp}(S) = \max_{a \in \mathcal{A}} \sum_{i \in S} \bar{\Pi}_i(a)$. By Definition (5.1), we have the following:

$$W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} (\Pi_0(0) - \Pi_1(0)),$$

$$W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} (\Pi_1(1) - \Pi_0(1)), \quad W^{rp}(\emptyset) = 0.$$

The cooperative game so-defined is again *strongly subadditive* since

$$W^{rp}(\{01\}) < W^{rp}(\{0\}) + W^{rp}(\{1\}) \Leftrightarrow \Pi_1(0) - \Pi_0(0) < \Pi_1(1) - \Pi_0(1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp})$ lies on the Pareto frontier of the set defined by the three following constraints:

$$(A.22) \quad \bar{\Pi}_0^{rp} \leq W^{rp}(\{01\}) - W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} (\Pi_1(1) - \Pi_0(1)),$$

$$(A.23) \quad \bar{\Pi}_1^{rp} \leq W^{rp}(\{01\}) - W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} (\Pi_0(0) - \Pi_1(0)),$$

$$(A.24) \quad \bar{\Pi}_0^{rp} + \bar{\Pi}_1^{rp} \leq W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Strong sub-additivity implies that (A.24) necessarily holds when (A.22) and (A.23) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp}) = \left(\frac{1}{2} \sum_{i=0,1} \Pi_i(0) - \frac{1}{2} (\Pi_1(1) - \Pi_0(1)), \Pi_1(0) \right).$$

Observe that F_1 's profits with or without resale are the same (see (4.9) and (5.3)). Then A 's profit in the unique truthful resale-proof equilibrium is greater than that in the unique truthful

equilibrium without resale since

$$\frac{1}{2} \left(\Pi_0(0) + \Pi_1(1) - \Pi_1(0) - \Pi_0(1) \right) > \Pi_1(1) - \Pi_1(0).$$

This concludes the proof. \square

PROOF OF PROPOSITION 5.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that $b_{-0} = (1, 1)$ and $b_{-1} = (0, 0)$ where $b_{-i} = \arg \max_{b \in \mathcal{A}_*^2} \bar{T}_{-i}(b)$. From (4.6), it follows that

$$(A.25) \quad \bar{T}_0(1, 1) = \bar{T}_1(0, 0) = 0.$$

From (4.5), we write the following:

$$(A.26) \quad \bar{T}_0(\bar{b}) + \bar{T}_1(\bar{b}) = \bar{T}_1(1, 1) = \bar{T}_0(0, 0).$$

From (4.3), any equilibrium allocation must satisfy the following:

$$(A.27) \quad \sum_{i=0,1} \Pi_i(\bar{b}) + \sum_{i=0,1} \bar{T}_i(\bar{b}) \geq \sum_{i=0,1} \Pi_i(0, 0) + \sum_{i=0,1} \bar{T}_i(0, 0).$$

Inserting (A.25) and (A.26) into (A.27) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\bar{b}) \geq \sum_{i=0,1} \Pi_i(0, 0).$$

Since $(0, 0)$ is constrained-efficient, we necessarily have $\bar{b} = (0, 0)$; which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria.

LEMMA A.2. *The whole set of bids in monotonic equilibria is defined by the following inequalities:*

$$(A.28) \quad \bar{T}_0(0, 0) = \bar{T}_1(1, 1) \in [\Pi_1(1, 1) - \Pi_1(0, 0), \Pi_0(0, 0) - \Pi_0(1, 1)],$$

$$(A.29) \quad \bar{T}_0(1, 1) = \bar{T}_1(0, 0) = 0,$$

$$(A.30) \quad \begin{aligned} \bar{T}_0(0, 1) &\in [0, \bar{T}_0(0, 0) + \Pi_1(0, 0) - \Pi_1(0, 1)], \\ \bar{T}_0(1, 0) &\in [0, \bar{T}_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 0)], \end{aligned}$$

$$(A.31) \quad \begin{aligned} \bar{T}_1(0, 1) &\in [0, \Pi_0(0, 0) - \Pi_0(0, 1)], \\ \bar{T}_1(1, 0) &\in [0, \Pi_0(0, 0) - \Pi_0(1, 0)], \end{aligned}$$

$$(A.32) \quad \max\{\bar{T}_0(0, 1) + \bar{T}_1(0, 1), \bar{T}_0(1, 0) + \bar{T}_1(1, 0)\} \leq \bar{T}_0(0, 0) = \bar{T}_1(1, 1).$$

PROOF OF LEMMA A.2.

Necessity. Since in any equilibrium $\bar{b} = (0, 0)$, the first equality in (A.28) follows from (4.5) and (4.6).

Also F_1 should not want to deviate to induce the alternative allocation $a = (1, 1)$. The corresponding incentive constraint becomes as follows:

$$(A.33) \quad \Pi_1(0, 0) \geq \max_{T_1(1,1) \text{ s.t. } T_1(1,1) \geq \bar{T}_0(0,0)} \Pi_1(1, 1) - T_1(1, 1) = \Pi_1(1, 1) - \bar{T}_0(0, 0).$$

Similarly, F_1 should not induce a deviation towards $a = (0, 1)$, which requires the following:

$$(A.34) \quad \Pi_1(0, 0) \geq \max_{T_1(0,1) \text{ s.t. } T_0(0,1)+T_1(0,1) \geq \bar{T}_0(0,0)} \Pi_1(0, 1) - T_1(0, 1) = \Pi_1(0, 1) + T_0(0, 1) - \bar{T}_0(0, 0);$$

thus, the first condition in (A.30) follows.

F_1 should also not induce a deviation towards $a = (1, 0)$, which requires the following:

$$(A.35) \quad \Pi_1(0, 0) \geq \max_{T_1(1,0) \text{ s.t. } T_0(1,0)+T_1(1,0) \geq \bar{T}_0(0,0)} \Pi_1(0, 1) - T_1(0, 1) = \Pi_1(0, 1) + T_0(0, 1) - \bar{T}_0(0, 0);$$

thus, the second condition in (A.30) follows.

Let us now turn to F_0 's incentives to abide to the equilibrium strategy rather than to induce $a = (1, 1)$. Such deviation is simply obtained with a null bid since $a_{-0} = (1, 1)$. F_0 's incentive constraint is thus written as follows:

$$(A.36) \quad \Pi_0(0) - \bar{T}_0(0, 0) \geq \Pi_0(1, 1).$$

Putting together (A.33) and (A.36) yields the second condition in (A.28).

Also, F_0 should not induce a deviation towards $a = (0, 1)$. Since, A is indifferent between $\bar{a} = (0, 0)$ and $a = (1, 1)$ (from the first equality in (A.28)), this requires avoiding that A switches to $(1, 1)$ in case F_0 no longer offers $\bar{T}_0(0, 0)$. We write this incentive constraint as follows:

$$(A.37) \quad \Pi_1(0, 0) - \bar{T}_0(0, 0) \geq \max_{T_0(0,1) \text{ s.t. } T_0(0,1)+\bar{T}_1(0,1) \geq \bar{T}_1(1,1)} \Pi_0(0, 1) - T_0(0, 1) = \Pi_0(0, 1) + \bar{T}_1(0, 1) - \bar{T}_1(1, 1).$$

Again taking into account the first equality in (A.28), (A.37) simplifies as follows:

$$\Pi_1(0, 0) - \bar{T}_0(0, 0) \geq \Pi_0(0, 1) + \bar{T}_1(0, 1) - \bar{T}_0(0, 0).$$

Therefore, the upper bound for $\bar{T}_1(0, 1)$ in (A.31) immediately follows.

Finally, F_0 should also not induce a deviation towards $a = (1, 0)$. Replicating lines above, it must be that

$$(A.38) \quad \Pi_0(0, 0) - \bar{T}_0(0, 0) \geq \max_{T_0(1,0) \text{ s.t. } T_0(1,0)+\bar{T}_1(1,0) \geq \bar{T}_1(1,1)} \Pi_0(1, 0) - T_0(1, 0) = \Pi_0(1, 0) + \bar{T}_1(1, 0) - \bar{T}_1(1, 1).$$

Again taking into account the first equality in (A.28), (A.38) simplifies as follows:

$$\Pi_0(0, 0) - \bar{T}_0(0, 0) \geq \Pi_0(1, 0) + \bar{T}_1(1, 0) - \bar{T}_0(0, 0).$$

Therefore, the upper bound for $\bar{T}_1(1, 0)$ in (A.31) immediately follows.

Sufficiency. Any pair of bidding schedules (\bar{T}_0, \bar{T}_1) that satisfy conditions (A.28) also satisfy F_0 , F_1 and A 's incentive constraints from the above analysis, which defines an equilibrium. \square

The expression of profits immediately follows from the existing bounds on bids characterized in Lemma A.2. \square

PROOF OF PROPOSITION 6. Truthful strategies are monotonic; thus, any putative equilibrium with truthful bidding schedules implements a constrained-efficient outcome. We again adapt the methodology already used in Propositions 3 and 4 to a scenario with multiple rights. For any arbitrary coalition $S \subseteq N = \{F_0, F_1, A\}$, we may define the coalitional payoff as $W(S) = \max_{b \in \mathcal{A}_*^2} \sum_{i \in S} \tilde{\Pi}_i(a)$. We immediately check that

$$\begin{aligned} W(\{01\}) &= \Pi_0(0, 0) + \Pi_1(0, 0), & W(\{0\}) &= \Pi_0(0, 0), \\ W(\{1\}) &= \Pi_1(1, 1), & W(\emptyset) &= 0. \end{aligned}$$

The cooperative game with the so-defined coalitional payoffs is again *strongly sub-additive* since

$$W(\{01\}) < W(\{0\}) + W(\{1\}) \Leftrightarrow \Pi_1(0, 0) < \Pi_1(1, 1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp})$ lies on the Pareto frontier of the set defined by the following three constraints:

$$(A.39) \quad \bar{\Pi}_0 \leq W(\{01\}) - W(\{1\}) = \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1),$$

$$(A.40) \quad \bar{\Pi}_1 \leq W(\{01\}) - W(\{0\}) = \Pi_1(0, 0),$$

$$(A.41) \quad \bar{\Pi}_0 + \bar{\Pi}_1 \leq W(\{01\}) = \Pi_0(0, 0) + \Pi_1(0, 0).$$

Strong sub-additivity implies that (A.41) necessarily holds when (A.39) and (A.40) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0^b, \bar{\Pi}_1^b) = (\Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1), \Pi_1(0, 0)).$$

Gathering everything, we finally obtain the expressions of equilibrium profits $\bar{\Pi}_0^b$, $\bar{\Pi}_1^b$ and $\bar{\Pi}_a^b$ in the unique truthful equilibrium which are given in (6.5). \square

PROOF OF PROPOSITION 7.

BIDS. Inserting the additivity requirement (6.6) into constraints (A.28) to (A.32) gives a characterization of the whole set of equilibrium transfers on split markets as those satisfying the following set of linear constraints:

$$(A.42) \quad 2\bar{T}_0(0) = 2\bar{T}_1(1) \in [\Pi_1(1, 1) - \Pi_1(0, 0), \Pi_0(0, 0) - \Pi_0(1, 1)],$$

$$(A.43) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0,$$

$$(A.44) \quad \bar{T}_0(0) \geq \Pi_1(0, 1) - \Pi_1(0, 0) \geq 0,$$

$$(A.45) \quad 0 \leq \bar{T}_1(1) \leq \Pi_0(0, 0) - \Pi_0(0, 1).$$

The set of possible values for $\bar{T}_0(0) = \bar{T}_1(1)$ satisfying all those constraints is non-empty whenever

$$(A.46) \quad \max\{2(\Pi_1(0, 1) - \Pi_1(0, 0)); \Pi_1(1, 1) - \Pi_1(0, 0)\} \leq \min\{2(\Pi_0(0, 0) - \Pi_0(0, 1)); \Pi_0(0, 0) - \Pi_0(1, 1)\}.$$

Using Assumption 2, the left-hand side of (A.46) is

$$(A.47) \quad \max\{2(\Pi_1(0, 1) - \Pi_1(0, 0)); \Pi_1(1, 1) - \Pi_1(0, 0)\} = \Pi_1(1, 1) - \Pi_1(0, 0).$$

Still using Assumption 2, the right-hand side of (A.46) is

$$(A.48) \quad \min\{2(\Pi_0(0,0) - \Pi_0(0,1)); \Pi_0(0,0) - \Pi_0(1,1)\} = \Pi_0(0,0) - \Pi_0(1,1).$$

Gathering (A.47) and (A.48) and inserting into (A.46), the set of possible values for $\bar{T}_0(0) = \bar{T}_1(1)$ is non-empty whenever

$$(A.49) \quad \Pi_0(1,1) + \Pi_1(1,1) \leq \Pi_0(0,0) + \Pi_1(0,0),$$

but again, this property follows from the fact that $(0,0)$ maximizes the industry's profit.

Finally, observe that conditions (A.42) to (A.45) are necessary but also sufficient for an equilibrium with additive bidding schedules. Sufficiency follows from the existence of bids satisfying those constraints.

PROFITS. From (A.42), we deduce the following characterization of F_0 , F_1 and A 's profits as follows:

$$(A.50) \quad \begin{aligned} \Pi_0(1,1) &\leq \bar{\Pi}_0^s \leq \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1), \\ \bar{\Pi}_1^s &= \Pi_1(0,0), \\ \Pi_0(0,0) + \Pi_1(0,0) - \bar{\Pi}_0^s &= \bar{\Pi}_a^s \geq \Pi_1(1,1) - \Pi_1(0,0). \end{aligned}$$

This ends the proof. □

REFERENCES

- Aghion, P. and P. Bolton, 1987, "Contracts as a Barrier to Entry," *The American Economic Review*, 77: 388-401.
- Anderson, S. and S. Coate, 2005, "Market Provision of Broadcasting: A Welfare Analysis," *The Review of Economic Studies*, 72: 947-72.
- Armstrong, M., 1999, "Competition in the Pay-TV Market," *Journal of the Japanese and International Economies*, 13: 257-280.
- Aseff, J. and H. Chade, 2008, "An Optimal Auction with Identity-Dependent Externalities," *The RAND Journal of Economics*, 39: 731-746.
- Bachelor, B., K. Lent, M. Cragg and J. Keyte, 2020, "Convergence and Divergence in Assessing the Broadcasting Practices of Professional Sports Leagues," *Business Law International*, 21(2): 101-120.
- Bedre-Defolie, Ö. and G. Biglaiser, 2022, "Competition for Exclusivity of a Superior Product and Quality Implications," Available at <https://ssrn.com/abstract=4278448> or <http://dx.doi.org/10.2139/ssrn.4278448>.
- Bernheim, D. and M. Whinston, 1986a, "Menu Auctions, Resource Allocation, and Economic Influence," *The Quarterly Journal of Economics*, 101: 1-31.
- Bernheim, D. and M. Whinston, 1986b, "Common Agency," *Econometrica*, 54: 923-942.
- Bernheim, D. and M. Whinston, 1990, "Multimarket Contact and Collusive Behavior," *The RAND Journal of Economics*, 21: 1-26.
- Bernheim, D. and M. Whinston, 1998, "Exclusive Dealing," *Journal of Political Economy*, 106: 64-103.
- Brocas, I., 2013, "Optimal Allocation Mechanisms with Type-Dependent Negative Externalities," *Theory and Decision*, 75: 359-387.

- Calzolari, G. and V. Denicolò, 2015, "Exclusive Contracts and Market Dominance," *The American Economic Review*, 105: 3321-51.
- Chiesa, G. and V. Denicolò, 2009, "Trading with a Common Agent under Complete Information: A Characterization of Nash Equilibria," *Journal of Economic Theory*, 144: 296-311.
- Choné, P. and L. Linnemer, 2015, "Nonlinear Pricing and Exclusion: I. Buyer Opportunism," *The RAND Journal of Economics*, 46: 217-240.
- Crémer, J. and M. Riordan, 1987, "On Governing Multilateral Transactions with Bilateral Contracts," *The RAND Journal of Economics*, 18: 436-451.
- Doganoglu, T. and J. Wright, 2006, "Multihoming and Compatibility," *International Journal of Industrial Organization*, 24: 45-67.
- Dukes, A. and E. Gal-Or, 2003, "Negotiations and Exclusivity Contracts for Advertising," *Marketing Science*, 22: 222-47.
- Esö, P., V. Nocke and L. White, 2010, "Competition for Scarce Resources," *The RAND Journal of Economics*, 41: 524-548.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2001, "Press Advertising and the Ascent of the "Pensée Unique?"," *European Economic Review*, 45: 645-51.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2002, "Press Advertising and the Political Differentiation of Newspapers," *Journal of Public Economic Theory*, 4: 317-34.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2004, "Programming and Advertising Competition in the Broadcasting Industry," *Journal of Economics and Management Strategy*, 13: 657-69.
- Gal-Or, E. and A. Dukes, 2003, "Minimum Differentiation in Commercial Media Markets," *Journal of Economics and Management Strategy*, 12: 291-325.
- Hagiu, A. and R. Lee, 2011, "Exclusivity and Control," *Journal of Economics and Management Strategy*, 20: 679-708.
- Harbord, D. and M. Ottaviani, 2001, "Contracts and Competition in the Pay-TV Market," University Library of Munich, Germany.
- Hart, O. and J. Tirole, 1990, "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics*, 205-276.
- Horn, H. and A. Wolinsky, 1988, "Bilateral Monopolies and Incentives for Merger," *The RAND Journal of Economics*, 19: 408-419.
- Jéhiel, P. and B. Moldovanu, 2000, "Auctions with Downstream Interaction Among Viewers," *The RAND Journal of Economics*, 31: 768-791.
- Kagel, J. and D. Levin, 2005, "Multi-Unit Demand Auctions with Synergies: Behavior in Sealed-Bid Versus Ascending-Bid Uniform-Price Auctions," *Games and Economic Behavior*, 53: 170-207.
- Laussel, D. and M. Le Breton, 2001, "Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency," *Journal of Economic Theory*, 100: 93-128.
- Martimort, D. 1996. "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory," *The RAND Journal of Economics*, 27: 1-31.
- Martimort, D., 2007, "Multi-Contracting Mechanism Design," in R. Blundell, A. Newey and T. Persson, eds., *Advances in Economic Theory Proceedings of the World Congress of the Econometric Society*, Cambridge University Press.

- Martimort, D. and J. Pouyet, 2020, "Downstream Mergers In Vertically-Related Markets with Capacity Constraints," *International Journal of Industrial Organization*, 72: 102643.
- Martimort, D. and L. Stole, 2012, "Representing Equilibrium Aggregates in Aggregate Games with Applications to Common Agency," *Games and Economic Behavior*, 76: 753-772.
- Maskin, E. and J. Moore, 1999, "Implementation and Renegotiation," *The Review of Economic Studies*, 66: 39-56.
- McAfee, R. and M. Schwartz, 1994, "Opportunism in Multilateral Vertical Contracting: Non-Discrimination, Exclusivity, and Uniformity," *The American Economic Review*, 84: 210-230.
- McAfee, R. and M. Schwartz, 1995, "The Non-Existence of Pairwise-Proof Equilibrium," *Economics Letters*, 49: 251-259.
- Milgrom, P., 2007, "Package Auctions and Exchanges," *Econometrica*, 75: 935-965.
- Molnar, J. and G. Virag, 2008, "Revenue Maximizing Auctions with Market Interaction and Signaling," *Economics Letters*, 99: 360-363.
- O'Brien, D. and G. Shaffer, 1992, "Vertical Control with Bilateral Contracts," *The RAND Journal of Economics*, 23: 299-308.
- Peitz, M. and T. Valletti, 2008, "Content and Advertising in the Media: Pay-TV versus Free-To-Air," *International Journal of Industrial Organization*, 26: 949-65.
- Rey, P. and T. Vergé, 2004, "Bilateral Control with Vertical Contracts," *The RAND Journal of Economics*, 35: 728-746.
- Segal, I. and M. Whinston, 2003, "Robust Predictions for Bilateral Contracting with Externalities," *Econometrica*, 71: 757-791.
- Shaked, A. and J. Sutton, 1982, "Relaxing Price Competition through Product Differentiation," *The Review of Economic Studies*, 49: 3-13.
- Shaked, A. and J. Sutton, 1983, "Natural Oligopolies," *Econometrica*, 51: 1469-1483.
- Sonnac, N., 2012, "Médias Audiovisuels et Concurrence. Le Cas de la Télévision Payante," *Revue d'Economie Industrielle*, 137: 109-129.
- Spagnolo, G., 1999, "On Interdependent Supergames: Multimarket Contact, Concavity and Collusion," *Journal of Economic Theory*, 89: 127-139.
- Stennek, J., 2014, "Why Exclusive Distribution May Benefit the TV-Viewers," *Information Economics and Policy*, 26: 42-57.
- Toft, T., 2003, "Football: Joint Selling of Media Rights," *Competition Policy Newsletter*, 3: 47-52.
- Varma, G., 2002, "Standard Auctions with Identity-Dependent Externalities," *The RAND Journal of Economics*, 33: 689-708.
- Weeds, H., 2016, "TV Wars: Exclusive Content and Platform Competition in Pay TV," *Economic Journal*, 126: 1600-1633.
- Wilson, R., 1979, "Auctions of Shares," *The Quarterly Journal of Economics*, 94: 675-689.