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## Introduction. Symbolic logic and scientific philosophy Paola Cantù and Georg Schiemer

### The Peano School, the *Révue de Métaphysique et de Morale* and the Vienna Circle

The turn of the last century, i.e., from the nineteenth to the twentieth century, was a key transitional period for the development of symbolic logic and "scientific philosophy". The Peano school (with its members G. Peano, G. Vailati, A. Padoa, C. Burali-Forti, M. Pieri, and G. E. E. Vacca) and the Vienna Circle (H. Hahn, K. Menger, R. Carnap, and K. Gödel, among others) are generally mentioned as champions of this transformation of the role of logic in mathematics as well as in the sciences. The change, often reconstructed as a key element in the history of analytic philosophy, was neither uniform nor consensual. Moreover, the philosophical conceptions associated with these research groups are generally presented as forms of (empirical) logicism, without taking into account relevant differences between the members of the groups, including the types of foundational problems they were interested in, the influence of classical traditions of thought (Leibniz, Kant, British and German empiricism), their historical interactions at international conferences and in the edition of journals, and finally, the peculiar collaborative and interdisciplinary dimensions of the two groups.

The editors of this volume co-directed an international scientific project to better understand how the modern conception of logic developed by these groups emerged from interactions with classical axiomatics and the Kantian, Leibnizian, and empiricist philosophical traditions as well as an epistemological consequence of collaborative and interdisciplinary undertakings. The project was entitled "The effect of interdisciplinary collaboration on early twentieth-century epistemologies. A comparison between the Peano school, the Vienna Circle, and the editorial board of the *Revue de Métaphysique et de Morale*: proto-structuralism and proto-pluralist logicism" (INTEREPISTEME) and funded by the French Scientific Center for Research.<sup>1</sup>

The reasons for the misrepresentation of the Peano group's and the Vienna Circle's epistemologies (and of their inner variants) as a coherent logicist understanding of mathematics are various. First, there was an overestimation in the literature of the three major 'isms' in the foundations of mathematics, an overestimation that was partly a result of the activities of the Vienna Circle itself, as they devoted the 1930 conference on *The Epistemology of the Exact Sciences* to logicism, formalism, and intuitionism. Second, there were tendencies to assimilate several different positions to a unique, clearly stated point of view, as in the case of Peano's *Formulario*,<sup>2</sup> Russell's remarks on Peano, and Neurath's efforts to present a unitary scientific perspective of the Vienna Circle in the redaction of the 1929 manifest. Third, these research groups have so far mainly been investigated in isolation, without a systematic analysis of their reciprocal connections and their interactions with classical philosophy.

The research project aimed to contribute to a richer historical and philosophical understanding of the three groups and research areas in Italy, France and Austria, as can be seen from the articles collected in this volume (see the next section for a detailed overview). The project also aimed to contribute to the following topics: a detailed investigation of the relation between structuralism and geometry and the different notions of definition and interpretation at the turn of last century (see, e.g., Schiemer 2020, Schiemer and Giovannini 2021); a closer

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<sup>2</sup> Recent investigations have instead shown that Peano, Padoa, Burali-Forti, Vailati and Pieri had different viewpoints on the relation between mathematics and logic (see Luciano 2017 and Cantù 2022).

understanding of the relation between symbolic logic and previous traditions such as syllogistics (Cantù forthcoming) and of the relation between logic, history and didactics in the Peano School (Cantù and Luciano 2021).

A separate focus in the project concerned the philosophical contributions of the editorial group linked to the *Revue de Métaphysique et de Morale* which acted more as an aggregator and disseminator of new scientific ideas than as a standard research center. Documented are the contributions of Lois Couturat (Luciano 2012) and Maximilien Winter (see Alunni 2015). In the present volume, we include an analysis of the contributions by Léon Brunschvicg, as well as the first publication of an English translation of his 1909 article devoted to the philosophy of Henri Poincaré. Further insights into Brunschvicg's conception also emerge from a comparison with Émile Borel (see Mazliak and Sage, this volume) which highlights a general resistance to a philosophy of mathematics focused on an overly abstract and dogmatic conception of set theory and too centered on a preliminarily given, logical classification of concepts.

Focusing on the originality of the epistemological and methodological approaches of these collaborative and interdisciplinary groups, the project took as a starting point the study of the origins and evolution of “scientific philosophy”, a notion that was in fact polysemous, as it included different institutional projects and different philosophical traditions (e.g., Helmholtz, Brentano, Tannery, the Italian journal *Rivista di filosofia scientifica*, Russell, Husserl, neo-Kantism, American pragmatism, the Berlin and Vienna circles, Federigo Enriques, or Gaston Bachelard). The focus on the notion of scientific philosophy is not only important for understanding the relationship between the new conception of logic and the legacy of positivism. It is also relevant for clarifying the origin of some issues that are still at the center of a lively contemporary debate, such as the autonomy of philosophy from science (generally defended in the form of an anti-naturalism) and the role of axiomatics in the conceptual analysis of science. Scientific philosophy was not born suddenly in the 1930s but is rather a response to methodological questions common to different philosophical traditions, emerging already in the second half of the nineteenth and the beginning of the twentieth century. Moreover, logical empiricism was not the only viable approach to scientific philosophy. The discussion that took place during the 1936 international conference (see Bourdeau et al. 2018) shows how different groups had diverging views at the time. For instance, for Enriques, symbolic logic was not even part of the method of philosophical analysis of science, whereas for Hahn and others, it played an essential role.

The study of the relationship between the views defended by individual authors and the ideas expressed in the philosophical manifesto of the Vienna Circle finds an analogue in the difference between the variety of contrasting views expressed in the Peano school and the relatively uniform picture presented in the *Formulario*. The present volume paves the way for analyzing the extent to which the dynamics of collective and interdisciplinary interaction of knowledge has modified or influenced individual dynamics of scientific research, providing concrete examples to answer a question that is often debated in an abstract way in social ontology: to what extent does a group's research differ from the sum of individual contributions?

To investigate the relation between logicism and structuralism, or at least some form of proto-structuralism in the philosophy of mathematics, two approaches proved particularly fruitful: on the one hand, the analysis of an element too often neglected in the study of modern axiomatics, namely different types of definitions; on the other hand, the analysis of structures and of the relation between their definitions and applications. The study of definitions involved comparing different forms of definitions used in the Peano school: implicit, explicit, proper, improper, direct, indirect, by abstraction, etc. (Cantù 2021), but also the aim to reach a better understanding the relationship between implicit definitions and axioms in the works of Enriques and Schlick (see Biagioli and Wagner in this volume). The attention given to the analysis of definitions does not only derive from efforts to make the foundations of mathematics more rigorous or to reduce mathematical concepts to logical concepts, as in the standard formulations

of logicism. It is also motivated by the metatheoretical question of the relation between axioms and theorems. Definitions, far from being exclusively abbreviated writings or logical truths, reflect quite different practices and objectives: if implicit definitions play the role of principles of an axiomatic system, explicit definitions are classified according to their logical form and the criteria they must satisfy in order to guarantee certain metatheoretical properties of an axiomatic system.

The analysis of the use of structures shows that they are conceived differently, depending on whether they derive from a logico-linguistic analysis, from a logico-arithmetic development of the notion of order, or from an attention to physical applications. For instance, Peano's, Schlick's, and Carnap's respective notions of structure not only have different origins, but also play quite different roles in the construction of an axiomatic system. The study of the relationship between definability, the construction of axiomatic systems and the structural analysis of mathematics has highlighted the importance of the unpublished reflections of one of the members of the Vienna Circle, namely Kurt Gödel. The transcription and edition of some of Gödel's unpublished notebooks (the MaxPhil), to which the INTEREPISTEME project has contributed with financial support, has provided several interesting clues for a better understanding of Gödel's philosophy of mathematics.

### Overview of the collection

The research articles in this volume investigate the historical development of and the interconnections between the different philosophical schools from various perspectives, including essays on Peano and Enriques, Borel and Brunschvicg, and the Vienna Circle. The contributions of Joan Bertran-San Millán, Laurent Mazliak and Marc Sage and Frédéric Patras focus on different key contributions to the foundations of mathematics around the turn of the last century: they investigate deep interactions between empiricism and the development of abstract mathematics, showing the role of deductivism, but also of probability and conventionalism. Bertran-San Millán's "*Peano's geometry: from empirical foundations to abstract development*" develops a critical discussion of Giuseppe Peano's foundational work on the axiomatic presentation of projective geometry. By focusing on the Peano's two central writings on the topic, namely *Principii di Geometria* (1889b) and '*Sui fondamenti della Geometria*' (1894), Bertran-San-Millán investigates a critical tension between two poles in Peano's account: on the one hand, the view that the basic components of geometry must be founded on intuition, and, on the other, Peano's advocacy of the axiomatic method and an abstract understanding of the axioms. By studying his empiricist remarks and his conception of the notion of mathematical proof, Bertran-San-Millán argues that these two poles can be understood as compatible stages of a single process of construction rather than conflicting options. Mazliak's and Sage's article "*Altered states. Borel and the probabilistic approach to reality*" focuses on Émile Borel's contributions to probability theory and what the authors call the "probabilistic shift" in his work around 1905. Specifically, they examine the transition from Borel's studies of the structure of real numbers and a certain rejection of Cantor's abstract vision in the foundations of set theory, to the study of the calculus of probabilities. Moreover, Mazliak and Sage give an informative discussion of Borel's views on the applicability and usefulness of probabilities in scientific methodology, in particular, in the field of statistical mechanics as well as in sociology.

Frédéric Patras translates a text by Brunschvicg on Poincaré that was first published in the *Revue de Métaphysique et de Morale* in 1909. Patras's introduction highlights the scientific role played by the RMM in France and sketches some previously unseen similarities between Brunschvicg's and Poincaré's philosophy, relating to the defense of an anti-positivist form of rationalism, the centrality of mathematics and criticism rather than logic in the scientific method, and the focus on reality and physics. Brunschvicg's article introduces Poincaré's philosophy with the aim of proving that scientific hypotheses, while conventions, are not arbitrary. To clear

Poincaré of the charge of nominalism, Brunschvicg cites numerous passages on truth and on the relationship between convenience, logical simplicity and applicability to the external world, but also the analysis of the continuum, and the use of probability theory in the study of the kinetic theory of gases.

Several articles contained in the volume focus on the philosophy of the Vienna Circle as well as its relation to other philosophical traditions. Massimo Ferrari's article "*Leibniz and the Vienna Circle*" focuses on the hitherto neglected influence of Leibniz and Leibnizianism both on the origins and development of the Vienna Circle. As Ferrari argues, this background suggests a re-assessment of the roots of Logical Empiricism beyond the dominant narrative, which has mainly overlooked the role of Leibniz in shaping the scientific world conception. The article starts by focusing on the significance of Leibniz for the Austrian philosophical tradition, which Otto Neurath has emphasized in order to better understand the rise of Viennese empiricism. Ferrari then turns to the debate about Leibniz's metaphysics and logic at the very beginnings of twentieth century, specifically by Giuseppe Peano and his school, Louis Couturat, and Bertrand Russell. This research has strongly motivated the anti-Kantianism of the Vienna Circle. Moreover, Ferrari argues that the ambitious project of the Encyclopedia endorsed by the late Vienna Circle can be considered, to some extent, in connection with Leibniz's dream of a *scientia generalis*, although carried out from the point of view both of Neurath's and Carnap's physicalism.

Julien Bernard compares two different ways of specifying the scientific status of philosophy: Schlick, like many of the philosophers closed to the Vienna Circle, claimed the rise of a "scientific philosophy", while Husserl wanted to make philosophy a "rigorous science". Arguing that these expressions hide conceptions of science and of its relationship to philosophy that are in sharp opposition, the paper analyzes the polemic focusing on Schlick and Husserl but also on Weyl. After presenting the context of the polemic from the Weyl-Schlick correspondence, and highlighting the opposed role assigned by Husserl and Schlick to intuition and lived experience (*Erlebnis*) in the constitution of a scientific philosophy, Bernard also shows how Weyl, in contrast with Schlick's demands, retains a role for the synthetic a priori within the foundations of science, thereby accounting for the historicity of science.

The contributions by Pierre Wagner and Francesca Biagioli closely connect to the articles on the intellectual context of the Vienna Circle mentioned above. Both articles focus on different contributions to the method of implicit definitions in mathematics and in scientific knowledge more generally. In her article, "*Federigo Enriques and the philosophical background to the discussion of implicit definitions*", Biagioli aims to draw further insights on implicit definitions from the development of this notion from its first occurrence in German language in Enriques's "Principles of Geometry" (1907) to Schlick's *General Theory of Knowledge* (1918). Biagioli argues that Enriques offers one way to counter some of the classical objections against the early twentieth-century conceptualization of implicit definitions. Specifically, Enriques did not conflate the distinct notions that had been identified as implicit definitions in the recent history of mathematics, but he tried to offer an account of the process leading to structural definitions. The paper points out, furthermore, that Enriques's account differs significantly from Schlick's. The scientific interpretations of implicit definitions in Schlick's theory of knowledge depend on the coordination of the terms of abstract mathematical structures with physical realities. By contrast, Enriques addressed the problem of bridging the gap between abstract and concrete terms by identifying patterns within mathematics that provide a clarification of conceptual relations, and so also serve the purposes of applied mathematics. In his article, "*Schlick and Carnap on definitions*", Wagner develops a critical comparison of Carnap's and Schlick's respective accounts of the notion of definition. In the 1920s, both philosophers both made an important use of definitions in their main publications: Schlick, in his *Allgemeine Erkenntnislehre* (1918) and Carnap in *Der Logische Aufbau der Welt* (1928). Wagner's paper provides an analysis of the kinds of definitions which are distinguished

in these books and a few other papers and then proposes a systematic comparison of Schlick's and Carnap's diverging conceptions of definitions in the 1920s, relating them, in both cases, to their respective philosophical projects in the *Allgemeine Erkenntnislehre* and in the *Aufbau*.

The contributions by Paola Cantù and Frédéric Patras as well as by Gabriella Crocco also investigate different aspects of Carnap's philosophy of mathematics. Cantù's and Patras' article "*Russell and Carnap or Bourbaki? Two ways toward Structures*" focuses on early contributions to mathematical structuralism. Specifically, they analyze a central difference between a logical notion of structure that can be traced back to the writings of Bertrand Russell and Rudolf Carnap, and a mathematical notion of structure, exemplified in the works by Bourbaki. As they argue, this coexistence gives rise to a fundamental ambiguity that affects contemporary structuralism. Philosophically, in one case the attention is rather centered on a foundational and reductionist perspective, as featured by the Whitehead-Russell *Principia* and the Carnapian project of the *Aufbau*: the scientific construction of the world around the idea of structure. In the other, the focus is on epistemological and dynamical issues, as exemplified by two key issues in Bourbaki's treatise: understanding the architecture of mathematics, offering a tool-kit to mathematicians. Crocco's article "*Carnap and Gödel, again*", re-addresses the analysis of Carnap's conception of logic and mathematics in Gödel's famous drafts of 'Is mathematics Syntax of Language?'. She critically responds to a recent defense of Gödel's arguments against Carnap's position developed in work by Greg Lavers, pointing out three important differences between her own understanding of Gödel's argument and Lavers's interpretation of it. These differences concern the appreciation of a) Gödel's strategy of using, in any critical examination of his opponents, only arguments that can be accepted by them; b) Gödel's analysis of Carnap's position in the 1950s; c) Gödel's understanding of Carnap's philosophical project. Crocco argues that, contrary to Lavers's opinion, Gödel takes seriously the details of Carnap's original conception and does not overlook the novelty of its solutions in the 1930s and 1950s.

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