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**Laterality in simple multiplication:  
Assessing hemispheric specialization of arithmetic fact retrieval  
in a visual hemifield paradigm**

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**AUTHOR CONTRIBUTIONS:**

Conceived research: SJ, EK. Methodology: SJ, EK. Data Curation: SJ, RJ. Analysis: RJ, SJ.

Writing-Original Draft: SJ, EK. Writing-Review and Editing: All authors.

**PRE-REGISTRATION AND OPEN DATA:**

Pre-registration of the study is available at <https://aspredicted.org/blind.php?x=yc248x>). Our data, syntax, and supplemental materials can be found at <https://osf.io/a4m7y>.

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## **Abstract**

The prominent view in numerical cognition is that highly overlearned arithmetic facts are verbally mediated and unilaterally processed in a left-hemispheric network, including language areas. However, this view has recently been challenged.

Here, we evaluated the hemispheric specialization of verbal arithmetic fact retrieval and number magnitude processing using two verification tasks (i.e., multiplication and number magnitude comparison) in a divided hemifield paradigm in healthy participants ( $n = 35$ ).

In the *number comparison task*, we replicated bilateral processing advantages for the unit-decade compatibility effect. Magnitude processing was facilitated after bilateral compared with a unilateral presentation. In the *multiplication task*, lateralized presentation to the left hemisphere yielded processing advantages in arithmetic fact retrieval when the required inter-hemispheric transitions from the input, processing, and output stages were considered. Crucially, we observed a systematic processing advantage in left visual hemifield stimuli presentation (i.e., initial right-hemisphere processing).

Our findings corroborate the assumption that arithmetic fact retrieval is subserved by left-lateralized verbal/linguistic processing. Thereby, they suggest a distinction between unilateral left-hemispheric linguistic processing of arithmetic fact retrieval and bilateral number magnitude processing. Interestingly, however, our data present right-hemispheric processing advantages in identifying early processed visual symbolic numerical stimuli.

**Word count: 192**

**Keywords:** hemispheric specialization, numerical cognition, arithmetic fact retrieval, multiplication

## **Public Significance Statements**

- This study strongly suggests an involvement of the two hemispheres in the brain in solving numerical tasks.
- This study observed right-hemisphere processing for the early identification of visually presented numbers in all tasks. In number magnitude comparison, bilateral processing was advantageous. In subsequent simple multiplication, left-hemispheric processing advantages were found when considering the minimum number of inter-hemispheric transitions to solve the task, favoring an association of arithmetic facts and language processing.
- The study results suggest a distinction between unilateral processing of arithmetic fact retrieval and bilateral number magnitude processing.

## **1 Introduction**

In the human brain, neural networks in the left and the right hemisphere serve for different perceptual, emotional, motor, and cognitive functions (Michel, 2021), which are processed either via inter-hemispheric cooperation or differentially specialized (predominantly) either in one or in the other hemisphere (Hervé et al., 2013). Hemispheric specialization [i.e., a (relative) functional asymmetry] or ‘dominance’ was shown for various cognitive processes (Corballis & Haeberling, 2017, for a review). Language processing, for instance, involves mostly unilateral processing in a large-scale network in the left hemisphere (Vigneau et al., 2006; for a meta-analysis; but see Mitchell & Crow, 2005 or Vingerhoets, 2019; for right-hemispheric language processing). Instead, the right hemisphere shows a relative dominance for spatial processing (Vogel et al., 2003; for a meta-analysis). For number processing, however, the specific contribution of both hemispheres has not yet been fully established.

### **Hemispheric specialization in number processing**

Number processing reliably engages a variety of brain regions. According to the predominate model of number processing, the triple code model (Dehaene & Cohen, 1995, 1997), number magnitude processing (e.g., the mental manipulation of numerical quantities like  $87 - 34 = 53$ ), is supposed to be located in a bilateral frontoparietal brain network (for meta-analyses, see Arsalidou & Taylor, 2011; Hawes et al., 2019; Siemann & Petermann, 2018). In particular, the bilateral intraparietal sulcus in the parietal cortex between the superior parietal lobule and the inferior parietal lobule serves as the core of this quantity code (Dehaene et al., 2003). Arithmetic fact retrieval (i.e., one-digit  $\times$  one-digit multiplication), instead, has been associated with unilateral left-lateralized processing in the brain (Dehaene & Cohen, 1995, 1997; Dehaene et al., 2003; but see Amalric & Dehaene, 2016, 2018, 2019. Thereby,

arithmetic facts are assumed to be mediated in a verbal code in left-hemispheric perisylvian language areas and the left angular gyrus (Dehaene et al., 2003).

Evidence favoring left-hemispheric specialization of arithmetic fact retrieval comes from numerous studies using neuropsychological, neuroimaging, and behavioral methods. Various neuropsychological single-case studies reported deficits in arithmetic fact retrieval following lesions to left-hemispheric perisylvian language areas (Dehaene & Cohen, 1995; Warrington, 1982; Zaunmueller et al., 2009). Neurofunctional evidence comes primarily from studies that pursued the acquisition of arithmetic facts using drill training of complex multiplication problems (e.g.,  $43 \times 9$ ; Bloechle et al., 2016; Delazer et al., 2003; Ischebeck et al., 2006). Here, a switch from bilateral magnitude-related procedural processing to left-hemispheric arithmetic fact retrieval was observed. Supporting evidence also comes from studies investigating dissociations in bilinguals (Dehaene et al., 1999) as well as (double)-dissociations between subtraction and multiplication in stroke patients (Dehaene & Cohen, 1997) and the blind (Crollen et al., 2019). In addition, several neuroimaging evidence suggested overlapping networks for language and arithmetic (e.g., Artemenko et al., 2018; Berteletti et al., 2014; Demir et al., 2014; Prado et al., 2014). These findings suggest that mental arithmetic partly relies on linguistic functions processes as both processes recruit overlapping cortical brain circuits. Finally, on a behavioral level, Moeller and colleagues (2011) showed that concurrent articulation (i.e., reciting “pataka pataka) in a number bisection task disturbed the access to phonologically stored multiplication facts.

Evidence against mere left-hemispheric specialization of arithmetic fact retrieval comes from more recent neuroimaging (Amalric & Dehaene, 2016, 2018, 2019; Amalric et al., 2018; for native and early blind single cases) and brain stimulation studies (Salillas et al., 2021; Semenza & Benavides-Varela, 2018). Amalric and Dehaene (2016; 2018; 2019) proposed that numbers and mathematical facts are processed independently of left-hemispheric language

processing based on their studies in high-level professional mathematicians and a control group of non-mathematician academics with comparable education levels. In brain stimulation studies during awake surgery, Salillas et al. (2012, 2021) showed differential involvement of both hemispheres in arithmetic fact retrieval. Inhibition of the right parietal hemisphere, which means processing favoring the left hemisphere, provoked operand errors (e.g.,  $6 \times 4 = 20$  or 28). Inhibition of the left hemisphere, which means processing favoring the left hemisphere, induced close-miss errors (e.g.,  $6 \times 4 = 22$ ), resulting close to the target. This qualitative analysis of errors suggested that each hemisphere has its specific role in arithmetic fact retrieval (Salillas et al., 2021)

In sum, hemispheric specialization in arithmetic fact retrieval is not unambiguous: Neuropsychological single-case studies suggested mere left-hemispheric linguistic processing of arithmetic facts (Dehaene & Cohen, 1995, 1997). More recent functional magnetic resonance imaging (fMRI) studies pointed towards a more extensive bilateral network (e.g., Bloechle et al., 2016); however, this network may not be causally involved in the required cognitive process to solve the task (Price & Friston, 2002). Brain stimulation studies also indicated bilateral processing, with a different contribution of each hemisphere (Semenza & Benavides-Varela, 2018), by analyzing different multiplication errors qualitatively (i.e., table errors and close-miss errors (Salillas et al., 2021). Such a qualitative analysis of multiplication errors related to hemispheric specialization seems a promising approach that can also be pursued at the behavioral level, for instance, using a divided hemifield paradigm.

## **1.2 The divided hemifield paradigm approach**

The divided visual hemifield paradigm has been used to demonstrate hemispheric specialization, for instance, for language (Brysbaert, 1994; Geffen, Bradshaw, & Wallace, 1971) and number processing (Boles, 1986; Dimond & Beaumont, 1971; Hatta et al., 2002;

Jung et al., 2020; Ratinckx et al., 2006). This approach is based on the idea that visual input is immediately available to both hemispheres after the bilateral presentation of visual stimuli. After the unilateral presentation, visual input is initially only transferred into the contralateral hemisphere (e.g., Brysbaert, 1994). If the information is initially directed to the less specialized hemisphere, an interhemispheric transfer to the specialized hemisphere occurs. Response latency differences point to such an interhemispheric transfer. Outcomes of the visual hemifield paradigm strongly correlate with results on the neurofunctional level (Hunter & Brysbaert, 2008; for language processing).

In number processing, left-hemispheric specialization (i.e., after stimulus presentation to the right visual hemifield) was observed for number words, but not for Arabic digits as assessed in number comparison, parity judgment, and number estimation (Boles, 1986). Right-hemispheric specialization (i.e., after stimuli presentation to the left visual hemifield) was also found for well-established numerical effects such as the unit-decade compatibility effect (Ratinckx et al., 2006). The unit-decade compatibility effect describes processing disadvantages (i.e., longer response times and higher error rates) for incompatible (e.g., 26\_43; when separate comparisons of the decade and unit digits do not result in the same decision) number comparisons as opposed to compatible (e.g., 25\_37) number comparisons. This effect was linked to right-hemispheric parietal processing in neuroimaging studies (Nuerk et al., 2011, for a review; Wood et al., 2006).

In a recent study (Jung et al., 2020), we replicated the hemispheric specialization of the unit-decade compatibility effect. We also replicated the multiplicativity effect in a number bisection task. In this task, participants evaluate whether the central number of a triplet (e.g., 14 in 12\_14\_16) corresponds to the arithmetic integer mean, defined by the two outer numbers. Whenever a triplet is part of a multiplication table (16\_24\_32, which is  $2 \times 8$   $3 \times 8$   $4 \times 8$ ), a processing advantage occurs in contrast to non-multiplicative triplets

(17\_21\_23, cf. Nuerk et al., 2002). However, in our study, lateralized stimuli presentation did not modulate the effect of multiplicativity. Therefore, we could not conclude that arithmetic fact retrieval is verbally mediated via left-hemispheric neural correlates (Wood et al., 2008). We assumed that both the considerable task difficulty and increased verbal working memory load prevented us from detecting left-hemispheric processing advantages (i.e., left-hemispheric specialization) for multiplicative triplets. Accordingly, we suggested investigating the hemispheric specialization of the multiplicativity effect in tasks that directly address the verbal code, such as arithmetic fact retrieval in one-digit by one-digit multiplication.

### **1.3 The present study**

In the present pre-registered study (AsPredicted.org on December 13th, 2019), we investigated whether arithmetic facts are processed lateralized in the left hemisphere of the human brain by applying the divided visual hemifield paradigm approach in two experiments, namely a *number comparison task* and a *multiplication task*). Additionally, a *control task* (multiplication) without the visual hemifield paradigm was conducted. In the first experiment, we replicated the lateralization of the unit-decade compatibility effect, as reported by Ratinckx and colleagues (Ratinckx et al., 2006), using half of their stimulus set in a *number comparison task*. The replication served as a proof of concept to verify that the divided hemifield paradigm was valid for detecting lateralization differences in processing. In the second experiment, we assessed the hemispheric specialization of arithmetic fact retrieval in a *number comparison task* using a similar verification paradigm as in our earlier study (Jung et al., 2020).

To compare the two experimental tasks in the present study directly and with our earlier findings, we presented a maximum of two numbers in the left or right visual hemifield (i.e., left or right of the fixation cross). Therefore, we presented the target first and then the two operands in the *multiplication task*. This presentation also entailed the following experimental

advantage: We expected that the target might prime the retrieval of its potential divisors. For instance, the target ‘24’ has eight divisors (i.e., 1, 2, 3, 4, 6, 8, 12, 24), whereas the target 63 has six divisors (i.e., 1, 3, 7, 21, 9, 63). Tie problems have only three divisors (i.e., 1, operand, result), and a prime number has two divisors (i.e., result and 1). Such early bottom-up priming effects have been considered necessary for demonstrating lateralized processing in a divided visual hemifield paradigm (Jung et al., 2020).

Inspired by more recent evidence suggesting specific involvement of each hemisphere in arithmetic fact retrieval (Salillas et al., 2021; Semenza & Benavides-Varela, 2018), we had the following expectations in addition to our pre-registration: First, we expected the left hemisphere to be involved, particularly in processing automatically activated table-related errors and correct responses. The right hemisphere, in contrast, was assumed to be involved in the processing of close-miss errors in our behavioral paradigm. Therefore, we included both error types as variables in our analysis. Second, building on this assumption, we assumed an sophisticated effect of left-hemispheric specialization on the target’s divisors, which we expected to prime multiplication tables.

## **2. Methods**

### **2.1 Transparency and Openness**

The study outline, data collection, and analysis plans were pre-registered (<https://aspredicted.org/blind.php?x=yc248x>). We report how we determined our sample size, all data exclusions, manipulations, and measures in the study. All data, the analysis code, and research materials have been made publicly available at the open science framework (OSF) at <https://osf.io/a4m7y>.

The study followed the latest version of the Code of Ethics of the World Medical Association (Declaration of Helsinki, 2014). The local Ethics Committee (082/2018BO2)

approved the study methods and procedures. All participants gave their informed written consent before the study and received a compensation payment.

## 2.2 Participants

As announced in the pre-registration, we recruited  $N = 36$  healthy right-handed volunteers (16 males, mean age = 24.75 years;  $SD = 3.72$ , 35 with more than ten years of formal education) via public announcements. However, we had to exclude one participant (P25, see the open data batch) due to an error in the individual blocks' randomization in the *multiplication task*. Thus, complete data sets of  $n = 35$  participants entered the analyses (16 males, mean age: 24.80,  $SD = 3.76$ ). All participants had normal or corrected to normal vision and reported no history of neurological or psychiatric disorders. According to the Edinburgh Handedness Inventory (Oldfield, 1971), right-handedness was determined for 34 participants by applying a cut-off criterion of Lateralization Quotient  $> +50$  (i.e., first decile or higher). The mean Lateralization Quotient was 77.36 ( $SD = 34.22$ ). One participant had ten years of formal education; 34 participants had more than ten years. We also assessed eye dominance for distance vision (right:  $n = 23$ , left:  $n = 10$ , and not defined:  $n = 2$ ) to control individual differences.

For the power analysis, we used findings from previous studies to anticipate the effect size for laterality in numerical tasks (e.g., Ratinckx et al., 2006; Jung et al., 2020). This analysis builds on data reported by Jung et al. (2020), who investigated the effect of lateralization in a simple *number comparison task* and a much more sophisticated number bisection task, using a combined performance measure of RT and accuracy, namely the Rate Correct Score (RCS; Woltz & Was, 2006). We applied the same measure of performance. We assumed the effect of lateralization, if observed, to be comparably large in number comparison

and simple multiplication. Accordingly, we suspected the intra-individual and inter-individual standard deviations in both tasks to be comparable.

We then re-analyzed Jung et al.'s (2020) data using a mixed model approach to understand fixed effect sizes and standard deviations (i.e., random effects). Model results showed relatively low variance in RCS (i.e., intra-individual  $SD = .20$  and inter-individual  $SD = .165$ ) in Jung et al.'s (2020) participants. Therefore, we doubled the individual variance to follow a more conservative approach. Simultaneously, the estimated beta values for the effect of lateralization turned out to be huge (i.e., right:  $\beta = 0.29$  and left:  $\beta = 0.07$ ), as was the interaction between lateralization and compatibility (i.e.,  $\beta = 0.11$ ). We divided these values by two, again following a conservative approach in our subsequent power analysis, which employed a repeated measure within-participant design and, thus, only one experimental group.

To simplify the power analysis, we only included the factors lateralization (i.e., left, right, bilateral presentation) and error type (i.e., true problem, close-miss error, table error). According to the adjusted parameter of the re-analysis, we estimated our model with a variance between subjects of  $SD = 0.40$  and residual variance of  $SD = 0.33$ . For effect sizes, we expected the following  $\beta$  parameters (in the form of dummy variables, bilateral and true problems being zero): right presentation ( $\beta = 0.14$ ), left presentation ( $\beta = 0.035$ ) as well as close miss error ( $\beta = 0.08$ ) and table error ( $\beta = 0.08$ ): We also anticipated interaction effects: right  $\times$  close miss and left  $\times$  table error at  $\beta = 0.055$  each, as well as left  $\times$  close miss and right  $\times$  table error at  $\beta = -0.055$ .

Using these values, we generated a data set of 36 participants and simulated the respective data. We then compared a mixed model including the factor lateralization with a model which disregards lateralization. This comparison was iterated 10.000 times. From these values, we would achieve an effect size of  $d > 0.92$  with an alpha error probability of  $p = .05$

and a within-subject correlation of  $r = .80$  in RCS. Accordingly, we are confident that we have enough power to detect lateralization effects in our study design.

### **2.3 Procedure**

Data collection took place in individual 90-minutes testing sessions. Within one session, we conducted two experimental tasks examining verification in a *multiplication task* (e.g.,  $12 = 3 \times 4$ ) and in a *number comparison task* (i.e., replication of the study by Ratinckx et al. [2006], e.g.,  $27 < 42$ ) in the divided visual hemifield paradigm. The order of both tasks was counterbalanced across participants to minimize order effects. A *control task* assessing multiplication fact retrieval in the standard form (e.g.,  $3 \times 4 = 12$ ) followed the experimental tasks. Task instructions emphasized both speed and accuracy in all tasks. We recorded stimulus presentation, response times, and accuracy using Presentation software version 20.03 (Neurobehavioral Systems Inc., Albany, CA, USA).

### **2.4 Experimental design**

All tasks were presented on a 17" screen at a resolution of  $1920 \times 1200$  pixels and a 60Hz refreshing rate. Participants were seated at a viewing distance of 60cm from the screen. We ensured the consistency of the viewing distance by using a head and chin rest. The experimenter was sitting across from the participant to control eye fixation. To this end, we presented an acoustic signal, which only the experimenter could hear via in-ear headphones to indicate the beginning of each trial. In case a participant lost fixation, the experimenter reminded the participant to fixate the center of the screen.

### **2.5. The divided visual hemifield paradigm**

In the divided visual hemifield paradigm (i.e., for the *number comparison task* and the *multiplication task* only), items were presented tachistoscopically in four different visual hemifield conditions (i.e., two unilateral conditions for the left/right visual hemifield and two

bilateral conditions) within three subsequent blocks. In the bilateral condition, we split the item set in half (i.e., bilateral A: numerical items appeared in the upper left and lower right corner, bilateral B: numerical items appeared in the lower left and upper right corner; Fig. 1A). Both visual hemifield paradigm tasks started with the presentation of a fixation cross (i.e., plus [+] sign, font: Arial, size: 30) at the center of the screen, which extended to a visual angle of  $0.5^\circ$  of horizontally and vertically (cf. Fig. 1).

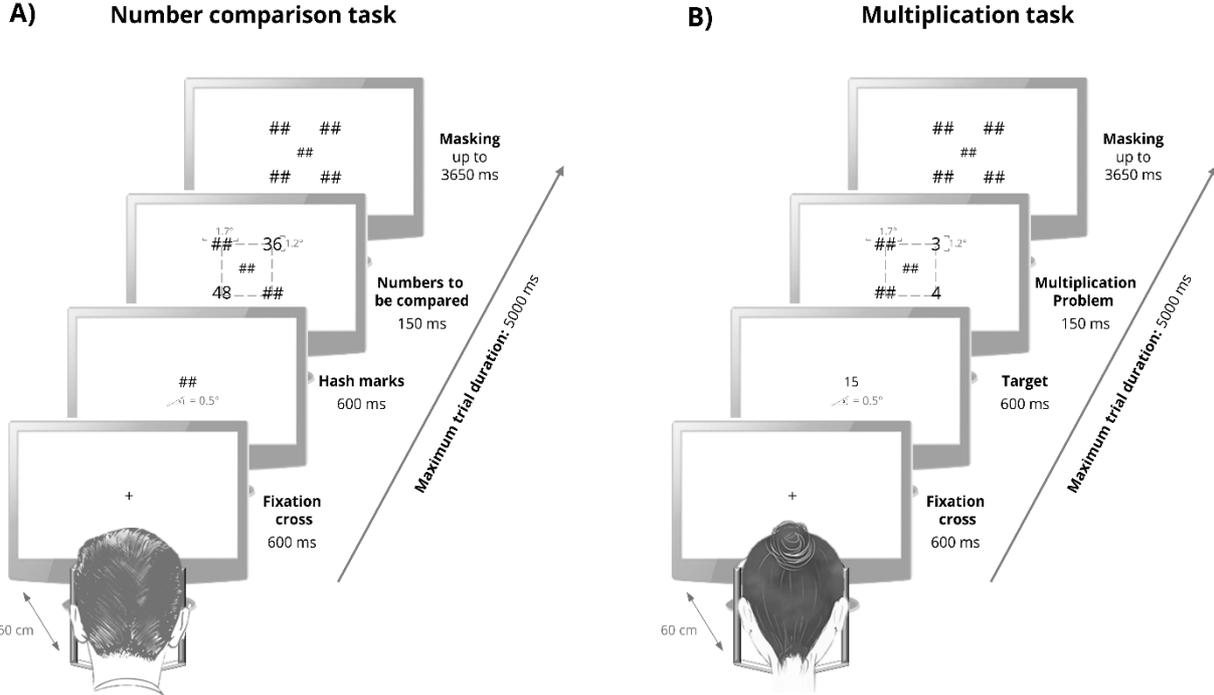


Fig.1: Experimental setup and example of the multiplication task: A) *Number comparison task* showing a bilaterally presented number pair; B) *Multiplication task* showing unilateral item presentation to the right visual hemifield

An imaginary square (i.e.,  $5^\circ$ - $5^\circ$ ) surrounded the fixation stimulus. The diagonal projection between the fixation cross and the corner of the imagery square measured an angle of  $45^\circ$ . We provided Arabic numbers ranging from 1 to 99 (extending to  $1.2^\circ$  vertically and  $1.7^\circ$  horizontally) at the imaginary square corners. Each numerical stimulus filled one corner of the imaginary square in all conditions (i.e., unilateral and bilateral conditions). We

presented hash marks ("###") to keep the perceptual load constant across conditions.

Stimulus presentation was very similar in both experimental tasks. First, the fixation cross appeared for 600ms accompanied by an acoustic signal, audible only to the examiner through the headphones. This acoustic signal was an indication for the examiner that a new trial had started. The examiner could not observe the screen but monitor the participants' eye movements by sitting opposite (in front of) the participant. In the *number comparison tasks* (Fig. 1A), hash marks ("###") were shown in the screen center for 600ms. Subsequently, the two-digit numbers to be compared were tachistoscopically presented for 150ms and again followed by "###". Finally, all positions were covered by "###" for a maximum of 3650ms or until a response key was pressed. In total, one trial lasted up to 5000ms.

In the *multiplication task* (Fig. 1 B), a target number appeared central at the fixation cross for another 600ms. Then "###" replaced the target, and the multiplication problem was tachistoscopically presented for 150ms at two corners of the imaginary square. Again, all positions were covered by "###" for a maximum of 3650ms or until a response key was pressed. Participants were instructed to fixate the screen's center throughout the entire trial and not move their eyes in both experimental tasks.

In the *control task*, we did not present the multiplication problems in the visual hemifield paradigm (cf. Clemens et al., 2013). Multiplication problems (e.g.,  $7 \times 8 = 52$ ) appeared at the center of the screen for a maximum presentation time of 3000ms. Individual presentation time for each trial was variable (i.e., the multiplication problem disappeared from the screen immediately after the participant responded). To avoid afterimages, we presented a mask ("#####") for 500ms after each multiplication problem.

## 2.6 Task and stimuli

The experimental and the *control task* were verification tasks. Participants responded using a DirectIn Button Box (Empirisoft, New York, NY). In the *multiplication and control task*, the response box was placed horizontally in front of the participants. The participants were required to give "Yes" or "No" responses by pressing either the outer left (i.e., "No" response) or the outer right response button (i.e., "Yes" response) with their left and right index finger, respectively. In the *number comparison task*, we rotated the response box to lay vertically in front of the participants. Participants were required to press the upper button when the above number was larger and press the lower button when the above number was smaller with the right and left index fingers.

In all tasks, problems were presented in pseudo-randomized order to prevent a direct repetition of the same problem. We manipulated the stimulus sequence such that no more than three correct or false trials, respectively, were presented in a row. Additionally, we allowed items to be repeated only after ten more trials.

### 2.6.1 Number comparison task

For the *number comparison task*, we adopted the stimulus set used by Ratinckx et al. (2006), which contained 480 two-digit number pairs between 20 and 99. In the original set, the authors matched the stimuli according to problem size, overall distance, unit distance (small: 29\_47 vs large 62\_79), and decade distance (small: 21\_39 vs large 21\_83) as well as unit-decade compatibility (compatible vs incompatible).

From these stimuli, we created two disjoint stimulus sets (e.g., sets A and B) with 240 number pairs to shorten the experimental time by maintaining this stimulus matching. Each stimulus set was presented right, left, and bilateral in the visual hemifield paradigm (e.g.,

set\_A\_right, set A\_left, set A\_bilateral). We further defined three different pseudo-randomised orders (i.e., stimulus arrays) for each lateralization condition (e.g., for presentation to the right visual hemifield: set\_A\_right\_1, set\_A\_right\_2, and set\_A\_right\_3). This way, each participant was assigned one of the two stimulus sets, with the stimulus arrays being always presented in a different order (e.g., set\_A\_right\_1, set\_A\_left\_2, set\_A\_bilateral\_3). We presented three different stimulus arrays with 240 items, resulting in a total of 720 trials. Before the experimental trials, eight randomly chosen number pairs had to be completed as practice trials.

Participants had to decide whether the upper number was larger than the lower number by pressing either the upper response button (i.e., "Yes" response: upper number larger) or the lower response button (i.e., "No" response: lower number larger). The experiment was paused after 50 trials to allow participants to take a self-defined break.

### **2.6.2. Verification of arithmetic facts**

Verification of arithmetic facts was investigated in both a divided hemifield paradigm varying the lateralization of stimulus presentation (i.e., *multiplication task*) and a standard *control task*.

In the *multiplication task*, we considered all possible multiplication problems (e.g.,  $32 = 4 \times 8$ ) between 1 and 99 if the product was smaller than 100 to create simple multiplication stimuli. In particular, we neither included problems with multiples of five (except for the prime number 5) nor ten. We also excluded multiplication problems requiring carry operations, which occurred when one factor was larger than 10 (e.g.,  $4 \times \underline{16}$ ;  $4 < 10$  but  $\underline{16} < 10$ ) and when the product of the units in the same multiplication problem was also larger than ten ( $4 \times 6 = 24$ ,  $24 > 10$ ). We reduced the number of rule problems (i.e., multiples of 1) to nine stimuli. For each multiplication problem, we categorized three different types of targets within

one item set: First, the target was the correct product of both factors (e.g.,  $12 = 4 \times 3$ ). Second, the target included an operand error (i.e., any multiple operands, such as  $15 = 4 \times 3$ ) within the same decade of the correct multiplication problem, if possible. Third, the target included a close-miss error (see also Campbell & Graham, 1985; McCloskey et al., 1991; Siegler, 1988), in which we added or subtracted 2 from the correct result (e.g.,  $14 = 4 \times 3$ ). Only in problems with multiples of 2, we added or subtracted 4 from the correct results to avoid a confound with operand errors. We randomly added to or subtracted 10 from the correct product if a problem required a decade transition in the target for operand errors or a prime number in the close-miss errors. This procedure was applied in seven trials. We also included 7 prime numbers (i.e., 2, 5, 7, 11, 47, 59, 89). Subsequently, we created three pseudo-randomized stimulus sets of all possible multiplication problems, one for each condition (i.e., right, left, and bilateral). In each item set, we presented the multiplication problem with the correct target twice (i.e., 134 items) as well as once with an operand error (i.e., 67 items) and once with a close-miss error (i.e., 67 items). In total, 264 simple multiplication problems were presented, preceded by eight practice trials.

Participants had to decide whether the central target, presented first, was the correct result of the two other operand numbers. They indicated their decision by pressing either the left (i.e., "No" response) or the right response button (i.e., "Yes" response) of the DirectIn Button Box. The experiment allowed participants to take a self-defined break after 50 trials each. As we presented the target first, we controlled the number of divisors of the result (e.g., a prime number can always be divided by 1 and by itself, a product of 6 can be divided by 1, 2, 3, and 6). We considered the number of potential divisors to influence task performance. In particular, we assumed that participants might conclude from the presented target (i.e., a product of the simple multiplication problem) to the potential operands of a simple

multiplication problem. The fewer divisors exist for a multiplication product; the faster participants might differentiate between correct and incorrect results.

In the *control task*, we examined the verification of 180 single-digit multiplication problems (i.e., 90 with a correct and 90 with an incorrect target covering an operand range from 0 to 10), using the same experimental setup used by Jung et al. (2020). The multiplication problems were given in the Arabic number format (e.g.,  $7 \times 5 = 35$ ) and included 96 standard problems, 72 rule problems (with 24 problems using 0, 1, and 10 as multiplicand, respectively), and 12 tie problems. Additionally, as practice trials, we used 30 different multiplication problems (15 with a correct and 15 with an incorrect target). An overview of the *control tasks'* stimulus set is provided in Jung et al.'s (2020) study.

## **2.7. Data Analysis**

Statistical analysis was conducted in R (R Core Team, 2020) using the packages 'lme4' to compute the mixed-effect models (Bates et al., 2014), subsequently 'lmerTest' (Kuznetsova et al., 2017) to test effects, and 'BayesFactor' (Morey, 2019) to evaluate null effects (Masson, 2011). We excluded trials with an RT of less than 125ms from data analysis and removed outliers from the data (i.e., RTs faster or slower than three standard deviations of the mean). Incorrect or missing responses were not considered. Mixed-effects models were calculated on the rate correct score (RCS) as introduced by Woltz and Was (2006) and further recommended by Vandierendonck (2017). The RSC reflects a combined reaction time (RT) and accuracy measure of performance. However, before specifying the RCS as the variable of interest in our mixed-effects models, we conducted correlation analyses to investigate whether our data justified the use of the RCS as a combined measure of performance.

### 2.7.1. *Number comparison task*

The *number comparison task* served as a proof of concept to verify that the divided hemifield paradigm was valid for detecting lateralization differences in processing, as previously shown by Ratinckx (2006) and Jung et al. (2020). Therefore, we replicated the identical analyses of variance (ANOVAs) on arcsine transformed accuracy rates and RT (cf. Ratinckx et al., 2006) and the RCS (cf. Jung et al., 2020). Please note that we used the arcsine-transformation only because Ratinckx et al. (2006) applied this method, and we aimed to exclude any effects due to the type of analysis.

In the subsequent mixed-effect models, we used two different variables in the mixed-effect models to investigate the influence of hemispheric specialization on the unit-decade compatibility effect in the *number comparison task*. First, we defined the variable unit-decade compatibility categorically analogous to the previous papers (Ratinckx et al., 2006; Jung et al., 2020), namely, as a binary variable (compatible vs. incompatible). Second, we used a parametric approach on the behavioral data similar to the one applied by Wood et al. (2006) on neuroimaging data. Thereby, decade distance (e.g., 3\_ vs 5\_:  $|3-5|*10 = 20$ ) and enhanced compatibility-based unit distance (e.g., \_6 vs \_1:  $|6-1| = 5$ ) processing indicated the presence of the unit-decade compatibility effect.

We further included the variable ‘block factor’ (i.e., the number of blocks that have already been completed) to consider training effects across blocks. This block factor was built in as dummy variables (i.e., block 1, block 2, block 3), which means that we fitted the model first for block 1. Then block two and block three were adjusted until the smallest square sum was obtained. By this approach, we could observe differences between the blocks, but not learning effects themselves.

In essence, we included the variables block factor, compatibility, decade-distance, unit-distance, and laterality in the mixed-effect model. We further added the interaction between compatibility and unit-distance, the interaction between compatibility and decade-distance, and the interactions between compatibility and the type of lateralization to the mixed model.

### ***2.7.2. Verification of arithmetic facts***

In the data analyses of arithmetic fact retrieval, we included a factor referring to the number of the target's divisors in the mixed models for *multiplication and control tasks*. For example, target 24 has eight divisors: 1, 2, 3, 4, 6, 8, 12, 24. A prime number instead has only two divisors (itself and 1). The number of divisors was of particular interest as the target presentation varied in the *multiplication and control tasks*. The target (i.e., o) was presented first in the *multiplication task*, and then the two factors (i.e.,  $m \times n$ ) were tachistoscopically presented. The more possible divisors a target has, the more multiplication tables are possible, making the task more difficult. For example, if the target was a prime number, it could directly prepare the participant for the correct answer. However, if the target had many divisors, it was uncertain which operands would occur. In the *control task*, we simultaneously presented the two operands of a multiplication problem with the target ( $m \times n = o$ ). Thus, the target's divisors should not have primed a specific multiplication table in the *control task*.

Accordingly, we included the following variables in the mixed effect model for analyzing the *multiplication task*: problem size and the number of the target's divisors, block factor to account for potential sequencing effects, and type of lateralization (i.e., right, left, and bilateral presentation). In addition, we added operand errors and close-miss errors in terms of qualitative analysis. Finally, we also included the pairwise interactions between the type of lateralization and operand errors, close-miss errors, and the number of the target's divisors. Since the estimated parameters in the mixed models turned out to be relatively small, we multiplied the regression parameter by 100 to simplify the presentation of the results.

### 3. Results

**Table 1.** Descriptive statistics of the *multiplication task* and the *number comparison task*.

		Left	right	bilateral
<b><i>Multiplication tasks</i></b>				
Overall	RT (SD)	984 (250)	1026 (245)	979 (244)
	Accuracy (SD)	0.87 (0.06)	0.87 (0.05)	0.86 (0.06)
	RCS (SD)	0.94 (0.24)	0.90 (0.23)	0.94 (0.22)
True trial	RT (SD)	944 (236)	979 (218)	935 (220)
	Accuracy (SD)	0.87 (0.08)	0.86 (0.08)	0.87 (0.09)
	RCS (SD)	0.97 (0.24)	0.94 (0.25)	0.97 (0.24)
Operand error	RT (SD)	1017 (258)	1058 (275)	1008 (270)
	Accuracy (SD)	0.86 (0.10)	0.87 (0.10)	0.84 (0.11)
	RCS (SD)	0.89 (0.24)	0.89(0.24)	0.85 (0.22)
Close-miss error	RT (SD)	992 (258)	1039 (242)	993 (241)
	Accuracy (SD)	0.88 (0.08)	0.90 (0.07)	0.87 (0.09)
	RCS (SD)	0.95 (0.26)	0.92 (0.25)	0.93 (0.24)
<b><i>Number comparison Task</i></b>				
Overall	RT (SD)	586 (86)	599 (87)	578 (91)
	Accuracy (SD)	0.93 (0.04)	0.92 (0.04)	0.93 (0.03)
	RCS (SD)	1.61 (0.23)	1.56 (0.25)	1.65 (0.26)
Small DD, small UD	RT (SD)	615 (89)	631 (93)	597 (90)
	Accuracy (SD)	0.88 (0.06)	0.87 (0.07)	0.88 (0.07)
	RCS (SD)	1.45 (0.21)	1.41 (0.25)	1.51 (0.26)
Small DD, large UD	RT (SD)	613 (96)	626 (88)	608 (100)
	Accuracy (SD)	0.88 (0.06)	0.88 (0.07)	0.89 (0.07)
	RCS (SD)	1.47 (0.23)	1.43 (0.27)	1.51 (0.27)
Large DD, small UD	RT (SD)	557 (86)	570 (88)	548 (86)
	Accuracy (SD)	0.98 (0.02)	0.97 (0.04)	0.98 (0.02)
	RCS (SD)	1.80 (0.27)	1.73 (0.27)	1.82 (0.27)
Large DD, large UD	RT (SD)	559 (80)	571 (88)	558 (92)
	Accuracy (SD)	0.96 (0.03)	0.95 (0.04)	0.97 (0.03)
	RCS (SD)	1.75 (0.24)	1.70 (0.27)	1.79 (0.26)
Decade-unit-compatible	RT (SD)	575 (87)	587 (89)	570 (90)
	Accuracy (SD)	0.95 (0.03)	0.94 (0.04)	0.94 (0.03)
	RCS (SD)	1.65 (0.25)	1.64 (0.26)	1.69 (0.26)
Decade-unit-incompatible	RT (SD)	597 (87)	612 (87)	585 (93)
	Accuracy (SD)	0.90 (0.05)	0.89 (0.06)	0.92 (0.05)
	RCS (SD)	1.54 (0.22)	1.50 (0.26)	1.62 (0.26)

*Note.* Close-miss errors: target  $\pm 2$  off the correct result; DD: decade distance; UD: unit distance (absolute difference of digits); distances of 0-3 are considered small and 4-8 large. Means and SDs were calculated across participants' averages in each condition; RTs, accuracies, and the RCS were rounded.

Before the main analyses, we conducted an initial correlation analysis to justify using the RCS as a suitable combined measure of performance in the subsequent analyses. Results revealed significant correlations between RT and accuracy in the *multiplication task*  $r(2790) = -.35, p < .001$  and the *number comparison task*  $r(4326) = -.20, p < .001$ , indicating the trade-off between increased correct response behaviour and slower reaction times. However, the correlation of both measurements in the *control task* was not significant  $r(186) = -0.62, p = .40$ . Although the latter analysis might not justify the RCS usage in this particular task, separate analyses, including RT and accuracy only, led to similar results. Therefore, we used the RCS for the subsequent analyses to control for the speed-accuracy trade-off.

Table 1 summarizes the descriptive statistics (i.e., percentage correct, RT, and the RCS) of the *multiplication* and *number comparison tasks*.

**Table 2.** Descriptive statistics of the *control task*

Effect		Yes	no
Correct problem	RT ( <i>SD</i> )	1003 (259)	1118 (285)
	Accuracy ( <i>SD</i> )	.94 (.09)	.92 (.11)
	RCS ( <i>SD</i> )	1.0 (0.27)	0.88 (0.25)
Tie problem	RT ( <i>SD</i> )	1051 (279)	1061 (278)
	Accuracy ( <i>SD</i> )	.91 (.14)	.93 (.09)
	RCS ( <i>SD</i> )	0.92 (0.27)	0.94 (0.27)
Rule problem	RT ( <i>SD</i> )	962 (183)	1099 (300)
	Accuracy ( <i>SD</i> )	.91 (.11)	.94 (.10)
	RCS ( <i>SD</i> )	0.98 (0.20)	0.92 (0.29)
multiplication by 10	RT ( <i>SD</i> )	959 (237)	1101 (283)
	Accuracy ( <i>SD</i> )	.99 (.05)	.91 (.11)
	RCS ( <i>SD</i> )	1.09 (0.26)	0.88 (0.25)
Larger factor first	RT ( <i>SD</i> )	1066 (283)	1056 (275)
	Accuracy ( <i>SD</i> )	.94 (.09)	.93 (.11)
	RCS ( <i>SD</i> )	0.95 (0.28)	0.94 (0.26)
Normal Problem	RT ( <i>SD</i> )	1262 (288)	980 (229)
	Accuracy ( <i>SD</i> )	.90 (.08)	.94 (.11)
	RCS ( <i>SD</i> )	0.76 (0.25)	1.01 (0.22)

*Note.* Values have been rounded. The table presents mean RTs, accuracy, and standard deviations (*SD*) for the individual factors we included in the mixed model analysis.

Table 2 provides the descriptive statistics of the *control task*. Both tables also present the individual factors which we included in the subsequent data analysis of each task.

### 3.1 Number Comparison Task

Descriptively, results indicated better performance for mean RTs [present study: right: 599ms, left: 586ms, bilateral: 578ms; Ratinckx et al. (2006): right: 658 left: 656, bilateral: 637; Jung et al. (2020): right: 914ms, left: 916ms, bilateral: 932] and accuracy rates [present study: right: 92%, left: 93%, bilateral: 93%; Ratinckx et al. (2006): right: 87%, left: 88%, bilateral: 90%; Jung et al. (2020): right: 82%, left: 83%, bilateral: 85%] in the present study.

#### 3.1.1 Replication of previous findings

First, we ran the Ratinckx et al. (2006)  $2 \times 2 \times 2 \times 3$  ANOVA on RT and arcsine transformed accuracy rates, with the categorical within-subject factors of unit-decade compatibility (compatible, incompatible), decade digit distance (large, small), unit digit distance (large, small), and laterality (right, left, bilateral). We observed a significant main effect of lateralization in RTs,  $F(2, 34) = 15.94, p < .001, \eta^2 p = 0.05$  and in arcsine-transformed accuracies,  $F(2, 34) = 5.70, p = .03, \eta^2 p = 0.06$ . In the bilateral condition, participants responded faster (right:  $M = 581.16, SD = 167.75$ ; left:  $M = 575.10\text{ms}, SD = 151.23$ ; bilateral:  $M = 567.77\text{ ms}, SD = 149.61$ ) and more correctly (right:  $M = 1.83, SD = 0.42$ ; left:  $M = 1.84, SD = 0.42$ ; bilateral:  $M = 1.87, SD = 0.43$ ) to compatible trials as compared to incompatible trials' response times (right:  $M = 606.94, SD = 162.70$ ; left:  $M = 601.87\text{ms}, SD = 151.55$ ; bilateral:  $M = 587.03\text{ ms}, SD = 159.95$ ) and accuracy rates (right:  $M = 1.75, SD = 0.41$ ; left:  $M = 1.76, SD = 0.42$ ; bilateral:  $M = 1.81, SD = 0.44$ ).

We also found a main effect for the unit-decade compatibility effect in RTs  $F(1, 34) = 74.14, p < .001, \eta^2 p = 0.69$  and arcsine-transformed accuracies  $F(2, 34) = 72.88, p < .001, \eta^2 p =$

= 0.68, as previously reported by Ratinckx et al. (2006). Participants responded faster and more correctly to unit-decade compatible trials as compared to unit-decade incompatible trials. Laterality and unit-decade compatibility interacted in arcsine-transformed accuracies,  $F(2, 34) = 5.65, p = .003, n^2p = 0.02$ , but not in RTs  $F(2, 34) = 1.51, p = .222, n^2p = 0.005$ . For the RT, we evaluated the null effect. We conducted a Bayesian analysis to estimate the Bayes factor which we interpreted in line with Jeffreys interpretation scale (1998). Bayesian analysis indicated anecdotal evidence in favor of an effect of lateralization ( $BF_{10} = 1.71$ ), which points in the same direction as the accuracy results.

Second, we used a 2×3 ANOVA with compatibility (compatible vs incompatible pairs) and lateralization (right vs left vs bilateral) on the RCS to compare our data with the results reported by Jung et al. (2020). We observed a significant main effect for unit-decade compatibility,  $F(1, 204) = 5.03, p = .02, n^2p = 0.03$ , indicating more correctly solved items per second for compatible trials ( $M = 1.85, SD = 0.42$ ) as compared to incompatible trials ( $M = 1.77, SD = 0.43$ ). However, we neither found a significant main effect for lateralization  $F(2, 204) = 0.76, p = .47, n.s., n^2p = 0.01$ , nor a significant interaction between the unit-decade compatibility and laterality  $F(2, 204) = 1.02, p = .90, n^2p = 0.00$ . Bayesian analysis revealed moderate to strong evidence against an effect of lateralization (and thus in favor of  $H_0$ ) when comparing a mixed model including compatibility only against a model containing compatibility and laterality (Bayes factor  $BF_{10} = 0.10$ ). Compared with a model that also contains an interaction parameter between compatibility and laterality, the odds were even smaller ( $BF_{10} = 0.01$ ).

### ***3.1.2 Hemispheric specialization of the unit-decade compatibility effect***

We defined the following variables for the subsequent mixed model analysis: unit distance and decade distance as continuous variables, compatibility (i.e., is the presented number pair compatible, yes or no) as a dichotomous dummy variable, and the block order,

and lateralization. We also considered pairwise interactions between compatibility and unit-distance, compatibility and decade-distance, and interactions between compatibility and the type of lateralization. Table 3 provides model estimates, confidence intervals, and  $p$ -values for the *number comparison task*.

**Table 3:** Mixed model for the *number comparison task* predicting RCS for items

Independent Variable	$\beta$ [ $CI_{95\%}$ ] * 100	$p$
<b>Participant <math>SD^a</math></b>	<b>23.77 [18.24, 29.30]</b>	---
<b>Residual <math>SD^a</math></b>	<b>34.66 [34.33, 35.08]</b>	---
<b>Intercept</b>	<b>161.40 [153.30, 169.59]</b>	<b>&lt;.001</b>
<b>Unit distance</b>	<b>1.47 [1.09, 1.84]</b>	<b>&lt;.001</b>
<b>Decade distance</b>	<b>4.98 [5.51, 5.46]</b>	<b>&lt;.001</b>
Compatibility <sup>a</sup>	3.38 [-0.28, 7.04]	.070
<b>Block 2<sup>a</sup></b>	<b>3.78 [2.48, 5.10]</b>	<b>&lt;.001</b>
<b>Block 3<sup>a</sup></b>	<b>7.61 [6.31, 8.92]</b>	<b>&lt;.001</b>
<b>Left presentation<sup>a</sup></b>	<b>-2.49 [-4.13, -0.07]</b>	<b>.007</b>
<b>Right presentation<sup>a</sup></b>	<b>-3.17 [-4.99, -1.35]</b>	<b>.001</b>
<b>Unit distance<math>\times</math>compatibility<sup>a</sup></b>	<b>-3.63 [-4.18, -3.08]</b>	<b>&lt;.001</b>
Decade distance $\times$ compatibility <sup>a</sup>	0.33 [-0.04, -1.03]	.339
<b>Laterality left<sup>a</sup><math>\times</math>compatibility<sup>a</sup></b>	<b>-3.06 [-5.66, -0.49]</b>	<b>.002</b>
<b>Laterality right<sup>a</sup><math>\times</math>compatibility<sup>a</sup></b>	<b>-3.69 [-6.31, -1.01]</b>	<b>.006</b>

*Note.* <sup>a</sup>dummy variables; confidence intervals were estimated via bootstraps;  $p$ -values are for  $t$ -statistics using Satterthwaite's method for denominator degrees of freedom. Compatibility refers to unit-decade compatibility.

The table shows that unit and decade distance significantly predicted a higher RCS, indicating a better task performance with increasing distance between the unit ( $\beta = 1.47$ ) and decade digits ( $\beta = 4.98$ ). In this context, decade distance is closely associated with the numerical distance effect in two-digit number processing (Wood et al., 2008), demonstrating a significant numerical distance effect. Crucially, the categorical variable compatibility ( $\beta = 3.38$ ) was not significant in this model. The significant interaction of unit distance and

compatibility ( $\beta = -3.63$ ) indicated that the unit distance might have only been relevant for task performance in incompatible number pairs. Moreover, block-order significantly predicted a higher RCS (block 2:  $\beta = 3.78$ ; block 3:  $\beta = 7.61$ ). The difference between blocks 2 and 3 is significant when the beta value of block 3 (i.e.,  $\beta = 7.61$ ) is outside the confidence interval of block 2 (i.e., CI = 2.48, 5.10), which was the case.

Regarding the effect of lateralization, presentation to the left and right visual hemifield significantly reduced task performance (left:  $\beta = -2.49$ ; right:  $\beta = -3.17$ ) compared to a bilateral presentation. Crucially, no significant difference was observed between the left and right visual hemifield presentation. Contrary to our ANOVA presented above [but in line with the results of Jung et al. (2020)], the interaction of lateralization and unit-decade compatibility was significant in our more nuanced mixed model.

### **3.2. Verification of arithmetic facts**

In the *control task* (which was not presented in a visual hemifield paradigm), we included the following variables in the mixed model to investigate participants' verification performance of arithmetic facts: first, whether or not the target represented the correct result (correct problem); second, whether or not both operands were identical (tie problem, like  $3 \times 3$  or  $4 \times 4$ ); third, whether or not an arithmetic rule had to be applied (rule problem, like  $1 \times 8$  or  $0 \times 8$ ), and fourth, whether or not one of the operands was 10 (multiplication by 10, like  $4 \times 10$ ), and whether or not the larger operand was given first.

Additionally, we included problem size and the number of the target's divisors (e.g., 24 has eight divisors: 1, 2, 3, 4, 6, 8, 12, 24) in the model. Table 4 provides model estimates, confidence intervals, and *p*-values for the *control task*. Again, regression parameters were multiplied by 100. As can be seen in the table, the presentation of the correct problem ( $\beta = 17.55$ ) significantly predicted a higher RCS. The same applied to tasks in which a tie problem

( $\beta = 17.91$ ) or a rule problem ( $\beta = 18.23$ ) occurred. The most considerable effect appeared for tasks in which one factor was multiplied by 10 ( $\beta = 43.28$ ).

**Table 4.** Mixed model results for the *control task* predicting RCS for items

Independent Variable	$\beta$ [ $CI_{95\%}$ ] * 100	$p$
<b>Participant SD<sup>b</sup></b>	<b>18.02 [14.20, 23.00]</b>	---
<b>Residual SD<sup>b</sup></b>	<b>32.77 [32.12, 33.39]</b>	---
<b>Intercept</b>	<b>88.36 [81.63, 95.09]</b>	< .001
<b>Correct problem<sup>b</sup></b>	<b>17.55 [15.74, 19.35]</b>	< .001
<b>Tie Problem<sup>b</sup></b>	<b>17.91 [13.88, 21.94]</b>	< .001
<b>Rule Problem<sup>b</sup></b>	<b>18.23 [15.53, 20.92]</b>	< .001
<b>Multiplication by 10<sup>b</sup></b>	<b>43.28 [40.48, 46.08]</b>	< .001
<b>Target (Problem size)</b>	<b>-0.42 [-0.49, -0.36]</b>	< .001
The target's divisors	-0.44 [-0.98, 2.19]	.106
Larger factor presented first <sup>b</sup>	-0.34 [-1.50, 2.19]	.717

*Note.* <sup>a</sup> random-effects; <sup>b</sup> dummy variables; confidence intervals were estimated via bootstraps;  $p$ -values are for  $t$ -statistics using Satterthwaite's method for denominator degrees of freedom.

However, the problem size (i.e., target magnitude) of the presented result ( $\beta = -0.42$ ) significantly predicted a lower RCS. Notably, the target's divisors and the order at which operands appeared did not significantly impact the RCS (e.g.,  $5 \times 8$  had the same RCS as  $8 \times 5$ ).

In the mixed model for the *multiplication task*, we included the following fixed effects: type of problem (i.e., true trials, operand errors, and close-miss errors), problem size, number of target's divisors, lateralization (i.e., bilateral, right- and left-lateralized presentation), block order as well as the interaction of lateralization and type of problem. Table 5 provides model estimates, confidence intervals, and  $p$ -values of the mixed model results for the *multiplication task*. The model estimates for lateralization were of particular interest in our study. Crucially, we neither observed a significant effect of lateralization nor significant interactions between model coefficients for the type of errors and lateralization and between the target's divisors

and lateralization. Contrary to our expectations, the visual hemifield presentation of simple arithmetic facts did not affect participants' performance. In the subsequent Bayesian analysis, we compared a model with problem type (i.e., true problems, close-miss error, operand error), laterality (i.e., left, right, bilateral presentation), and their interaction. There was strong evidence for a model that only contained the factor of problem type in comparison to a (more complex) model that contained the factors of problem type and laterality ( $BF_{10} = 0.06$ ). There was even less evidence for a model that also included an interaction ( $BF_{10} < 0.01$ ).

**Table 5.** Mixed model results for the *multiplication task* predicting RCS for items.

Independent Variable	$\beta$ [ $CI_{95\%}$ ] * 100	$p$
<b>Participant <math>SD^a</math></b>	<b>20.87 [15.97, 25.78]</b>	---
<b>Residual <math>SD^a</math></b>	<b>46.52 [45.95, 47.10]</b>	---
<b>Intercept</b>	<b>124.11 [116.18, 132.03]</b>	<b>&lt;.001</b>
Operand error <sup>b</sup>	-1.52 [-4.99, 1.95]	.480
<b>Close-miss error <sup>b</sup></b>	<b>-4.23 [-7.67, -0.79]</b>	<b>.002</b>
<b>Problem size</b>	<b>-0.26 [-0.30, -0.23]</b>	<b>&lt;.001</b>
<b>Target's divisors</b>	<b>-2.93 [-3.50, -2.36]</b>	<b>&lt;.001</b>
<b>Block 2 <sup>b</sup></b>	<b>13.91 [11.91, 15.91]</b>	<b>&lt;.001</b>
<b>Block 3 <sup>b</sup></b>	<b>24.12 [22.12, 26.13]</b>	<b>&lt;.001</b>
Left presentation <sup>b</sup>	-1.63 [-7.20, 3.95]	.692
Right presentation <sup>b</sup>	-1.12 [-4.45, 6.70]	.567
Operand error*left presentation	0.41 [-4.53, 5.35]	.871
Operand error*right presentation	-2.27 [-7.21, 2.68]	.369
Close miss error*left presentation	0.77 [-4.05, 5.59]	.753
Close miss error*right presentation	-0.27 [-5.09, 4.56]	.914
Target's divisors*left presentation	0.10 [-0.69, 0.87]	.810
Target's divisors *right presentation	-0.34 [-1.12, 0.44]	.388

*Note.* <sup>a</sup> random-effects; <sup>b</sup> dummy variables; confidence intervals were estimated via bootstraps;  $p$ -values are for  $t$ -statistics using Satterthwaite's method for denominator degrees of freedom.

With our planned and preregistered analyses presented above, we found an effect of lateralization only in the *number comparison task* and not in the *multiplication task*. This fact

raises the question of the validity of our experimental setup [consisting of input (i.e., hemifield stimulation) and output (i.e., motor response)]. We have already shown that our setup on the input side is valid for detecting hemispheric specialization. On the output side, there is a fixed assignment of the response hand in both paradigms. However, this assignment is coupled with a condition (correct multiplication) only in the *multiplication task*, which we can evaluate by the following alternative analysis.

### **3.3 Alternative analysis considering the number of hemispheric transitions**

In the *multiplication task*, the fixed assignment of multiplicativity with the response hand was as follows: Whenever the given target was the correct product of the operands (i.e., multiplication correct: yes), the answer had to be given with the right hand. Tasks with non-matching operands (i.e., multiplication correct: no) had to be answered with the left hand. Thus, multiplicative items were answered with the right hand, non-multiplicative items with the left hand. This response behavior was identical across all three presentation types (i.e., input side: left, right and bilateral). Depending on the combination of the input side, the hypothesized processing side (i.e., primarily left, right, and bilateral processing), and the output side (i.e., either the left or the right hand), there may be differences in the total hemispheric transitions required to solve the task that should be taken into account<sup>1</sup>.

In our experimental setup, the processing side was always known for two of the three processing steps: the input side and the output side. From both, we can infer the minimum number of inter-hemispheric transitions from perceiving the stimuli (i.e., input), processing the multiplication problem (hypothesized processing), and entering the response (i.e., output). The minimum number of hemispheric transitions allows us to perform an alternative analysis: We hypothesized an individual pattern of processing duration for left, right, and bilateral

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<sup>1</sup> We wish to thank an anonymous reviewer for pointing this out to us.

processing following item presentation in each input side. This is based on the assumption that more hemispheric transitions lead to a longer processing time. We then compared this hypothesized pattern of processing duration to the reaction time pattern of the actual data obtained. In the following, we explain our procedure at the example of the left-hemispheric processing of arithmetic facts (see Table 6).

**Table 6.** Scoring system of the hypothesized pattern of processing duration

Hypothesized processing of arithmetic facts						
	(I) LH		(II) RH		(III) Bilateral	
Right visual hemifield (Input LH)	(a) Mult. Output: LH <b>SCORE 0</b>	(b) Non-Mult. Output: RH <b>SCORE 1</b>	(a) Mult. Output: LH <b>SCORE 2</b>	(b) Non-Mult. Output: RH <b>SCORE 1</b>	(a) Mult. Output: LH <b>SCORE 0</b>	(b) Non-Mult. Output: RH <b>SCORE 1</b>
Left visual hemifield (Input RH)	(c) Mult. Output: LH <b>SCORE 1</b>	(d) Non-Mult. Output: RH <b>SCORE 2</b>	(c) Mult. Output: LH <b>SCORE 1</b>	(d) Non-Mult. Output: RH <b>SCORE 0</b>	(c) Mult. Output: LH <b>SCORE 1</b>	(d) Non-Mult. Output: RH <b>SCORE 0</b>
Bilateral to both visual hemifields	(e) Mult. Output: LH <b>SCORE 0</b>	(f) Non-Mult. Output: RH <b>SCORE 1</b>	(e) Mult. Output: LH <b>SCORE 1</b>	(f) Non-Mult. Output: RH <b>SCORE 0</b>	(e) Mult. Output: LH <b>SCORE 0</b>	(f) Non-Mult. Output: RH <b>SCORE 0</b>

*Note.* RH = processed by the right hemisphere; LH = processed by the left hemisphere; Mult. = multiplicative items, Non-Mult. = non-multiplicative items, Score = minimum number of inter-hemispheric transitions

The scoring was derived as follows: First, we assume that arithmetic facts are processed primarily in the left hemisphere (I).

- When presented in the right visual hemifield, the input was transported directly to the left hemisphere. Accordingly, no hemispheric crossing was necessary from the input side (i.e., right) to primarily left-hemispheric processing. (a) In problems with multiplicative items (i.e., a yes-answer), the response was given with the right hand,

i.e., the output was processed in the left hemisphere. This means, for conducting all three processes, no hemispheric crossing was necessary. We scored this “no-hemispheric crossing” condition with a score of 0. (b) In non-multiplicative items (i.e., a no-answer), participants answered with the left hand. As at least one transition to the right hemisphere was necessary to give a no-answer after processing the multiplication problem in the left hemisphere, we scored this condition with 1.

- When presented in the left visual hemifield, the input was transported directly to the right hemisphere. If we again assume that arithmetic facts are processed in the left hemisphere, a hemispheric transfer is necessary from input to processing. (c) In problems with multiplicative items, the answer was given with the right hand. The right-handed motor response is processed in the left hemisphere, which requires no further inter-hemispheric crossing (i.e., Score 1). (d) At least one further hemisphere transition into the right hemisphere was required in non-multiplicative items (i.e., left-hand response = right-hemispheric processing of the motor output; i.e., Score 2).
- Finally, during the bilateral presentation, the input was transmitted directly into both hemispheres. Thus, in the case of left-hemispheric processing of arithmetic facts, no hemisphere change would be necessary. (e) In problems with multiplicative items (i.e., right-handed response = left-hemispheric processing of the motor output), no additional inter-hemispheric crossing was necessary (i.e., Score 0). However, (f) in non-multiplicative items (i.e., left-hand response = right-hemispheric processing of the motor output), at least one transition to the right hemisphere would be required (i.e., Score 1).

Scores for right-hemispheric or bilateral processing were determined similarly (see Table 6). Please note, we do not claim that the number of inter-hemispheric transitions reflects the maximum possible absolute number of inter-hemispheric transitions.

We considered the relative differences between the conditions, which predicted an overall specific pattern for processing duration. Panel A of Figure 2 shows a schematic representation of these hypothetical patterns for the three possible processing types of arithmetic fact retrieval. Panel B depicts the actual reaction times as observed in these six conditions.

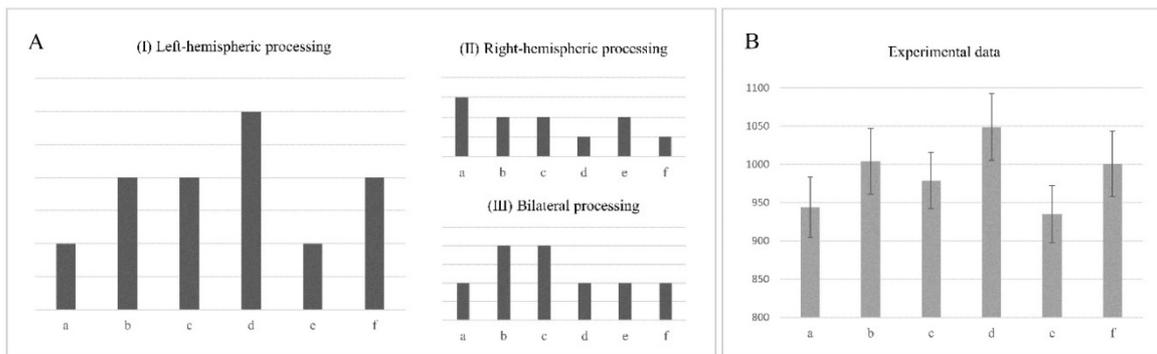


Fig. 2. Schematic overview of processing duration patterns for the three possible processing types underlying arithmetic fact retrieval (left-hemispheric, right-hemispheric, and bilateral processing). For better comparison with the experimental data, we added 1 to each Score. This way, the bars do not start at point 0 but at point 1. Panel B depicts the actual reaction times as observed in the respective conditions. Error bars depict standard errors of the mean.

The comparison between the hypothesized patterns (A) and the empirically assessed pattern of RT (B) suggests that arithmetic fact processing should most likely be situated in the left-hemispheric processing [A(I)]. This assumption becomes most evident in comparing condition a (= multiplication, input directly to the left-hemisphere) with condition d (= no multiplication, input directly to the right-hemisphere). Condition (a) should only be processed significantly faster than condition (d) if arithmetic fact retrieval is primarily processed left-hemispheric. We compared the differences in reaction times directly between both conditions using a subsequent *t*-test. Indeed, participants responded faster to multiplicative trials directly processed in the left hemisphere than to non-multiplicative trials directly processed in the right hemisphere as the one-tailed *t*-test revealed,  $t(34) = -2.94, p = .002$ . A two-sided *t*-test came down to a similar result ( $p = .005$ ).

Taken together, we initially did not consider the different inter-hemispheric transitions necessary from the input side to process the task and give a motor output. However, the

additional analysis considering all inter-hemispheric transitions suggested left-hemispheric processing advantages in the *multiplication task*, which we could not observe by our experimental setup and the previous analysis.

## 4 Discussion

The present study aimed to evaluate whether there is a unilateral left-hemispheric specialization for processing verbal arithmetic fact retrieval, which contrasts bi-hemispheric redundant processing for numerical magnitude as postulated in the triple code model (Dehaene and Cohen, 1995, 1997; Dehaene et al., 2003). For this purpose, we investigated arithmetic fact retrieval in a *multiplication task* and number magnitude processing in a *number comparison task* in a divided hemifield paradigm, building upon the studies by Jung et al. (2020) and Ratinckx et al. (2006).

In particular, we evaluated the lateralization of the unit-decade compatibility effect (Ratinckx et al. 2006) and the distance effect within the parametric analysis of unit-decade compatibility (cf. Wood et al., 2006) in the *number comparison task*. We further intended to show the differential involvement of both hemispheres in the processing of table-related errors and correct responses (i.e., left-hemispheric processing) and close-miss errors (i.e., right-hemispheric processing). Such hemispheric specialization has been observed by Salillas et al. (2021) in a brain stimulation study. In line with this observation, we expected lateralization effects on the number of the target's divisors only in the *multiplication task*, when the target was presented before the factors, but not in the *control task*.

Before discussing the results in detail, we will address a rather unexpected result: a systematic processing disadvantage in all experimental tasks when the stimuli were presented in the right visual hemifield (i.e., initially processed in the left hemisphere).

### 4.1. Systematic processing advantage of the presentation in the left visual hemifield

In all experimental tasks, we observed processing advantages of the presentation in the left visual hemifield (i.e., processing in the right hemisphere first) compared to the right visual hemifield (i.e., processing in the left hemisphere first) when comparing the unilateral conditions with each other. This observation is counterintuitive at first sight, as the left hemisphere undoubtedly has a crucial role in number processing (Semenza & Benavides-Varela, 2018, Vigneau et al., 2006). Recognizing Arabic digits has long been associated with processing in the visual number form area in the bilateral occipitotemporal cortex (Dehaene & Cohen, 1995; Hannagan et al., 2015; for a review, see Merkley et al., 2016).

However, and more recently, right-hemispheric processing has been proposed to be mandatory in (early) visual number processing (Grotheer et al., 2016). In particular, the visual number form area in the right inferior temporal gyrus has been identified to be specifically involved in number processing (e.g., Abboud et al., 2015, for congenitally blind participants; Hannagan et al., 2015). In line with these results, it seems plausible that left-lateralized presentation yielded processing advantages because early visual identification of the presented Arabic digits is directly processed in the right hemisphere. All experimental tasks require visual identification of the numerical stimuli. Consequently, we observed a systematic processing advantage of the presentation in the left visual hemifield, which suggested a relative processing advantage of the right hemisphere in processing numerical symbols (Abboud et al., 2015; Grotheer et al., 2016).

Alternatively, various explanations are possible to account for the right-sided visual hemifield disadvantage: First, reading direction in left-to-right readers, as in our German-speaking sample, was associated with left hemifield preference for faces (e.g., Vaid & Singh, 1989), objects (e.g., Låg et al., 2006), and words (e.g., Siéhoff & Slama, 2018). For example, Siéhoff & Slama (2018) showed that reading direction mainly influenced visual orientation to the left visual hemifield when reading words. This left hemifield preference leads to a relative

disadvantage of the right visual hemifield for word material, processed linguistically in the left hemisphere. In this context, Smigasiewicz and colleagues (2010) reported a tendency to shift attention back to the starting point of the reading direction in the processing of Latin letters. Such a shift of attention might have also occurred in the present study. We presented our stimuli separately block-wise in either the left or right hemifield. However, the four corners at the imaginary square were filled with visual stimuli (i.e., two hash marks ‘##’ and two Arabic digits). The participant might have assumed relevant stimuli to appear on the not relevant visual hemifield. Second, the right hemisphere is more active in mediating the shift of attention (Corbetta et al., 1993). In particular, the right hemisphere is supposed to shift attention to both visual hemifields. The left hemisphere directs attention only to the contralateral hemifield (M.-M. Mesulam, 1990; M. -Marchsel Mesulam, 1981). Accordingly, any shift of attention is associated with right-hemispheric processing and a left processing advantage. Third, representations of stimuli presented in the left visual hemifield in the brain were stronger than representations of the right hemifield (Siman-Tov et al., 2007). Consequently, numerical processing could be faster despite interhemispheric transfer.

In sum, there are several explanations for the systematic processing advantage of the presentation in the left hemifield. One explanation considers recent findings pointing to more right-lateralized processing of visually presented numerical stimuli in the visual number form area. Other explanations are attributed to more general cognitive processing in the divided hemifield paradigm.

#### **4.2 Lateralization in multi-digit magnitude comparison**

In the *number comparison task*, we replicated the modulation of the unit-decade compatibility effect by lateralizing the input presentation. Compared with Ratinckx et al. (2006), we focused on RT and accuracy rates running precisely the same ANOVA.

Descriptively, participants performed slightly better in the present study for both RTs and accuracy rates. We can therefore assume that the visual item presentation in all four locations (i.e., at each corner, either one of the two numbers or "##" as a mask) did not substantially increase task difficulty (cf. Jung et al., 2020; Poole & Kane, 2009). We replicated the unit-decade compatibility effect showing faster and more accurate unit-decade compatible responses than to unit-decade incompatible trials.

Furthermore, we observed processing advantages when the stimuli were presented bilaterally to both visual hemifields rather than unilaterally presented in the left or right visual hemifield. Bilateral processing advantages in two-digit number processing have been previously described (e.g., Dimond & Beaumont, 1971; Hatta et al., 2002) and would also be in line with our finding regarding the systematic processing advantages of the presentation in the left visual hemifield. While early visual numerical stimuli are processed in the right-hemispheric visual number form area (Abboud et al., 2015; Grotheer et al., 2016), the subsequent numerical comparison involves both hemispheres, reflected in processing advantages during a bilateral presentation. Bilateral processing is in line with the propositions of the triple code model, suggesting inter-hemispheric cooperation in interfering tasks (Ratinckx et al., 2006). Contrary to Ratinckx et al.'s (2006) study, we found a significant main effect of lateralization. Differences in statistical power might explain this observation: We included almost twice as many participants ( $N = 35$  vs.  $N = 16$ ). However, post-hoc analyses suggested a highly bilaterally organized processing for two-digit Arabic number comparison, i.e., the number magnitude representation (Dehaene et al., 2003).

For comparison with Jung et al. (2020), we used the RCS (i.e., a combined RT and accuracy measure) in an ANOVA over unit-decade compatibility and laterality. While our results confirmed the unit-decade compatibility effect, they did not provide evidence of lateralized processing or interaction between both effects. Differences in the experimental

design, such as a modified item presentation (i.e., randomized instead of block-wise) and response behavior (i.e., response keys arranged left and right vs. up and down), might explain these contradicting findings.

However, we observed uni- and bilateral processing differences for compatible and incompatible number pairs in the *number comparison task* in our mixed effect model approach. Following Wood et al.'s (2006) approach, we modeled the unit-decade compatibility effect parametrically by including unit distance and decade distance (e.g.,  $36 < 55$  and  $36 < 51$  are both incompatible, but  $36 < 55$  is assumed to be more difficult than  $36 < 51$ ) as continuous variables in our mixed model analysis. We observed that the numerical distances between units and decades significantly predicted higher performance. With increasing decade distance, the decision about which number is larger becomes more straightforward, reflected by a larger RCS. This observation closely resembles a significant numerical distance effect in two-digit *number comparison tasks* (Dehaene et al., 1990), more pronounced for the decade distance (cf. Wood et al., 2006).

Additionally [and different from Wood and colleagues (2006)], we also included unit-decade compatibility as a binarily coded dummy variable. Having both variables together in one model also allowed investigating their interaction. Unit and decade distance significantly predicted participants' performance (i.e., RCS), whereas the binarily coded variable compatibility did not significantly contribute to the model. On an empirical level, this finding implies that considering the parametric unit distance and decade distance allows examining the unit-decade compatibility effect's influence more directly. This finding is in line with neuroimaging data by Wood et al. (2006), who presented the neurofunctional correlates for unit-decade compatibility through unit distance and decade distance only. On a theoretical level, the current data provide further evidence favoring decomposed representations of tens and units (Moeller et al., 2011).

In this vein, higher mean RCS for left-lateralized presentation indicated decomposed processing in the *number comparison task*. This observation corresponds to the literature: in the left-lateralized presentation, stimuli are directly transmitted to the right hemisphere via transcallosal fiber pathways (Chaumillon et al., 2018), where tens and units are integrated into the place-value structure of the Arabic number system (Wood et al., 2006). Based on the present results, we considered our experimental setting as valid for detecting hemispheric specialization.

### **4.3 Lateralization in the verification of arithmetical facts**

In the *multiplication task*, we investigated whether the processing of simple multiplication facts (e.g.,  $12 = 3 \times 4$ ) was superior (i.e., higher RCS) when the visual input was transmitted to the left hemisphere only (i.e., presentation to the right visual hemifield). As discussed above, lateralized stimuli presentation yielded processing advantages in the left visual hemifield in all tasks. Following the triple code model suggesting unilateral left-hemispheric processing of arithmetic facts (Dehaene and Cohen, 1995, 1997; Dehaene et al., 2003; see also Berteletti, Prado & Booth, 2014; Demir, Prado & Booth, 2014; Prado, Mutreja & Booth, 2014), we expected processing advantages of the right visual hemifield. Considering the systematic processing disadvantage in the right visual hemifield, we assumed this disadvantage is reduced in the *multiplication task*. Mixed-model results, however, indicated neither of both. Additionally, we could not find differential contributions of the two hemispheres when analyzing the type of multiplication errors (i.e., operand errors, close-miss errors), as reported by Salillas and colleagues (2021). For automatically activated table-related errors and correct responses, left-hemispheric processing has been proposed. In contrast, right-hemispheric processing has been suggested to be associated with rather unexpected close-miss errors (Salillas et al., 2021). However, close-miss errors and the number of the target's

divisors, which automatically activates multiplication tables, influenced participant's performance, indicated by significant beta values in our mixed effect model.

However, as already outlined in the results section, the discrepancy that the expected effect of lateralization was only observed in the *number comparison task* but not in the *multiplication task* can be explained by a fixed association of response hand with one of the conditions in the *multiplication task*. If this association was considered (which was possible), we found evidence for lateralized processing of multiplication facts. However, before discussing this analysis in more detail, we briefly recapitulate why we are confident that there is no error in further methodological aspects. To this end, the following methodological aspects need to be considered.

First, we are confident that we assessed arithmetic fact retrieval directly with our *multiplication task* as participants evaluated one-digit×one-digit multiplication problems (e.g.,  $72 = 9 \times 8$ ). Jung et al. (2020), who did not observe lateralized processing in multiplication, suggested that participants might have failed to observe hemispheric specialization because phonological recall of the multiplication problem superimposed the effect of lateralization. The authors used an NBT in which verbally-mediated arithmetic fact retrieval is supposed to facilitate the evaluation of multiplicative triplets (De Brauwer et al., 2006; but see Domahs et al., 2006) associated with retrieval-specific activation of the left angular gyrus (Wood et al., 2008). Nevertheless, information about multiplicativity is only indirectly used to perform this task. Participants are not directly asked to solve multiplication problems but evaluate whether the middle number of a number triplet is identical with the arithmetic mean. Furthermore, the authors suggested that evaluating simple multiplication problems such as  $8 \times 3 = 24$  should concisely address arithmetic fact retrieval. This task seems easier than evaluating triplets such as 21\_24\_27, especially when the numbers are presented only briefly due to the visual hemifield. Thus, we are confident that we specifically addressed

arithmetic fact retrieval using simple multiplication problems in the present study (Dehaene, 1995; Dehaene et al., 2003).

Second, we designed the current stimulus set to contain homogeneous multiplication problems. More specifically, we excluded multiplications by 0, 5 (except the prime number 5 itself), and 10, from which we know that they are processed faster (Campbell, 1995). For incorrect targets, we included only operand errors and close-miss errors. We further excluded multiplication problems requiring additional numerical operations, such as carry operations in problems with one operand larger than 10. Carrying a digit from the unit digits to the next higher power of ten demands place-value integration (Klein et al., 2010; Ratinckx et al., 2006; Wood et al., 2006). Thus, in these problems, we would not expect an advantage of left-hemispheric processing. Finally, we added some prime numbers to this homogenized item set. As prime numbers are straightforward to solve, they may better detect priming effects and indicate early bottom-up processes (Jung et al., 2020). However, although we maximized the homogeneity of the multiplication problems, results did neither indicate a significant effect of lateralization nor significant interactions between model coefficients for the type of errors and lateralization.

Third, the target was presented before the two operands in the *multiplication task*. We assumed that this kind of presentation primes retrieving those multiplication tables associated with the target, which is, then again, associated with left hemispheric specialization (Salillas et al., 2021). Accordingly, we included the number of the target's divisors in our mixed effect model, additionally to the target itself (i.e., problem size). We did not anticipate such a priming-based solution strategy when the target and factors appeared simultaneously on the screen in the *control task*. Like previous models for arithmetic fact retrieval, we assumed a more considerable influence of the problem size than for the target's number of divisors in this standard paradigm. Again, we included both variables in the model to investigate possible

differences in their influence on arithmetic fact retrieval. Results indicated a significant effect of the target's number of divisors only for the *multiplication task* but not for the *control task*. Whenever the target is presented before the operands, the number of the target's divisors is a better predictor for arithmetic fact retrieval performance than problem size. This finding corresponds to the fact that operand errors were not more challenging to reject than other errors. When the operands do not occur before the target, operand table errors probably were not activated similarly strong than in the standard presentation sequence. These results let us assume that we provoked early bottom-up processes, which theoretically allow us to detect hemispheric processing differences in healthy participants (Jung et al., 2020) when transcallosal fiber pathways allow for interhemispheric processing within only a few milliseconds (e.g., Chaumillon et al., 2018).

In sum, we selected the stimulus material in the *multiplication task* designed to reveal theoretically proposed left-lateralized processing of arithmetic facts. Therefore, we are confident that the significant difference between the response behavior in the *number comparison* and *multiplication task* may account for the lack of hemispheric specialization in the latter task. In particular, participants pressed superimposed buttons in the *number comparison task*. They had to press adjacent keys (i.e., right: when the target was the correct product of the operands; left: in tasks with non-matching operands) with the right and the left index finger in the *multiplication task*. Therefore, there was a fixed assignment of multiplicativity with the answer hand in the latter task. This fixed assignment caused the total number of inter-hemispheric transitions to vary between the tasks, potentially preventing us from detecting a effect of laterlization.

We conducted an alternative analysis considering the minimum number of inter-hemispheric transitions relevant to solving the task to disclose this potential confounding. Thereby, we inferred from the input side and the output side, which was known for each

condition, the required inter-hemispheric transitions. Results indicated that participants responded faster to multiplicative tasks directly processed to the left hemisphere (i.e., input in the right-visual hemifield) than to non-multiplicative trials directly processed in the right hemisphere (i.e., input in the left visual hemifield). This observation suggested a left-hemispheric specialization in the *multiplication task* if we consider inter-hemispheric transitions.

Thus, our results are consistent with the previous findings from neuropsychological single-case studies on brain-lesioned patients (e.g., Dehaene & Cohen, 1997; Zaunmueller et al., 2009), who showed severe selective impairments in multiplication following left-hemispheric stroke. Results also align with various fMRI training studies providing evidence for left-hemispheric verbally mediated arithmetic fact retrieval (Bloechle et al., 2016; Delazer et al., 2005; Delazer et al., 2003; Grabner et al., 2009; Ischebeck et al., 2006) and a recent study investigating white matter disconnections in stroke patients (Smaczny et al., submitted). Importantly, these studies cannot provide sufficient evidence for isolated left-hemispheric processing because they often observed bilateral activation of perisylvian language areas (Grabner et al., 2009; Klein, Moeller, & Willmes, 2013; Klein, Moeller, Glauche, et al., 2013; Stanescu-Cosson et al., 2000). Against these findings, Amalric and colleagues (2016, 2018, 2019) proposed that numbers and mathematical facts are processed independently of left-hemispheric language processes by recent data from neuroimaging in expert mathematicians. The authors presented mathematical and non-mathematical statements to high-level professional mathematicians and a control group of non-mathematician academics with comparable education levels. Only in mathematicians, the mathematical statements (but not the non-mathematical statement) led to an amplification of brain activity in the bilateral fronto-parietal math responsive network. Crucially, left-hemispheric perisylvian language areas were not activated. However, it has been argued that these findings might be an artifact

of comparing a highly homogeneous group of experts in a specific domain with a very heterogeneous control group (Knops, 2019), and a somewhat unusual task to assess arithmetic fact retrieval. Although our data indicate left-hemispheric processing in our *multiplication task*, we cannot decide on the present behavioral study whether arithmetic fact retrieval is verbally mediated or is processed independently of language.

#### **4.4. Limitations**

When interpreting the results of the current study, some limiting aspects need to be considered. First, even though we were able to show processing differences depending on the input lateralization in the *number comparison task*, fixed assignment of multiplicativity with the answer hand most likely prevented us from detecting lateralized processing in the *multiplication task*. However, considering the number of inter-hemispheric transitions allows detection of processing differences depending on the side of the presentation. Future studies would be desirable to investigate the relation between inter-hemispheric transitions and lateralized processing in more detail.

Second, deviations from the required eye fixations were monitored by the examiner sitting opposite of the participant instead of using, for instance, eye-tracking protocols. The lateralized stimuli were located 5 degrees from central fixation. This location goes beyond the saccade amplitude's critical distance, which can be detected with the examiner's naked eye (Bock et al., 2008; Gamer & Hecht, 2007). Accordingly, we are confident that this procedure prevented unwanted fixation loss. Nevertheless, eye tracking protocols could be sufficient for detecting microsaccades, which the examiner cannot observe visually. As microsaccades are closely correlated with the dynamics of covert spatial attention (Rima et al., 2020), their detection would allow more insights into primary visual processing.

## 5 Conclusion

The current study adds to the discussion of hemispheric specialization in number processing in the human brain. In line with assumptions of the triple code model, we found bilateral processing advantages in the *number comparison task*, suggesting that number magnitude processing is processed bi-hemispheric. Considering the minimum number of inter-hemispheric transitions in the *multiplication task*, we observed strong indications that lateralized processing to the left hemisphere yields processing advantages in retrieving arithmetic facts. The observation for left-hemispheric specialization continues to favor linguistically processed arithmetic facts. This conclusion is considered in light of the systematic processing advantage of the presentation in the left visual hemifield that we observed in our divided visual hemifield approach, which we associate with early right-hemispheric identification of visual symbolic numerical stimuli.

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# Appendix

m	n	x	T	OE	DIV
1	4	5	0	1	2
1	4	4	1	0	3
1	6	8	0	0	4
1	8	7	0	1	2
1	8	8	1	0	4
1	8	8	1	0	4
1	47	49	0	0	3
1	47	47	1	0	2
1	47	47	1	0	2
1	56	58	0	0	4
1	56	55	0	1	4
1	56	56	1	0	8
1	62	63	0	1	6
1	62	62	1	0	4
1	74	76	0	0	6
1	74	75	0	1	6
2	3	2	0	0	2
2	3	4	0	1	3
2	3	6	1	0	4
2	3	6	1	0	4
2	6	14	0	1	4
2	6	12	1	0	6
2	7	18	0	0	6
2	7	16	0	1	5
2	7	14	1	0	4
2	7	14	1	0	4
2	8	12	0	0	6
2	8	14	0	1	4
2	9	18	1	0	6
2	11	24	0	1	8
2	11	22	1	0	4
2	12	28	0	0	6
2	12	24	1	0	8
2	12	24	1	0	8
2	13	24	0	1	8
2	13	26	1	0	4
2	14	24	0	0	8
2	14	28	1	0	6
2	21	44	0	1	6
2	21	42	1	0	8
2	21	42	1	0	8
2	22	46	0	1	4
2	23	42	0	0	8
2	23	44	0	1	6
2	24	44	0	0	6
2	24	48	1	0	10
2	32	68	0	0	6
2	32	66	0	1	8
2	32	64	1	0	7
2	33	66	1	0	8
2	34	68	1	0	6
2	34	68	1	0	6
2	41	82	1	0	4

m	n	x	T	OE	DIV
2	41	82	1	0	4
2	42	88	0	0	8
2	42	86	0	1	4
2	43	84	0	1	12
2	43	86	1	0	4
2	44	88	1	0	8
3	3	7	0	0	2
3	3	6	0	1	4
3	3	9	1	0	3
3	3	9	1	0	3
3	4	14	0	0	4
3	4	15	0	1	4
3	4	12	1	0	6
3	6	16	0	0	5
3	6	15	0	1	4
3	7	11	0	0	2
3	7	24	0	1	8
3	8	26	0	0	4
3	8	27	0	1	4
3	8	24	1	0	8
3	9	25	0	0	3
3	9	24	0	1	8
3	9	27	1	0	4
3	9	27	1	0	4
3	11	36	0	1	9
3	11	33	1	0	4
3	12	33	0	1	4
3	12	36	1	0	9
3	12	36	1	0	9
3	13	49	0	0	3
3	21	65	0	0	4
3	21	63	1	0	6
3	23	66	0	1	8
3	23	69	1	0	4
3	31	95	0	0	4
3	31	93	1	0	4
3	32	96	1	0	12
3	33	96	0	1	12
3	33	99	1	0	6
4	1	6	0	0	4
4	1	4	1	0	3
4	3	12	1	0	6
4	4	18	0	0	6
4	4	12	0	1	6
4	4	16	1	0	5
4	4	16	1	0	5
4	6	24	1	0	8
4	7	26	0	0	4
4	7	24	0	1	8
4	7	28	1	0	6
4	7	28	1	0	6
4	8	36	0	1	9
4	8	32	1	0	6

m	n	x	T	OE	DIV
4	8	32	1	0	6
4	9	36	1	0	9
4	11	46	0	0	4
4	11	44	1	0	6
4	12	44	0	1	6
4	12	48	1	0	10
4	21	86	0	0	4
4	21	88	0	1	8
4	21	84	1	0	12
4	22	86	0	0	4
6	1	5	0	1	2
6	1	6	1	0	4
6	1	6	1	0	4
6	2	16	0	0	5
6	2	12	1	0	6
6	3	18	1	0	6
6	3	18	1	0	6
6	4	26	0	0	4
6	4	28	0	1	6
6	4	24	1	0	8
6	6	38	0	0	4
6	6	30	0	1	8
6	6	36	1	0	9
6	6	36	1	0	9
6	7	48	0	1	10
6	7	42	1	0	8
6	8	46	0	0	4
6	8	48	1	0	10
6	9	56	0	0	8
6	9	54	1	0	8
6	11	66	1	0	8
7	3	21	1	0	4
7	3	21	1	0	4
7	6	44	0	0	6
7	6	42	1	0	8
7	7	39	0	0	4
7	7	42	0	1	8
7	7	49	1	0	3
7	7	49	1	0	3
7	8	58	0	0	4
7	9	65	0	0	4
7	9	56	0	1	8
7	9	63	1	0	6
7	11	75	0	0	6
7	11	77	1	0	4
8	1	6	0	0	4
8	2	16	1	0	5
8	2	16	1	0	5
8	3	24	1	0	8
8	4	34	0	0	4
8	6	42	0	1	8
8	6	48	1	0	10
8	7	63	0	1	6

m	n	x	T	OE	DIV
8	7	56	1	0	8
8	7	56	1	0	8
8	8	66	0	0	8
8	8	56	0	1	8
8	8	64	1	0	7
8	8	64	1	0	7
8	9	74	0	0	4
8	9	72	1	0	12
8	9	72	1	0	12
9	2	14	0	0	4
9	2	16	0	1	5
9	2	18	1	0	6
9	4	38	0	0	4
9	4	32	0	1	6
9	4	36	1	0	9
9	6	48	0	1	10
9	6	54	1	0	8
9	7	63	1	0	6
9	8	64	0	1	7
9	9	91	0	0	4
9	9	72	0	1	12
9	9	81	1	0	5
9	9	81	1	0	5
9	11	89	0	0	2
9	11	90	0	1	12
9	11	99	1	0	6
9	11	99	1	0	6
11	2	26	0	0	4
11	2	22	1	0	4
11	3	35	0	0	4
11	3	33	1	0	4
11	4	48	0	1	10
11	4	44	1	0	6
11	6	68	0	0	6
11	6	60	0	1	12
11	6	66	1	0	8
11	7	70	0	1	8
11	7	77	1	0	4
11	8	86	0	0	4
11	8	80	0	1	10
11	8	88	1	0	8
11	8	88	1	0	8
12	2	26	0	1	4
12	3	38	0	0	4
12	4	46	0	0	4
12	4	48	1	0	10
13	2	22	0	0	4
13	2	26	1	0	4
13	3	36	0	1	9
13	3	39	1	0	4
13	3	39	1	0	4
14	2	26	0	1	4
14	2	28	1	0	6

m	n	x	T	OE	DIV
21	2	46	0	0	4
21	3	66	0	1	8
21	3	63	1	0	6
21	4	84	1	0	12
22	2	48	0	0	10
22	2	44	1	0	6
22	2	44	1	0	6
22	3	68	0	0	6
22	3	63	0	1	6
22	3	66	1	0	8
22	3	66	1	0	8
22	4	84	0	1	12
22	4	88	1	0	8
22	4	88	1	0	8
23	2	46	1	0	4
23	2	46	1	0	4
23	3	59	0	0	2
23	3	69	1	0	4
24	2	46	0	1	4
24	2	48	1	0	10
31	2	66	0	0	8
31	2	64	0	1	7
31	2	62	1	0	4
31	2	62	1	0	4
31	3	96	0	1	12
31	3	93	1	0	4
32	2	64	1	0	7
32	3	98	0	0	6
32	3	93	0	1	4
32	3	96	1	0	12
33	2	62	0	0	4
33	2	64	0	1	7
33	2	66	1	0	8
33	3	89	0	0	2
33	3	99	1	0	6
34	2	64	0	0	7
34	2	66	0	1	8
41	2	86	0	0	4
41	2	84	0	1	12
42	2	84	1	0	12
42	2	84	1	0	12
43	2	82	0	0	4
43	2	86	1	0	4
44	2	84	0	0	12
44	2	86	0	1	4
44	2	88	1	0	8
47	1	46	0	1	4
56	1	56	1	0	8
62	1	64	0	0	7
62	1	62	1	0	4
74	1	74	1	0	4
74	1	74	1	0	4

Note: m: Factor 1; n: Factor 2; : Result; T = True problem (1 = yes / 0 = no); OE: Operand error (1 = yes / 0 = no), DIV: Number of potential divisors