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Symmetry and financial markets

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Abstract

It is hard to overstate the importance that the concept of symmetry has had in every field of physics, a fact alluded to by the Nobel Prize winner P.W. Anderson, who once wrote that “physics is the study of symmetry”. Whereas the idea of symmetry is widely used in science in general, very few (if not almost no) applications has found its way into the field of finance. Still, the phenomenon appears relevant in terms of for example the symmetry of strategies that can happen in the decision making to buy or sell financial shares. Game theory is therefore one obvious avenue where to look for symmetry, but as will be argued, also technical analysis and long term economic growth could be phenomena which show the hallmark of a symmetry.

JEL classification: G14; C73

Keywords: Agent-based modelling; Game theory; Ginzburg-Landau theory; financial symmetry

Introduction. - For physicists the idea of symmetry is familiar, used in all sub-disciplines of the field. For example, properties of particles originate from the symmetries of laws of physics [1]. The concept of symmetry also plays a fundamental role in other sciences for example mathematics, biology, chemistry, and neuroscience.

In the traditional branch of finance, however, the concept of symmetry has not yet been adopted. Still various interdisciplinary approaches have put forward various ideas of how to apply the idea of symmetry to financial markets, even though a large consensus about this idea yet has to emerge [2,3]. Not surprisingly physicists have mostly been responsible for applying the concept in various contexts. Zumbach [4] showed in a study that empirical time series clearly are not time reversal invariant. The interesting point put forward in [4] was to use time reversal invariance in financial time series, as a selection tool, by discarding financial models/processes that cannot reproduce the stylized facts related to the time irreversibility observed in empirical time series. As pointed out by Zumbach [4] it is not completely surprising that financial market are not symmetrically with respect to time: markets are driven by humans, who are clearly not time reversal invariant. In particular, market participants remember the past, and this memory creates an asymmetry in the time direction. In [5] Lillo and Mategna considered the asymmetry of the price return probability distributions on rally and crash days. They found a change in the shape and symmetry of the probability distribution function of the daily price returns, during market days characterized by extreme absolute returns. In addition to the asymmetry of the probability distribution functions, their study suggests that the correlation properties between stocks may change during such extreme market events. Symmetry considerations were also used in the work of J.-P. Bouchaud and R. Cont [6]. They introduced a nonlinear Langevin equation as a model for price fluctuations based on the different processes influencing the demand and supply. In their model risk aversion led to an “up-down” symmetry breaking term responsible for crashes, with “panic” self-reinforcing due to feedback loops. The “up-down” symmetry breaking term was responsible for the sudden collapse of speculative bubbles, and crashes in their model appeared as rare events, with an exponentially small probability of occurrence. Sornette [7] identified a fundamental parity symmetry of stock prices, which results from the relative direction of payment flux compared to commodity flux. He

suggested that a company's risk-adjusted growth rate discounted by the market interest rate would behave as a control parameter for the observable price. This insight led to the recognition of a low growth rate phase described by firm foundation theory, and a large growth rate phase with a regime of speculation and crowd behavior. In practice, his symmetry breaking speculation theory was able to account for the apparent over-pricing and the high volatility of fast growing companies on the stock markets.

More recent work where the idea of symmetry, and symmetry breaking, has been applied to quantum theory of financial markets can be found in [8] and [9]. In [8] Arraut et al. considered how a spontaneous symmetry breaking in quantum finance could be used as a starting point in the Black-Scholes and the Merton-Garman equations expressed in a Hamiltonian form. In their scenario the martingale condition (state) corresponds to the vacuum state which becomes degenerate when the symmetry of the system is spontaneously broken. In such a point of view, the martingale condition is equivalent to the ground state of the system, and it will be only modified if the information in the system flows or changes. Their main findings are that the symmetry under changes of prices is spontaneously broken via the flow of information to the market. They use this insight to produce a couple of interesting relations between the volatility and the price of options. In [9] Halperin and Dixon proposed a simple non-equilibrium model of investment portfolios and of financial markets as open systems with a possible exchange of money with an outside world and market frictions incorporated into asset price dynamics via a feedback mechanism. The feedback mechanism was inspired by reinforcement learning and physics. The resulting price dynamics was developed using physics motivated analyses of symmetries. In their model defaults and market crashes were seen to be associated with dissipative tunneling events, and correspond to instanton (saddle-point) solutions.

In the following we will argue, that the concept of symmetry, and symmetry breaking, can capture fundamental properties of the dynamics of financial markets. At the individual level – the investor – symmetry breaking originates in the cognitive system and can be described as an anchoring. At the social level symmetry breaking is caused by social influence – dependence of others when making investment decisions. At the societal level symmetry breaking is due to feedback loops among different

institutional actors who shape financial markets. The idea in the following will then be to argue, that the concept of symmetry can appear in a variety of financial contexts. As such, the hope is it should give new insight to old problems appearing in finance. Specifically, the concept of symmetry/symmetry break will be applied to such examples as technical analysis, long term growth dynamics of financial markets, as well as symmetry of trading strategies applied to buying/selling assets. One of the most interesting aspect of symmetries, is the new understanding one gets, when a symmetry is either broken or restored.

Symmetry in price formation due to technical analysis: Breaking of symmetry from an unstable support level to a stable support level. - Technical analysis uses past price histories to predict future price movements, whereas fundamental analysis instead uses future estimates of earnings to find the right, fundamental, price at any given time. Below will be described then, how the rise of either unstable support levels, or stable support levels in the price formation introduces symmetries. The understanding of when those symmetries are broken will be shown in certain cases to led to arbitrage possibilities.

Sometimes it can happen the price formation in financial markets can show oscillatory price movements between so-called support levels, for an example, see Figure 1.

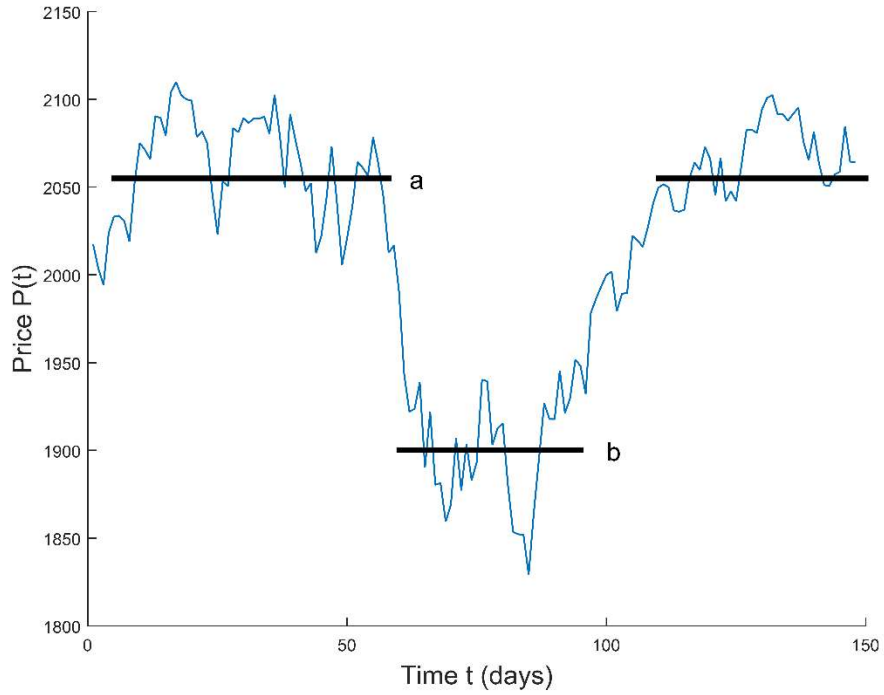


Figure 1: Symmetry of one stable, and one un-stable support level. *Illustration of daily price variations, and support levels, of the SP500 index over the period 12/10/2015-13/05/2016. The level **a** corresponds to a stable technical support level, whereas the level **b** corresponds to an unstable technical support level.*

The reason for such behavior can be manifold, but one reason could be because of the behavioral trait called “anchoring” [10]. Anchoring describes the tendency for humans to “anchor” on irrelevant information in their decision making. For various works discussion the topic of anchoring see e.g. [11-14]. A simple description taking into account support levels, can be modeled by the equation:

$$\frac{dP(t)}{dt} = \alpha [P(t) - a] [P(t) - b] \quad (1)$$

Here a and b are the support levels, and $P(t)$ is the price of the asset at time t . $\frac{dP(t)}{dt}$ is the “price velocity” of the asset (for a longer discussion of early warning signals for transitions between support levels, see an excellent article [15]). (1) describes a price velocity that goes to zero when either of the

two support levels, a or b , are approached, so the two support levels are equilibrium solutions for $P(t) = a$, respectively $P(t) = b$. As can be seen from (1), assuming $\alpha < 0$, $a > b$, then if $P(t) > a$ the price velocity is negative and the prices are “pushed” down towards a . If on the other hand $P(t) < b$ then the right hand side of (1) is negative, and so prices are “pushed” down and away from the support level b . For prices in between the two price support levels $a > P(t) > b$, the price velocity is positive so prices are pushed away from b towards a .

One can formally see the appearance of a stable, respectively unstable equilibrium by using linear response theory. Let us call the two equilibria solutions $\overline{P}_1 = a$, and $\overline{P}_2 = b$. One can then study what happens to a small fluctuation ε around that price equilibrium. Call $\widetilde{P}_1 = \overline{P}_1 + \varepsilon$, from (1) we have

$$\frac{d\widetilde{P}_1}{dt} = f(\widetilde{P}_1) \quad (2)$$

Where the function $f(\widetilde{P}_1) = \alpha (\widetilde{P}_1 - a)(\widetilde{P}_1 - b)$ is given by the right-hand side of (1). Linearizing via a Taylor expansion:

$$\frac{d(\overline{P}_1 + \varepsilon)}{dt} = f(\overline{P}_1 + \varepsilon) \quad (3)$$

$$\approx f(\overline{P}_1) + \left. \frac{\partial f}{\partial P} \right|_{P=\overline{P}_1} \varepsilon \quad (4)$$

Comparing (2)-(4) we therefore get the equation that describes the behavior of the fluctuation ε

$$\frac{d\varepsilon}{dt} = \left. \frac{\partial f}{\partial P} \right|_{P=\overline{P}_1} \varepsilon \quad (5)$$

$$\frac{d\varepsilon}{dt} = \mu_1 \varepsilon \quad (6)$$

With $\mu_1 \equiv \left. \frac{\partial f}{\partial P} \right|_{P=\overline{P}_1} = \left. \frac{\partial f}{\partial P} \right|_{P=a} = \alpha (a - b)$. Similarly one find that $\mu_2 \equiv \alpha (b - a)$. From (6) we then see that for $\alpha < 0$ and $a > b$ any fluctuation around the solution $P(t) = a$ decreases exponentially, i.e. the solution is stable. Similarly any fluctuation around the solution $P(t) = b$ increases exponentially, i.e. this solution is unstable.

It should be noted that the transition from b to a , as one can see in Figure 1 cannot be described by (1), so external economic news, or another cause, would be needed in describing such a transition.

The presence of support levels in financial markets are not limited to equity markets, but can appear in all classes of assets like commodities, currencies and bond markets. One particular case which led to the appearance of several stable support levels, happened in currency markets, and was politically imposed in the 1970's. After the dissolution of the Bretton Woods system, which had kept the values of currencies fixed to gold, most of the countries of the EEC (the organization prior to the EU) agreed in 1972 to maintain stable exchange rates of their currencies, by allowing for currency fluctuations of at most 2.25%. The fixture was called the European "currency snake". In 1979 the "currency snake" was then replaced by the EMS, the European Monetary System, where most member states linked their currencies in order to prevent large fluctuations relative to one another. The introduction of multiple stable support levels via the EMS was in place for over a decade, until continued attacks on the system, due to different countries different economic policies, rendered the system obsolete. For a longer discussion of the symmetry/symmetry break of several support levels, see [10,16,17]. In [10] a trading algorithm using the symmetry/symmetry break of support levels was introduced. The basic idea is for example to go long (i.e. buying) asset i whenever a favorable fluctuation happens, defined by a price fluctuation below the support level. Next the idea is to wait for another favorable price fluctuation to show up, in a new configuration of price levels. When such a new price configuration happens, one then switch to a new asset j , long or short, depending of the price level of asset j . This illustrates how the presence of support levels, e.g. politically imposed like the "currency snake", opens up for profit possibilities (given certain conditions are fulfilled), by letting a patient trader wait and act when the right price fluctuation happens. Whether such strategy is actually profitable, and how risky it would be, was analyzed in [10]. To see how a trading algorithm with induced symmetry break can be applied to support levels of real data, consider Figure 2. The figure illustrates an example of a (market neutral) support level trading algorithm applied to the Dow Jones Industrial Average, as well as the CAC40 stock index. As can be seen from the figure, the trading algorithm is capable of excellent returns with a high Sharpe ratio. For more information, see [10].

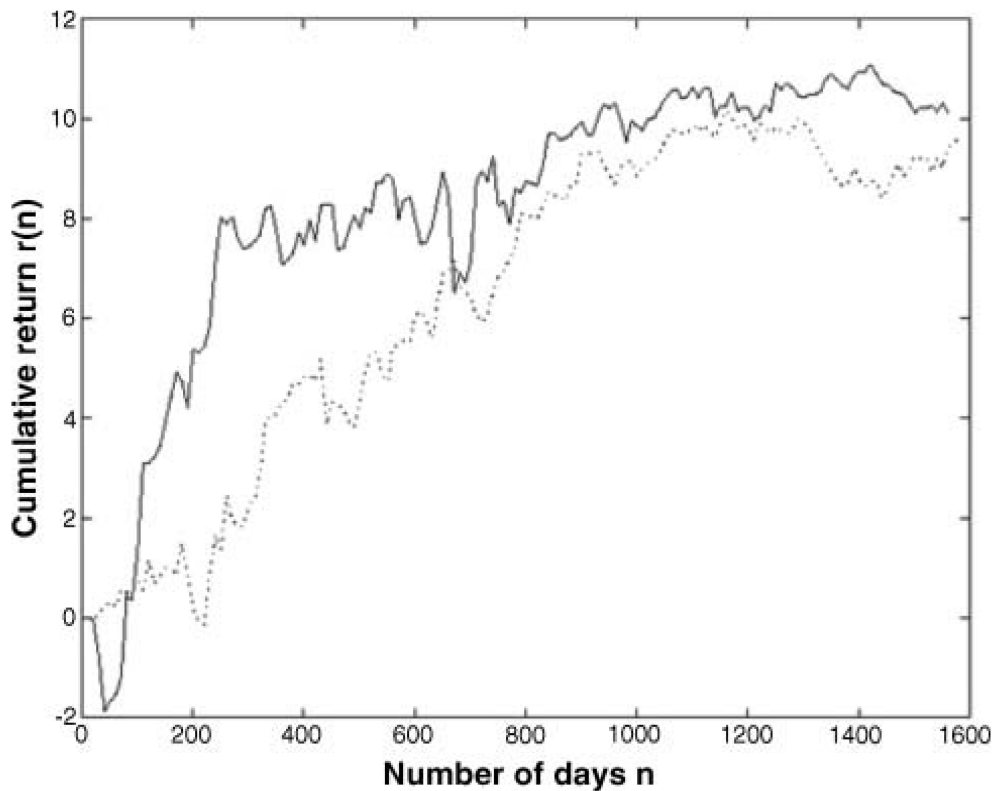


Figure 2: Trading algorithm of support levels applied to a real market situation. *Cumulative return of a (market neutral) support level algorithm applied to daily price data of the Dow Jones stock index (dotted line), as well as the CAC40 stock index (solid line) over the period March 1, 2000–February 5, 2006. First half of the time period for the Dow Jones Industrial Average was used in sample to determine the best choice among three values of the length of support level $m = 5, 10, 15$ days. Second half of the time period for the Dow as well as full period of the CAC40 index was done out of sample with $m = 10$. As a measure of the performance of the algorithm, the Sharpe ratio was found to be 2.0 and 2.94 for the Dow, respectively, CAC40 price time series. A trading cost of 0.1% was included for each transaction. As is clear from the figure the support level trading algorithm is able to generate returns with high Sharpe ratios.*

Symmetry break in trading strategies due to collective decision making. - One can then consider, how certain speculative transitions in financial markets can be ascribed to a symmetry break, that happens in the collective decision making.

Traditionally economics considers investors to be rational, meaning that in their decision making on whether to buy or sell shares, they use all available information concerning future dividends and future interest rates, to calculate the proper, *fundamental*, price of an asset. A large part of the financial industry therefore has analysts trying to deduce future earnings of companies, recommending to sell if they deduce the present price is too high, or to buy if the present price is deemed too low when considering future earnings prospective. However, in practice investors use decision making based not only on fundamental price, but use a variety of investment strategies. Technical analysis considers past price performance in order to predict future performance. Investors, and the decision making in their trading of assets, should therefore rather be considered as a dynamically changing pool of investment strategies that depend both on the present price (fundamental value investment strategies) and past price history (technical analysis). In such a setting, markets can be considered as complex systems. As will be seen, the payoff of various investment strategies thereby reflects an intrinsic financial symmetry that generates equilibrium in price dynamics (fundamentalist state), until eventually the symmetry is broken, which then leads to scenarios of bubble or anti-bubble formation (speculative state). One can eventually give a behavioral “twist” to such a tale: the financial symmetry breaking corresponds to the financial system moving from one state (or mood), where randomness in price movements obey no-arbitrage conditions, into another state in which the price takes the route towards bubble or anti-bubble formation giving rise to arbitrage possibilities. The first mood is the *fundamental state*, while the second mood is the *speculative state*. A description can be proposed, that considers such behavior transitions in a micro-to-macro scheme. In such a picture a market “temperature” then modulates the fundamental-to-speculative state transitions of the market.

An analogy can be made of markets behaving like thermodynamically systems. What characterizes such systems is the struggling forces between energy (order) and entropy (disorder). As can be argued, similar market forces exist forcing the markets to go from one state to another based on their inner “temperature”, for more detailed explanations, see [18]. Using such a characterization one solution then corresponds to the case of complete order, with all used trading strategies taking the same direction (buy or sell), thereby giving rise to the formation of a bubble or an anti-bubble. This is the pure speculative state, with up or down price movements leading to a bubble or anti-bubble state, breaking the financial symmetry with agents getting positive profits by going long or short in the market. On the other hand, another solution exists which corresponds instead to a financial system with disorder. In this case roughly half of the population use strategies that take one action, and the remaining taking the opposite action, thus giving rise to a price path around its fundamental value. This is the disordered state, in which randomness in price movements leads to a no-arbitrage condition.

The “*market temperature*”: One of the main implication of the above discussion is that one can introduce a Ginzburg-Landau-based theory of mood transitions in the market[18], showing the existence of a nontrivial transition from a “high temperature” symmetric state, where traders don’t create a trend over time, to a “low temperature” state, characterized by trend following with a definite trend in the price trajectories (up or down). In the following a “temperature” linked to the randomness of the agent’s actions will be suggested via agent based modeling. In [19] an agent based model was used in which the agents used fundamental analysis strategies, as well as technical analysis strategies. Randomness entered the modeling via initial conditions in the assignments of s technical analysis strategies to N traders in the game. The technical analysis strategies were created by randomly assigning either a -1 (meaning sell) or a 1 (meaning buy) for each of the 2^m different price histories, that one gets from considering only the last m up (1) or down (0) price movements. In such a modeling the total pool of strategies increases as 2^{2^m} versus m . However, many of these strategies are closely related, and refs. [20-21] showed how to construct a small subset of size 2^m of independent strategies out of the total pool of 2^{2^m} strategies. As suggested in Refs. [22-24], a qualitative understanding of this problem can be obtained by considering the parameter $\alpha = \frac{2^m}{N}$. Along the same line of reasoning, Ref. [25] pointed out,

however, that the ratio $\alpha = \frac{2^m}{N \times S}$ seems more intuitive, since this quantity describes the ratio of the total number of relevant strategies to the total number of strategies held by the traders. In similar vein a “market temperature” was introduced in [19] via the expression:

$$T = \frac{2^{m+1}}{N \times S}, \quad (7)$$

where the “+1” in the numerator is because of the fundamental strategy in addition to the 2^m uncorrelated speculative strategies.

From (7) one can then see how the mechanism behind a “temperature” that can break the symmetry and shift the market between different moods/states: when the total pool of strategies held by the N traders is small with respect to the total pool of relevant strategies (low denominator of T), this corresponds indeed to the large fluctuations, large temperature case. Vice versa a large pool of strategies held by the N traders (increasing denominator of T), therefore corresponds to a small temperature case.

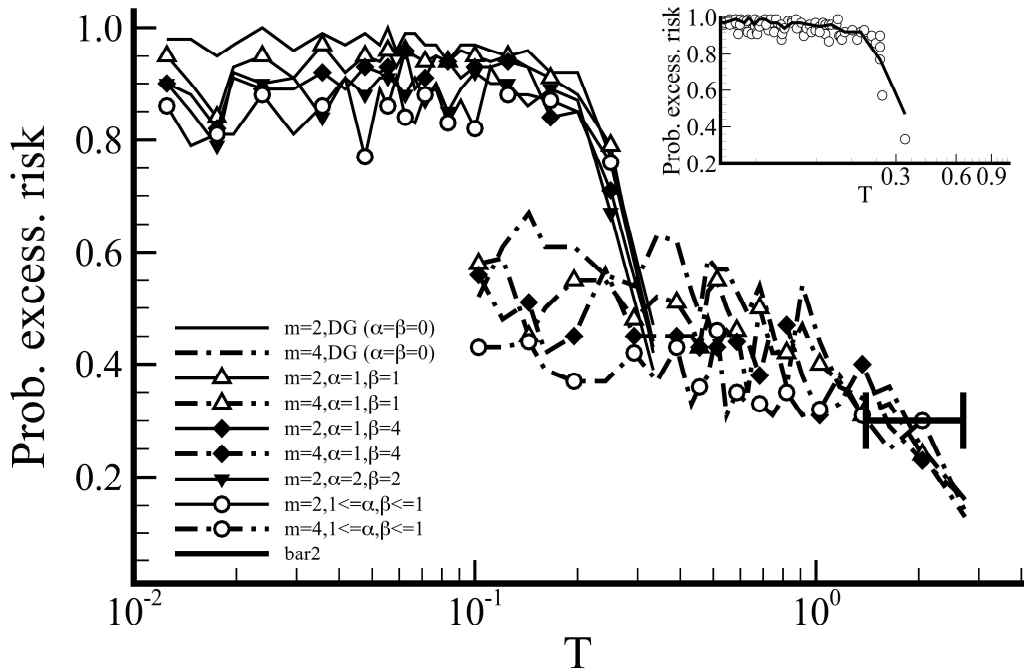


Figure 3: The importance of T for symmetry breaking: probability of excessive risk-taking versus T for groups with different risk profiles. Collective speculative price behavior, defined as occurring when the final price is more than twice the fundamental price, is plotted against the control parameter $\frac{[3^{M+1}]}{(s+1)^N}$. Inset: simulations of games with different values of s and N, but for a fixed value of T and $M \equiv 2$. The different symbols correspond to simulations with populations having different risk profiles. As can be seen from the figure, the main parameters governing whether the agents are risk taking or not, is determined by the two parameters T and M. Different risk profiles (for a given fixed T and M) of the agents (represented by the different lines) is seen to play a minor role.

Figure 3 shows Monte Carlo simulations of an agent-based model [19] illustrating the relevance of the symmetry-breaking “temperature” introduced above. On the y-axis is plotted the probability of generating a collective speculative price behavior against the control parameter T, different curves correspond to different risk profiles of the agents. The first thing to notice is by considering the inset which shows that given *fixed* values of M and T lead to the *same* probability of creating excessive price behavior (see Figure 3 inset, which presents simulations of games with *different* values of s and N, but for *fixed* T and $M \equiv 2$).

The simulations shown in Figure 3 were carried out for agents using their total return as payoff function (the \$-Game [26]), but with different risk profiles (different curves). From Figure 3 one can see that different risk profiles gave similar behavior, but the temperature defined above could indeed switch the micro behavior of agent from speculative behavior (small T) to fundamental behavior (large T). The two different clusters of curves correspond to two different sizes in the price histories used by the agents (see [19]).

Symmetry and its role for the long term growth of financial markets. - The concept of symmetry in markets can be studied in a different setting, namely the phenomenon of long term growth of financial markets. It will be argued that in this case, the human decision making on whether to go long (buy a share) or go short (sell a share) introduces a symmetry relevant for long term behavior of markets. The question posed is: what conditions are needed for a long term sustainable growth or contraction in a

financial market? It will then be argued, that the concept of symmetry is relevant in the answer to this question, because of an inherent asymmetry between the role of traditional market players of long only mutual funds, versus the role of hedge funds, who take both short and long positions. If one considers the traditional composure of market participants, then by far a majority of participants (pension funds, insurance companies, retail investors, ...) are only allowed to take long (buy) positions. However, since the beginning of the 1990 a steadily increasing percentage of market participants are hedge funds. They do not have the same limitations as traditional funds, and can take up long as well as short positions. The question that arises is then, as to what effect this “broken” symmetry of a historically surplus of long-only market participants could mean for the long term behavior of financial markets as well as the economy?

The asymmetric case: one large dominating fund (a mean-field solution). Let us therefore, consider the case of a given financial market, which for simplicity will be assumed to be controlled by just one large investor/fund that tries to maintain a certain constant growth rate of the market. If one call the wealth of the fund at time t , for $W(t)$:

$$W(t) = n(t)P(t) + C(t) \quad (8)$$

$n(t), P(t)$ are the number of shares held by the fund and the price of the shares at time t , respectively. $C(t)$ is the cash possessed at time t by the fund. Since the aim of the fund is to ensure a constant growth rate α of the market, the fund at every time step needs to keep on buying a certain number of shares $n(t)$. In order to see how many shares $n(t)$, are needed at time t , let us call the excess demand of shares created by the constant buying for $A(t)$. In the following it will be taken a constant in time, $A(t) \equiv A$. Therefore:

$$\frac{d n(t)}{dt} = A; \quad n(t) = A t \quad (9)$$

Since the return of the price, $R(t)$, is proportional to the excess demand one has:

$$R(t) = \ln \left(\frac{P(t+1)}{P(t)} \right) = \frac{A(t)}{\lambda} ; \quad P(t) = e^{\frac{At}{\lambda}} \quad (10)$$

With λ the liquidity of the market.

Apart from the expenses of the fund to keep on buying shares, the fund get an income from the interest rate $r(t)$ on its cash supply $C(t)$, as well as an income from the dividends of the shares $d(t)$. The balance equation for the cash supply $C(t)$ reads:

$$\frac{dC(t)}{dt} = -\frac{dn(t)}{dt}P(t) + C(t)r(t) + n(t)d(t) + C_{flow}(t, r(t), d(t), P(t), \dots) \quad (11)$$

Where $C_{flow}(t, r(t), d(t), P(t), \dots)$ takes into account the additional cash inflow/outflow which, as indicated, could for example depend on the time, the return, the dividends or the price of the market.

Since the idea is to study the ability of the fund to maintain a certain growth rate $\alpha \equiv \frac{A}{\lambda}$, it makes sense to rewrite (11) in the following way:

$$\frac{d\tilde{C}(t)}{dt} = -\alpha e^{\alpha t} + \tilde{C}(t)r(t) + \alpha t d(t) \quad (12)$$

In (12) the cash is now renormalized in terms of the market liquidity $\tilde{C} \equiv \frac{C}{\lambda}$, the interest rate is assumed constant, $r(t) \equiv r$, and for simplicity the cash flow term, C_{flow} , has been left out in (12).

In [34] it was shown that for the case of constant dividends, $d(t) \equiv d$, the fund could not maintain a “super-interest” growth, $\alpha > r$, but in certain cases it was found that the fund could enable “sub-interest” growth, $\alpha < r$. For further elaboration on the topic see [34]. In order to obtain a “super-interest” growth, as seen in the booming 1990s and more recently, one would need to include the impact of the so-called wealth effect. In (12) this is taken into account by assuming a positive feedback of the stock price on the earnings of a firm, so that the dividend is assumed to increase proportional to the price $d(t) = \frac{d_0 P(t)}{P(t=0)}$. Using this assumption (12) takes the form:

$$\frac{d\tilde{C}(t)}{dt} = -\alpha e^{\alpha t} + \tilde{C}(t)r(t) + \alpha t e^{\alpha t} d_0 \quad (13)$$

With the solution

$$\tilde{C}(t) = \alpha e^{\alpha t} \left\{ \frac{t d_0 - 1}{\alpha - r} - \frac{d_0}{(\alpha - r)^2} \right\} + e^{\alpha t} \left\{ \frac{-r\alpha + \alpha^2 + \alpha d_0}{(\alpha - r)^2} + \tilde{C}(t=0) \right\} \quad (14)$$

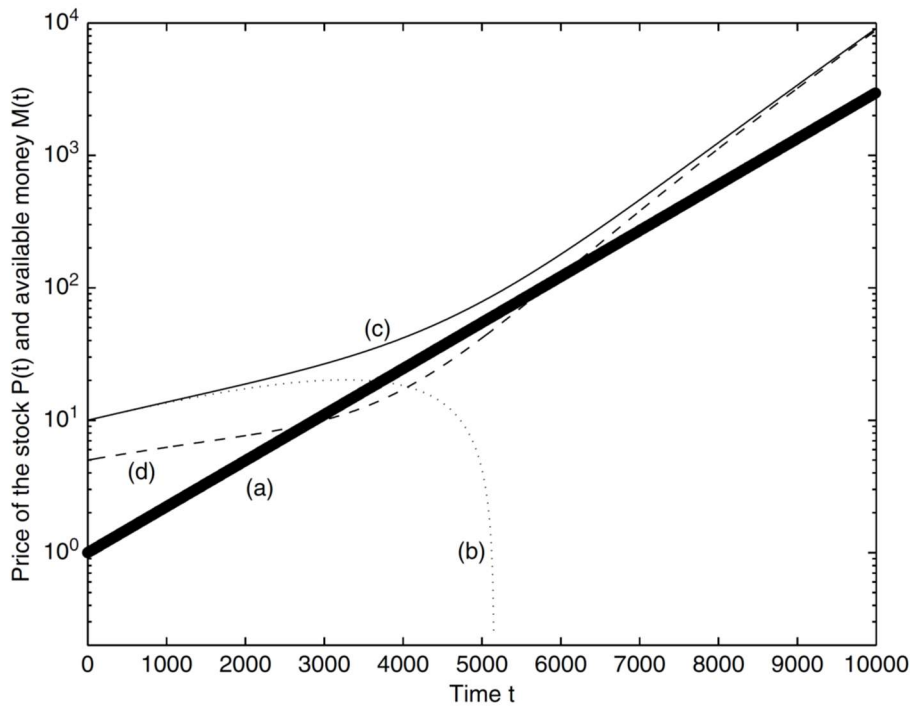


Figure 4: When is long term growth of markets possible? *The figure illustrates when cash solutions of (13) is either able, or unable, to sustain a certain fixed price growth. a) Price $P(t)$ assuming constant growth rate α . b) Cash solution of (13) which isn't able to sustain the growth of curve a) (the price of the market at some point in time becomes larger than the cash available). c) sufficient initial amounts of cash can however enable constant growth. For a detailed description see [34].*

In order to see how a positive feedback of the stock price on the earnings of a firm, could make super-interest growth possible consider first curve b) in Figure 4. The curve illustrates a solution of (21) where

a given amount of initial cash, initial dividends and an interest rate r of 10%, is *not* enough to sustain a super-interest growth of the stock market, since as can be seen, the fund runs out of cash. If on the other hand one considers curve c), which is for market behavior with the same conditions as b) except having a higher initial dividend d_0 of 8%, then a super interest growth of 20% of the markets *does* become sustainable (this is seen since the cash as a function of time is always larger than the price of the market, $C(t) \geq P(t) \forall t$). Sufficient initial amount of cash is however required, as illustrated by curve d), where a high initial dividend is not sufficient to avoid that the investor runs out of money ($C(t) < P(t)$ for $t \approx 2800$).

The exact same exercise can be done for the other asymmetric case of a fund that instead tries to profit from keeping pushing the market down by a steady rate of short selling of shares, leading to a negative growth $-\alpha$ of the market. For a longer discussion of this case, see [34]. To summarize: the solution of (13) illustrates how a broken symmetry created by having an excess of investors trying either to push up/push down the market is not only possible, but also profitable for such investors. Considering price evolution of financial markets as a long term growth phenomenon, (13) illustrates how a broken symmetry of market participants taking up more long, or more short positions, could lead to a long term growth/decline of the markets. As argued above, financial markets have always been in the situation of having a broken symmetry tilted towards buying market participants, due to the mere investment strategies of a majority of market participants. The question then is how a restoration of this broken symmetry, by the introduction of more and more short selling agents, could influence the market price formation on the long term?

Concluding remarks. - In this article we have suggested to put forward using the idea of symmetry as a tool to apply in a financial context. The concept of symmetry has had a huge impact in the way physicist think about problems in most sub branches of physics, and presenting this article, it is our hope the concept could be of similar use for practitioners/theoreticians working in the field of finance.

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