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Philippe Woloszyn. IS FRACTAL ESTIMATION OF A GEOMETRY WORTH FOR ACOUSTICS ?. Miroslav M Novak (Kingston University, UK). Emergent nature: Patterns, Growth and Scaling in the Sciences: Conference on Fractals 2002 Granada, Spain, 17 – 20 March 2002, World Scientific Publishing Company, pp.423-424, 2002, 978-981-02-4910-6. 10.1142/4926 . halshs-04252280

**HAL Id: halshs-04252280**

**<https://shs.hal.science/halshs-04252280v1>**

Submitted on 20 Oct 2023

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# IS FRACTAL ESTIMATION OF A GEOMETRY WORTH FOR ACOUSTICS ?

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Through collaboration in an interdisciplinary CNRS research program with the LCPC, the LAUM and the CRESSON Laboratories, we are intended to constitute a simulation tool for complex urban soundscapes. For this purpose, we aim at quantifying the diffusivity of the urban morphology. The built urban structure is considered as a non-entire dimensional network, taking the volume structure and the multiscale distribution of specular surfaces within the urban space into account. For this distribution, acoustical diffusion is considered. Its multiscale characterization leads us to use a fractal technique for modeling the complexity of urban surfaces. To do so, we develop a method to quantify scaling morphology of the town with a capture procedure based on Minkowski's operators. The generalization of this measurement technique through the three dimensions of the urban shape provides an angle-dependent diffusive polar response for urban frontages, through computation of the diffusion volume of the structure. This new multiscale analysis tool of urban structure has been applied to a 19th urban morphology. Considering these results pertinent for implementing a raw diffusion model, fractal computing of diffusive geometry will be able to provide angular acoustical diffusion parameters, in order to generalize this method to any complex diffusion system.

## 1 Introduction: Problematic and purpose

The exterior frontage of a typical urban building does not reflect noise in a purely specular manner. Because the dimensions of the irregularities (decorative elements, windows, balconies, ...) are comparable to the sound wavelengths, the major type of reflexions on the buildings is scattering, inducing a global diffusion behavior of sound in an urban street. Consequently, irregular surfaces like urban frontages produces an anomalous back-scattered region, with the creation of an acoustic interference field in its neighborhood. Thus, in order to be able to detect that scattered energy's minima and maxima, we have to take into account both incidence angle and multiscale characterization for fractal evaluation of urban surfaces through mathematical morphology techniques.

## 2 A Touch of Acoustics

### 2.1 Frequency characterization of a regularly indented structure

Considering a simple surface configuration constituted with regular indentations of depth  $\Lambda$ , the frequency we use is spatial, and defined with the obstacle's average number per length unit as:

$$\nu = 1 / \Lambda .$$

Near the limit of the diffusion process, the recovering measure of a complex urban frontage provides the fractal dimension of the diffusive structure as:

$$D = 1 - \frac{S_x}{P \nu} = 1 - \frac{\Lambda S_x}{P}$$

with  $S_x$  as the diffusion section of the structure.

If , for a given scale, we consider the value  $\Lambda$  as the upper limit that verifies the acoustic diffusion law, the specular limit of the acoustic propagation phenomenon is expressed with the limit frequency. In that case, its corresponding wavelength:

$$\lambda_{\Lambda} = c / f_{\Lambda}$$

is close to the depth dimensions  $\Lambda$  of the structure. This is the lowest propagation mode, in relationship with this roughness limit. The corresponding frequency  $f_{\Lambda}$  is provided with an analytical expression of the first destructive interference of the surface- and bottom-reflected waves as:

$$(2n + 1) \frac{\lambda}{2} = 2 \Lambda \Rightarrow f = (2n + 1) \frac{c}{4\Lambda}$$

This classical acoustic law [4], called "  $c / 4 \Lambda$  " law, is expressed figure 1 (half-dashed lines) as a function of the roughness limit of the diffusion domain  $\Lambda$ , which is called *localization length*.

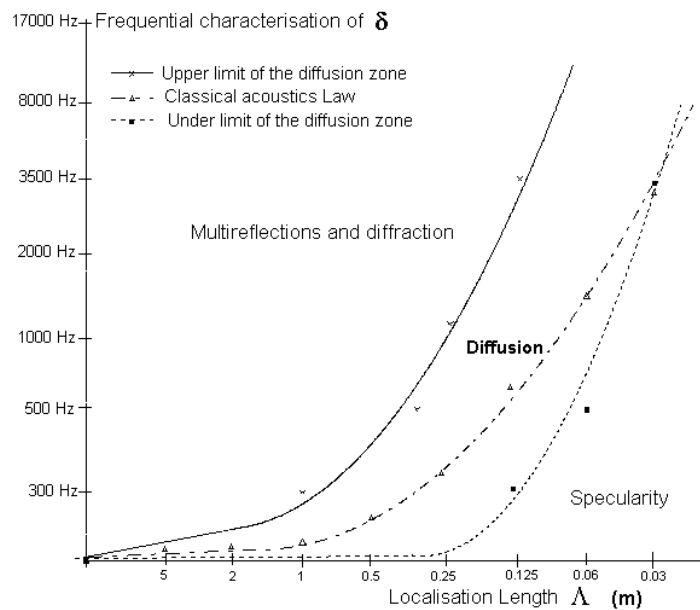


Figure 1: Frequency characterization of diffusion and specular domains for a regular indentation

For acoustic modes at frequencies greater than the wavelength  $\lambda_{\Lambda}$ , acoustical propagation remains specular in the structure, which is considered rigid. In this case, our entity is assimilated to a non-diffusive structure with a diffusion coefficient  $\delta = 0$ . Nevertheless, this simple model represents the acoustic behavior of a specified surface geometry for a normal receiving point. As sound scattering cannot be defined as an unlocalized phenomenon, the aim of next step of this research will take into account the angular distribution of the scattered sound radiation.

### 2.1.1 Diffusion through oblique incident wave: first approximation with the Rayleigh criterion

Historically, the first attempt at determining the scattering amplitudes was made in 1893 by Lord Rayleigh, who assumed a unique solution for the wave equation for the whole

boundary of a  $\Lambda$ -corrugated surface [4]. Concerning the inferior diffusion limit frequency, Rayleigh's work proposes a phase grating calculation between two acoustic rays related in [1], which takes into account the source incidence angle. The walk difference  $d$  between two rays with wavelength  $\lambda$  and incidence angle  $\alpha$  regarding a surface with depth  $\Lambda$ , the phase grating calculation between the two rays is :

$$\Delta\alpha = d (2\pi / \lambda) = \cos \alpha (4 \pi \Lambda / \lambda)$$

with the walk difference :  $d = 2 \Lambda \cos \alpha$ . For a weak walk difference  $d$ , rays are coherent and the acoustic wave is specularly reflected. Increasing of  $d$  will interfere rays, till to phase opposition ( $d = \pi$ ), so that no energy is displayed in the specular direction : sound energy is diffuse. Rayleigh criterion defines the limit between specular and diffuse behavior of an incident source, corresponding to the depth irregularities  $\Lambda < 1 / 8 \cos \alpha$ , as represented figure 2:

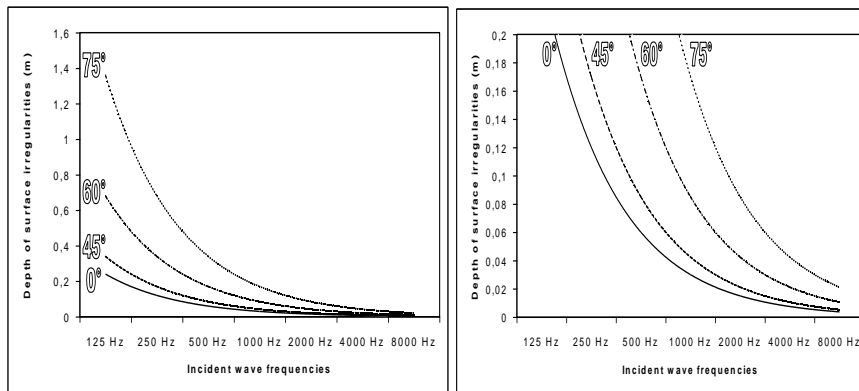


Figure 2 : Rayleigh criterion: Quantification of the dimensional limit between specularity and diffusion, as a function of frequency and incidence angle (m and dm).

The specular reflexion zone is defined under the minimal values of the frontages depths irregularities, taking the frequency and the angle of the incident wave into account.

### 2.1.2 The polar response of an indented plane

Following those non-specularity conditions, the propagation directions specified by the unit vectors  $v(d) = (\phi_d, 0, \gamma_d)$  for a given regular plane division, repeating  $n$  times a spatial unit of width  $\Lambda$  are defined as the *characteristic directions of scattering* associated with the localization length  $\Lambda$ :

$$\sin \alpha_d = \frac{n \Lambda}{p \lambda} - \sin \alpha, n = 0, +1, \dots$$

where  $\alpha_d$  is the grazing diffraction angle made by vectors  $v(d) = (\phi_d, 0, \gamma_d)$  in the direction  $Ox$ , with  $\phi_d = \cos \alpha_d$  and  $\gamma_d = \sin \alpha_d$ , and  $p$  the diffraction order. For  $\Lambda = 0$ , equality between incident and reflected angle remaining true (specular conditions).

The two-dimensional polar response of a given indented surface can be expressed through the diffraction orders ( $p, q$ ), taking the angles of incidence and diffraction into account [2]:

$$\sqrt{p^2 + q^2} = \Lambda \frac{\sin \alpha_d + \sin \alpha}{\lambda}$$

The characteristic directions are those along which the wavelets emanating from the individual localization lengths  $A_n$  (periods of the surface) are exactly in phase. This constructive interference condition is conditioned with the a-dimensional modulus  $A_n/\lambda$ , which quantifies the energy of non-evanescent scattering losses, represented by the area of scattering intensity pattern lobes. Indeed, this modulus is conditioning the solutions of the previous equation, as  $A_n/\lambda = 0$  traduces specular reflection (as the diffraction order is null through the limit of the diffusion), and as this modulus value conditions its number of real solutions, corresponding to the diffraction directions (lobes of the surface's radiated energy).

## 2.2 Diffraction on multiscale rough boundary surfaces

For all other directions, the reflected wavelets will destructively interfere, resulting in complete cancellation for a self-similar periodic structure. For non-or prefractal structures as urban frontages, the scattered field will show mainlobes in the characteristic directions, and sidelobes elsewhere for a done sound frequency.

Taking the multiscale evolution of a D-dimensioned prefractal structure into account, localization length  $A$  suits the following expression, related to the Hurst coefficient:

$$A_n \rightarrow A^{(d-D)} \propto \frac{S_x (d-D)A}{P_v} = (d-D)^2 A = H^2 A$$

This leads us to reconsider the evolution of the diffraction orders with a multiscale reformulation of the polar response of a D-dimensioned structure as:

$$\sqrt{\sum_0^{\Lambda_{\max}} k^2} = H^2 A \frac{\sum_0^{\Lambda_{\max}} \sin \alpha_d + \sin \alpha}{\lambda}$$

with  $k$ , the cumulative number of the diffraction orders, and  $\Lambda^H/\lambda \propto H^2 \Lambda/\lambda$ , the adimensional modulus of the local constructive interference conditions.

## 3 An Application: Fractal dimension evaluation of an urban frontage

### 3.1 The urban frontage model

The spatial configuration we measure here is a numerical 3-D model of a neoclassical frontages of an urban street of Nantes, France, *the rue d'Orléans*, belonging to a 19<sup>th</sup> urban morphology type, with windows, doors, and freestone casting off. One of the main characteristics of this type of architecture is the relative exuberance of its frontage structure, following neo-classical composition. This frontage is considered as a tridimensional object situated in an ortho-normed space, rotating around the Z-axis (figure 3).

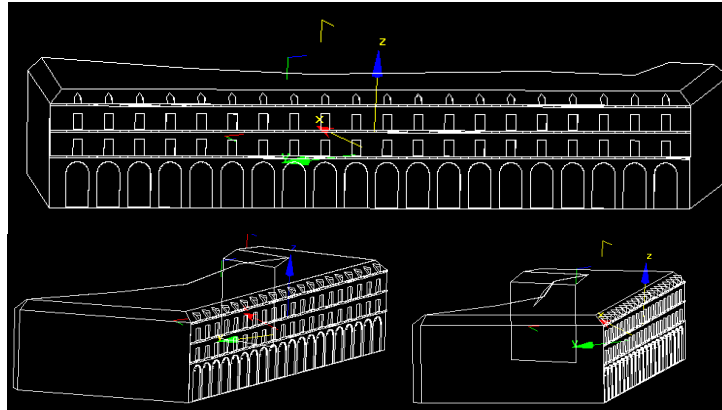


Figure 3: orthographical rotative analyze of a frontage in the *rue d'Orléans*, Nantes.

### 3.2 The Minkowski measurement technique

In order to characterize the multiscale behavior of the volume of the frontage, we apply a fractal Minkowski operator, called Minkowski sausage, to evaluate the vertex multiscale densitometry distribution at each incidence angle. This operator consists into replacing each point of the vertexes of the urban geometry with a sphere, as seen figure 4.

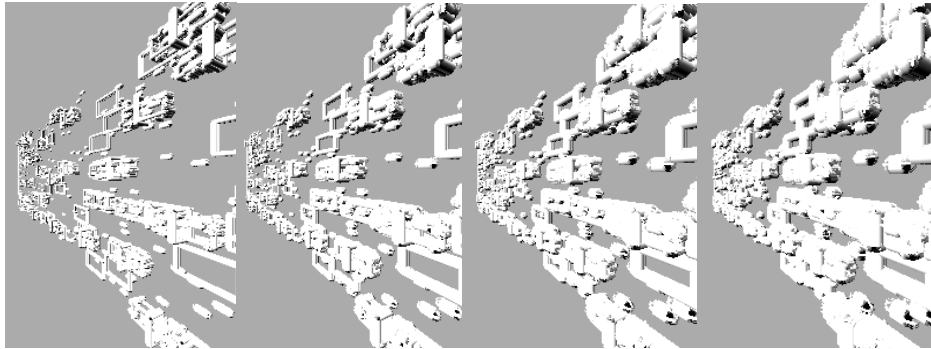


Figure 4: Tridimensional dilation of the urban scene vertexes (perspective views)

This transformation of the urban geometry corresponds to the *dilation* operation in the morpho-mathematical context [6]. The union of all spheres is called the *3-D Minkowski sausage*. The variation of their diameter gives us successive approached perimeter/surface ratios at each vision angle, which regression evaluates the fractal dimension of the structure, in a specified validity domain.

#### 3.2.1 Frontage fractal dimensioning

The spatial multiscale evolution of the perimeter-surface ratio  $P/S$  defines the profile's *Shape spectrum* of the frontage [12]. This spectrum defines the multiscale relationship between the radius evolution of the spherical structuring element and the "mass" of the structure, for each the angular measure. As readable in the following figure, the specular domain is illustrated with a strong decrease of the  $P/S$  ratio, which corresponds to

localization length limit  $\Lambda$  for the radius of the structuring element. For this domain, the Euclidean dimension  $d$  and the fractal dimension  $D$  reach the same value. This break in the frontage indentations shape spectrum behavior occurs for the structuring element value  $\Lambda_{max}$  (figure 5):

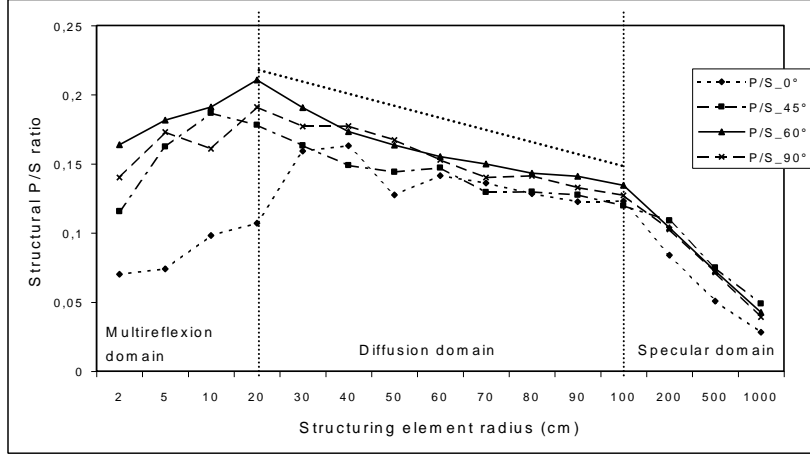


Figure 5: “Shape spectrums” of the urban frontages with a spherical recovering element, at different incident angles.  $D = 3-H$ , its fractal dimension inside the diffusion domain (given by the slope of the dashed regressive line).

### 3.2.2 Local neighborhood volume estimation

The urban structure recovered with this Minkowski operator provides, for each angle of scrutiny, a ribbon surface constituting a neighborhood area, which is called, by extension, *neighborhood volume*. This volume depends on the diameter  $r$  of the structuring element, and of its  $D$ -dimensioned multiscale behavior. The neighborhood, term used by [7], defines quantitatively the surface of the Minkowski sausage from the recovered vertexes, under the condition of continuity.

As our tridimensional urban structure is covered with structural elements of radius  $r$ , their number evolves like  $r^{-D}$ . The neighborhood volume  $V_d(r)$  verifies then:

$$V_d(r) = V_D \gamma (d - D) r^{(d - D)}$$

where  $\gamma$  is the structural element volume. For a spherical recovering,  $\gamma = 4/3\pi$ , then:

$$V_d(r) = V_D (d - D) 4/3\pi r^{(d - D)}$$

$V_D$  represents here the Minkowski contents, that means, the recovering ribbon surface for a specified angular viewpoint of the structure after the Minkowski operation:

$$V_D = \lim_{r \rightarrow 0} [V_d(r) r^{D - d} / 4/3\pi (d - D)]$$

The unitary limit of  $V_D$  describes the unit vertexes of the structure with the specified Minimum one-directional Hausdorff distance from  $x$  to the structure, in a  $D = d - H$  dimensional space:

$$V_D = \max_{r_0 \in ]0,1]} \min_{\Lambda \in ]0,1]} (r_0, \Lambda) \propto \Lambda^H = H^2 \frac{\Lambda}{\lambda}$$

At this limit, the Hausdorff distance  $V_D$  defines the multiscale distribution of the scattering vertexes under the structure translation. The minimum value of the Hausdorff distance provides the monodimensional distance  $[r_o, \Lambda_{max}]$  at each step of the Minkowski measure. This formulation of neighborhood distance  $V_D$  is a one-dimensional estimation of the *neighborhood distance* of the urban frontage structure, proceeded step by step with a local measure, involving the adimensional modulus  $H^2 \Lambda_{max} / \lambda$ . This modulus constitutes the pre-factor of the polar response of the structure, through evaluation of  $V_D$ , directly obtained towards the corresponding localization length value of  $\Lambda_{max}$  of the polar response at each step of the urban structure scrutiny with the Minkowski sausage technique.

### 3.2.3 Frontage neighborhood volume characterization

Neighborhood volume  $V_D$  is locally characterized through the Minkowski measure of the frontage, providing polar response with an angular increment of 5 degrees, for localization lengths from 5 cm to 10 m, which corresponding frequency domain is 25 Hz to 8 kHz. Global polar responses for growing localization lengths shows a decreasing diffusivity, as readable on the following figure 6. This decrease is confirmed by the figure 1 showing the frequency characterization of the diffusion coefficient, respecting to both the “ $c / 4 A$ ” law and the Rayleigh principle.

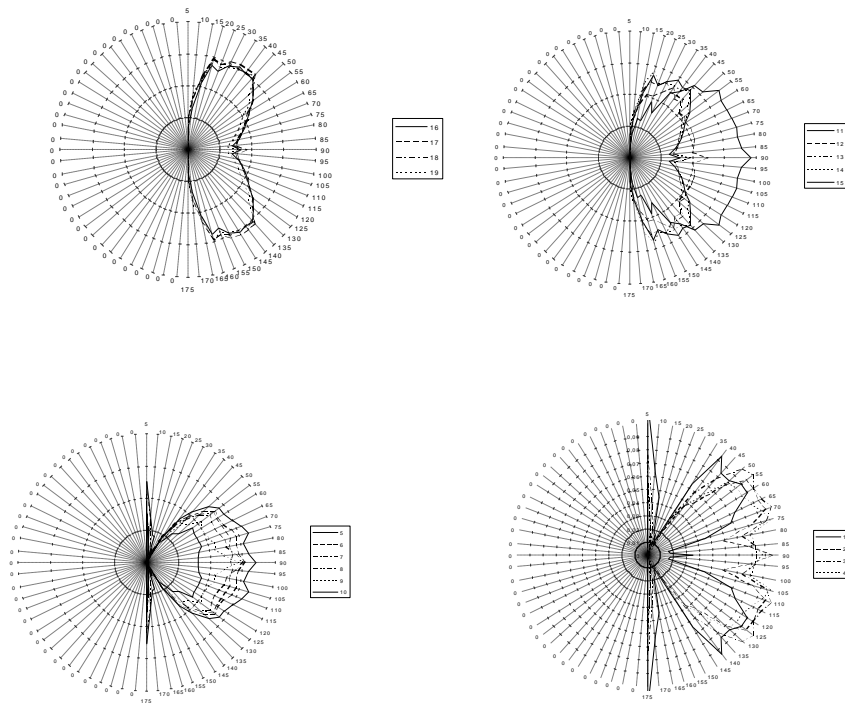


Figure 6: 5° polar responses of the urban frontage applied to the urban scene. Measures for localization length values of, respectively, 1 to 10 m, 0.5 to 1 m, 0.2 to 0.5 m, and 0.05 to 0.2 m.



This multiscale evaluation of the vertexes distribution shows evolving azimuthal densitometries of the window corners and the freestone casting. Those densitometries corresponds to the characteristic directions of scattering, which has to be calculated using the *scattering pressure function* along the lateral active diffraction zone.

### 3.3 Scattering pressure function for diffusion

Quantification of the sound pressure of the scattered acoustic field is provided with the specific angle's frequential polar response, according to an inhomogeneity parametering of the Lamberts law [5]. This model has been largely discussed into both acoustician and optician communities, as the definition of this "scattering model" is assumed to represent the physical behavior of sound or light after reflection on an ideally diffusing surface. The two models discussed into those communities are shared between uniform and Lambertian scattering. The first model is the simplest one, as it assumes a constant directional diffuse intensity radiated by the surface. The second one, related as the "Lambert's Law", was initially defined for the thermal radiation of a "Black Body", stating the variation of the intensity of the diffuse radiation as a function of the cosine of the angle between the direction of observation and the normal to the surface in the observation point. Although the uniform scattering model seems to provide a finest statistical behavior of a perfectly diffusion surface in optics, the Lambert's model is more accurate in geometry acoustics, as it provides the angular energy distribution for the characteristic directions of scattering with respect of the geometrical polar response of a plane. Calculation of the mean square diffracted sound pressure  $P_d$  by the whole surface of area  $S$  is given by [5]:

$$P_d^2 = \sigma \iint_S \frac{\cos \alpha \cos \alpha_d}{\pi r_0^2} P^2 dS$$

where  $r_0$  represents the distance from the receiver to the structure, and  $P$  the incident waves pressure. The angular repartition of the sound energy is computed through the pre-factor  $\sigma$ , which represents the *scattering pressure function* of the surface.

This function can be obtained with the density distribution of the sonic particles  $P(r,t)$ , with the following relationship [9]:

$$\sigma(k_d, i\omega) = \int dr \exp(ik_d r) \int_0^\infty dt \exp(-i\omega t) P(r,t)$$

using the imaginary part of the wave number  $\omega$ , the fractional diffusion order  $k_d$ , and the density distribution of the sonic particles  $P(r,t)$ , which is the probability for a sonic particle to walk to location  $r$  during the time  $t$  into a *fractional Brownian walk* in a  $D$ -dimensional space [3], [6]:

$$P(r,t) \rightarrow \frac{1}{(4\pi\delta t)^{D/2}} \exp\left(-\frac{r^2}{4\delta t}\right), N \rightarrow \infty$$

where  $\delta$  is the diffusion coefficient of the  $D$ -dimensional space. This function can be evaluated using a Laplace transform [8]. After integrating over the angles, density distribution becomes:

$$\sigma(k_d, i\omega) = \frac{2\pi G}{i\omega^{1-D/d_w}} \int dr r^{D-1} \frac{\sin(k_d r)}{k_d r} * \frac{1}{(i\omega r^{1/d_w})^{D-1/2}} \exp(-i\omega r^{1/d_w} / G)$$

where  $G$  is the *angular distribution factor* along the characteristic directions of scattering and  $D$  the fractal dimension of the structure.

As suggested by the theoretical arguments based on the continuous time random walks [1], it is possible to find a spatial argument to define  $P(r,t)$  for high values of  $t$ . Consequently, the *diffusion length* can be obtain from the average quadratic path, and from the probability  $P_i$ :

$$\sum_i r_i^2 P_i(t) \propto t^{2/d_w}$$

The mean path's quadratic average, expressed as the square of the diffusion length evolves as the particle's path time. This symmetry between number of reflections in the structure and mean path times affords us to reconsider the classical acoustics energy conservation laws, by considering a probability conservation law [8]. The upper limit of the diffusion domain conditioning the specular condition is then obtained with the asymptotic behavior of  $P(r,t)$  when  $\Lambda/r_0$  goes to infinite, that means when the radius of the Minkowski sausage's structural element tends to zero, for little values of localization length in multi-reflection conditions [10]:

$$\sigma(k_d, \omega) \propto \frac{G^2 k_d^2}{\omega^{1+2/d_w}}$$

with  $k_d$ , the concerned *fractional diffusion order*, and  $d_w$ , the associated Brownian walk dimension.

Fractional diffusion order  $k_d$  can be locally expressed with the corresponding  $k$  diffraction orders, involving the polar response of the structure as:

$$k_d \propto \sqrt{\sum_0^{\Lambda_{max}} k^2} = V_D \sum_0^{\Lambda_{max}} \sin \alpha_d + \sin \alpha$$

Considering the analysis of a spherical tridimensional structure, scrutated with the Minkowski technique involving a structuring element of radius  $\Lambda$ , and a global neighborhood volume  $V_{d(r_0)}$  of radius  $r_0$ , which corresponds to the acoustic receiving distance  $r_0$ , we can reformulate the previous equation with a reduced variable  $x = r_0/\Lambda_{max}$ . Then, it is possible to find an expression of the *diffusion volume*  $V_x$ , corresponding to a volume measure of the structure involving the global measure  $V_{d(r_0)}$ , the local measure  $V_{d(\Lambda_{max})}$  and the generalized pre-factor  $F_x$ , which defines the global behavior of the structure, in reference to its Euclidean nature [11]:

$$V_x \pi \frac{V_{d(\Lambda_{max})}}{V_{d(r_0)}} = \left( \frac{\Lambda_{max}}{r_0} \right)^{(d-D)} = x^H = \frac{H^2 \Lambda_{max}}{F_x \lambda}$$

This volume expression of a structure's neighborhood is a measure of the *diffusion volume* of the structure, which formulation has to be done simultaneously with a local and a global measure.

At each step of the measure of the structure, this angular scrutation gives successive diffusion sections  $S_x$ , which traduces the behavior of the scattering pressure function  $\sigma(k_d)$  for a given wavenumber  $\omega$  of the entire structure, with taking both the diffusion volume  $V_x$  evaluation, and the spatial positioning of the receiver  $r_0$ , into account:

$$\sigma(k_d) \propto \frac{F_x}{V_x} r_0 \sum_0^{\Lambda_{max}} \sin \alpha_d + \sin \alpha$$

## 4 Conclusion

The fractal technique we develop for modeling the complexity of urban building geometries quantifies discrete morphologies with an automatic capture procedure based on Minkowski's operators. Developed in the aim of architectural design tools for urban acoustics, the frontal angular diffusivity factor allows a good evaluation of the acoustical local reaction of an urban frontage. With discerning the morphological attributes of main types of architectures, we will be able to compute their specific angular acoustical behavior at every frequency, directly from fractal analysis of the numerical 3-D model of their geometry.

This Minkowski technique provides a quantification of the scattering pressure function at each incidence angle through the determination of the neighborhood volume, providing the polar response of a specified urban neo-classical frontage. A further development of this model will allow a calculation and representation of the surfacic angular polar response in real three dimensions, by rotating the 3-D model of the frontage around the  $Y$  axis.

Additionally, scattering experimental results will be compared to the described scattering pressure function at each incidence angle, with using an MLS multisensor method of measurement, in order to perform an experimental validation of this angle dependent scattering fractal-characterization model.

This research work will lead us to define the diffusion process as a geometrical-dependant phenomenon, confirming the multiscale influence of the built structure on urban acoustics.

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