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Vindicating the Future

A Dialogical Stance on Belnap's Approach to Tomorrow's Sea Battle

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Abstract. In several texts, some authored by himself alone and some in collaboration N. D. Belnap proposed a pragmatist approach to predictions and further speech acts such as promising, betting, and wondering, in an indeterministic setting within a branching structure that shapes the future course of events. In the joint paper “Future Contingents and the Battle Tomorrow”, M. Perloff and N. E. Belnap discuss Aristotle’s famous example in the context of STIT-logic. In particular, the paper studies the pragmatics of predictions of future contingents under the background of the 480 BCE battle of Salamis: the general commanding the Greek Athenian fleet predicts that a battle will be fought on the sea while the Spartan general denies it. According to Perloff and Belnap’s analysis, this kind of prediction is neither true nor necessary at the moment of utterance, but can be *vindicated* or *impugned*, retrospectively. The central thought of the present paper is that *vindication* is a dialogical process associated to *statements made*. In the particular case that the statement expresses a prediction, the dialogical process involves plays on the *settled past truth* of the predicted contingent future. This way of analysing Tomorrow’s Sea Battle highlights the interplay of the ontological perspective and the linguistic perspective, where vindication or impugnement of the predictions are under scrutiny.

1 In honour of Göran Sundholm

It is a pleasure to dedicate the present paper to Göran Sundholm. This paper touches historical and philosophical topics that gave rise to longstanding and prolific conversations on knowledge, meaning, and logic between him and the dialogical community.

As *Dialogicians* of Lille are fond of recalling, such conversations have been flowing since at least 1998, when Shahid Rahman met Göran Sundholm for the first time in a train for Greifswald taking them to a workshop on Hugh McColl, organized by Stephen Read and Michael Astroh. The dialogue enriched substantially through the years not only by sharing advisory duties of master and PhD students, by participating at common seminars and workshops – particularly during Sundholm’s visiting Professorship at the University of Lille in 2012 –, but also by the joining of junior and senior colleagues, whose inputs led to a new direction for the development of dialogical logic known as *Immanent Reasoning*.¹ A public acknowledgement of the academic debt the group of Lille owes to Sundholm has recently been secured through his nomination as *honorary professor* of the University of Lille.

In fact, Sundholm’s historical and philosophical insights on Proof Theory, and his reflections on assertion and judgment in particular, influenced new developments on the dialogical stance in the

¹ See in particular Rahman et al. (2018).

crossing of epistemology, philosophy of language, and argumentation that reaches far beyond the initial work at Erlangen and Saarbrücken.²

2 Introduction

It is a quite common conception that while the past is fixed and determined, the future is open: while only one set of events actually takes place, before it becomes fixed in the actual course of events, there are many real possibilities that could take place. Thus, if Themistocles and Eurybiades argue over there being or not a sea battle the next day, at the moment of utterance there are two real possibilities for the following day: either there will be a sea battle or there will not. Only one will happen, but on the previous day, the two options are equally possible. While the past is fully determined, the future is open – at least to a degree compatible with free will and some form of causal indeterminism.

Although such a conception seems quite natural to our common sense, the analysis of assertions involving future contingents in such a setting gives rise to challenging logical and philosophical questions which have triggered, from the days of ancient philosophy to our contemporary developments, heated debates regarding the nature of time and of temporality in diverse fields of knowledge.³

In several texts, N. D. Belnap and co-authors have strived towards answering these challenges, focusing mainly on assertions involving future events, such as predictions, promises and bets. Their work combines speech-act theory with a model-theoretical semantics congenial with *local* indeterminism, i.e., the indeterminism that results from considering only the transition between one initial set of events to a second one, disregarding past transitions.⁴

As we will see, regarding predictions, their core idea is that, since the future is open, predictions cannot be assessed at the moment of utterance, they require a second time reference for their assessment. At that second time reference, the prediction can be vindicated or impugned, depending on the actual course of events.

In this paper, we pursue Belnap et al.'s approach to predictions but we contend that the pragmatic and normative dimensions of the speech acts of predicting and vindicating are best rendered in a dialogical framework. Taking Perloff and Belnap's (2011) interpretation of the sea battle prediction, we first recall the main lines of their construal before developing a dialogical reconstruction of their scenario.

3 Belnap et al.'s STIT framework for vindicating predictions

3.1 The sea battle scenario

In their 2011 paper “Future Contingents and the Battle Tomorrow”, M. Perloff and N. D. Belnap focus on predictions. Taking inspiration from Aristotle's problem regarding future contingents (*De Interpretatione*, 9), they provide a scenario to flesh out their theory: in 480 BCE, the Athenian Themistocles, in command of the Greek fleet fighting the Persians, predicts that a battle will be fought

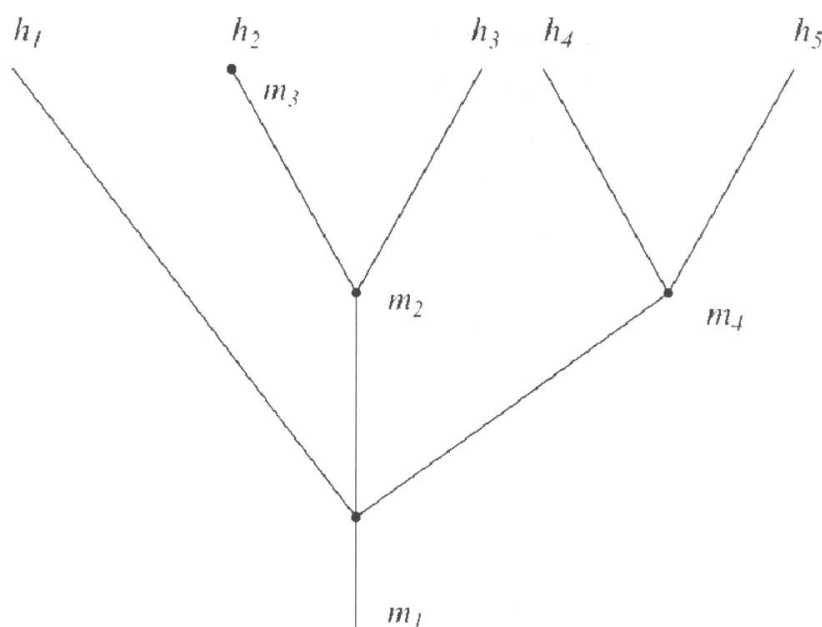
² Cf. among others, Keiff (2007), Marion (2010), Redmond (2011), Ginzburg (2012), Fontaine (2013), Marion & Rückert (2016), Shafiei (2018), Dutilh Novaes & French (2018), Dutilh Novaes (2021), Rückert (2019), Crubellier et al. (2019), Fermüller (2021), McConaughy (2021), Sterling (2021), Clerbout & McConaughy (2022), Iqbal (2022), Dutilh Novaes (2022), Lion (2023), Klev (2023)-forthcoming.

³ Just to mention a few from a very large list, see Prior (1967), Thomason (1984), van Benthem (1991), Bobzien (1993), Gaskin (1995), Crubellier (2010, 2011), Coope (2005), Øhrstrøm & Hasle (2015), Goranko & Rumberg (2020). In the context of his analysis of the Master argument, Vuillemin (1984) even classifies philosophical systems with regard to the endorsement or rejection of the principles involved in the future contingents.

⁴ See Belnap and Green (1994), Perloff and Xu (2001), Belnap (2002).

the next day at sea, while the Spartan Eurybiades denies it. Under the conception of an indeterminate future, this kind of prediction is neither true nor necessary at the moment of utterance. In order to be able to evaluate such a prediction, a later moment is required at which the outcome of the prediction has been settled; the pragmatics of predictions thus requires a “double time reference” (Perloff and Belnap 2011, p. 593; Belnap 2002). Indeed, predictions cannot be assessed on the spot, as a mere description could, since they involve something that has not yet happened. Predictions thus call for a second moment, the moment of assessment, which comes once it is possible to determine whether or not the prediction was fulfilled. Thus, the prediction can, retrospectively, be vindicated or impugned.

Building on the branching tree account of indeterminism provided by Prior (1967) and Thomasson (1970, 1984), Perloff and Belnap (2011, pp. 597–588) propose a general model for speaking of future contingents, predictions, and assessments. This model is a branching tree in which histories branch to the future but not to the past; every node is a moment m_i that occurs, and an uninterrupted branch of moments constitutes a history h_i (a particular course of events). In the figure they provide, reproduced below in Figure 1, there are four (decisive) moments, m_1 – m_4 , which mark a branching in space-time between at least two different histories or alternative possible courses of events. In Figure 1, there are five histories, h_1 – h_5 .

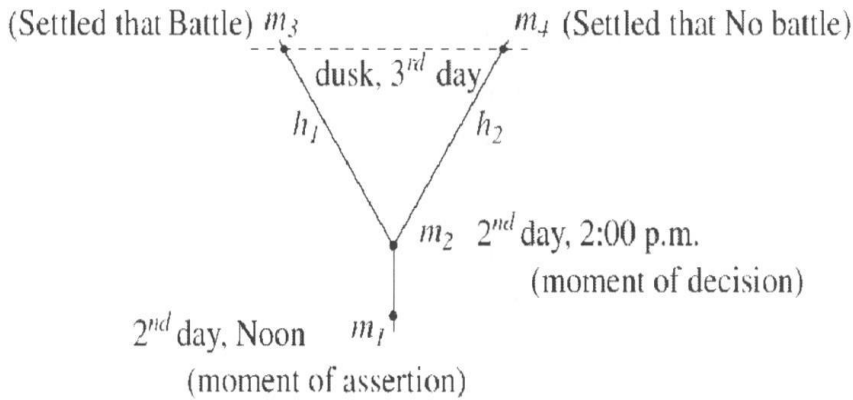


branching histories from Perloff and Belnap (2011, p. 587)

Figure 1: Tree of

The tree structure entails that a particular moment can be shared by many histories: m_1 is shared by all five histories, while m_2 is shared by h_2 and h_3 but by none of the others, and so on. What is more, “ m_2 is in the future of possibilities of m_1 , not simply in its future. A future of possibilities, unlike a future portion of a history, can contain incompatible moments” (Perloff and Belnap 2011, p. 587). In order to indicate that moment m_i occurs on history h_j , Perloff and Belnap use the notation “ m_i/h_j ”.

If we take a simplified version depicting the sea battle prediction, we obtain just one branching (Figure 2).



Perloff and Belnap (2011, p. 589)

Figure 2: The sea battle prediction from

Under this representation m_1 is the moment of assertion, where Themistocles claims that there will be a sea battle the next day while Eurybiades claims that there will not be a sea battle – thus, m_1/h_1 and m_1/h_2 , or more briefly, $m_1/(h_1, h_2)$. The moment m_2 (also shared by both histories) is the moment when the Persians decide whether or not to engage battle the next day, thus branching into two possible courses of events, one in which a sea battle occurs (branching towards the history h_1), the other in which no sea battle takes place (branching towards the history h_2). So at the moment of assertion m_1 , it is *possible* that there will be a sea battle (at m_3/h_1) or that there will not be a sea battle (at m_4/h_2). According to Perloff and Belnap, these are *real, objective* possibilities, they are not just linguistic or epistemic possibilities.

3.2 Truth and settled truth

An important notion in this construal is the distinction between mere truth and settled truth (Perloff and Belnap 2011, p. 591): a settled truth is what is true no matter what the future brings. Thus, at the time of assertion, it is only *possible* that there will be a sea battle since it is *also* possible that no sea battle will take place (we are dealing with future contingents). None of the two possibilities is a *settled truth*, none is true no matter what the future brings, they both depend on the future courses of events. However, at m_3 and m_4 , the matter is settled: it is a settled truth that there was a sea battle at m_3 , in history h_1 , while it is a settled truth that there was no sea battle at m_4 , in history h_2 . While retrospectively, since h_1 actually happened, it is correct to say that Themistocles was right, that it was true there would be a sea battle the next day, it is incorrect to say that it was a *settled truth* at that time. For there to be a settled truth of future contingents, the event must have occurred, we must be in a history in which the event occurred.

This notion of *settled truth* thus involves the use of histories. That p is a settled truth at a moment (i.e., that p is true in all the histories that cross the moment) is written **Sett**: p . This notion validates the implication from a fact p to its settled truth **Sett**: p :

$$(1) p \supset \mathbf{Sett}:p \quad \text{for elementary } p$$

As the authors point out, the notion of settled truth is not the notion of necessity in contemporary modal logic; it just amounts to the idea that the truth of p at a moment does not depend on what happens later. In other words, if p is elementary and true at m_1 then being settled true does not say more than that it is true when it is.

As a matter of fact, the notion of *settled truth* comes close to Aristotle’s notion of the actual necessity of events: if something has happened, then it is necessary; but we cannot infer from that that it was

absolutely necessary since at some point (e.g. the previous day), it was not necessary for the event to happen.

What is, necessarily is, when it is; and what is not, necessarily is not, when it is not. But not everything that is, necessarily is; and not everything that is not, necessarily is not. For to say that everything that is, is of necessity, when it is, is not the same as saying unconditionally that it is of necessity. Similarly, with what is not. And the same account holds for contradictories: everything necessarily is or is not, and will be or will not be; but one cannot divide and say that one or the other is necessary. I mean, for example: it is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for a sea-battle to take place tomorrow, nor for one not to take place – though it is necessary for one to take place or not to take place. (Aristotle, *De Interpretatione*, 19a23–32; transl. Ackrill)

Such a principle is known by Al-Fārābi (*Al-Taḥṣīl al-sa‘āda*), and Ibn Sīnā (*Al-Shifa’: al-Ilahiyyat*) after him, as *conditional necessity*.⁵ The implication (1) can be understood as a (propositional) interpretation of this Aristotelian notion of conditional necessity in a setting for models with branching histories.⁶

The framework developed by Perloff and Belnap (2011) includes two monadic propositional time operators, **Was** and **Will**, used to express respectively that something was true and that something will be true.⁷ Thus, given suitable definitions of these operators, the framework also validates another Aristotelian principle, namely, the necessity of the past, as opposed to the indeterminacy of the future.⁸

(2) **Was:** $p \supset$ **Sett:****Was:** p for elementary p

Note that the restriction to an elementary p prevents substituting a present or past p with a future p , which would destroy the necessity of the past and the indeterminacy of the future which the theory is precisely trying to safeguard.⁹ For instance, in (1) and (2), if we substitute p by **Will:** p , we obtain these two implications for which it is easy to find countermodels:

⁵See Miller (1986) and Adamson (2006). Ibn Sīnā introduces explicit intervals of time in order to shape modal syllogisms where conditional necessity occurs (Street 2006, 2008, 2013; Strobino 2015, 2016). William of Ockham later dubs this kind of necessity “accidental necessity”, which he thus justifies in his *Ordinatio*, I, Prologue, q. 6: “I add this because of propositions that are necessary *per accidens*, as is the case with many past tense propositions. They are necessary *per accidens*, because it was contingent that they be necessary, and because they were not always necessary” (quoted from Plantinga 1986, p. 245). This passage is discussed by Plantinga (1986, sections III and IV).

⁶The link between $p \supset$ **Sett:** p and Aristotle’s notion of conditional necessity was already established by Jules Vuillemin (1977, 1979, 1984), who introduces conditional necessity as an axiom in his reconstruction of the *Master Argument* of Diodorus Cronus, using the Polish notation $p \supset \mathbf{L}p$. However, since his formal rendering is not that accurate and the semantic is missing, Vuillemin’s reconstruction has triggered a harsh criticism; see Denyer (1998, 2009).

⁷“The truth-value of future-tensed sentences at a moment depends on what course history takes. [...] **Will:** A is true at a moment and history with respect to a model and a moment of use $(M, m_c, m/h \vdash \mathbf{Will}:A)$ just in case there is a moment m_i on h_i later than m at which A is true $(M, m_c, m_i/h \vdash A)$. With respect to the past tense, we have **Was:** A is true at a moment and history with respect to a model and a moment of use $(M, m_c, m/h \vdash \mathbf{Was}:A)$ just in case there is a moment m_i prior to m at which A is true $(M, m_c, m_i/h \vdash A)$. The idea is to shift along the present history, h_i , to earlier moments to check for A ” (Perloff and Belnap 2011, p. 592). Analogously one can define **Will-always:** A , and **Was-Always:** A , by quantifying universally over the moments in the relevant history.

⁸Besides the passages about future contingents in the *De Interpretatione*, the notion of deliberation shows how important the indeterminacy of the future is for Aristotle: as his saying goes, we do not deliberate on what cannot be otherwise, that is, since the past is determinate, we deliberate only on “what is future and contingent” (see *Nichomachean Ethics*, 1139b5–11).

⁹The restriction of the necessity to the past, to assertions that are “strictly” about the past, has been explicitly formulated by Ockham. For a thorough discussion on Ockham’s views on this restriction see Plantinga (1986).

(1*) **Will:** $p \supset$ **Sett:** **Will:** p

(2*) **Was:****Will:** $p \supset$ **Sett:****Was:****Will:** p

Less intuitive is the operator for possibility **Poss**, which, if defined as the dual of **Sett**, yields: **Poss:** p is true at m iff for at least one history that passes through m , p is true at m (Belnap, Perloff & Xu 2001, p. 32). This validates, for elementary p , the implication **Poss:** $p \supset p$, which amounts to stating that if the possibility of p is true at m on h , then p has been actualized at m (on h); this possibility then trivially implies p at the same moment on h . Thus, for elementary p , the truth of **Poss:** p follows the principle of plenitude.¹⁰ Moreover, in such a setting, for elementary p , **Poss:** p , **Sett:** p , and p , are indistinguishable.

However, in a framework for an open future, an alternative notion of possibility gives up the duality with **Sett**, defining the possibility fA as being true iff there is a history h passing through m such that A is true at some non-actual future n on h , $n > m$. See Øhrstrøm & Hasle (2015, section 4d, definition g) who add it to the Peircean framework for indeterminism.¹¹ In fact, their notation indicates that it constitutes an alternative to the notion of open future **F**, rather than an alternative way to define possibility. Accordingly, such a notion of possibility excludes not only the past (the possibility cannot be actualized in the past) but also the present time of utterance.¹²

3.3 Vindicating predictions and double time reference

An account of vindication and of impugment can be provided using (i) the double time reference embedded in the speech act of predictions, which distinguishes the moment of utterance and the moment of assessment, together with (ii) the semantic definitions of settled truth (**Sett**), of a future occurrence (**Will**), and of a past occurrence (**Was**). A first account is provided in the following way, which is however deemed inadequate because restricted to the sea battle example and not generalizable to cases of embedded future operators (Belnap 2002, p. 49).

(A) At the later moment, m_3 , Themistocles' assertion *There will be a sea battle tomorrow* made to Eurybiades at m_1 is *vindicated* just in case it is *settled* at m_3 that *There is a sea battle is true* (paraphrase of Perloff and Belnap 2011, p. 594).

Perloff and Belnap however propose this second account of vindication, which they consider to be the correct one.

¹⁰Vuillemin (1984) also attributes this principle to Aristotle, obtaining it from the dual of Aristotle's accidental necessity. This, however, seems to contradict Aristotle's explicit rejection of plenitude.

¹¹ As discussed in Rahman & Seck (forthcoming), a precursor of such an alternative approach to possibility can be found in the modal logic of the influential post-Avicenean Islamicate thinker Shihāb al-Dīn Suhrawardī (549 H/1155 CE) – cf. *al-Ishrāq* 1999, p. 16-17.

¹²A further notion of possibility within a framework for indeterminacy, based on identifying stretches of time across histories, was indicated to Rahman in a personal email by Tero Tulenheimo (Tampere). Tulenheimo drew our attention to Belnap and Green's (1994, p. 372) notion of *instant t*, i.e. an equivalence class of moments that results by correlating moments m' , totally disjoint from the moment m on the actual flow of time h . Then, for this other notion of possibility labeled $\langle \text{Poss} \rangle$, $\langle \text{Poss} \rangle : A$ is true at m iff there is at least an alternative flow of time whose now-point m' is correlated with m , and A is true at m' . Given this primitive, it makes sense to ask what might have happened at this very instant. However, eventually, Belnap and Green (1994) reject correlating moments across histories for being scientifically and intuitively unclear.

(B) At the later moment, m_3 , Themistocles' assertion made to Eurybiades at m_1 is vindicated just in case it is settled at m_3 that his assertion, (*) “There will be a sea battle tomorrow” was true – but by no means was it settled true – at m_1 . (Perloff and Belnap 2011, p. 595).

The predictions made by Themistocles and Eurybiades – and, as a matter of fact, any prediction whatsoever – are assertions that a certain event *will* or *will not* occur. The first account of vindication only considers whether or not the predicted event did occur in the course of events, i.e., whether or not it is a *settled truth*: the prediction says there *will* be such an event, the vindication says that it is *settled* there *was*. The second account of vindication does not bear on the fact as much as on the prediction: the prediction is vindicated at the moment of assessment when it is *settled* at that moment that it *was* the case that the event *will indeed* take place or not, depending on the prediction. This is not the same as saying that, say, Themistocles's, prediction *is true* when he makes his prediction, since such accounts generate paradoxes paving the way to determinism or merely epistemological open futures. It also differs from saying that at the time of prediction, Themistocles's prediction is *settled true*, since at that time the possible future branches towards h_1 or h_2 . The account states that *in the future* (the moment of assessment) it is settled that in the past it *would indeed* be the case. In the dialogical reconstruction below, we will keep this second account of vindication of predictions.

3.4 Recapitulation of Perloff and Belnap's construal

To sum up, according to Perloff and Belnap's analysis of the open future, predictions of the kind “There will be a sea battle tomorrow” are neither settled true nor necessary at the moment of utterance; they however call for a later moment of assessment at which such speech acts can be vindicated or impugned. Vindication and impugnement are thus produced in relation to (i) the original utterance moment, (ii) the actualized history h_i , and (iii) an assessment moment within h_i , at which the taking place or not of the predicted event has been *settled true* – i.e., true whatever the future brings.

This perspective, which still endorses the validity of the Excluded Middle in relation to the predictions, concurs in this respect with the family interpretations of Aristotle's view on the matter called, after Rescher (1963) and Adamson (2006), Al-Fārābī's solution-B.¹³

What characterizes Belnap et al.'s proposal is (a) the conception of a genuinely dynamic open future that precludes attaching to each utterance of a prediction a privileged history, known as the actual future or the thin red-line,¹⁴ which fixes the ways the future course of events will go; (b) the speech-act analysis of the notion of vindication and its role for assessing predictions. In relation to the latter,

¹³Belnap's approach contrasts on one side with the so-called *standard interpretation* of Aristotle's solution, according to which the law of Excluded Middle fails, and on the other with the *non-standard interpretation*, which actually consists in a family of several variants that endorse the validity of that law. The main point of the non-standard interpretations is that the components of the disjunction of future contingents are true or false *indefinitely* – in the sense that for the moment, no state of affairs obtains that makes it have this or that truth-value. Such interpretations that hold the validity of Excluded Middle have been attributed, among others, to Boethius and Abelard (Mignucci 1998; Seel 2001a, b, c). Tulenheimo (2020a,b) penned a formal semantics for the Boethius–Abelard approach. More precisely, as Tulenheimo (2001, p. 66) already pointed out, his own approach can be understood as a model-theoretic interpretation of the position that Vuillemin (1984, chapter 8) and Seel (2001, p. 28) identify with one of Carneades (214–129/8 BCE), who seems to have been the first philosopher to have held the view that future contingent propositions are made true by future facts, not by present facts about a future event – the latter might be attributed to Ammonius (Seel 2001c, pp. 239–246; cf. Seel (ed.) 2001; 2001a, p. 36; 2001b, p. 233). Belnap and Green (1994) also accept the law of Excluded Middle, but contest the non-standard solutions known as *Ockhamist Theories*. Ockhamist theories assume a branching course of events but either do not distinguish actual courses of events from purely possible ones, or take it that each utterance time brings with it one privileged actual future – conceived in a rather static manner –, called the *thin-red-line* (Øhrstrøm&Hasle 2015, section 5.3).

¹⁴Cf. Øhrstrøm (1981), Malpass & Waver (2012), Borghini & Torrenco (2013).

a predictive assertion has deontic consequences for the speaker: the speaker deserves credit if the prediction obtains, and discredit if it does not obtain.

3.5 An issue with the semantization of pragmatics in Perloff and Belnap

The issue we have with Belnap et al.'s construal is that they separate the pragmatist analysis from the semantic levels, so that the normative dimension of the speech acts they study (predictions, promises, bets) is either studied from the pragmatist perspective of the speech acts, or from the perspective of semantics, which then seeks to reduce the pragmatic and normative dimension to the ontological model expressing the prediction situation.¹⁵ The study of vindication and impugment, we submit, should be rooted in the intertwining of normativity and meaning.

A clear case of the pragmatization strategy is the following text, where the authors suggest to leave the semantics of operators unchanged but switch to a pragmatist perspective that stops the syntactic analysis as soon as assertions of the form **Will**: α are reached.

We are now in a position to see that on the present account of assertion, it makes sense to talk of asserting “*Will*:(the coin lands heads)” exactly because assertion constitutes a way of closing the history parameter—not indeed semantically (the semantics of the asserted sentence is unchanged), but pragmatically, by the very act of assertion. (Belnap et al. 2001, p. 174).

The main motivation of this strategy is that it also allows to deal with embedded temporal operators:

There is, however, a more general and more satisfactory account of vindication, one which makes even better use of double time references within the doctrine of branching histories and its accompanying semantics. This account no longer requires syntactic unpacking of the assertion. We use, instead, the rich semantic content of the assertion (*) at the moment of utterance $m_l (= mc)$. Since we are committed to the relativization of future-tensed assertions to moment/history pairs, the question becomes, for which moments and for which histories should we be evaluating Themistocles' assertion? (Perloff and Belnap 2011, p. 594)

However, the latter text announces a second strategy, that proposes to give a semantic account of the speech acts under consideration, where, according to the authors, the introduction of a “double time reference” and a model theoretical rendering of *settled truth* allows a general approach on speech acts in the presence of indeterminism. They recapitulate the merits of their model in the following fashion.

Among the advantages to be gained in employing a semantics that treats assertion as publicly normative, calling for vindicating or impugning, we count our ability to understand assertion as one of a variety of speech acts, including asserting, betting, promising, all of which, in the presence of indeterminism, are illuminated by similar considerations. Just as an assertion about an undetermined future is vindicated or impugned at a later moment, when the assertion is settled true or false, a promise to see

¹⁵Taking up the terminology introduced by Lorenz (1987, pp. 65–66; reprint 2010, pp. 71–72), the first pitfall consists in a pragmatization of semantics, while the second, which concerns Perloff and Belnap's (2011) construal, consists in a semantization of pragmatics. Because the semantic perspective is not generalizable for cases with infinite chains, at that moment they switch to the purely pragmatic perspective, without bringing the two perspectives in one unified theory of meaning.

to something is either kept or not kept only at a later moment, when the matter is settled. In like manner, a wager made about an undetermined future cannot come due for settlement until a later moment. (Perloff and Belnap 2011, p. 596; our italics)

And later, on the next page,

We suggest that a semantic account invoking double time references is applicable, with suitable adjustments, for dealing with the whole range of speech acts whose assessment must wait, in the presence of indeterminism, for an appropriate moment. Betting, promising, and even the speech-act-like case of wondering, which might be thought of as wanting to know about a not yet settled future, *all require for satisfactory assessment the earlier moment of the original speech act as well as the later moment of assessment*. On this account, every history through the moment of assertion receives equal treatment. *Assertions are vindicated or impugned, promises fulfilled or not, bets resolved one way or the other, and wonderings laid to rest at the later moment, a moment when the matter is settled one way or the other*. (Perloff and Belnap, 2011, p. 597; our italics)

In these passages, a descriptive shift occurs: the normative dimension of speech acts, clearly recognized (p. 590, again p. 596), is transformed into semantic characteristics within the model. The first step is to reduce speech acts to acts, and thus to moments within the model, on the same level as any other moment; the second step is to recognize the normative aspect of speech acts depending on future contingents, which calls for a later moment of assessment, and to semanticize this aspect through the double time reference: a speech act such as a prediction is linked to a second moment in the model, the moment of assessment. The normative dimension of speech acts is thus reduced to the semantic model, in what can be called after Lorenz (1987, p. 65; reprint 2010, p. 71) a semantization of pragmatics. Yet, the vindication of a prediction must consist in a public recognition that the prediction was correct; there is no private vindication, the moment must be public and made public through a speech act. There is thus an intimate link between the speech act of prediction and the speech act of vindication, a link that cannot be reduced to a reference (the assessment refers to the prediction and to the content of the prediction): these two “moments” are of the same nature, and they are of a different nature from other “moments” such as the moment of decision (the Persians decide to attack) or the moment of happening (there is a sea battle). The semantic model thus captures the *meaning* embedded in the normative dimension of predictions and their vindication or impugment, but this semantization loses the *normative* aspect.

This paper’s proposal consists in adopting the framework of dialogical logic in order to study the link between a prediction made and its vindication or impugment, while keeping fully alive the normative and pragmatic dimension of these speech acts. What needs to be explicitly added in a *descriptive* manner to the model-theoretic construct is already embedded in the dialogical framework in a *normative* way, i.e., without transforming a normative dimension into a descriptive one (which reduces the dynamics of norms to a static description of them). While the semantic model expresses the ontology but semanticizes (and thus loses) the normative aspect, as will be shown below, the dialogical approach to the vindication of predictions immediately carries a normative aspect in the dialogue between the two players while still allowing to recuperate the ontological model of indeterminate futures.

4 Vindicating predictions in the dialogical approach

The dialogical approach to predictions in the spirit of Belnap et al.’s construal consists in a public assessment of a prediction depending on the actual course of events. A *prediction made* is a statement expressing a future contingent, thus taking the form **Will:***A* or **Will:** $\neg A$, made by an agent at some

point in time. This disjunction does not express the law of Excluded Middle, but in the particular case of *tomorrow's* sea battle, the internal or external negation does not make a difference. In order to focus on predictions, we will adopt the internal negation (the dialogues can however be adapted to the external one for a focus on the Excluded Middle).

The public assessment compares the merits of an opposite pair of predictions made depending on the actual course of events. In the confrontation between the opposite predictions and the actual course of events, the prediction that stands the test is vindicated, while the other is impugned. The dialogical framework provides a natural setting for this confrontation between predictions and facts: in the public assessment of predictions, someone, called the Proponent (he, by convention), will argue for one of the predictions while someone else, called the Opponent (she, by convention), engages her responsibility regarding what the actual course of events has been. In this setting, sharing the burden of proof warrants the impartiality of the assessment: the person who is responsible for vindicating a prediction and impugning its opposite is not the person responsible for determining what the actual course of events was.

In this framework, the Proponent will have to defend a prediction made depending on the course of events provided by the Opponent. The “second time reference” does not have to be a particular time of assessment defined in the model (such as m_3 or m_4), since the assessment time is *de facto* given by carrying out the public assessment in the dialogical form: the “second time reference” that is required in order to assess the prediction can occur, in the dialogical setting, at any time once the outcome of the prediction has been settled. Thus, in the case of the sea battle prediction, m_1 is the moment of utterance, m_3 is the moment from which it is *settled true* that a sea battle took place the day after m_1 , and m_4 is the moment from which it is *settled true* that no sea battle took place on that day. What is required at the time of assessment, whatever that time may be, is a reference to a time where the outcome has been settled, so m_3 or m_4 or any time after that in the ensuing histories (thanks to the principle of the determinate past). If the Proponent is capable of defending one of the predictions made regardless of the way the Opponent defends which course of events took place and regardless of the way she challenges his claims (we can imagine a series of Opponents defending and challenging the Proponent in all possible ways), then the Proponent is said to have a “winning strategy” and the prediction made is vindicated while the other prediction made is impugned.

4.1 An informal presentation of the dialogue and key concepts

Adopting Perloff and Belnap’s second account of vindication of a prediction, vindicating requires that *it is settled* that *there was a moment* at which it is true that *there will (or will not) be* a certain event. The dialogical approach makes this requirement the Opponent’s responsibility. The Proponent will have to vindicate one of the two predictions based on the course of events warranted by the Opponent. Informally, the Proponent starts the public assessment (or play), by claiming that *either the prediction of Themistocles is vindicated, according to which there will be A the next day, or the prediction of Eurybiades is vindicated, according to which there will not be A the next day, provided that the Opponent determines whether the next day it is settled that it was the case at the moment of prediction that A would happen, or that it would not happen*. This claim is the object of the public assessment and can be expressed in the following way, which will be detailed below.

! Will: $A@tmr(m_1) \vee$ Will: $\neg A@tmr(m_1)$ [f:(Sett:Was:Will: $A@tmr(m_1)) \vee$ f:(Sett:Was:Will: $\neg A@tmr(m_1))]$

If the Proponent can defend Themistocles’s statement that a sea battle would be fought the next day by using the Opponent’s claim that the course of events is such that it is settled that a sea battle would be fought the next day, then Themistocles’s prediction is vindicated and Eurybiades’s is impugned. The same goes, *mutatis mutandis*, for vindicating Eurybiades’s prediction.

The evaluation starts with Opponent accepting her role, i.e., accepting to defend either that it is a settled truth that it was the case there will be A the next day or the other disjunct; the Proponent then

accepts his own role, i.e., to defend the first disjunction and thus to vindicate one of the two predictions. Once the roles have been clearly distributed, the Opponent brings up a challenge to the Proponent's claims; he can then either respond to the challenge, or challenge one of the Opponent's own claims. The public assessment thus proceeds with the Opponent and the Proponent reacting in turn to the other's actions by either challenging a claim made by the other or answering to a challenge raised against one of their own claims. The way the challenges and answers can be carried out is defined by the *particle rules*. These rules determine how any logical constant stated can be challenged and how such a challenge can be answered. The particle rules thus define, at once, what kind of action and reaction is appropriate in a dialogue game (here, the public assessment), and what the meaning of the logical constants is. The dialogical framework is thus a pragmatist framework whereby syntax and semantics are set at the same time.

The particle rules (specified below for **Will**, **Was**, **Sett** and **f**) define the appropriate challenges and answers for each logical constant; they go from what is more complex, the statement with the logical constant, to what is less complex, what is required in order to be entitled to make a claim. There is thus a contextual notion of assertion at play, which is rooted in Wittgenstein's language games and has been one of the main tenets of Paul Lorenzen, who conceived dialogical logic by the turn of 1958, and of Kuno Lorenz after him.¹⁶ As Mathieu Marion (2010) has shown, the core principles of the dialogical framework come very close to Robert Brandom's (2017, 2002) inferentialism, which provides rich insights on assertion understood as the most basic move in the public space for conceptual thinking enacted by games of giving and asking for reasons. More precisely, such a move (an assertion) engages further moves following the interaction of obligations and rights this move commits and entitles to.

What is more, the fact that in a dialogue game the interaction between the players breaks a complex claim down into simpler claims (of which the burden of proof may be distributed between the two players) conforms with the process of vindicating a prediction: the entire prediction made is taken as object of scrutiny, and the public assessment consists in making sure the Proponent is indeed warranted to hold each of the claims that must be held for the prediction to be vindicated depending on the course of events warranted by the Opponent's role in the exchange. The evaluation starts with the predictions made in their entirety and goes down to check the details.

4.1.1 An informal dialogical account of vindication

In the case of the sea battle scenario, we obtain two rounds for a full public assessment. We will first develop one round in an informal fashion, point out how the second round proceeds, and then introduce the formal notation in the dialogical framework before carrying out the plays using the formal notation.

The public assessment really starts when the Opponent has claimed that *it is settled that it was the case that there will be A the next day or it is settled that it was the case that there will not be A the next day* (move 1), and the Proponent has claimed in turn that *there will be A the next day (Themistocles) or there will not be A the next day (Eurybiades)* (move 2). A visual representation of the first round is provided in Figure 3 below. The answers to the challenges are written on the same line as the challenge, the order of the moves needs to be followed using the indications in the outer columns.

Move 3. It is the Opponent's turn, she challenges Proponent's claim of move 2 by asking who is right, i.e., which of the two disjuncts Proponent will try to vindicate.

Move 4. It is the Proponent's turn, instead of answering the Opponent immediately, he first challenges her claim of move 1. He will come back to her challenge only much later, at move 12. Challenging claim 1, the Proponent asks the Opponent to choose which disjunct she claims to be true.

¹⁶ See the seminal paper of Lorenzen (1960) and further developments in Lorenzen & Lorenz (1978).

Move 5. The Opponent answers the challenge by choosing the first disjunct (in this round; the second disjunct is chosen in the second round), i.e., she claims that at m_3/h_1 , it is settled that it was the case that there will be A the next day.¹⁷

Move 6. The Proponent challenges the previous move, asking whether it was settled for the history (or course of events) h_1 .

Move 7. The Opponent answers by saying yes, allowing her to delete the settled clause and simply claim that at m_3/h_1 it was the case that there will be A the next day.

Move 8. The Proponent challenges the previous move by asking when it was the case.

Move 9. The Opponent answers by saying that at the moment m_1 common to the histories h_1 and h_2 ($m_1/(h_1, h_2)$),¹⁸ there will indeed be A the next day.

Move 10. The Proponent challenges that claim by asking when it will be the case.

Move 11. The Opponent answers by saying that at the moment m_3 in the history h_1 (m_3/h_1), A happened.

Move 12. The Proponent now knows the actual course of events. He answer the pending challenge on his claim 2, and chooses which prediction he will vindicate. Here, since the Opponent gave her word that A happened at m_3 , actualizing history h_1 , the Proponent decides to vindicate Themistocles's prediction. He thus answers the challenge by saying that at $m_1/(h_1, h_2)$, Themistocles is right that there will be A the next day.

Move 13. The Opponent challenges this claim by asking when it will happen.

Move 14. The Proponent can then recall what the Opponent said move 11: that A happened at m_3/h_1 .

The Opponent has nothing more to say, so the Proponent has successfully vindicated the prediction he chose, namely, in this case, Themistocles' prediction. However, if we look at the choices the Opponent made during the public assessment, we can notice that she had to make a decision for move 5: she had to choose which disjunct was actually the case, and she chose the first disjunct. Thus, a second round of public evaluation can take place in which the Opponent chooses the second disjunct. In that case, the Proponent would not be able to vindicate Themistocles's prediction, he would not have the means to ground this vindication in the actual course of events guaranteed by the Opponent. However, if the Opponent chooses the second disjunct, by choosing to defend Eurybiades's prediction at move 12, the Proponent can successfully vindicate that prediction and impugne Themistocles's.

4.1.2 Normative dimension and dialogical meaning explanations

It is now clear that the dialogical approach to the vindication of predictions is set in a pragmatic and normative exchange which however reflects the ontological structure represented in the tree structure of Perloff and Belnap's (2011) model: the indeterminacy of the future is embedded in the choice the Opponent must make, but the determinacy of the present and past is embedded in the fact that the Opponent must actually choose one alternative. Her *various options* correspond to the branching possibilities, but her actual *choices* correspond to the course of events. The normative dimension is safeguarded (i) in the public dimension of dialogues, (ii) in the fact that the meaning of the logical constants is defined through the particle rules defining the appropriate interaction, and especially (iii) in the fact that the Proponent can try to vindicate a prediction that has not been fulfilled (eg. Eurybiades's prediction when the Opponent gave her word the sea battle did take place), in which case he will be unable to ground his choice of prediction, thus losing the play and failing to successfully vindicate his choice of prediction.

The level of plays, i.e., the level of actual dialogues between the players, reflects the public recognition of a vindication when it is successful: the Proponent wins the play, making it clear to all that the vindication was successful. When he does not win, it is clear that there is no public recognition of his claim. The distinction between the play level and the strategy level, which takes all the relevant plays into consideration, is a characteristic feature of the dialogical framework that makes it fall

¹⁷ We adopt here Perloff and Belnap's notation m_i/h_j mentioned above in section 3.1 for indicating that moment m_i occurs on history h_j .

¹⁸ As mentioned above in section 3.1 $m_i/(h_j, h_k)$ stands for m_i/h_j and m_i/h_k .

neither in the pragmatization of semantics, nor the semantization of pragmatics flagged out by Lorenz (1987, 2010).¹⁹

move	Opponent	Challenged move	Proponent	move
			Thesis for vindicating a prediction	0
1	Claim at $m_1/(h_1, h_2)$ It is settled that it was the case that there will be A the next day OR it is settled that it was the case that there will not be A the next day	0	Claim at $m_1/(h_1, h_2)$ There will be A the next day (Themistocles) OR there will not be A the next day (Eurybiades)	2
3	Challenge at $m_1/(h_1, h_2)$ Who is right?	2	Claim at $m_1/(h_1, h_2)$ Themistocles is right, there will be A the next day (choice of the first disjunct)	12
5	Claim at m_3/h_1 It is settled that it was the case that there will be A the next day (choice of the first disjunct)	1	Challenge at $m_2/(h_1, h_2)$ Which of the two disjuncts claimed at 1 is the case?	4
7	Claim at m_3/h_1 Yes, it was the case (at this moment) that there will be A the next day.	5	Challenge at m_3/h_1 Is this the case at h_1 ?	6
9	Claim at $m_1/(h_1, h_2)$ There will be A the next day	7	Challenge at m_3/h_1 When was this the case?	8
11	Claim at m_3/h_1 A	9	Challenge at $m_1/(h_1, h_2)$ When will this be the case?	10
13	Challenge at $m_1/(h_1, h_2)$ When will this happen?	12	You said with your own move (11) that this A will happened at m_3/h_1	14

¹⁹If in the standard dialogical framework the normative dimension is present but implicit, it can be explicitly expressed in the object language in the variant Immanent Reasoning.

4.2 Vindicating predictions made in the dialogical framework

Let us now express this informal exchange in a formal way, using the tools of the dialogical framework. We will assume to some extent a basic knowledge of the dialogical framework, as presented for instance in the first sections of the *Stanford Encyclopedia of Philosophy* entry on “Dialogical Logic” (Clerbout, McConaughey 2022) or in the present volume’s contribution by Rahman and Gallois. Before carrying out the play and building the strategy, we will present elements required for the vindication of predictions which are not present in the standard framework.

4.2.1 Time and Modality

We will present the following three key elements:

1. adverbials for the indexical tomorrow,
2. dialogical contexts,
3. moves involving the operators **Sett**, **f**, **Was**, and **Will**.

Adverbials

As we have seen, Perloff and Belnap express predictions with the monadic operator **Will**. We wish here to express the fact that the prediction refers to *the day after* the enunciation, i.e., to “tomorrow”. Departing from Perloff and Belnap’s (2011) work, as well as the more technical formulation of Belnap (2002), we will use a notation for *tomorrow* based on Aarne Ranta (1994, chapter 5.9) for temporal indexical expressions.

One way to deal with temporal conditions shaping statements is to introduce them as contextual parameters that can be also made explicit at object-language level by means of adverbial expressions in order to *enrich* an assertion that has already content.²⁰ This contrasts not only with the view that temporal operators are first-order quantifiers with metalogical truth-conditions, but also with the model-theoretical approaches to temporal logic of the Priorian style.

In our notation, the expression “@” occurring in “@*tmr*(m_1)”, stands for Ranta’s (1994, p. 103 & p. 106) monadic propositional operator **AT**, used for rendering temporal (locative) adverbial constructions such as *Fighting on 24 Feb. 2023*, *Fighting now*, *Fighting tomorrow*. The expression “*tmr*(m_1)” within the scope of the adverbial “@*tmr*(m_1)” stands for the span $\langle m_1, 1 \rangle$ which begins with the moment $m_1 \in Day$ and has 1-day-unit as length.²¹ In the present paper we will use adverbials only for the indexical *tomorrow*. Notice that, though m_3 (on history 1) and m_4 (on history 2) both refer to the day after m_1 , which we would expect to be the same day, in Perloff and Belnap’s (2011) construal of indeterminate futures, they are strictly speaking different: according to the authors, a moment is a set of events, so that the set m_3 , which includes the taking place of the sea battle, differs from the set m_4 , which precludes it.

The main use of temporal parameters introduces them as *dialogical contexts* as in the dialogical framework for standard modal logic (see Rahman & Rückert 1999, rep. 2011). The point is that this stresses Perloff and Belnap’s own use of the *context of use*, which in fact privileges the Priorian

²⁰ We owe the expression *enrichment* to Recanati (2017).

²¹ In fact, if we introduce the expression *tmr*(m_i) within the framework of Ranta’s *Type Theoretical Grammar* (1994, p. 115), it can be postulated as being some element d of the set of spans $span(Day) = Day \times \mathbb{N} \in set$, whereby Day is a finite set defined as a scale on *Time*. So, $d \in span(Day)$ is a pair where a moment $m_i \in Day$ is defined as its beginning and a natural number n indicates its length.

analysis of temporal operators. Of course, the price to pay is that the semantics of the temporal operators shifts to the metalanguage. In a dialogical setting, however, temporal contexts are made explicit during the development of the vindication process, while defending or challenging modal and temporal operators. The operators *Was*, *Will*, and *Sett*, involving temporal dialogical contexts, can be introduced in a quite straightforward manner.

Temporal Dialogical Contexts

Temporal Dialogical Contexts follow the notation of modal dialogical contexts (see for instance the entry “Dialogical Logic” of the SEP). Moreover, every statement in a play will be assumed to have been set in a dialogical temporal context of the form m_i/h_j (with i and j standing for natural numbers). In the dialogues below

- The thesis stated by the Proponent is set at the temporal dialogical context $m_1/(h_1, h_2)$, i.e., at the context of the predictions’s utterance.

We combine the use of temporal dialogical contexts with adverbials in the following way

- When the actual course of events is known, and the outcome of the adverbial indexical and the temporal context become identical, we substitute the indexical $tmr(m_1)$ occurring in $A@tmr(m_1)$ (stated in the dialogical context m_3/h_1) or $\neg A@tmr(m_1)$ (stated in the dialogical context m_4/h_2) by the exact moment after m_1 referred to by that indexical, namely, $A@m_3/h_1$ or $\neg A@m_4/h_2$ and we delete in the dialogue the dialogical temporal context.

The Operators *Sett*, *f*, *Was*, *Will*

Sett. From a dialogical point of view, the operator **Sett** has universal force on the set of histories passing through a moment m_i . Thus, the particle rule prescribes that it is *the challenger* who can choose any one of those histories, say h_j , and require that the defender states the rest of the proposition precisely at the same moment m_i but on the history h_j , i.e. at m_i/h_j .

Statement	Challenge	Defence
m_i/h_i X ! Sett:A	m_i/h_i Y ? Sett h_j	m_i/h_j X ! A

In the sea battle scenario there is no real choice since only one history, h_1 , passes through m_3 , and only one other, h_2 , passes through m_4 .

f. The thesis for the sea battle scenario includes the operator **f**. This is the possibility operator **f** of Øhrstrøm & Hasle (2015, section 4d). From the dialogical point of view, **f** has existential force on both the set of histories passing through a moment m_i and on the sequence of moments occurring on those histories. Thus, the particle rule prescribes that it is *the defender* who can choose a moment *after* the moment of utterance along the chosen history.

Statement	Challenge	Defence
m_i/h_i X ! f:A	m_i/h_i Y ?_f	m_j/h_j X ! A Where $m_j > m_i$

In the given scenario, the choices are reduced to m_3/h_1 if the left of the disjunction is chosen, or m_4/h_2 if the right one is. Actually, in the dialogue, a kind of shortcut is made. In order to avoid prolonging an already long dialogue, instead of stating one side of the disjunction first and then defending the **f** operator, we simply require that the defender of a disjunction composed of settled truths chooses the dialogical context for stating the chosen disjunct.

Was. From the dialogical point of view, the **Was** operator has existential force on the backward sequence of moments occurring in a given history. Thus, the particle rule prescribes that it is the defender who can choose a moment before the moment of utterance of **Was**, along the given history.

Statement	Challenge	Defence
m_i/h_i X ! Was:A	m_i/h_i Y ?Was	m_j/h_i X ! A Where $m_i > m_j$

In the sea battle scenario, the moment in the past of **Was**, stated at either at m_3/h_1 or m_4/h_2 , is the time of utterance of the predictions under the scope of this operator, i.e., $m_1/(h_1, h_2)$ – recall that in the sea battle scenario m_3 and m_4 share the same past, thus, m_1 is on both, h_1 and h_2 .

Will. From the dialogical point of view, the **Will** operator also has existential force but on the forward sequence of moments occurring in a given history. Thus, the particle rule prescribes that it is the defender who can choose a moment after the moment of utterance of **Will**, along the relevant history.

Statement	Challenge	Defence
m_i/h_i X ! Will:A	m_i/h_i Y ?Will	m_j/h_i X ! A Whereby $m_i > m_j$

In the sea battle scenario, the moment in the future of **Will** stated at $m_1/(h_1, h_2)$ is either m_3/h_1 or m_4/h_2 .

4.2.2 Repetition Ranks and Plays

As mentioned above, the main thesis of the dialogue can be formulated as the following thesis

$$\begin{aligned} & ! \text{Will:}A@tmr(m_1) \vee \text{Will:}\neg A@tmr(m_1) \\ & [f:(\text{Sett:Was:Will:}A@tmr(m_1)) \vee f:(\text{Sett:Was:Will:}\neg A@tmr(m_1))]^{22} \end{aligned}$$

The dialogical (structural) rule for challenging a thesis with premises, which are in square brackets, prescribes that the Opponent states the premises, i.e., $f:(\text{Sett:Was:Will:}A@tmr(m_1)) \vee f:(\text{Sett:Was:Will:}\neg A@tmr(m_1))$, to which the Proponent has to answer by stating the first part of the thesis, i.e., $\text{Will:}A@tmr(m_1) \vee \text{Will:}\neg A@tmr(m_1)$.

According to the standard (structural) rules of the dialogical framework, after setting the thesis the players have to choose a repetition rank, which bounds the number of times a player can challenge a claim made by the interlocutor, and the number of times a player can answer to the same challenge made by the interlocutor. In the dialogue below, the Opponent chooses rank $\rho := 1$, so she can challenge any claim only once and, more importantly, she can also defend a disjunction only once during a play.

Notice that if the defence of **Will** requires considering an infinite chain of moments (clearly excluded in case of tomorrow's sea battle scenario) two options are available:

1. follow the standard procedure for dealing with them in the dialogical setting, namely, build a strategy-tree constituted by an infinite number of finite plays – see Clerbout (2014a,b);
2. implement Perloff and Belnap's (2011, p. 594) proposal to stop the analysis of statements of the form **Will:** α (for complex α), mentioned above, by a formulation of the Socratic Rule

²² For writing this kind of thesis with premises see McConaughey (2021).

specific to the global meaning of the **Will**-operator in the context of games for the vindication of predictions – see next section.

4.2.3 Questions, the Socratic Rule, Statements, and Assertions on Predictions made

Questions and Predictions made

A consequence of focusing only on assertions is that, as Ranta (1994, pp. 137–142) has pointed out, more often than not, other important speech acts such as questions, orders, and instructions (the kind constituting recipes found in cookery books, for instance) are neglected. Relevant to the present study are the so-called *propositional questions* which have the form $A \mid B$ (Ranta 1994, p. 138). Noticing that Ranta’s analysis of propositional questions comes very close to the (local) meaning explanation of disjunctions in standard dialogical logic, it can be adapted to the dialogical framework to deal with the specific case of *predictions made*. A *prediction made* is a statement regarding a future contingent which has the name of the agent attached. Thus, when a player has stated a disjunction regarding *predictions made* by agents X and Y at a temporal dialogical context, the interlocutor may challenge with a propositional question of the form $X\text{-left} \mid Y\text{-right}$ at the same temporal dialogical context. For instance, with the sea battle scenario, we have:

$$m_1/(h_1, h_2)$$

? Themistocles-*left* | Eurybiades-*right*

In order to answer this challenge, the player must state one of the predictions made involved in the challenged disjunction:

! Themistocles **Will:** $A@tmr(m_1)$

Or

! Eurybiades **Will:** $\neg A@tmr(m_1)$

The Socratic Rule

As mentioned above, the Socratic Rule prescribes which of the Opponent’s moves entitle the Proponent to ground his statements based solely on her previous statements; these moves are not further analyzed and are declared to be unchallengeable. In standard formal dialogues, the unchallengeable statements are elementary. In material dialogues, the Socratic Rule might be applied to other kinds of statements, in order to set the *global meaning* of some expression. In fact, one way to look at the rules for global meaning, and in particular at the Socratic Rule, is to consider them as setting the use of the rules for local meaning within the development of a play shaped by the context of the targeted system of argumentation.

This is precisely the case for plays which aim at vindicating or impugning predictions. Despite the fact that the local meaning of **Will** and of negation are set, the global meaning of the negation of an elementary proposition within the scope of a **Will embedded in a prediction made** (on the Proponent’s side) and *on the settled past truth of the future contingent* (on the Opponent’s side), prescribe those statements as governed by the Socratic Rule, and are therefore handled as non-challengeable. In other words, in the sea battle scenario, A and $\neg A$ are not further analyzed and, once stated by the Opponent **O**, they can be stated by the Proponent **P**. This constitutes the global meaning of predictions.²³

This results in the following two possibilities, which guarantee the Proponent’s successful vindication of the relevant prediction.

²³The importance of this point can be easily seen in the more general case of embedded temporal operators: the prediction that is to be vindicated is marked by the first Will, even if it is a prediction about a prediction. So *in this context*, **Will:** A and \neg **Will:** A will be declared unchallengeable in order to vindicate or impugne the predictions. In this case, the thesis will be: ! **Will:****Will:** $A \vee$ **Will:** \neg **Will:** A [f:(Sett:Was:Will:Will: A) \vee (f:(Sett:Was:Will: \neg Will: A)], whereby the disjunction can be further analyzed into the predictions made !_X **Will:**Will: A , !_Y **Will:** \neg Will: A

- **P** $you(n) \varepsilon A@m_3/h_1$ for the vindication of Themistocles’s prediction made $!_{Themistocles}$
Will: $A@tmr(m_1)$;
- **P** $you(n) \varepsilon \neg A@m_4/h_2$ for the vindication of Eurybiades’s prediction made $!_{Eurybiades}$
Will: $\neg A@tmr(m_1)$.

These admit the glosses:

You, the Opponent, just conceded at move n that at m_3/h_1 a sea battle has been fought;

You, the Opponent, just conceded at move n that at m_4/h_2 a sea battle has not been fought.

The notation “ $a \varepsilon \alpha$ ” is a variant of “ $a: \alpha$ ” introduced in *Immanent Reasoning* whereby “ a ” stands for a *local reason* for α .²⁴ The special case “ $you(n) \varepsilon \alpha$ ” indicates that the Proponent appeals to the Socratic rule for producing a local reason for α , namely, the Opponent’s own statement α at move n .

In order to deal with the more general case of embedded operators, the Socratic rule entitles the Proponent to state **Will:** α , (for complex α), in the context of vindicating a prediction made, iff the Opponent has previously stated the same expression while defending **Sett:****Will:** α . Thus, we can speak here of a double contextualization of the local rule for **Will**, namely the context of vindicating a *prediction made* and the context of the sequence of moves that leads the Opponent to state **Will:** α . It is this double contextualization that entitles the Proponent to state **Will:** α and that turns it into a unchallengeable statement.

SOCRATIC RULE FOR “WILL: α ” IN THE CONTEXT OF VINDICATING A PREDICTION ON A COMPLEX α □	
Statement	Entitlement
m_i/h_i O ! Will: α (move n_i) Given O ! Sett:Will: α (move $n_j < n_i$)	m_j/h_i P $you(n_i) \varepsilon$ Will: α (move k_i)

The structural rules for developing plays involving temporal operators can be found in the appendix.

4.3 The formal dialogues vindicating the sea battle predictions

As mentioned above, the dialogical approach to vindicating the predictions in the sea battle scenario yields two plays: the Opponent’s choice of disjunct she defends calls for a second play in which she chooses the second disjunct; the Proponent wins both plays if he chooses to vindicate the appropriate prediction made.

Opponent	Proponent
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²⁴ See Clerbout & McConaughy (2022, section 3.2). Lorenz already used such a notation in several texts reprinted in Lorenz (2011) in order to express the most basic form of predication. In Lorenz & Mittelstrass (1967, p. 8), this form of predication is called $\delta\nu\omicron\mu\acute{\alpha}\zeta\epsilon\iota\nu$, i.e. naming (what it is).

		$m_1/(h_1, h_2)$				$m_1/(h_1, h_2)$			
						! Will: $A@tmr(m_1) \vee$ Will: $\neg A@tmr(m_1)$			
						[f:(Sett:Was:Will: $A@tmr(m_1)) \vee$ f:(Sett:Was:Will: $\neg A@tmr(m_1))]$			
0.1	$\rho = 1$					$\rho' = 2$			0.2
		$m_1/(h_1, h_2)$				$m_1/(h_1, h_2)$			
1			0						2
		! f:(Sett:Was:Will: $A@tmr(m_1)) \vee$ f:(Sett:Was:Will: $\neg A@tmr(m_1))$! Will: $A@tmr(m_1) \vee$ Will: $\neg A@tmr(m_1)$			
		$m_1/(h_1, h_2)$				$m_1/(h_1, h_2)$			
3			2						12
		? Themistocles- <i>left</i> Eurybiades- <i>right</i>				! Themistocles Will: $A@tmr(m_1)$			
		m_3/h_1				$m_1/(h_1, h_2)$			
5				1					4
		! Sett:Was:Will: $A@tmr(m_1)$? \vee f			
		m_3/h_1				m_3/h_1			
7				5					6
		! Was:Will: $A@tmr(m_1)$? Sett h_1			
		$m_1/(h_1, h_2)$				m_3/h_1			
9				7					8
		! Will: $A@tmr(m_1)$? Was			
		$m_1/(h_1, h_2)$				$m_1/(h_1, h_2)$			
11				9					10
		!_A $@m_3/h_1$? will			
		$m_1/(h_1, h_2)$							
13			12						14
		? will				you (11) ε $A@m_3/h_1$			

Figure 4: Play 1 for the sea battle scenario

Opponent		Proponent	
		$m_1/(h_1, h_2)$	
		! Will: $A@tmr(m_1) \vee$ Will: $\neg A@tmr(m_1)$	
		[f:(Sett:Was:Will: $A@tmr(m_1)) \vee$ f:(Sett:Was:Will: $\neg A@tmr(m_1))]$	
0.1	$\rho = 1$	$\rho' = 2$	0.2

1	$m_1/(h_1, h_2)$ 0 ! f:(Sett:Was:Will:A@tmr(m ₁))∨f:(Sett:Was:Will:¬A@tmr(m ₁))	$m_1/(h_1, h_2)$ 2 ! Will:A@tmr(m ₁) ∨ Will:¬A@tmr(m ₁)
3	$m_1/(h_1, h_2)$ 2 ? Themistocles-left Eurybiades-right	$m_1/(h_1, h_2)$ 12 ! Eurybiades Will: ¬A@tmr(m ₁)
5	m_4/h_2 ! Sett:Was:Will: ¬A@tmr(m ₁)	$m_1/(h_1, h_2)$ 1 4 ?∨f
7	m_4/h_2 ! Was:Will: ¬A@tmr(m ₁)	m_4/h_2 5 6 ?Sett h ₂
9	$m_1/(h_1, h_2)$! Will: ¬A@tmr(m ₁)	m_4/h_2 7 8 ?Was
11	!¬A@m ₄ /h ₂	$m_1/(h_1, h_2)$ 9 10 ?Will
13	$m_1/(h_1, h_2)$ 12 ?Will	$m_1/(h_1, h_2)$ 14 you(11) ε ¬A@m ₄ /h ₂

Figure 5: Play 2 for the sea battle scenario

Notice that these dialogues do not represent the general case with embedded temporal operators. This more general case can however be built either following the indications mentioned in the section on the Socratic Rule for a non “unpacked” solution, or by building a strategy tree constituted by an infinite number of finite plays (see Clerbout 2014a,b).

Note on material dialogues

In the context of material dialogues, each play constitutes a winning strategy; we thus obtain an *assertion made* for the predictions on the future contingents. This differs from a *statement made*, which is the entire sequence of moves constituting each of the plays.

P’s statement made and Themistocles’s Prediction made

The result of vindicating the prediction made by Themistocles amounts to recording all the sequences of pairs of moves that end with a victory for the Proponent in a play in which the Proponent decided to vindicate Themistocles’s prediction as thesis. In the plays above this yields the following sequence in which even numbers correspond to moves by the Proponent and odd numbers to moves by the Opponent. Since the last number is pair, the play is won by the Proponent.

$\langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Themistocles}} \text{Will}:A@tmr(m_i)$ for the vindication of Themistocles's prediction

If we are provided with a code for substituting the numbers by actual moves we obtain that move 1 corresponds to the Opponent's challenge to the thesis by stating the premise; move 2 corresponds to the Proponent's answer stating the conclusion of the thesis; move 6 (say) corresponds to the challenge of the **Was**-operator stated by the Opponent in her move 5 – and so on, we will not present the code here. The codes can thus be read as a sequence of instructions bestowing a *blue print* for reproducing the play.²⁵

The important point here linked to the notion of statement made is that by reproducing the sequence following the instructions, we reproduce exactly the moves of the play as developed. Notice that the moves corresponding to the order could have been changed, for example move 4 could have been the Proponent defending his own disjunction rather than challenging the one of the Opponent. The following of the sequences of instructions yields the exact play played by the Proponent while vindicating Themistocles's prediction. So we distinguish the *prediction made* and the Proponent's statement made: the latter is the result of vindicating Themistocles's prediction. Thus, strictly speaking, we should use a double agentive notation

$P ! \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Themistocles}} \text{Will}:A@tmr(m_i)$ for the vindication of Themistocles's prediction

The same holds for the Proponent's vindicating Eurybiades's prediction.

From assertions made to assertions

If the plays constitute a winning strategy we obtain the following assertion made

$f:(\text{Sett:Was:Will}:A@tmr(m_i))\forall f:(\text{Sett:Was:Will}:\neg A@tmr(m_i))\vdash P ! \text{Will}:A@tmr(m_i)\vee \text{Will}:\neg A@tmr(m_i)$

Letting " β " stand for " $f:(\text{Sett:Was:Will}:A@tmr(m_i))\forall f:(\text{Sett:Was:Will}:\neg A@tmr(m_i))$ ", the assertion can be further analyzed as

$\beta \vdash_P \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Themistocles}} \text{Will}:A@tmr(m_i)$

for the vindication of Themistocles's prediction

$\beta \vdash_P \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Eurybiades}} \text{Will}:\neg A@tmr(m_i)$

for the vindication of Eurybiades's prediction

In order to abstract away from the *assertion made*, we need to abstract from the concrete player and from the number of moves. In short, (i) we express the assertion without attaching to it any concrete player, (ii) we delete the number of the moves, and (iii) we substitute the codes by general strategic objects – see Rahman et al. (2018, pp. 175–179). Now, this abstraction requires insights regarding the particle and structural rules that set the meaning of each expression involved – the insight in this case requires noticing that substituting the Proponent's move by a defensive one, rather than by a counterattack, has no logical consequences.

5 Conclusion

Research on future contingents has of course a long and venerable history and continues to offer new challenges and perspectives. Perloff & and Belnap's (2011) contribution to the matter combines the idea that speech acts involving future contingents can (and perhaps even should) be included within the family of assertions, and that those assertions obey logical laws.

In relation to the first idea their point is to include speech acts involving future contingents within the realm of assertions. In fact, though in their paper they target predictions, the general approach includes other speech acts such as betting, promising and commanding – see Belnap et al. (2001). Moreover,

²⁵This is closely linked to the notion of *state of a dialogue* in Redmond & Fontaine (2011).

in order to classify predictions as assertions they put at work the notion of vindication they borrow from Brandom's normative analysis of assertion – explicitly referred to in Belnap & Green (1994, footnote 21). Vindication is then understood as a pragmatist feature that leads to the credit or discredit of the predictor – or, to use Brandom's (2000, chapter 3) terms, to determine if he or she is reliable or unreliable. Their notion of vindication, which they call a pragmatist one, aims at avoiding the syntactic (and semantic) “unpacking” of embedded temporal operators that might occur in the prediction under scrutiny.

However, because predictions are assertions after all, and assertions should obey the laws of logic, they conjugate the pragmatist conception of vindication with a model-theoretic framework for temporal logic expanded with a tree structure in the way mentioned above. The resulting temporal logic follows Carneades's view that the formal truth²⁶ of the law of Excluded Middle can be upheld if we are prepared to accept that the disjunction is made true by events happening in the future.

This strategy neglects the fact that, as Brandom (2000, chapter 8) points out, declarations of reliability are the result of an attribution process that takes the form of a language game shaped by games of giving and asking for reasons. This takes us to our own perspective, whereby the process of vindicating is understood as a dialogical interaction.

As discussed, the dialogical perspective does not understand predictions by adding a (model-theoretical) semantics to the pragmatics specific to the speech acts involved; rather, it understands them by associating the process of vindicating to a set of rules that bestow the dialogical meaning explanation of predictions.

More generally, the main philosophical tenet of the dialogical approach rejects the usual reductionist ways to understand the interface between semantics and pragmatics. Kuno Lorenz's general dialogical principle is that the interface between semantics and pragmatics should be understood in the following way:

neither as the result of the semantization of pragmatics—where deontic, epistemic, ontological, and temporal constraints become truth-functional operators; nor as the result of the pragmatization of semantics—where a propositional kernel, when put into use, is complemented by moods yielding assertions, questions, commands and so on. (Lorenz 2011, pp. 519–520)

In short, Lorenz's view (2010, pp. 71–79) is that the differentiation of semantic and pragmatic layers is the result of the articulation within one and the same utterance: each utterance displays in principle both features, as it signifies (semantic layer) and communicates (pragmatic layer).

Per Martin-Löf (2014) also proposed an analysis of Aristotle's Sea Battle challenge which stresses, like Carneades, an analogy between situations where the truth of a past event is indeterminate and situations involving future contingent events.²⁷ His reconstruction has been developed within his own Constructive Type Theory (CTT), which, as he pointed out in recent texts, can be linked to a dialogical interpretation of the notions of epistemic assumption and assertoric knowledge.²⁸ Martin-Löf's analysis highlights the hypothetical nature of this kind of assertion. The proposal amounts to analysing the case of future (and past) contingents as a special case of the truth of empirical propositions, where the notion of proof is extended by including *empirical quantities*. According to such an approach, sentences such as *There was a sea battle* and *There will be a sea battle*, express

²⁶ For a thorough discussion of Carneades's position and his distinction between factual and formal truth see Vuillemin (1984, chapter 11). We owe this reference to Thomas Bénatouil (Lille U. STL).

²⁷ Martin-Löf (2014) discusses here Michael Dummett's (1976, p. 114) example “There was either an even or an odd number of geese which cackled on the Capitol”, which refers to the date of the 18th of July in 390 BCE, the day the Gauls attacked Rome. This example resembles those discussed by Carneades, who, as mentioned above, pointed out that in such kinds of situation, which can be placed not only in the past but also in the future, we cannot in the present determine the truth of the event involved since the (causal) traces are either lost or not yet available.

²⁸ Cf. Martin-Löf's (2017, 2019) – for a detailed development of this interpretation see Klev (2023).

both an empirical quantity – more precisely, a non-canonical proof-object within the set *Bool*. Still, despite the proof-theoretical insights on empirical propositions such a reconstruction provides, it is unlikely that this corresponds to the views of ancient or medieval philosophy on empirical propositions.

Be that as it may, the CTT framework allows to render the hypothetical structure of future contingents, which when set in the dialogical framework of Immanent Reasoning, opens the door to the study of the speech acts targeted by Belnap et al., which go beyond just predictions. This is still part of an ongoing work in progress, which also displays the intertwining of historic and systematic studies in philosophy of logic, so cherished by Göran Sundholm, and that also shapes the main traits of the research at the Ecole Dialogique de Lille.

Let us finish our essay by paraphrasing a beautiful remark by Steedman (1997, p. 898) in his chapter on temporality:

The large and ramifying literature on the matter in very diverse fields will sometimes require to focus in some of them and there might be a danger of briefly losing sight of others. However, the different perspectives will meet up again as the enquiry proceeds, for historians of philosophy, philosophers of science, logicians, and linguists are linked together in this collective endeavour towards understanding the manifold ways temporality affects assertion.

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Appendix

SR0 (starting rule): A play starts with a player stating a proposition called the thesis; that player becomes the Proponent (**P**) and the move is labelled move 0.

The other player—the Opponent (**O**)—chooses a repetition rank determining how many times she is allowed to challenge or defend any move in a play. It is usually enough for **O** to choose a repetition rank of 1 ($m := 1$), it is move 1. **P** then chooses a repetition rank: 2 is usually enough ($n := 2$); it is move 2.

SR1i (intuitionistic game-playing rule): After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move, in accordance with the rules for **local** dialogical meaning explanations. Each player can challenge the same previous move at most n times, where n is the player's repetition rank, or defend against the adversary's *last unanswered* challenge.

SR2 (formal Rule for plays involving Belnap's temporal operators)

1. **P** may state an elementary proposition A at the temporal context m_i/h_j , only if **O** stated first A at m_i/h_j or at m_i/h_k provided histories h_j and h_k cross the same moment m_i .
2. **P** may choose any moment m_k/h_j , such that $m_i < m_k/h_j$ for the future or $m_k < m_i/h_j$ for the past, whereby m_i is the utterance moment at which the choice has been taken (transitivity of the temporal order with regard to the future and the past).
3. In the case of material dialogues some specific moments and histories might have been stated by **O** from the outset: SR2.1 and SR.2.2 apply to these moments and histories.

SR3 (winning rule): The play ends when it is a player's turn to make a move but (s)he has no available move left. That player loses and the other player wins.

In the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for **X** means that for any choice of moves by **Y**, **X** has at least one possible move at his disposal such that (s)he (**X**) wins:

- **Validity (definition):** A proposition is valid in a certain dialogical system iff **P** has a winning strategy for this proposition set as thesis of the dialogue

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